Julie Kumar, Final Project

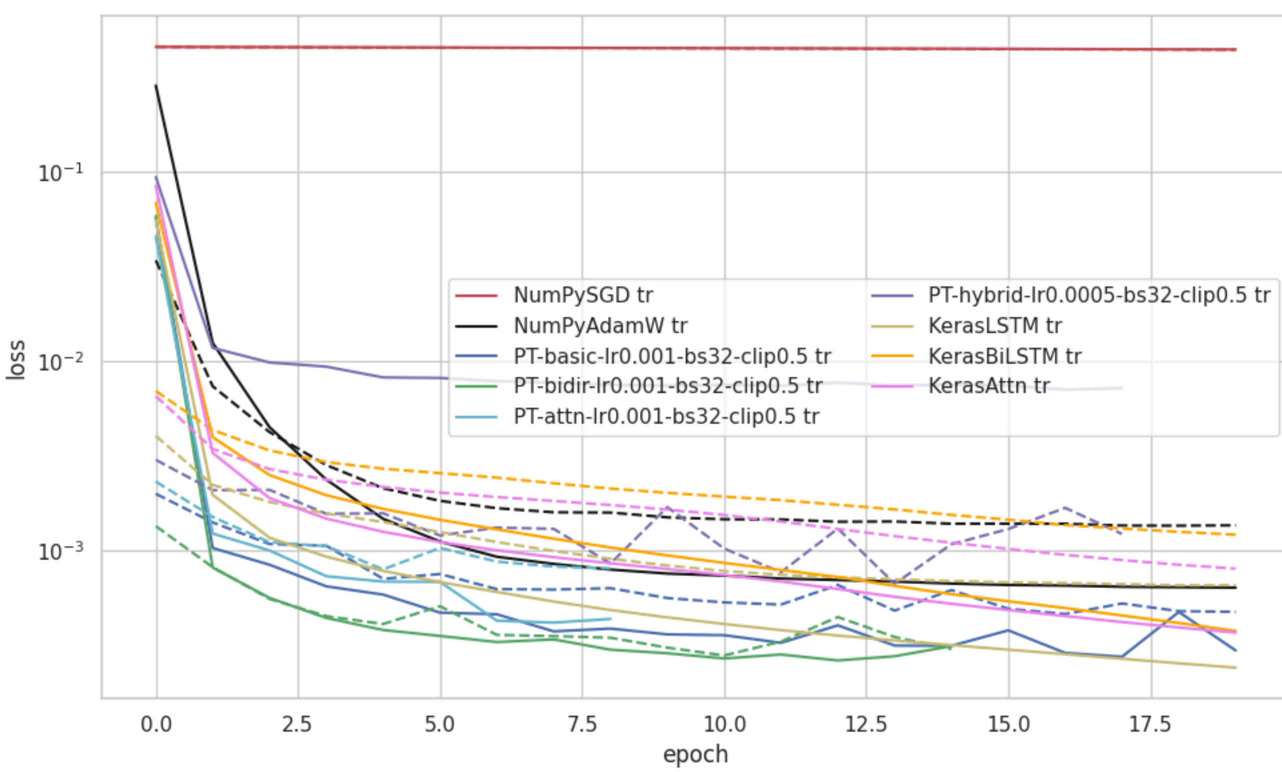
Hypothesis:

The recent history (62 time steps) of a car's movement characteristics (including position, velocity, acceleration, etc.) contains predictable patterns that can be learned by a sequence model like various LSTMs to forecast its near-future state (specifically, 'Local_X', 'Local_Y' for the next 5 time steps). The project also implicitly tests the hypothesis that different implementations (NumPy vs. optimized libraries like PyTorch/Keras) and architectural variations (unidirectional LSTM vs. BiLSTM vs. LSTM+Attention) will exhibit different learning efficiencies and predictive performances on this specific task.

