Ph 20 Lab3 Write Up

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1 Part 1

1.1 The Explicit Euler Method

The following plots show the numerically integrated solution to a simple harmonic oscillator with initial conditions x = 1 and v = 0. For the explicit Euler method, the errors vary periodically, with increasing amplitude. The amplitude of errors appears to increase exponentially (not picture, but for many cycles this becomes more apparent). The long range trend for the total energy, which is proportional to $x^2 + v^2$ is a linear increase with time. Since we know that energy is conserved in the analytic solution, this increase in energy must be a result of the accumulation of errors due to small errors in the numerical integration.
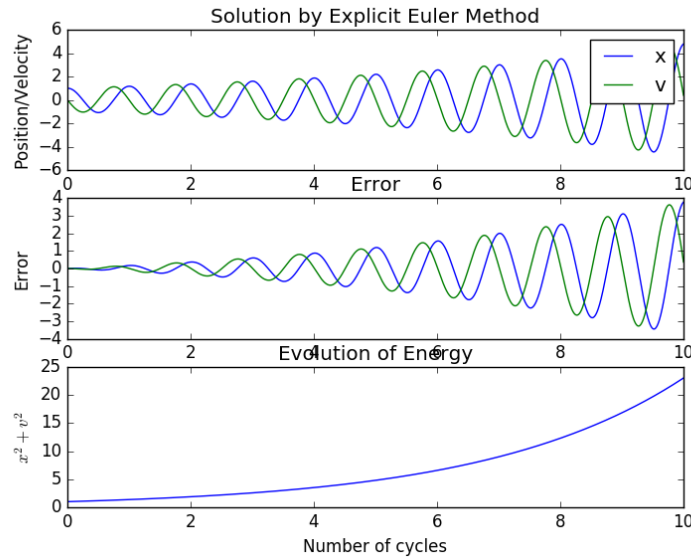

Figure 1: The first subplot shows the solutions for x and v obtained via numerical integration by the Explicit Euler method. The second subplot shows the errors that accumulate. The third plot shows the evolution of the energy in the system, which increases linearly with time due to the accumulation of errors. The value of h used while integrating was .01

1.2 Truncation Error

The truncation error is plotted as a function of h in figure 2. Note that the x-axis goes from larger h to smaller h, showing that the error decreases as h becomes smaller. The error becomes much better as h becomes smaller, as is roughly proportional to h.
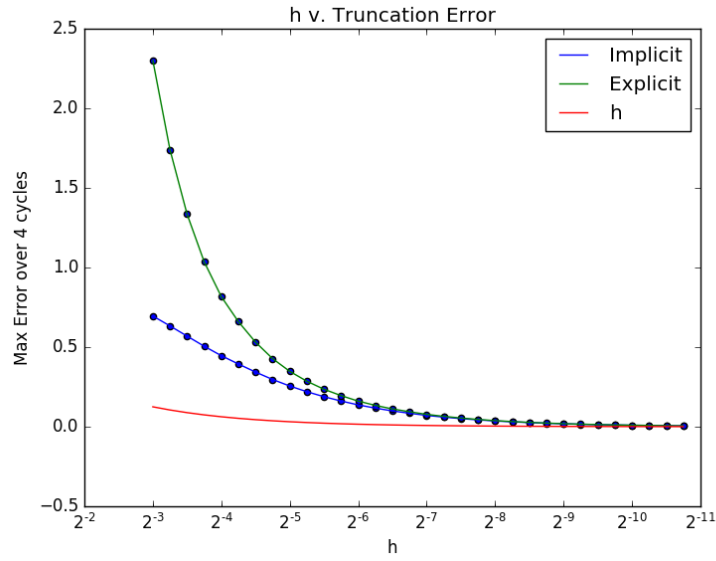
Figure 2: The truncation error associated with implicit and explicit Euler methods, as a function of h. Note that as h becomes small, the truncation error goes approximately as h.

