

Question 1:

a) Consider the following objective function

$$\min f(x) = (x-1)^2(x-2)(x-3), \quad 0 \leq x \leq 4$$

i) Use the method of Newton-Raphson to compute x^* that minimizes $f(x)$ for the following three initial values of $x_0 = 0.2, 1.5$ and 2.5 with 10^{-10} tolerance. (Write matlab m-file code)

ii) Plot the change of the x versus iteration number, the alternation of the objective function by the evolution of x and gradient information versus iteration number. (Write matlab m-file code)

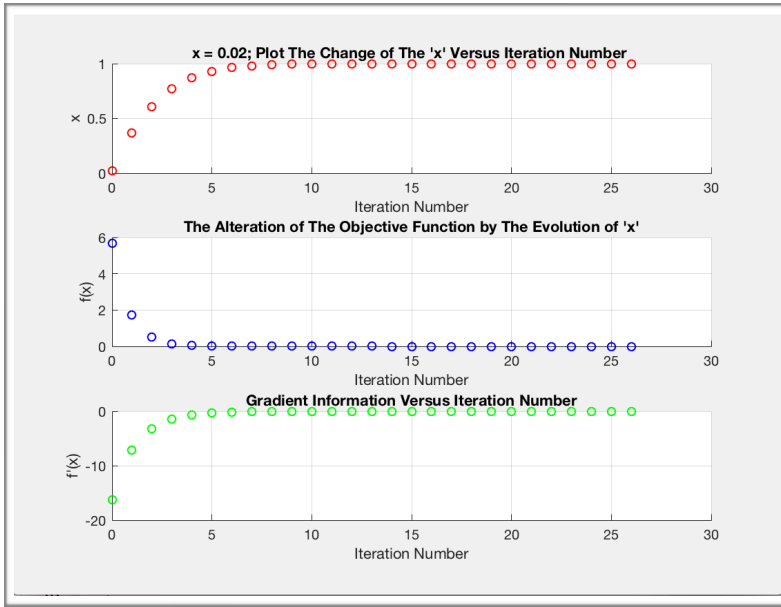
Algorithm:

Step 1: Initialize x_0 , ε and $k = 0$

Step 2: Calculate $\Delta x_k = -\frac{f'(x)}{f''(x)}$

Step 3: Update $x_{k+1} = x_k + \Delta x_k$

Step 4: If $|f'(x_{k+1})| < \varepsilon$ terminate iteration, else go to **Step 2**.



x_start= 0.02 Iteration 0: x=0.020, err=0.980
 x_start= 0.02 Iteration 1: x=0.367, err=0.632
 x_start= 0.02 Iteration 2: x=0.607, err=0.392
 x_start= 0.02 Iteration 3: x=0.768, err=0.231
 x_start= 0.02 Iteration 4: x=0.869, err=0.130
 x_start= 0.02 Iteration 5: x=0.929, err=0.070
 x_start= 0.02 Iteration 6: x=0.963, err=0.036
 x_start= 0.02 Iteration 7: x=0.981, err=0.018
 x_start= 0.02 Iteration 8: x=0.990, err=0.009
 x_start= 0.02 Iteration 9: x=0.995, err=0.004
 x_start= 0.02 Iteration 10: x=0.997, err=0.002
 x_start= 0.02 Iteration 11: x=0.999, err=0.001