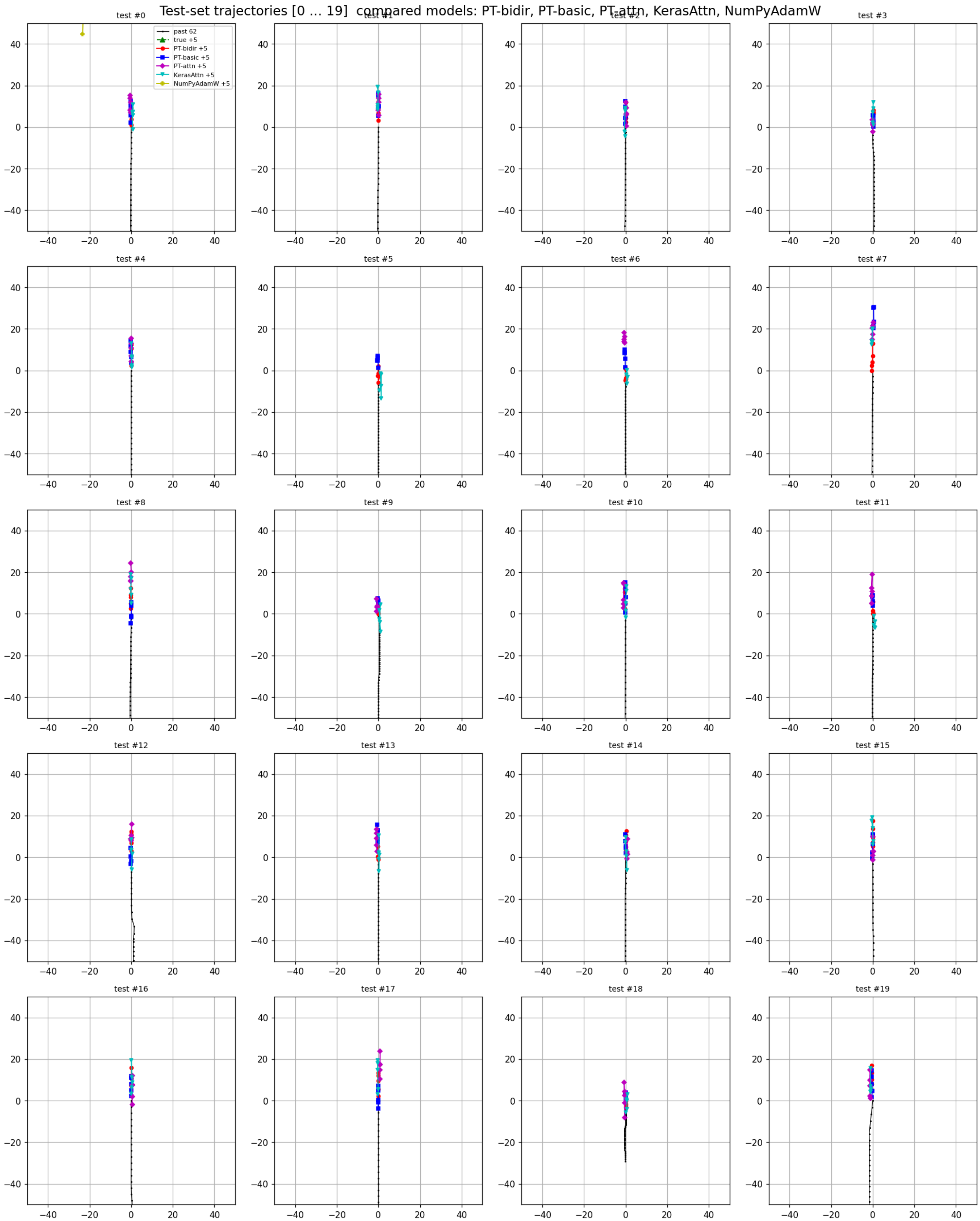
Obtained results

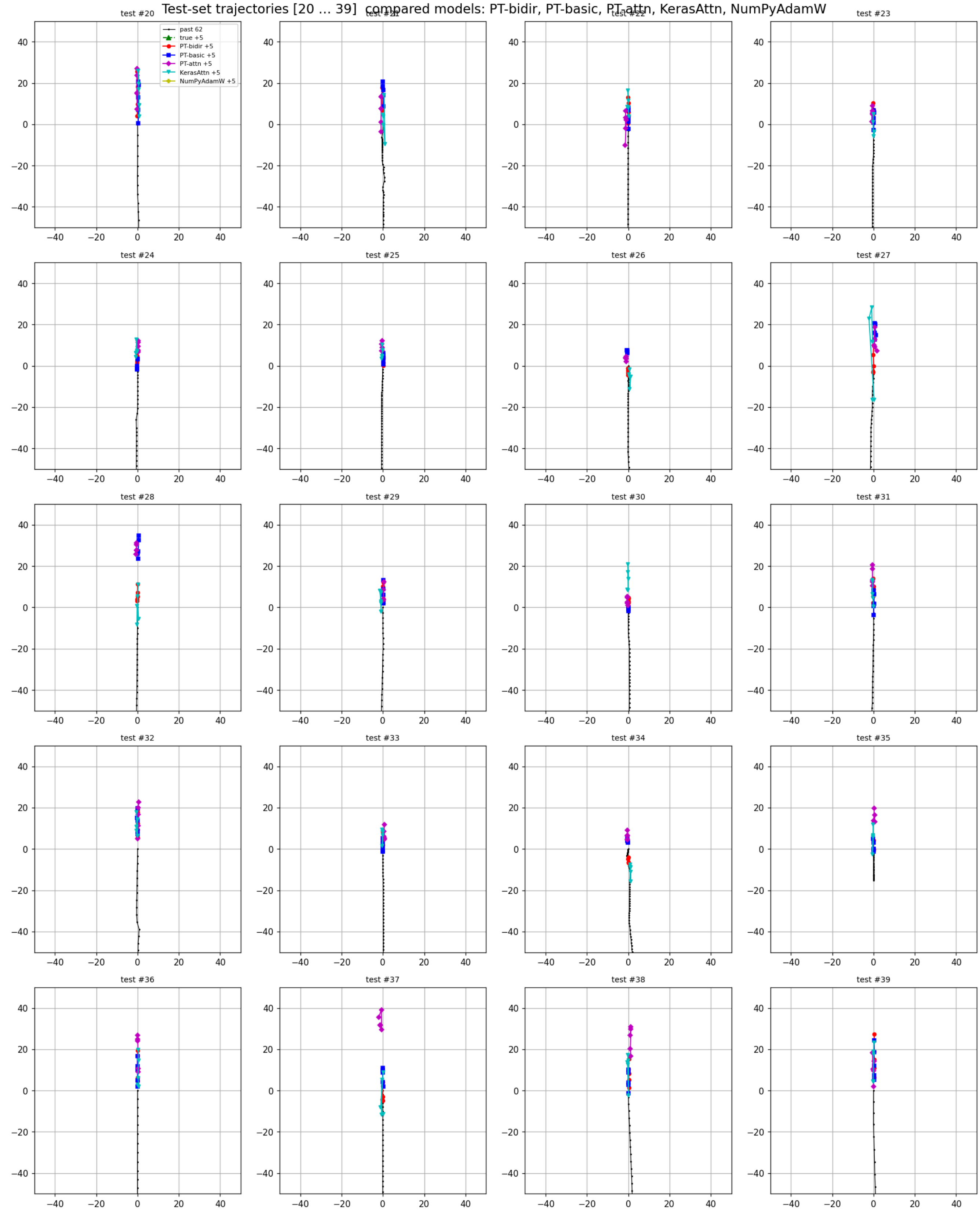
Table 1: Model Performance Metrics Sorted by MSE Ascending
detect-family, detect-shape, detect-weight

Model / Configuration	MSE	RMSE	MAE	R^2
PT-bidir-lr0.001-bs32-clip0.5	20.44	4.52	1.31	0.9997
PT-basic-lr0.001-bs32-clip0.5	39.82	6.31	2.03	0.9996
PT-hybrid-lr0.0005-bs32-clip0.5	73.91	8.60	5.07	0.9990
PT-attn-lr0.001-bs32-clip0.5	90.29	9.50	3.47	0.9985
KerasLSTM	150.16	12.25	3.02	0.9988
NumPyAdamW	239.79	15.49	2.09	0.9980
KerasAttn	308.04	17.55	3.00	0.9975
KerasBiLSTM	374.98	19.36	3.40	0.9972
NumPySGD	78965.45	281.01	180.05	0.0387



Testing results


Test-set trajectories [0 … 19] compared models: PT-bidir, PT-basic, PT-attn, KerasAttn, NumPyAdamW
test #0
past 62
true +5
PT-bidir +5
PT-basic +5
PT-attn +5
KerasAttn +5
NumPyAdamW +5
test #1
test #2
test #3
test #4
test #5
test #6
test #7
test #8
test #9
test #10
test #11
test #12
test #13
test #14
test #15
test #16
test #17
test #18
test #19



Best / lowest validation-set losses

Model	Loss Function	Lowest Validation Loss
KerasLSTM	`tf.keras.losses.Huber(delta=1.0)`	0.0006573
KerasBi	Huber	0.0012155
KerasAttn	Huber	0.0008049