1.3   The Implicit Euler Method

The solution to the propagation of the implicit Euler method was found by taking the inverse of the matrix found in Equ 7 in the notes. The errors appear to increase at the same rate, but flipped about the y-axis. This indicates that the small accumulations will decrease the amplitude of the estimate. This is confirmed by the Total energy plot, which decreases with the number of cycles.
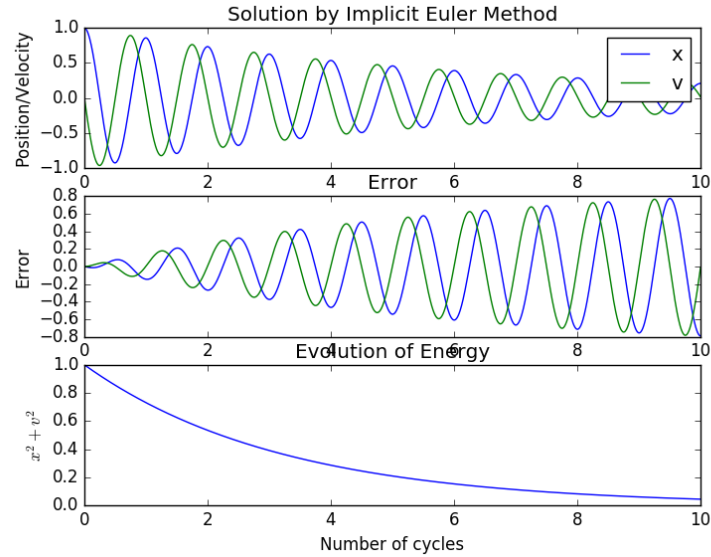

Figure 3: The first subplot shows the solutions for x and v obtained via numerical integration by the Implicit Euler method. The second subplot shows the errors that accumulate. The third plot shows the evolution of the energy in the system, which decreases linearly with time due to the accumulation of errors. The value of h used while integrating was .01

2 Part 2

2.1 Phase Trajectories of the Explicit and Implicit Euler methods

A perfect solution to the SHO would have a perfectly circular trajectory in phase space, given our in initial conditions (k, m = 1). We see that the numerically integrated solutions deviate from this. The energy of the system is proportional to the area enclosed by is trajectory in phase space. Note that this area decreases for the implicit method and increases for the explicit method as the trajectories spiral inwards or outwards.
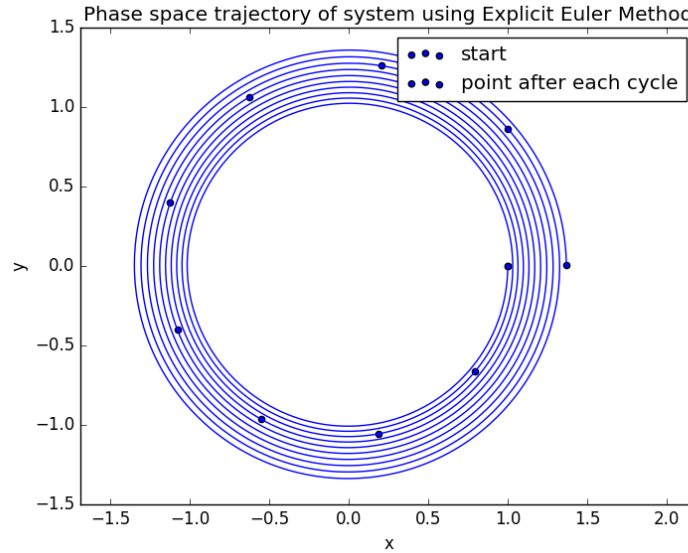

Figure 4: The plot of the first 4 cycles of the SHO in phase space, using the numerical solution obtained by the explicit Euler method. Note how the trajectory spirals outward. If energy were conserved, we would expect the trajectory to be closed. The starting point is noted with a blue dot. The interval h used was .01

Figure 5: The plot of the first 4 cycles of the SHO in phase space, using the numerical solution obtained by the implicit Euler method. Note how the trajectory spirals inward. Again, the system is losing energy. The starting point is noted with a blue dot. The interval h used was .01

2.2 The Symplectic Euler Method

Here we see that the amplitudes are conserved, as the energy oscillates about 1 with a frequency roughly equal to the frequencies of x and v. However, upon closer examination, the frequency of oscillation lags slightly compared with that of the perfect solution. In figure[], the analytic and symplectic solutions with h=.1 after 100 cycles are plotted on the same axis. The sypmletic solution is almost 180 out of phase with the analytic solution!
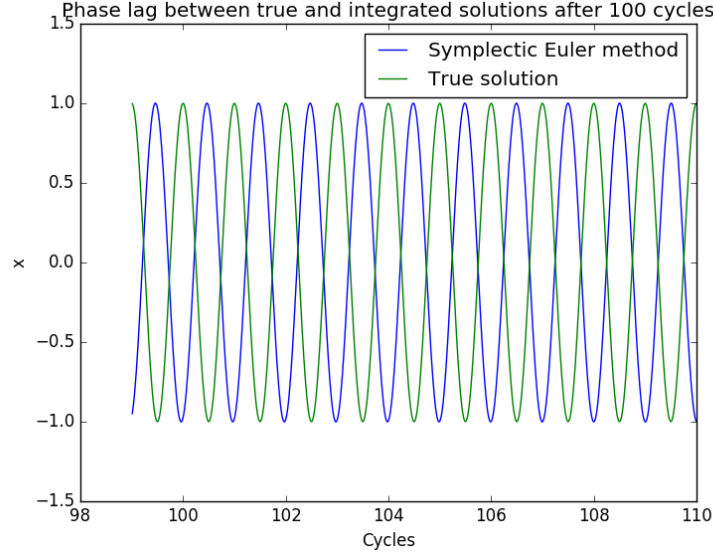
Figure 6: The lag between the numerical and analytical solutions after 100 cycles. The are almost completely out of phase. The integration was done using h = 0.1