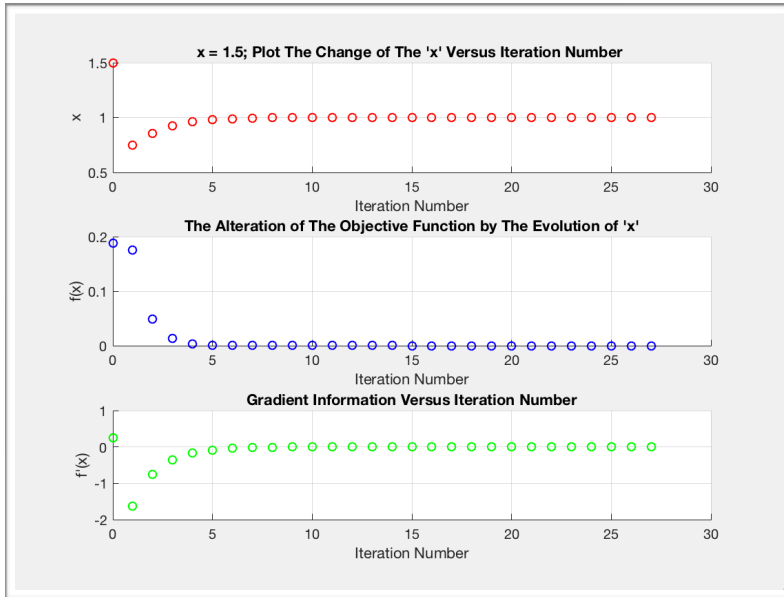
x_start= 0.02 Iteration 27:
x=0.99999997617604330369,
err=0.0000002382395669631

$$\epsilon = 10^{-10}$$

x_start= 1.5 Iteration 0: x=1.50, err=-0.50
 x_start= 1.5 Iteration 1: x=0.750, err=0.250
 x_start= 1.5 Iteration 2: x=0.858, err=0.142
 x_start= 1.5 Iteration 10: x=0.999, err=0.001
 err=0.00000065583238395561

x_start= 1.5 Iteration 28:
x=1.00000001062910159888,
err=-0.00000001062910159888

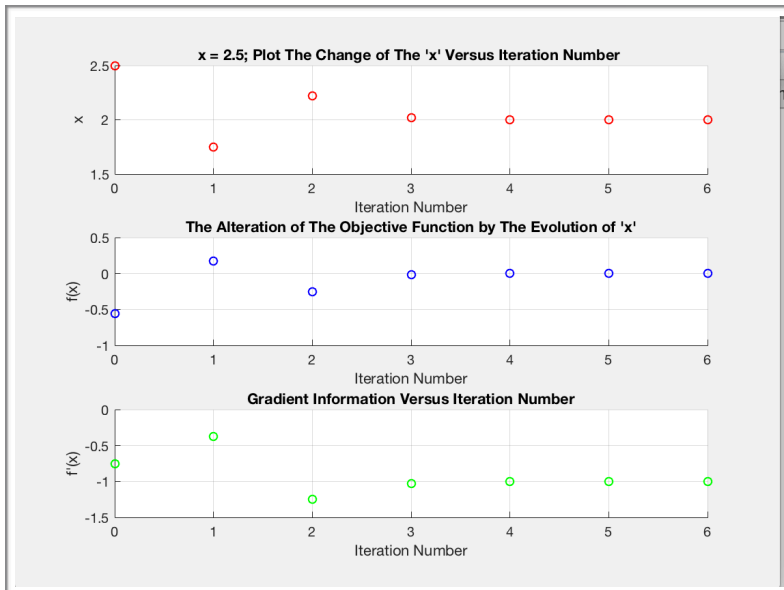
$$\epsilon = 10^{-10}$$



x_start= 2.5 Iteration 0: x=2.50, err=-0.50
 x_start= 2.5 Iteration 1: x=1.750, err=0.250
 x_start= 2.5 Iteration 2: x=2.218, err=-0.2187
 x_start= 2.5 Iteration 3: x=2.016, err=-0.016
 x_start= 2.5 Iteration 4: x=2.0002, err=-0.00024
 x_start= 2.5 Iteration 5: x=2.0001, err=-0.0009

x_start= 2.5 Iteration 6:
x=2.0000000000000001332268,
err=-0.0000000000000001332268

$$\epsilon = 10^{-10}$$



b)