Project code

LSTM implementations

```
class MyLSTM:
    def __init__(self,inp_dim,hid,fh,td,lr=1e-3,clip=.5,drop=0.):
        self.i_dim,self.hid,self.fh,self.td = inp_dim,hid,fh,td
        self.lr,self.clip,self.drop = lr,clip,drop
        def xavier(sh): lim=math.sqrt(6/sum(sh)); return np.random.uniform(-lim,lim,sh).astype(np.float32)
        self.Wih=xavier((4*hid,inp_dim)); self.Whh=xavier((4*hid,hid))
        self.b=np.zeros(4*hid,np.float32)
        self.b[hid:2*hid]=+1; self.b[:hid]=-3; self.b[3*hid:]=-1
        self.Wy=xavier((fh*td,hid)); self.by=np.zeros(fh*td,np.float32)
        self.parameters=[self.Wih,self.Whh,self.b,self.Wy,self.by]

    def forward(self,seq,training=False):
        h=np.zeros(self.hid,np.float32); c=np.zeros_like(h)
        if training: self.cache=[]
        for x in seq:
            z=self.Wih@x + self.Whh@h + self.b
            i,f,g,o=np.split(z,4); i,f,o=map(SIG,(i,f,o)); g=TAN(g)
            if training and self.drop:
                mask=(np.random.rand(*h.shape)>self.drop).astype(np.float32)
                h*=mask/(1-self.drop)
            c=f*c+i*g; h=o*TAN(c)
            if training: self.cache.append((x,h.copy(),c.copy(),i,f,g,o))
        logit=self.Wy@h + self.by
        return logit.reshape(self.fh,self.td)

    def backward(self,t,pred):
        dl=dloss_np(t,pred).flatten()
        dWy=np.outer(dl,self.cache[-1][1]); dby=dl.copy()
        dh_n=self.Wy.T@dl; dc_n=np.zeros(self.hid,np.float32)
        dWih=np.zeros_like(self.Wih); dWhh=np.zeros_like(self.Whh); db=np.zeros_like(self.b)
        for x,h,c,i,f,g,o in reversed(self.cache):
            tan=TAN(c)
            dc=dh_n*o*(1-tan**2)+dc_n
            di=dc*g*i*(1-i); df=dc*c*f*(1-f); dg=dc*i*(1-g**2); do=dh_n*tan*o*(1-o)
            dz=np.concatenate([di,df,dg,do])
            dWih+=np.outer(dz,x); dWhh+=np.outer(dz,h); db+=dz
            dh_n=self.Whh.T@dz; dc_n=dc*f
        for d in (dWih,dWhh,db,dWy,dby): np.clip(d,-self.clip,self.clip,out=d)
        return [dWih,dWhh,db,dWy,dby]

    def step(self,grads):
        for p,g in zip(self.parameters,grads): p-=self.lr*g
```

```
class EnhancedMyLSTM(MyLSTM):
    def __init__(self,*a,dropout=.2,eps=1e-5,wd=1e-4,**kw):
        super().__init__(*a,drop=dropout,**kw)
        hid=self.hid
        self.peep_i=self.peep_f=self.peep_o=np.zeros(hid,np.float32)
        self.ln_gamma=np.ones(4*hid,np.float32); self.ln_beta=np.zeros(4*hid,np.float32)
        self.parameters+= [self.peep_i,self.peep_f,self.peep_o,self.ln_gamma,self.ln_beta]
        self.eps,self.wd=eps,wd
        self.m=[np.zeros_like(p) for p in self.parameters]
        self.v=[np.zeros_like(p) for p in self.parameters]
        self.b1,self.b2,self.t=0.9,0.999,0

    def _ln(self,z): m=z.mean(); v=z.var(); return (z-m)/np.sqrt(v+self.eps),m,v
    def forward(self,seq,training=False):
        h=np.zeros(self.hid,np.float32); c=np.zeros_like(h)
        if training: self.cache=[]
        for x in seq:
            z=self.Wih@x + self.Whh@h + self.b + \
                np.concatenate([self.peep_i*c,self.peep_f*c,np.zeros_like(c),self.peep_o*c])
            z_hat,μ,σ2=self._ln(z); z_ln=z_hat*self.ln_gamma+self.ln_beta
            i,f,g,o=np.split(z_ln,4); i,f,o=map(SIG,(i,f,o)); g=TAN(g)
            if training and self.drop:
                mask=(np.random.rand(*h.shape)>self.drop).astype(np.float32)
                h*=mask/(1-self.drop)
            c=f*c+i*g; h=o*TAN(c)
            if training: self.cache.append((x,h.copy(),c.copy(),i,f,g,o,z_hat,μ,σ2))
        return (self.Wy@h+self.by).reshape(self.fh,self.td)

    def _ln_bwd(self,dout,z_hat,μ,σ2):
        N=len(z_hat); dγ=(dout*z_hat).sum(); dβ=dout.sum()
        dz_hat=dout*self.ln_gamma
        dσ2=-.5*(σ2+self.eps)**-1.5*(dz_hat*z_hat).sum()
        dμ=-(dz_hat/np.sqrt(σ2+self.eps)).sum()-2*dσ2*z_hat.mean()
        dz=dz_hat/np.sqrt(σ2+self.eps)+dσ2*2*z_hat/N+dμ/N
        return dz,dγ,dβ
```

```
    def backward(self,t,pred):
        dl=dloss_np(t,pred).flatten()
        dWy=np.outer(dl,self.cache[-1][1]); dby=dl.copy()
        dh_n=self.Wy.T@dl; dc_n=np.zeros(self.hid,np.float32)
        dWih=np.zeros_like(self.Wih); dWhh=np.zeros_like(self.Whh); db=np.zeros_like(self.b)
        dpi=dpf=dpo=np.zeros(self.hid,np.float32)
        dγ=np.zeros_like(self.ln_gamma); dβ=np.zeros_like(self.ln_beta)
        for x,h,c,i,f,g,o,z_hat,μ,σ2 in reversed(self.cache):
            tan=TAN(c)
            dc=dh_n*o*(1-tan**2)+dc_n
            di=dc*g*i*(1-i); df=dc*c*f*(1-f); dg=dc*i*(1-g**2); do=dh_n*tan*o*(1-o)
            dz_ln=np.concatenate([di,df,dg,do])
            dz, dγ_t, dβ_t = self._ln_bwd(dz_ln,z_hat,μ,σ2)
            dγ+=dγ_t; dβ+=dβ_t
            di,dF,dg,do=np.split(dz,4)
            dpi+=di*c; dpf+=dF*c; dpo+=do*c
            dWih+=np.outer(dz,x); dWhh+=np.outer(dz,h); db+=dz
            dh_n=self.Whh.T@dz
            dc_n=dc*f+self.peep_i*di+self.peep_f*dF+self.peep_o*do
        grads=[dWih,dWhh,db,dWy,dby,dpi,dpf,dpo,dγ,dβ]
        grads=[clip_l2(g,self.clip) for g in grads]
        return grads

    def step(self,grads):
        self.t+=1
        for i,(p,g) in enumerate(zip(self.parameters,grads)):
            self.m[i]=self.b1*self.m[i]+(1-self.b1)*g
            self.v[i]=self.b2*self.v[i]+(1-self.b2)*(g*g)
            m_hat=self.m[i]/(1-self.b1**self.t)
            v_hat=self.v[i]/(1-self.b2**self.t)
            upd=m_hat/np.sqrt(v_hat+self.eps)
            if p.ndim==2: upd+=self.wd*p
            p-=self.lr*upd
```