2.2.1 Phase Trajectory

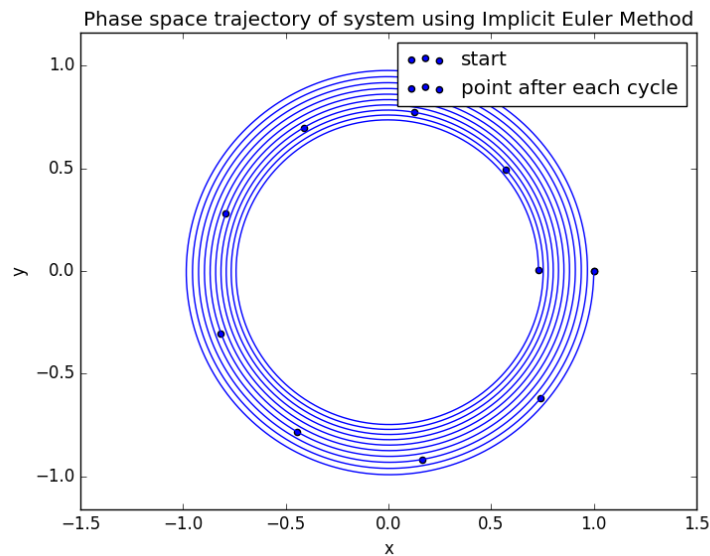As expected, the plot of the phase trajectory of the symplectic solution is closed.
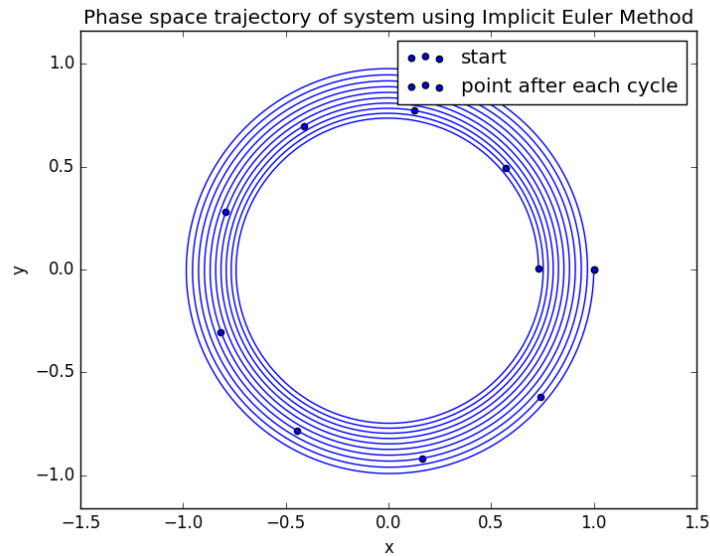
Figure 7: The plot of the first 4 cycles of the SHO in phase space, using the numerical solution obtained by the symplectic Euler method. Note how the trajectory appears closed. The starting point is noted with a blue dot. The interval h used was .01