Consider the following objective function

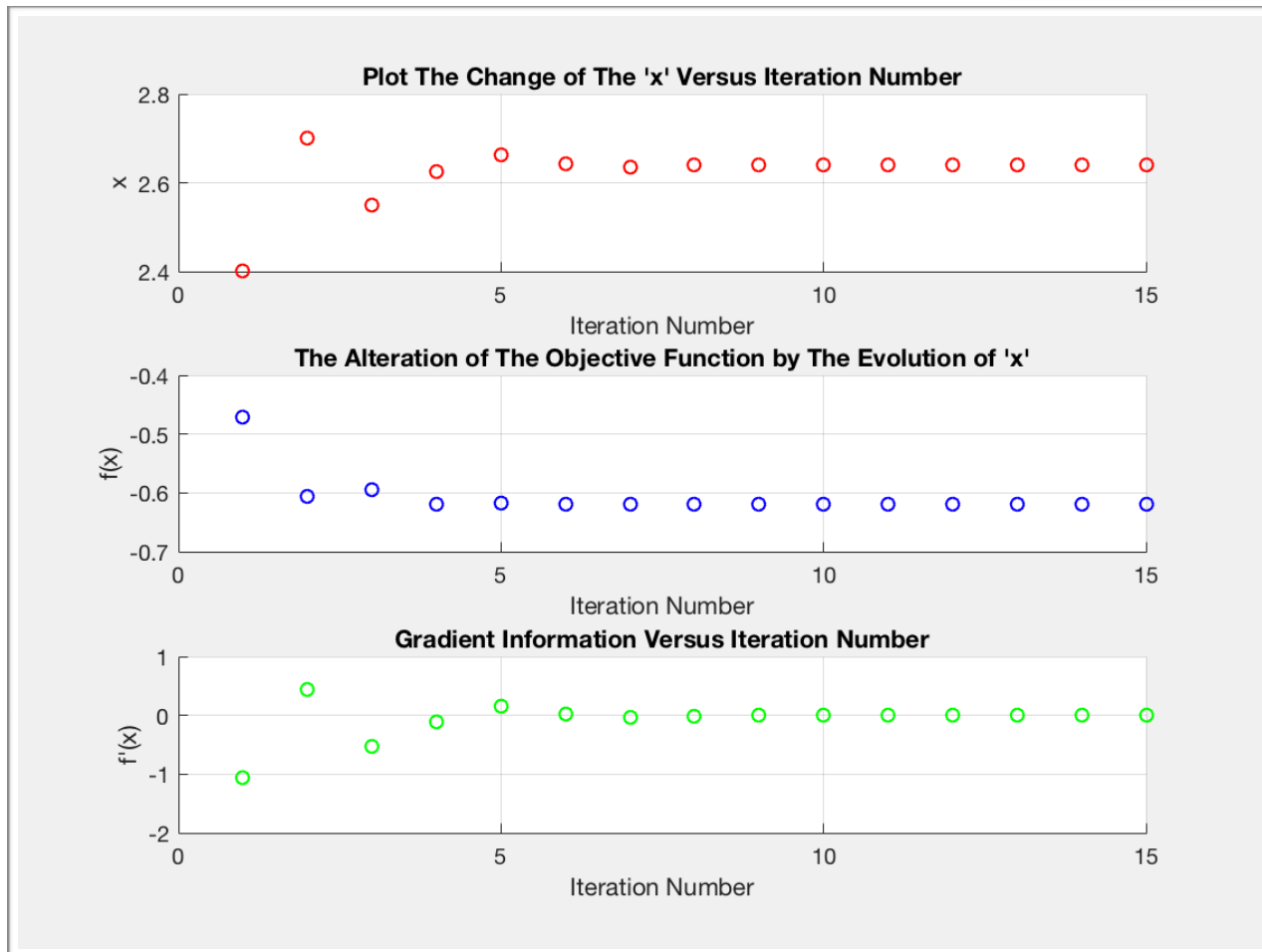
$$\min_x f(x) = (x-1)^2(x-2)(x-3)$$

$$0 \leq x \leq 4$$

i) Use the method of bisection to compute x^* that minimizes $f(x)$ for $\alpha_a = 1.8$, $\alpha_b = 3$.