Training

```
from google.colab import drive
drive.mount('/content/drive')
```

```
# 0. imports & global cfg
import os, random, math, copy, pathlib, warnings, itertools, shutil
import numpy  as np
import pandas as pd
```

```
import matplotlib.pyplot as plt; import seaborn as sns

from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score

import torch, torch.nn as nn, torch.optim as optim
from torch.utils.data import TensorDataset, DataLoader
from torch.optim.lr_scheduler import ReduceLROnPlateau

import tensorflow as tf; from tensorflow import keras
from tensorflow.keras.layers import (Input, LSTM, Dense, Reshape,
                                     Bidirectional, Attention, Concatenate)
warnings.filterwarnings("ignore"); sns.set(style="whitegrid")

SEED = 42
random.seed(SEED); np.random.seed(SEED)
torch.manual_seed(SEED); tf.random.set_seed(SEED)

INPUT_LEN, OUTPUT_LEN = 62, 5
HIDDEN_SIZE, INITIAL_LR = 128, 5e-4
BATCH_SIZE, CLIP_VALUE  = 64, .5
LOSS_FUNCTION, HUBER_DELTA = 'huber', 1.0
TRAIN_RATIO, VAL_RATIO = .70, .15
EPOCHS, EARLY_STOP     = 20, 4
DEVICE = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print("[INFO] device:", DEVICE)

# 1. data discovery / split
DATA_ROOT = "/content/drive/MyDrive/car_data"
SAVE_DIR   = "/content/drive/MyDrive/TrajectoryModels"
if not os.path.isdir(DATA_ROOT):
    raise FileNotFoundError(f"{DATA_ROOT} missing – mount Drive!")

csv_files = sorted(pathlib.Path(DATA_ROOT).rglob("*.csv"))
if not csv_files: raise RuntimeError("no CSVs found")
print(f"[INFO] {len(csv_files)} CSVs found")

with open(csv_files[0]) as f: header = f.readline().strip().split(',')
TARGET_COLS = ['Local_X', 'Local_Y']; TARGET_IDX=[header.index(c) for c in TARGET_COLS]
OUT_FEAT = len(TARGET_COLS); print("[INFO] targets:", TARGET_COLS)

random.shuffle(csv_files)
cut1=int(len(csv_files)*TRAIN_RATIO); cut2=int(len(csv_files)*(TRAIN_RATIO+VAL_RATIO))
train_files, val_files, test_files = csv_files[:cut1], csv_files[cut1:cut2], csv_files[cut2:]
print(f"[SPLIT] {len(train_files)} train | {len(val_files)} val | {len(test_files)} test")

# 2. window builder
INPUT_TOTAL = INPUT_LEN+OUTPUT_LEN
def make_windows(files, fit=False, sX=None, sY=None):
    Xs, Ys, infos = [], [], []
    for fp in files:
        arr = np.loadtxt(fp, delimiter=',', skiprows=1).astype(np.float32)
        L=len(arr)
        if L>=INPUT_TOTAL:
            for i in range(L-INPUT_TOTAL+1):
                Xs.append(arr[i:i+INPUT_LEN])
                Ys.append(arr[i+INPUT_LEN:i+INPUT_TOTAL, TARGET_IDX])
                infos.append({'file':fp.name,'start':i})
        else:
            need=INPUT_TOTAL-L; pad_past=max(0,INPUT_LEN-L); pad_fut=need-pad_past
            padded=np.vstack([np.repeat(arr[0:1],pad_past,0),arr,
                              np.repeat(arr[-1:],pad_fut,0)])
            Xs.append(padded[:INPUT_LEN])
            Ys.append(padded[INPUT_LEN:INPUT_TOTAL,TARGET_IDX])
            infos.append({'file':fp.name,'start':0,'padded':True})
    X,Y=np.stack(Xs),np.stack(Ys)
    if fit:
        sX=StandardScaler().fit(X.reshape(-1,X.shape[2]))
        sY=StandardScaler().fit(Y.reshape(-1,OUT_FEAT))
    X=sX.transform(X.reshape(-1,X.shape[2])).reshape(X.shape)
    Y=sY.transform(Y.reshape(-1,OUT_FEAT)).reshape(Y.shape)
    return X,Y,sX,sY,infos

print("[LOAD] building windows…")
X_train,Y_train,sX,sY,_ = make_windows(train_files,fit=True)
X_val,  Y_val,  _, _, _ = make_windows(val_files, sX=sX, sY=sY)
X_test, Y_test, _, _, it= make_windows(test_files,sX=sX,sY=sY)
```

```
print("[SHAPE]", X_train.shape, X_val.shape, X_test.shape)

def inv_scale(a): return sY.inverse_transform(a.reshape(-1,OUT_FEAT))\
                             .reshape(-1,OUTPUT_LEN,OUT_FEAT)
true_all = inv_scale(Y_test)

# 3. loss helpers
def mse_np(t,p):     return np.mean((p-t)**2)
def dmse_np(t,p):    return p-t
def huber_np(t,p,δ=HUBER_DELTA):
    e=p-t; a=np.abs(e); q=np.minimum(a,δ)
    return np.mean(0.5*q**2+δ*(a-q))
def dhuber_np(t,p,δ=HUBER_DELTA):
    e=p-t; a=np.abs(e); return np.where(a<=δ,e,δ*np.sign(e))
loss_np  = huber_np  if LOSS_FUNCTION=='huber' else mse_np
dloss_np = dhuber_np if LOSS_FUNCTION=='huber' else dmse_np

# 4. NumPy LSTM classes
_CLIP_ACT=5.0
def SIG(x): return 1/(1+np.exp(-np.clip(x,-_CLIP_ACT,_CLIP_ACT)))
def TAN(x): return np.tanh(np.clip(x,-_CLIP_ACT,_CLIP_ACT))
def clip_l2(g,l): n=np.sqrt((g**2).sum()); return g if n<=l else g*(l/(n+1e-12))

class MyLSTM:
    def __init__(self,inp_dim,hid,fh,td,lr=1e-3,clip=.5,drop=0.):
        self.i_dim,self.hid,self.fh,self.td = inp_dim,hid,fh,td
        self.lr,self.clip,self.drop = lr,clip,drop
        def xavier(sh): lim=math.sqrt(6/sum(sh)); return np.random.uniform(-lim,lim,sh).astype(np.float32)
        self.Wih=xavier((4*hid,inp_dim)); self.Whh=xavier((4*hid,hid))
        self.b=np.zeros(4*hid,np.float32)
        self.b[hid:2*hid]=+1; self.b[:hid]=-3; self.b[3*hid:]=-1
        self.Wy=xavier((fh*td,hid)); self.by=np.zeros(fh*td,np.float32)
        self.parameters=[self.Wih,self.Whh,self.b,self.Wy,self.by]

    def forward(self,seq,training=False):
        h=np.zeros(self.hid,np.float32); c=np.zeros_like(h)
        if training: self.cache=[]
        for x in seq:
            z=self.Wih@x + self.Whh@h + self.b
            i,f,g,o=np.split(z,4); i,f,o=map(SIG,(i,f,o)); g=TAN(g)
            if training and self.drop:
                mask=(np.random.rand(*h.shape)>self.drop).astype(np.float32)
                h*=mask/(1-self.drop)
            c=f*c+i*g; h=o*TAN(c)
            if training: self.cache.append((x,h.copy(),c.copy(),i,f,g,o))
        logit=self.Wy@h + self.by
        return logit.reshape(self.fh,self.td)

    def backward(self,t,pred):
        dl=dloss_np(t,pred).flatten()
        dWy=np.outer(dl,self.cache[-1][1]); dby=dl.copy()
        dh_n=self.Wy.T@dl; dc_n=np.zeros(self.hid,np.float32)
        dWih=np.zeros_like(self.Wih); dWhh=np.zeros_like(self.Whh); db=np.zeros_like(self.b)
        for x,h,c,i,f,g,o in reversed(self.cache):
            tan=TAN(c)
            dc=dh_n*o*(1-tan**2)+dc_n
            di=dc*g*i*(1-i); df=dc*c*f*(1-f); dg=dc*i*(1-g**2); do=dh_n*tan*o*(1-o)
            dz=np.concatenate([di,df,dg,do])
            dWih+=np.outer(dz,x); dWhh+=np.outer(dz,h); db+=dz
            dh_n=self.Whh.T@dz; dc_n=dc*f
        for d in (dWih,dWhh,db,dWy,dby): np.clip(d,-self.clip,self.clip,out=d)
        return [dWih,dWhh,db,dWy,dby]

    def step(self,grads):
        for p,g in zip(self.parameters,grads): p-=self.lr*g

class EnhancedMyLSTM(MyLSTM):
    def __init__(self,*a,dropout=.2,eps=1e-5,wd=1e-4,**kw):
        super().__init__(*a,drop=dropout,**kw)
        hid=self.hid
        self.peep_i=self.peep_f=self.peep_o=np.zeros(hid,np.float32)
        self.ln_gamma=np.ones(4*hid,np.float32); self.ln_beta=np.zeros(4*hid,np.float32)
        self.parameters+= [self.peep_i,self.peep_f,self.peep_o,self.ln_gamma,self.ln_beta]
        self.eps,self.wd=eps,wd
        self.m=[np.zeros_like(p) for p in self.parameters]
        self.v=[np.zeros_like(p) for p in self.parameters]
        self.b1,self.b2,self.t=0.9,0.999,0
```

```
    def _ln(self,z): m=z.mean(); v=z.var(); return (z-m)/np.sqrt(v+self.eps),m,v
    def forward(self,seq,training=False):
        h=np.zeros(self.hid,np.float32); c=np.zeros_like(h)
        if training: self.cache=[]
        for x in seq:
            z=self.Wih@x + self.Whh@h + self.b + \
                np.concatenate([self.peep_i*c,self.peep_f*c,np.zeros_like(c),self.peep_o*c])
            z_hat,μ,σ2=self._ln(z); z_ln=z_hat*self.ln_gamma+self.ln_beta
            i,f,g,o=np.split(z_ln,4); i,f,o=map(SIG,(i,f,o)); g=TAN(g)
            if training and self.drop:
                mask=(np.random.rand(*h.shape)>self.drop).astype(np.float32)
                h*=mask/(1-self.drop)
            c=f*c+i*g; h=o*TAN(c)
            if training: self.cache.append((x,h.copy(),c.copy(),i,f,g,o,z_hat,μ,σ2))
        return (self.Wy@h+self.by).reshape(self.fh,self.td)

    def _ln_bwd(self,dout,z_hat,μ,σ2):
        N=len(z_hat); dγ=(dout*z_hat).sum(); dβ=dout.sum()
        dz_hat=dout*self.ln_gamma
        dσ2=-.5*(σ2+self.eps)**-1.5*(dz_hat*z_hat).sum()
        dμ=-(dz_hat/np.sqrt(σ2+self.eps)).sum()-2*dσ2*z_hat.mean()
        dz=dz_hat/np.sqrt(σ2+self.eps)+dσ2*2*z_hat/N+dμ/N
        return dz,dγ,dβ

    def backward(self,t,pred):
        dl=dloss_np(t,pred).flatten()
        dWy=np.outer(dl,self.cache[-1][1]); dby=dl.copy()
        dh_n=self.Wy.T@dl; dc_n=np.zeros(self.hid,np.float32)
        dWih=np.zeros_like(self.Wih); dWhh=np.zeros_like(self.Whh); db=np.zeros_like(self.b)
        dpi=dpf=dpo=np.zeros(self.hid,np.float32)
        dγ=np.zeros_like(self.ln_gamma); dβ=np.zeros_like(self.ln_beta)
        for x,h,c,i,f,g,o,z_hat,μ,σ2 in reversed(self.cache):
            tan=TAN(c)
            dc=dh_n*o*(1-tan**2)+dc_n
            di=dc*g*i*(1-i); df=dc*c*f*(1-f); dg=dc*i*(1-g**2); do=dh_n*tan*o*(1-o)
            dz_ln=np.concatenate([di,df,dg,do])
            dz, dγ_t, dβ_t = self._ln_bwd(dz_ln,z_hat,μ,σ2)
            dγ+=dγ_t; dβ+=dβ_t
            di,dF,dg,do=np.split(dz,4)
            dpi+=di*c; dpf+=dF*c; dpo+=do*c
            dWih+=np.outer(dz,x); dWhh+=np.outer(dz,h); db+=dz
            dh_n=self.Whh.T@dz
            dc_n=dc*f+self.peep_i*di+self.peep_f*dF+self.peep_o*do
        grads=[dWih,dWhh,db,dWy,dby,dpi,dpf,dpo,dγ,dβ]
        grads=[clip_l2(g,self.clip) for g in grads]
        return grads

    def step(self,grads):
        self.t+=1
        for i,(p,g) in enumerate(zip(self.parameters,grads)):
            self.m[i]=self.b1*self.m[i]+(1-self.b1)*g
            self.v[i]=self.b2*self.v[i]+(1-self.b2)*(g*g)
            m_hat=self.m[i]/(1-self.b1**self.t)
            v_hat=self.v[i]/(1-self.b2**self.t)
            upd=m_hat/np.sqrt(v_hat+self.eps)
            if p.ndim==2: upd+=self.wd*p
            p-=self.lr*upd

# 5. trainers – NumPy
def fit_numpy(model,Xtr,Ytr,Xv,Yv,label,batch=32):
    best=np.inf; wait=0; hist={'train':[],'val':[]}
    best_state=[p.copy() for p in model.parameters]
    idx=np.arange(len(Xtr))
    for ep in range(1,EPOCHS+1):
        np.random.shuffle(idx); tr_loss=0.
        for bi in range(0,len(idx),batch):
            sel=idx[bi:bi+batch]
            grads=[np.zeros_like(p) for p in model.parameters]
            for j in sel:
                yp=model.forward(Xtr[j],training=True)
                g=model.backward(Ytr[j],yp)