3 Documents

3.1 code

```python
import matplotlib.pyplot as plt
import numpy as np
import pdb


#Returns the next iteration given the previous one
def Explicit(x, v, h):
    return x + h*v, v - h*x

def Implicit(x, v, h):
    return 1/(h**2 + 1) * (x + h*v), 1/(h**2 + 1) * (v - h*x)

def Symplectic(x, v, h):
    return  x + h*v,  1/(h**2 + 1) * (v - h*x)

def find(times, h):      #Analytic solutions
    x = np.cos(times)
    v = -np.sin(times)
    return x, v

def find_trunc(h, method, n):
    v = np.zeros(int(n/h))
    x = np.ones(int(n/h))
    for i in range(1, int(n / h)):
        x[i], v[i] = method(x[i-1], v[i-1], h)
    t= np.linspace(0, n, int(n/h))
    x_true, v_true = find(t, h)
    return np.max(np.abs(x_true - x)), np.max(np.abs(v_true - v))

def plot_method(tf, t0, h, method, cycles, name):
    plt.figure()
    t = np.arange(t0, tf, h)
    v = np.zeros(len(t))
    x = np.ones(len(t))
    for i in range(1, len(t)):       #using "method" method to find x, v
        x[i], v[i] = method(x[i-1], v[i-1], h)
    x_ax = np.linspace(0, cycles, len(t))
    plt.subplot(311)
    plt.title('Solution by '+ method.__name__ + ' Euler Method')
    plt.plot(x_ax, x, label="x")
    plt.plot(x_ax, v, label="v")
    plt.xlabel('Number of cycles')
    plt.ylabel('Position/Velocity')
    plt.legend()
    plt.subplot(312)
    plt.title('Error')
    x_true, v_true = find(t, h)
    plt.plot(x_ax, x-x_true)
    plt.plot(x_ax, v-v_true)
    plt.xlabel('Number of cycles')
    plt.ylabel('Error')
    plt.subplot(313)
    plt.title('Evolution of Energy')
    plt.plot(x_ax, x**2 + v**2)
    plt.xlabel('Number of cycles')
```

```python
    plt.ylabel(r'$x^2 + v^2$')
    plt.savefig(name)

def plot_phase(tf, t0, h, method, cycles, name):
    plt.figure()
    t = np.arange(t0, tf, h)
    v = np.zeros(len(t))
    x = np.ones(len(t))
    for i in range(1, len(t)):
        x[i], v[i] = method(x[i-1], v[i-1], h)
    plt.plot(x, v)
    plt.xlabel('x')
    plt.ylabel('y')
    plt.title('Phase space trajectory of system using '+ method.__name__ + ' Euler Method')
    plt.scatter(1, 0, label='start')
    cyc_idx = np.linspace(0, len(t)-1, cycles).astype(int)
    plt.scatter(x[cyc_idx], v[cyc_idx], label='point after each cycle')
    plt.axis('equal')
    plt.legend()
    plt.savefig(name)


#!/usr/bin/env python

import numpy as np
import matplotlib.pyplot as plt
import pdb

from functions import *

cycles = 10
tf =cycles * 2*np.pi
t0 = 0
h = .05

#Part 1
plot_method(tf, t0, h, Implicit, cycles, 'impl.png')
plot_method(tf, t0, h, Explicit, cycles, 'expl.png')


h0 = .125
h_list = h0 * 2.**np.arange(0., -8., -.25)
imp, exp = np.ones((2, len(h_list)))
n = 20

#Find and plot truncation errors
plt.figure()
for i, h in enumerate(h_list):
    imp[i] = find_trunc(h, Implicit, n)[0]
    exp[i] = find_trunc(h, Explicit, n)[0]
plt.semilogx(h_list, imp, basex=2, label='Implicit')
plt.semilogx(h_list, exp, basex=2, label='Explicit')
plt.semilogx(h_list, h_list, basex=2, label='h')
plt.scatter(h_list, imp)
plt.scatter(h_list, exp)
plt.gca().invert_xaxis()
plt.legend()
plt.xlabel('h')
plt.ylabel('Max Error over 4 cycles')
```

```
plt.title('h v. Truncation Error')
plt.savefig('t_err.png')

#Part 2

plot_phase(tf, t0, .01, Implicit, cycles, 'ph_i.png')
plot_phase(tf, t0, .01, Explicit, cycles, 'ph_e.png')
plot_phase(tf, t0, .1, Symplectic, cycles, 'ph_s.png')

plot_method(tf, t0, h, Symplectic, cycles, 'sympl.png')

#Plot phase lag after ~100 cycles
h = .1
cycles = 110
tf =cycles * 2*np.pi
plt.figure()
t = np.arange(t0, tf, h)
v = np.zeros(len(t))
x = np.ones(len(t))
for i in range(1, len(t)):
    x[i], v[i] = Symplectic(x[i-1], v[i-1], h)
plt.plot(t[int(-.1*len(t)):]/(2*np.pi), x[int(-.1*len(t)):], label='Symplectic Euler method')
plt.plot(t[int(-.1*len(t)):]/(2*np.pi), np.cos(t)[int(-.1*len(t)):], label = 'True solution')
plt.legend()
plt.xlabel('Cycles')
plt.ylabel('x')
plt.title('Phase lag between true and integrated solutions after 100 cycles')
plt.savefig('lag.png')
```

3.2 Makefile

```
# generate lab 3 writeup

all: lab3.pdf

pngs=*.png

lab3.pdf: main.tex $(pngs)
pdflatex main.tex

%.png: main.py
python main.py functions.py

.PHONY : clean
clean:
rm -f *.log
rm -f *.png
```