(Write matlab m-file code)

ii) Plot the change of the x versus iteration number, the alternation of the objective function by the evolution of x and gradient information versus iteration number. (Write matlab m-file code)**Algorithm:****Step 1:** Determine the interval α_a and α_b when $\alpha_a < \alpha_b$. $\varepsilon = 10^{-4}$ **Step 2:** Update $\alpha_k = \alpha_a + \frac{(\alpha_b - \alpha_a)}{2}$ **Step 3:** If $f'(\alpha_k) = 0$, terminate the iteration owing to convergenceelse and if $(\alpha_b - \alpha_a) < \varepsilon$, then terminate iteration due to tolerans value(ε)else and if $f'(\alpha_k)f'(\alpha_a) > 0$, then $\alpha_a = \alpha_k$ else $\alpha_b = \alpha_k$ and continue with **Step 2**.



Algorithm on the
Matlab

```

iteration_number = 0;
while 1
    %Update a_k (Step 2)
    a_k = a_a + (a_b - a_a)/2;

    %increase the iteration number
    iteration_number = iteration_number + 1;

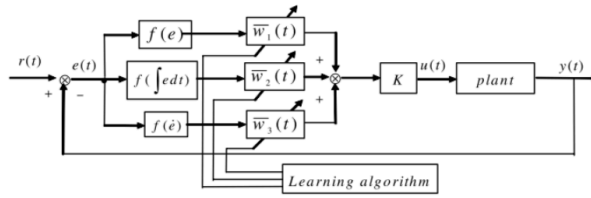
    %Plot the Figure 1: the change of the x versus iteration number
    plot(ax1, iteration_number, a_k, 'ro');

    %Plot the Figure 2: The Alteration of The Objective Function by The Evolution of x
    plot(ax2, iteration_number, f(a_k), 'bo');

    %Plot the Figure 3: Gradient Information Versus Iteration Number
    plot(ax3, iteration_number, f_derivative(a_k), 'go');

    if f_derivative(a_k) == 0
        fprintf('Terminated iteration owing the convergence\n');
        break;
    elseif (a_b - a_a) < Epsilon
        fprintf('Terminated iteration due to tolerans value(Epsilon)\n');
        break;
    elseif ( f_derivative(a_k) * f_derivative(a_a) ) > 0
        a_a = a_k;
    else
        a_b = a_k;
    end
end

```

Question 2:**Fig. 1** Neuron based Nonlinear PID Controller

The neuron based nonlinear PID Controller in figure is going to be used to control paper making process. The dynamic characteristics of the plant is as follows:

$$G(z) = \begin{cases} \frac{0.2719}{z(z-0.8187)} & , \text{ when } 80 \text{ g/m}^2 \text{ paper is made} \\ \frac{0.4484}{z(z-0.7788)} & , \text{ when } 100 \text{ g/m}^2 \text{ paper is made} \\ \frac{0.7087}{z(z-0.7165)} & , \text{ when } 120 \text{ g/m}^2 \text{ paper is made} \end{cases}$$

The controller parameters are selected as follows :

$$K = 1.1, \eta_1 = 15, \eta_2 = 1, \eta_3 = 10$$

$$\alpha_p = 1, \alpha_I = 0.5, \alpha_D = 0.5,$$

$$\delta_p = 0.1, \delta_I = 0.4, \delta_D = 0.3$$

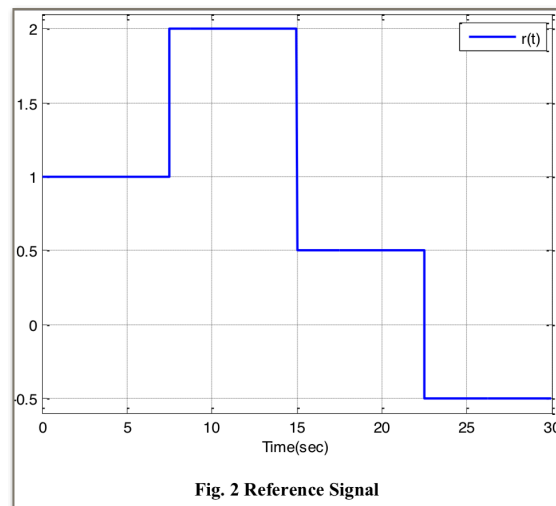
Set the initial values as follows :

$$u(0) = 0, y(0) = 0, w_1(0) = w_2(0) = w_3(0) = 0.005$$

$$e_{tr}(0) = 0, T_s = 0.01 \text{ sec (sampling time)}$$

a) Plot the response of the system and control signal versus time using the reference signal in figure 2. Plot the alternation of the controller parameters in terms of w_1, w_2, w_3 and also