                grads=[gk+dk for gk,dk in zip(grads,g)]
                tr_loss+=loss_np(Ytr[j],yp)
            grads=[g/len(sel) for g in grads]; model.step(grads)
        tr_loss/=len(Xtr)
        val_loss=np.mean([loss_np(y,model.forward(x)) for x,y in zip(Xv,Yv)])
        hist['train'].append(tr_loss); hist['val'].append(val_loss)
```

```
        print(f"[{label}] ep{ep:02d} tr={tr_loss:.3f} val={val_loss:.3f}")
        if val_loss<best: best,val_wait=val_loss,0; best_state=[p.copy() for p in model.parameters]
        else:
            val_wait+=1
            if val_wait>=EARLY_STOP: break
    for p,b in zip(model.parameters,best_state): p[...] = b
    preds=inv_scale(np.array([model.forward(x) for x in X_test]))
    return hist,preds,model

# 6. PyTorch models
class PT_LSTM(nn.Module):
    def __init__(self): super().__init__()
    def forward(self,x): ...

class PTBasic(nn.Module):
    def __init__(self): super().__init__()
    def forward(self,x): ...

class PT_Basic(nn.Module):
    def __init__(self):
        super().__init__()
        self.lstm=nn.LSTM(X_train.shape[2],HIDDEN_SIZE,batch_first=True)
        self.fc  =nn.Linear(HIDDEN_SIZE,OUTPUT_LEN*OUT_FEAT)
    def forward(self,x):
        h=self.lstm(x)[0][:,-1]
        return self.fc(h).view(-1,OUTPUT_LEN,OUT_FEAT)

class PT_Bi(nn.Module):
    def __init__(self):
        super().__init__()
        self.lstm=nn.LSTM(X_train.shape[2],HIDDEN_SIZE,batch_first=True,bidirectional=True)
        self.fc  =nn.Linear(HIDDEN_SIZE*2,OUTPUT_LEN*OUT_FEAT)
    def forward(self,x):
        h=self.lstm(x)[0][:,-1]
        return self.fc(h).view(-1,OUTPUT_LEN,OUT_FEAT)

class PT_Attn(nn.Module):
    def __init__(self):
        super().__init__()
        self.lstm=nn.LSTM(X_train.shape[2],HIDDEN_SIZE,batch_first=True)
        self.Wa  =nn.Linear(HIDDEN_SIZE,HIDDEN_SIZE,bias=False)
        self.fc  =nn.Linear(HIDDEN_SIZE*2,OUTPUT_LEN*OUT_FEAT)
    def forward(self,x):
        seq,(h,_) = self.lstm(x); h=h[-1]
        score=(seq*self.Wa(h).unsqueeze(1)).sum(-1)
        α=torch.softmax(score,dim=-1).unsqueeze(-1)
        ctx=(seq*α).sum(1)
        return self.fc(torch.cat([h,ctx],-1)).view(-1,OUTPUT_LEN,OUT_FEAT)

class PT_Hybrid(nn.Module):
    def __init__(self,hidden=HIDDEN_SIZE,mlp_hidden=256,dropout=.25):
        super().__init__()
        self.lstm=nn.LSTM(X_train.shape[2],hidden,batch_first=True)
        self.drop1=nn.Dropout(dropout)
        self.mlp1=nn.Linear(hidden,mlp_hidden)
        self.drop2=nn.Dropout(dropout)
        self.mlp2=nn.Linear(mlp_hidden,OUTPUT_LEN*OUT_FEAT)
    def forward(self,x):
        h=self.lstm(x)[0][:,-1]
        h=self.drop1(torch.relu(self.mlp1(h)))
        return self.mlp2(self.drop2(h)).view(-1,OUTPUT_LEN,OUT_FEAT)

# 7. PT runner & grid
def run_pt(model, lr, clip, batch, label, sched_cls=None):
    tr_ds = TensorDataset(torch.tensor(X_train), torch.tensor(Y_train))
    va_ds = TensorDataset(torch.tensor(X_val),   torch.tensor(Y_val))
    tr_dl = DataLoader(tr_ds, batch, shuffle=True, num_workers=0)
    va_dl = DataLoader(va_ds, batch,             num_workers=0)

    model.to(DEVICE)
    opt   = optim.Adam(model.parameters(), lr)
    crit  = nn.SmoothL1Loss()
    sched = sched_cls(opt, mode="min", patience=2) if sched_cls else None

    best = float("inf"); wait = 0; best_state = None
    hist = {"train": [], "val": []}

    for ep in range(1, EPOCHS + 1):
```

```
    for ep in range(1, EPOCHS + 1):
        # ---- training -----
        model.train(); tr = 0.0
        for xb, yb in tr_dl:
            xb, yb = xb.to(DEVICE), yb.to(DEVICE)
            opt.zero_grad()
            loss = crit(model(xb), yb)
            loss.backward()
            if clip:
                nn.utils.clip_grad_norm_(model.parameters(), clip)
            opt.step()
            tr += loss.item()
        tr /= len(tr_dl)

        # ---- validation ---
        model.eval(); val = 0.0
        with torch.no_grad():
            for xb, yb in va_dl:
                val += crit(model(xb.to(DEVICE)), yb.to(DEVICE)).item()
        val /= len(va_dl)

        hist["train"].append(tr); hist["val"].append(val)
        if sched:
            sched.step(val)

        if val < best:
            best, wait, best_state = val, 0, copy.deepcopy(model.state_dict())
        else:
            wait += 1
            if wait >= EARLY_STOP:
                break
        print(f"[{label}] ep{ep:02d} tr={tr:.3f} val={val:.3f}")

    # ---- test ----
    model.load_state_dict(best_state); model.eval()
    te_dl = DataLoader(torch.tensor(X_test), batch, num_workers=0)
    preds = []
    with torch.no_grad():
        for xb in te_dl:                         # ← FIXED LINE
            preds.append(model(xb.to(DEVICE)).cpu().numpy())
    preds = inv_scale(np.concatenate(preds, 0))
    return hist, preds, model, best

def grid_search_pt(name,make_model):
    best={'mse':np.inf}
    for lr in [1e-3,5e-4,1e-4]:
        for clip in [None,0.5]:
            for bs in [32,64]:
                tag=f"{name}-lr{lr:g}-bs{bs}-clip{clip}"
                h,p,m,_=run_pt(make_model(),lr,clip,bs,tag)
                mse=mean_squared_error(true_all.reshape(-1,OUT_FEAT),p.reshape(-1,OUT_FEAT))