3.3 Logs and Output

```
This is pdfTeX, Version 3.1415926-2.5-1.40.14 (TeX Live 2013/Debian) (format=pdflatex 2017.2.21)
entering extended mode
 restricted \write18 enabled.
 %&-line parsing enabled.
**main.tex
(./main.tex
LaTeX2e <2011/06/27>
Babel <3.9h> and hyphenation patterns for 78 languages loaded.
(/usr/share/texlive/texmf-dist/tex/latex/base/article.cls
```

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Document Class: article 2007/10/19 v1.4h Standard LaTeX document class
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File: size10.clo 2007/10/19 v1.4h Standard LaTeX file (size option)
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Package: babel 2013/12/03 3.9h The Babel package

(/usr/share/texlive/texmf-dist/tex/generic/babel-english/english.ldf
Language: english 2012/08/20 v3.3p English support from the babel system

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File: babel.def 2013/12/03 3.9h Babel common definitions
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\l@australian = a dialect from \language\l@british
\l@newzealand = a dialect from \language\l@british
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Package: ucs 2013/05/11 v2.2 UCS: Unicode input support

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File: uni-global.def 2013/05/13 UCS: Unicode global data
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Package: fontenc 2005/09/27 v1.99g Standard LaTeX package

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File: t1enc.def 2005/09/27 v1.99g Standard LaTeX file
LaTeX Font Info:    Redeclaring font encoding T1 on input line 43.
))
(/usr/share/texlive/texmf-dist/tex/latex/geometry/geometry.sty
```

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Package: geometry 2010/09/12 v5.6 Page Geometry

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Package: keyval 1999/03/16 v1.13 key=value parser (DPC)
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Package: ifpdf 2011/01/30 v2.3 Provides the ifpdf switch (HO)
Package ifpdf Info: pdfTeX in PDF mode is detected.
)
(/usr/share/texlive/texmf-dist/tex/generic/oberdiek/ifvtex.sty
Package: ifvtex 2010/03/01 v1.5 Detect VTeX and its facilities (HO)
Package ifvtex Info: VTeX not detected.
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(/usr/share/texlive/texmf-dist/tex/generic/ifxetex/ifxetex.sty
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Package: amsmath 2013/01/14 v2.14 AMS math features
\@mathmargin=\skip43

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Package: amstext 2000/06/29 v2.01

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Package: amsopn 1999/12/14 v2.01 operator names
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\dotsspace@=\muskip10
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\column@=\count102
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Package: graphicx 1999/02/16 v1.0f Enhanced LaTeX Graphics (DPC,SPQR)

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Package: graphics 2009/02/05 v1.0o Standard LaTeX Graphics (DPC,SPQR)

(/usr/share/texlive/texmf-dist/tex/latex/graphics/trig.sty
Package: trig 1999/03/16 v1.09 sin cos tan (DPC)
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File: graphics.cfg 2010/04/23 v1.9 graphics configuration of TeX Live
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Package: infwarerr 2010/04/08 v1.3 Providing info/warning/error messages (HO)
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Package: todonotes 2012/07/25

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Package: ifthen 2001/05/26 v1.1c Standard LaTeX ifthen package (DPC)
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Package: xkeyval 2012/10/14 v2.6b package option processing (HA)

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Package: everyshi 2001/05/15 v3.00 EveryShipout Package (MS)
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\c@pgf@countb=\count107
\c@pgf@countc=\count108
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File: pgfsyssoftpath.code.tex 2008/07/18  (rcs-revision 1.7)