K_p, K_I, K_D . (Write matlab m-file code)

**Fig. 2** Reference Signal

Control Algorithm:**Step 0:** Set $n = 1$ **Step 1:** Calculate tracking error $e_r(n) = r(n) - y(n)$ **Step 2:** Calculate inputs of the PID Controller

$$P(n) = e_r(n)$$

$$I(n) = I(n-1) + e_r(n)$$

$$D(n) = e_r(n) - e_r(n-1)$$

Step 3: Calculate outputs of $f(\cdot)$ nonlinear function

$$x_p(n) = f(e_r(n), \alpha_p, \delta_p)$$

$$x_I(n) = f\left(\int e_r(n) dn, \alpha_I, \delta_I\right)$$

$$x_D(n) = f(\dot{e}_r(n), \alpha_D, \delta_D)$$

Step 4: Calculate control signal

$$\begin{aligned} u(n) &= K \frac{w_1(n)x_p(n) + w_2(n)x_I(n) + w_3(n)x_D(n)}{w_1(n) + w_2(n) + w_3(n)} \\ &= K_p(n)x_p(n) + K_I(n)x_I(n) + K_D(n)x_D(n) \end{aligned}$$

Step 5: Apply control signal to the plant and obtain output of the system

$$y(n+1) = ?$$

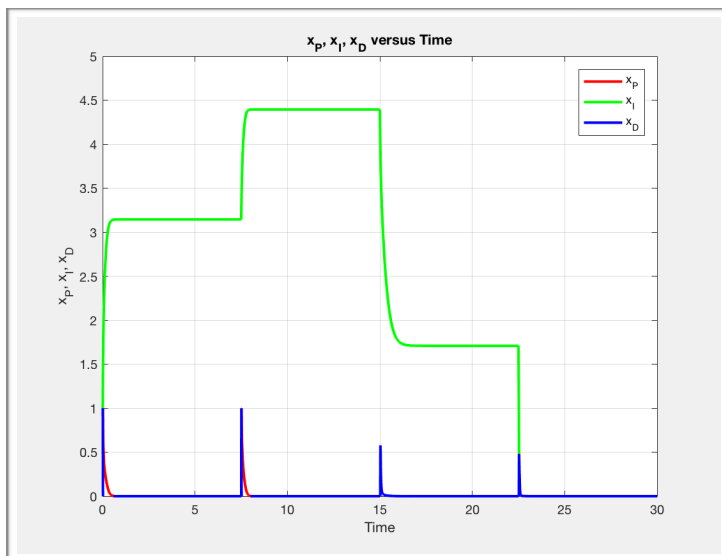
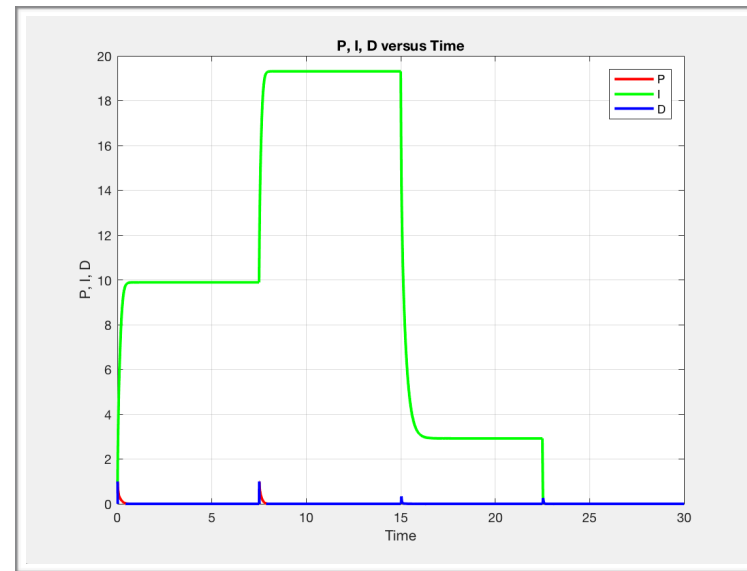
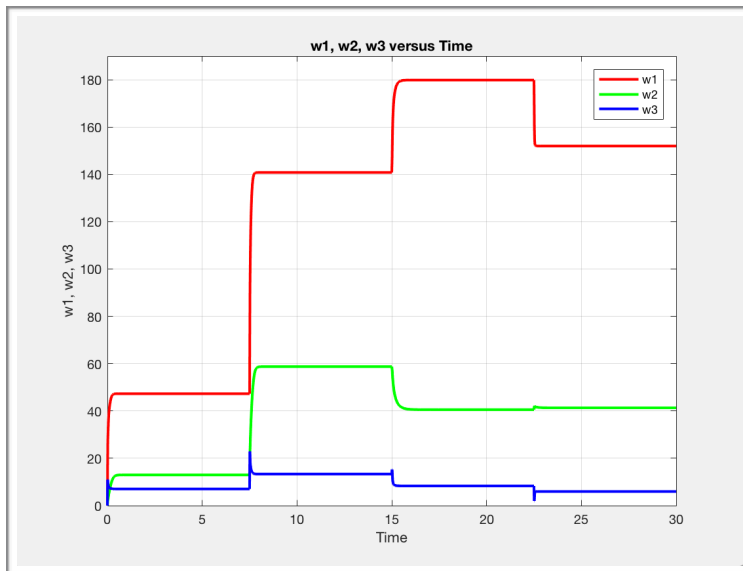
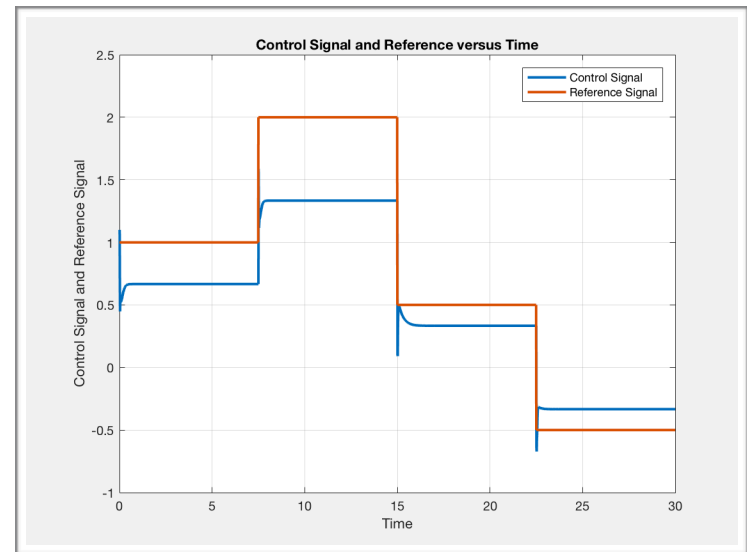
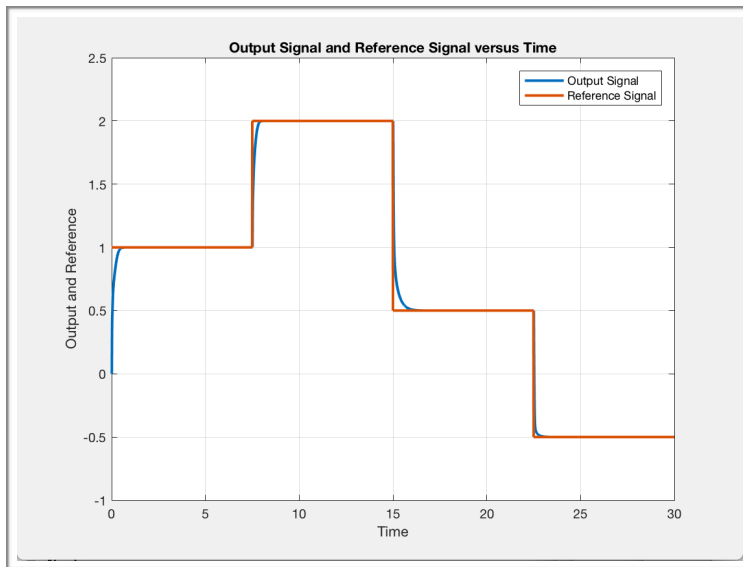
Step 6: Update controller parameters