                if mse<best['mse']: best.update(tag=tag,hist=h,pred=p,model=m,mse=mse)
                print(tag,"MSE",round(mse,3))
    print("[BEST]",name,best['tag'],best['mse'])
    return best['tag'],best['hist'],best['pred'],best['model']

# 8. train runs
print("\n=== NumPy ===")
hist_np,preds_np,np_lstm=fit_numpy(MyLSTM(X_train.shape[2],HIDDEN_SIZE,OUTPUT_LEN,OUT_FEAT,lr=1e-3),
                                   X_train,Y_train,X_val,Y_val,"NumPySGD",batch=32)
hist_enh,preds_enh,enh_lstm=fit_numpy(EnhancedMyLSTM(X_train.shape[2],HIDDEN_SIZE,OUTPUT_LEN,OUT_FEAT,
                                   lr=5e-4,dropout=.1),X_train,Y_train,X_val,Y_val,"NumPyAdamW",batch=32)

print("\n=== PT basic ===")
pt_basic_tag,pt_basic_hist,pt_basic_pred,pt_basic_model = grid_search_pt("PT-basic",PT_Basic)
print("\n=== PT Bi ===")
pt_bi_tag,pt_bi_hist,pt_bi_pred,pt_bi_model = grid_search_pt("PT-bidir",PT_Bi)
print("\n=== PT Attn ===")
pt_attn_tag,pt_attn_hist,pt_attn_pred,pt_attn_model = grid_search_pt("PT-attn",PT_Attn)
print("\n=== PT Hybrid ===")
pt_hyb_tag,pt_hyb_hist,pt_hyb_pred,pt_hyb_model = grid_search_pt("PT-hybrid",PT_Hybrid)

print("\n=== Keras ===")
def K_build(bidir=False,attn=False):
    inp=Input((INPUT_LEN,X_train.shape[2]))
    seq = Bidirectional(LSTM(HIDDEN_SIZE,return_sequences=attn))(inp) if bidir else \
          LSTM(HIDDEN_SIZE,return_sequences=attn)(inp)
    if attn:
```

```
        last=seq[:,-1:,:]; ctx=Attention()([last,seq])
        x=Concatenate()([last,ctx]); x=Reshape((2*HIDDEN_SIZE*(2 if bidir else 1),))(x)
    else: x=seq
    out=Dense(OUTPUT_LEN*OUT_FEAT)(x); out=Reshape((OUTPUT_LEN,OUT_FEAT))(out)
    return keras.Model(inp,out)

def fit_keras(model,name,lr):
    model.compile(keras.optimizers.Adam(lr),loss=tf.keras.losses.Huber(delta=HUBER_DELTA))
    es=keras.callbacks.EarlyStopping(patience=EARLY_STOP,restore_best_weights=True,verbose=0)
    h=model.fit(X_train,Y_train,validation_data=(X_val,Y_val),epochs=EPOCHS,
                batch_size=BATCH_SIZE,verbose=0,callbacks=[es]).history
    print(name,"best",min(h['val_loss']))
    return h,inv_scale(model.predict(X_test,BATCH_SIZE,verbose=0)),model

hist_kl,preds_kl,keras_lstm=fit_keras(K_build(),         "KerasLSTM", 1e-3)
hist_klb,preds_klb,keras_blstm=fit_keras(K_build(bidir=True),"KerasBi",5e-4)
hist_kla,preds_kla,keras_attn=fit_keras(K_build(attn=True),"KerasAttn",5e-4)

# metrics
print("\n───────── metrics ──────────────────────────")
for k, v in pred_dict.items():
    mse, rmse, mae, r2 = metr(true_all, v)
    print(f"{k:<18} MSE={mse:9.2f}  RMSE={rmse:7.2f}  MAE={mae:6.2f}  R²={r2:.4f}")

# loss-curve figure
plt.figure(figsize=(10, 6))
def _p(h, c, l): plt.plot(h['train'], c+'-', label=f"{l} tr"); plt.plot(h['val'], c+'--')
_p(hist_np,   'r', 'NumPySGD')
_p(hist_enh,  'k', 'NumPyAdamW')
_p(pt_basic_hist, 'b', pt_basic_tag)
_p(pt_bi_hist,    'g', pt_bi_tag)
_p(pt_attn_hist,  'c', pt_attn_tag)
_p(pt_hyb_hist,   'm', pt_hyb_tag)
plt.plot(hist_kl ['loss'],      'y-',      label='KerasLSTM tr')
plt.plot(hist_kl ['val_loss'],  'y--')
plt.plot(hist_klb['loss'],      'orange',  label='KerasBiLSTM tr')
plt.plot(hist_klb['val_loss'],  'orange',  linestyle='--')
plt.plot(hist_kla['loss'],      'violet',  label='KerasAttn tr')
plt.plot(hist_kla['val_loss'],  'violet',  linestyle='--')
plt.yscale('log'); plt.xlabel("epoch"); plt.ylabel("loss"); plt.legend(ncol=2)
plt.tight_layout(); plt.show()

# full-test scatter / residuals
for v, col in enumerate(TARGET_COLS):
    t = true_all[:, :, v].flatten()
    plt.figure(figsize=(5, 5))
    for k, p in pred_dict.items():
        plt.scatter(t, p[:, :, v].flatten(), s=8, alpha=.35, label=k)
    m = np.abs(t).max()
    plt.plot([-m, m], [-m, m], 'k--'); plt.title(col)
    plt.xlabel("true"); plt.ylabel("pred"); plt.legend(fontsize=6)
    plt.tight_layout(); plt.show()

    plt.figure(figsize=(6, 4))
    for k, p in pred_dict.items():
        plt.hist((p[:, :, v] - true_all[:, :, v]).flatten(),
                 bins=60, alpha=.35, density=True, label=k)
    plt.axvline(0, color='k'); plt.title("Residuals – " + col)
    plt.legend(fontsize=6); plt.tight_layout(); plt.show()

# save scalers
np.savez(
    os.path.join(SAVE_DIR, "scalers.npz"),
    x_mean=sX.mean_,  x_scale=sX.scale_,
    y_mean=sY.mean_,  y_scale=sY.scale_
)
print("[✓] scalers saved to", os.path.join(SAVE_DIR, "scalers.npz"))

# save models
np.savez(os.path.join(SAVE_DIR, "NumPySGD.npz"),   *np_lstm.parameters)
np.savez(os.path.join(SAVE_DIR, "NumPyAdamW.npz"), *enh_lstm.parameters)
torch.save(pt_basic_model.state_dict(), os.path.join(SAVE_DIR, f"{pt_basic_tag}.pt"))
torch.save(pt_bi_model.state_dict(),    os.path.join(SAVE_DIR, f"{pt_bi_tag}.pt"))
torch.save(pt_attn_model.state_dict(),  os.path.join(SAVE_DIR, f"{pt_attn_tag}.pt"))
torch.save(pt_hyb_model.state_dict(),   os.path.join(SAVE_DIR, f"{pt_hyb_tag}.pt"))
keras_lstm.save (os.path.join(SAVE_DIR, "KerasLSTM.keras"))
keras_blstm.save(os.path.join(SAVE_DIR, "KerasBiLSTM.keras"))
keras_attn.save(os.path.join(SAVE_DIR, "KerasAttn.keras"))
```

```
keras_attn.save(os.path.join(SAVE_DIR, "KerasAttn.keras"))
print("[✓] everything saved in", SAVE_DIR)
```