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\pgfsyssoftpath@bigbuffer@items=\count111
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(/usr/share/texmf/tex/generic/pgf/systemlayer/pgfsysprotocol.code.tex
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))
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Package: pgfcore 2010/04/11 v2.10 (rcs-revision 1.7)

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(/usr/share/texmf/tex/generic/pgf/math/pgfmathcalc.code.tex
(/usr/share/texmf/tex/generic/pgf/math/pgfmathutil.code.tex)
(/usr/share/texmf/tex/generic/pgf/math/pgfmathparser.code.tex
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\pgfmath@count=\count112
\pgfmath@box=\box29
\pgfmath@toks=\toks32
\pgfmath@stack@operand=\toks33
\pgfmath@stack@operation=\toks34
)
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(/usr/share/texmf/tex/generic/pgf/math/pgfmathfunctions.basic.code.tex)
(/usr/share/texmf/tex/generic/pgf/math/pgfmathfunctions.trigonometric.code.tex)
(/usr/share/texmf/tex/generic/pgf/math/pgfmathfunctions.random.code.tex)
(/usr/share/texmf/tex/generic/pgf/math/pgfmathfunctions.comparison.code.tex)
(/usr/share/texmf/tex/generic/pgf/math/pgfmathfunctions.base.code.tex)
```

```
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\c@pgfmathroundto@lastzeros=\count113
))
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorepoints.code.tex
File: pgfcorepoints.code.tex 2010/04/09  (rcs-revision 1.20)
\pgf@picminx=\dimen132
\pgf@picmaxx=\dimen133
\pgf@picminy=\dimen134
\pgf@picmaxy=\dimen135
\pgf@pathminx=\dimen136
\pgf@pathmaxx=\dimen137
\pgf@pathminy=\dimen138
\pgf@pathmaxy=\dimen139
\pgf@xx=\dimen140
\pgf@xy=\dimen141
\pgf@yx=\dimen142
\pgf@yy=\dimen143
\pgf@zx=\dimen144
\pgf@zy=\dimen145
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorepathconstruct.code.tex
File: pgfcorepathconstruct.code.tex 2010/08/03  (rcs-revision 1.24)
\pgf@path@lastx=\dimen146
\pgf@path@lasty=\dimen147
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorepathusage.code.tex
File: pgfcorepathusage.code.tex 2008/04/22  (rcs-revision 1.12)
\pgf@shorten@end@additional=\dimen148
\pgf@shorten@start@additional=\dimen149
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorescopes.code.tex
File: pgfcorescopes.code.tex 2010/09/08  (rcs-revision 1.34)
\pgfpic=\box30
\pgf@hbox=\box31
\pgf@layerbox@main=\box32
\pgf@picture@serial@count=\count114
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoregraphicstate.code.tex
File: pgfcoregraphicstate.code.tex 2008/04/22  (rcs-revision 1.9)
\pgflinewidth=\dimen150
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoretransformations.code.tex
File: pgfcoretransformations.code.tex 2009/06/10  (rcs-revision 1.11)
\pgf@pt@x=\dimen151
\pgf@pt@y=\dimen152
\pgf@pt@temp=\dimen153
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorequick.code.tex
File: pgfcorequick.code.tex 2008/10/09  (rcs-revision 1.3)
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoreobjects.code.tex
File: pgfcoreobjects.code.tex 2006/10/11  (rcs-revision 1.2)
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorepathprocessing.code.tex
File: pgfcorepathprocessing.code.tex 2008/10/09  (rcs-revision 1.8)
)
```

```
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorearrows.code.tex
File: pgfcorearrows.code.tex 2008/04/23  (rcs-revision 1.11)
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoreshade.code.tex
File: pgfcoreshade.code.tex 2008/11/23  (rcs-revision 1.13)
\pgf@max=\dimen154
\pgf@sys@shading@range@num=\count115
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoreimage.code.tex
File: pgfcoreimage.code.tex 2010/03/25  (rcs-revision 1.16)

(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoreexternal.code.tex
File: pgfcoreexternal.code.tex 2010/09/01  (rcs-revision 1.17)
\pgfexternal@startupbox=\box33
))
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorelayers.code.tex
File: pgfcorelayers.code.tex 2010/08/27  (rcs-revision 1.2)
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcoretransparency.code.tex
File: pgfcoretransparency.code.tex 2008/01/17  (rcs-revision 1.2)
)