$$w_1(n+1) = w_1(n) + d_1 e_r(n) u(n) x_p(n)$$

$$w_2(n+1) = w_2(n) + d_2 e_r(n) u(n) x_I(n)$$

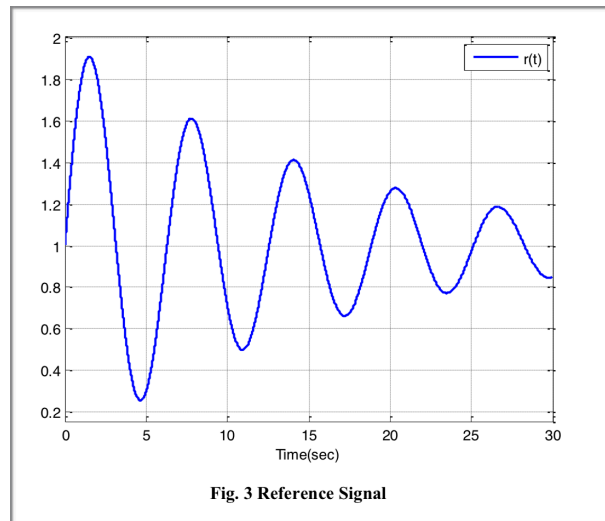
$$w_3(n+1) = w_3(n) + d_3 e_r(n) u(n) x_D(n)$$

Step 7: $n \leftarrow n+1$ and go to **Step 1**.



b) Plot the response of the system and control signal versus time using the reference signal in figure 3. Plot the alternation of the controller parameters in terms of w_1, w_2, w_3 and also

K_P, K_I, K_D . ($r(t) = 1 + e^{-\frac{2\pi}{100}t} \sin(t)$) (Write matlab m-file code)



Algorithm on the
Matlab Code

```

for n=1:3000

    %Step 1: Calculate tracking error  $e_{tr}(n) = r(n) - y(n)$ 
    e_tr(n) = r(n) - y(n);

    %Step 2: Calculate inputs of the PID Controller
    P(n) = e_tr(n);
    I(n) = integral_e(e_tr);
    D(n) = derivative_e(e_tr);

    %Step 3: Calculate outputs of f(.) nonlinear function
    x_p(n) = f_nonl(e_tr(n), alpha_p, delta_p);
    x_i(n) = f_nonl(I(n), alpha_i, delta_i);
    x_d(n) = f_nonl(D(n), alpha_d, delta_d);

    %Step 4: Calculate Control Signal
    u(n) = K*(w1(n)*x_p(n)+w2(n)*x_i(n)+w3(n)*x_d(n))/(w1(n)+w2(n)+w3(n));

    %Step 5: Apply control signal to the plant and obtain output of the system
    y(n+1) = y_out(u, y, m_paper);

    %Step 6: Update controller parameters
    w1(n+1) = w1(n) + eta1*e_tr(n)*u(n)*x_p(n);
    w2(n+1) = w2(n) + eta2*e_tr(n)*u(n)*x_i(n);
    w3(n+1) = w3(n) + eta3*e_tr(n)*u(n)*x_d(n);

    %Step7: n <-n+1 and go to Step 1.

end

```