Show hidden output

Testing

```
import math, matplotlib.pyplot as plt, numpy as np

#  the models
models_to_show = ["PT-bidir", "PT-basic", "PT-attn",
                  "KerasAttn", "NumPyAdamW"]   # ≤ 5
horizon        = 5              # how many steps each pred contains

# sanity-check
missing = [m for m in models_to_show if m not in pred]
assert not missing, f"predictions not found for: {missing}"

# colour/marker table
cmap = ["r", "b", "m", "c", "y"]
mmap = ["o", "s", "D", "v", "P"]   # circle, square, diamond …

# ── plotting constants ------------------------------------------------
N              = len(X_test)
samples_per_fig = 20
cols, rows     = 4, math.ceil(samples_per_fig/4)

for start in range(0, N, samples_per_fig):
    end  = min(start+samples_per_fig, N)
    fig, axs = plt.subplots(rows, cols,
                            figsize=(cols*4, rows*4),
                            squeeze=False)

    for k in range(rows*cols):
        r, c  = divmod(k, cols)
        ax    = axs[r][c]
        idx   = start + k
        if idx < end:
            # ------------- data in *absolute* coords --------------------
            past_xy = sX.inverse_transform(
                          X_test[idx].reshape(-1, X_test.shape[2])
                      )[:, :2]                       # (62,2)
            true_xy = true_all[idx, :horizon]        # (horizon,2)

            # anchor = last observed point
            anchor  = past_xy[-1]
            past    = past_xy - anchor
            true5   = true_xy - anchor

            ax.plot(past[:,0], past[:,1], 'k.-', lw=.8, ms=2, label='past 62')
            ax.plot(true5[:,0], true5[:,1], 'g^--', lw=1.2, ms=5, label='true +5')

            # ------------- overlay every chosen model -------------------
            for m_i, m_name in enumerate(models_to_show):
                col = cmap[m_i % len(cmap)]
                mk  = mmap[m_i % len(mmap)]
                pred_xy = pred[m_name][idx, :horizon] - anchor
                ax.plot(pred_xy[:,0], pred_xy[:,1],
                        color=col, marker=mk, lw=1.2, ms=4,
                        label=f"{m_name} +{horizon}")

            ax.set_title(f"test #{idx}", fontsize=9)
            ax.set_xlim(-50, 50); ax.set_ylim(-50, 50)   # zoom window
            ax.set_aspect('equal'); ax.grid(True)
        else:
            ax.axis('off')

    # legend once (upper-left)
    handles, labels = axs[0][0].get_legend_handles_labels()
    axs[0][0].legend(handles, labels, fontsize=7, loc='upper right')

    fig.suptitle(f"Test-set trajectories [{start} … {end-1}]  "
                 f"compared models: {', '.join(models_to_show)}",
```

```
            fontsize=15)
plt.tight_layout(); plt.show()
```

Show hidden output

Results

The results strongly support the primary hypothesis that the car's recent history contains predictable patterns learnable by sequence models. Most models achieved very low MSE and extremely high R^2 values , indicating they successfully learned to predict the near-future trajectory with high accuracy. The results also confirm that different implementations and architectures yield significantly different performance levels.

- The PyTorch Bidirectional LSTM achieved the lowest MSE (20.44) and highest R^2 (0.9997), indicating the best overall predictive accuracy on the test set.
- The basic PyTorch LSTM (MSE=39.82) and the hybrid PyTorch model (MSE=73.91) also performed very well. The PyTorch Attention model (MSE=90.29) was slightly less accurate but still effective.

- `KerasLSTM` (MSE=150.16), `KerasAttn` (MSE=308.04)& `KerasBiLSTM` (MSE=374.98) showed good R^2 values but had notably higher MSE compared to the best PyTorch configurations.
- The `NumPyAdamW` implementation performed well (MSE=239.79), demonstrating the effectiveness of the AdamW optimizer even with a from-scratch NumPy implementation. However, `NumPySGD` performed extremely poorly (MSE=78965.45, R^2=0.0387), highlighting the critical importance of the optimizer choice, especially for basic SGD without enhancements like momentum in this context.

The top-performing model was bidirectional (`PT-bidir`), suggesting that considering both past and "future" context (relative to each point in the input sequence) was beneficial for predicting the next 5 steps. While the Attention and Hybrid models performed well, they didn't surpass the Bidirectional or even the best Basic LSTM configuration in this specific test setup. This might indicate that for this short-term prediction task (5 steps), the added complexity didn't provide a significant advantage over a well-tuned BiLSTM or basic LSTM. The difference between the best PyTorch models and the Keras models, as well as the vast difference between `NumPyAdamW` and `NumPySGD`, underscores that both the underlying library/implementation details and hyperparameter tuning (optimizer, LR, batch size, clipping) are crucial.

The loss curves show rapid initial learning for most models using adaptive optimizers (Adam variants, RMSProp implied in Keras defaults) or the enhanced NumPy implementation (`NumPyAdamW`). `NumPySGD` shows extremely slow convergence, barely improving over the epochs shown. Most successful models appear to converge quickly, often within the first few epochs, with validation loss plateauing early, justifying the use of early stopping.

The plots comparing predictions from selected models (`PT-bidir`, `PT-basic`, `PT-attn`, `KerasAttn`, `NumPyAdamW`) against the true future path generally show very close tracking for most test trajectories. This visually confirms the low quantitative error metrics and the models' ability to capture the short-term vehicle dynamics effectively. Differences between the top models are subtle on these plots.

Future Work direction can be in exploring Transformers or Temporal Convolutional Networks