(/usr/share/texmf/tex/generic/pgf/basiclayer/pgfcorepatterns.code.tex
File: pgfcorepatterns.code.tex 2009/07/02  (rcs-revision 1.3)
)))
(/usr/share/texmf/tex/generic/pgf/modules/pgfmoduleshapes.code.tex
File: pgfmoduleshapes.code.tex 2010/09/09  (rcs-revision 1.13)
\pgfnodeparttextbox=\box34
)
(/usr/share/texmf/tex/generic/pgf/modules/pgfmoduleplot.code.tex
File: pgfmoduleplot.code.tex 2010/10/22  (rcs-revision 1.8)
)
(/usr/share/texmf/tex/latex/pgf/compatibility/pgfcomp-version-0-65.sty
Package: pgfcomp-version-0-65 2007/07/03 v2.10 (rcs-revision 1.7)
\pgf@nodesepstart=\dimen155
\pgf@nodesepend=\dimen156
)
(/usr/share/texmf/tex/latex/pgf/compatibility/pgfcomp-version-1-18.sty
Package: pgfcomp-version-1-18 2007/07/23 v2.10 (rcs-revision 1.1)
))
(/usr/share/texmf/tex/latex/pgf/utilities/pgffor.sty
(/usr/share/texmf/tex/latex/pgf/utilities/pgfkeys.sty
(/usr/share/texmf/tex/generic/pgf/utilities/pgfkeys.code.tex))
(/usr/share/texmf/tex/generic/pgf/utilities/pgffor.code.tex
Package: pgffor 2010/03/23 v2.10 (rcs-revision 1.18)
\pgffor@iter=\dimen157
\pgffor@skip=\dimen158
\pgffor@stack=\toks35
\pgffor@toks=\toks36
))
(/usr/share/texmf/tex/generic/pgf/frontendlayer/tikz/tikz.code.tex
Package: tikz 2010/10/13 v2.10 (rcs-revision 1.76)

(/usr/share/texmf/tex/generic/pgf/libraries/pgflibraryplothandlers.code.tex
File: pgflibraryplothandlers.code.tex 2010/05/31 v2.10 (rcs-revision 1.15)
\pgf@plot@mark@count=\count116
\pgfplotmarksize=\dimen159
)
\tikz@lastx=\dimen160
```

```
\tikz@lasty=\dimen161
\tikz@lastxsaved=\dimen162
\tikz@lastysaved=\dimen163
\tikzleveldistance=\dimen164
\tikzsiblingdistance=\dimen165
\tikz@figbox=\box35
\tikz@tempbox=\box36
\tikztreelevel=\count117
\tikznumberofchildren=\count118
\tikznumberofcurrentchild=\count119
\tikz@fig@count=\count120

(/usr/share/texmf/tex/generic/pgf/modules/pgfmodulematrix.code.tex
File: pgfmodulematrix.code.tex 2010/08/24  (rcs-revision 1.4)
\pgfmatrixcurrentrow=\count121
\pgfmatrixcurrentcolumn=\count122
\pgf@matrix@numberofcolumns=\count123
)
\tikz@expandcount=\count124

(/usr/share/texmf/tex/generic/pgf/frontendlayer/tikz/libraries/tikzlibrarytopat
hs.code.tex
File: tikzlibrarytopaths.code.tex 2008/06/17 v2.10 (rcs-revision 1.2)
)))
(/usr/share/texmf/tex/generic/pgf/frontendlayer/tikz/libraries/tikzlibraryposit
ioning.code.tex
File: tikzlibrarypositioning.code.tex 2008/10/06 v2.10 (rcs-revision 1.7)
) (/usr/share/texlive/texmf-dist/tex/latex/tools/calc.sty
Package: calc 2007/08/22 v4.3 Infix arithmetic (KKT,FJ)
\calc@Acount=\count125
\calc@Bcount=\count126
\calc@Adimen=\dimen166
\calc@Bdimen=\dimen167
\calc@Askip=\skip46
\calc@Bskip=\skip47
LaTeX Info: Redefining \setlength on input line 76.
LaTeX Info: Redefining \addtolength on input line 77.
\calc@Ccount=\count127
\calc@Cskip=\skip48
)
\c@@todonotes@numberoftodonotes=\count128
)
(/usr/share/texlive/texmf-dist/tex/latex/hyperref/hyperref.sty
Package: hyperref 2012/11/06 v6.83m Hypertext links for LaTeX

(/usr/share/texlive/texmf-dist/tex/generic/oberdiek/hobsub-hyperref.sty
Package: hobsub-hyperref 2012/05/28 v1.13 Bundle oberdiek, subset hyperref (HO)


(/usr/share/texlive/texmf-dist/tex/generic/oberdiek/hobsub-generic.sty
Package: hobsub-generic 2012/05/28 v1.13 Bundle oberdiek, subset generic (HO)
Package: hobsub 2012/05/28 v1.13 Construct package bundles (HO)
Package hobsub Info: Skipping package 'infwarerr' (already loaded).
Package hobsub Info: Skipping package 'ltxcmds' (already loaded).
Package: ifluatex 2010/03/01 v1.3 Provides the ifluatex switch (HO)
Package ifluatex Info: LuaTeX not detected.
Package hobsub Info: Skipping package 'ifvtex' (already loaded).
Package: intcalc 2007/09/27 v1.1 Expandable calculations with integers (HO)
```

```
Package hobsub Info: Skipping package 'ifpdf' (already loaded).
Package: etexcmds 2011/02/16 v1.5 Avoid name clashes with e-TeX commands (HO)
Package etexcmds Info: Could not find \expanded.
(etexcmds)                That can mean that you are not using pdfTeX 1.50 or
(etexcmds)                 that some package has redefined \expanded.
(etexcmds)                 In the latter case, load this package earlier.
Package: kvsetkeys 2012/04/25 v1.16 Key value parser (HO)
Package: kvdefinekeys 2011/04/07 v1.3 Define keys (HO)
Package: pdftexcmds 2011/11/29 v0.20 Utility functions of pdfTeX for LuaTeX (HO
)
Package pdftexcmds Info: LuaTeX not detected.
Package pdftexcmds Info: \pdf@primitive is available.
Package pdftexcmds Info: \pdf@ifprimitive is available.
Package pdftexcmds Info: \pdfdraftmode found.
Package: pdfescape 2011/11/25 v1.13 Implements pdfTeX's escape features (HO)
Package: bigintcalc 2012/04/08 v1.3 Expandable calculations on big integers (HO
)
Package: bitset 2011/01/30 v1.1 Handle bit-vector datatype (HO)
Package: uniquecounter 2011/01/30 v1.2 Provide unlimited unique counter (HO)
)
Package hobsub Info: Skipping package 'hobsub' (already loaded).
Package: letltxmacro 2010/09/02 v1.4 Let assignment for LaTeX macros (HO)
Package: hopatch 2012/05/28 v1.2 Wrapper for package hooks (HO)
Package: xcolor-patch 2011/01/30 xcolor patch
Package: atveryend 2011/06/30 v1.8 Hooks at the very end of document (HO)
Package atveryend Info: \enddocument detected (standard20110627).
Package: atbegshi 2011/10/05 v1.16 At begin shipout hook (HO)
Package: refcount 2011/10/16 v3.4 Data extraction from label references (HO)
Package: hycolor 2011/01/30 v1.7 Color options for hyperref/bookmark (HO)
)
(/usr/share/texlive/texmf-dist/tex/latex/oberdiek/auxhook.sty
Package: auxhook 2011/03/04 v1.3 Hooks for auxiliary files (HO)
)
(/usr/share/texlive/texmf-dist/tex/latex/oberdiek/kvoptions.sty
Package: kvoptions 2011/06/30 v3.11 Key value format for package options (HO)
)
\@linkdim=\dimen168
\Hy@linkcounter=\count129
\Hy@pagecounter=\count130

(/usr/share/texlive/texmf-dist/tex/latex/hyperref/pd1enc.def
File: pd1enc.def 2012/11/06 v6.83m Hyperref: PDFDocEncoding definition (HO)
)
\Hy@SavedSpaceFactor=\count131

(/usr/share/texlive/texmf-dist/tex/latex/latexconfig/hyperref.cfg
File: hyperref.cfg 2002/06/06 v1.2 hyperref configuration of TeXLive
)
Package hyperref Info: Option 'colorlinks' set 'true' on input line 4319.
Package hyperref Info: Hyper figures OFF on input line 4443.
Package hyperref Info: Link nesting OFF on input line 4448.
Package hyperref Info: Hyper index ON on input line 4451.
Package hyperref Info: Plain pages OFF on input line 4458.
Package hyperref Info: Backreferencing OFF on input line 4463.
Package hyperref Info: Implicit mode ON; LaTeX internals redefined.
Package hyperref Info: Bookmarks ON on input line 4688.
\c@Hy@tempcnt=\count132
```

```
(/usr/share/texlive/texmf-dist/tex/latex/url/url.sty
\Urlmuskip=\muskip11
Package: url 2013/09/16  ver 3.4  Verb mode for urls, etc.
)
LaTeX Info: Redefining \url on input line 5041.
\XeTeXLinkMargin=\dimen169
\Fld@menulength=\count133
\Field@Width=\dimen170
\Fld@charsize=\dimen171
Package hyperref Info: Hyper figures OFF on input line 6295.
Package hyperref Info: Link nesting OFF on input line 6300.
Package hyperref Info: Hyper index ON on input line 6303.
Package hyperref Info: backreferencing OFF on input line 6310.
Package hyperref Info: Link coloring ON on input line 6313.
Package hyperref Info: Link coloring with OCG OFF on input line 6320.
Package hyperref Info: PDF/A mode OFF on input line 6325.
LaTeX Info: Redefining \ref on input line 6365.
LaTeX Info: Redefining \pageref on input line 6369.
\Hy@abspage=\count134
\c@Item=\count135
\c@Hfootnote=\count136
)

Package hyperref Message: Driver (autodetected): hpdftex.

(/usr/share/texlive/texmf-dist/tex/latex/hyperref/hpdftex.def
File: hpdftex.def 2012/11/06 v6.83m Hyperref driver for pdfTeX
\Fld@listcount=\count137
\c@bookmark@seq@number=\count138

(/usr/share/texlive/texmf-dist/tex/latex/oberdiek/rerunfilecheck.sty
Package: rerunfilecheck 2011/04/15 v1.7 Rerun checks for auxiliary files (HO)
Package uniquecounter Info: New unique counter `rerunfilecheck' on input line 2
82.
)
\Hy@SectionHShift=\skip49
)
(/usr/share/texlive/texmf-dist/tex/latex/float/float.sty
Package: float 2001/11/08 v1.3d Float enhancements (AL)
\c@float@type=\count139
\float@exts=\toks37
\float@box=\box37
\@float@everytoks=\toks38
\@floatcapt=\box38
)
(/usr/share/texlive/texmf-dist/tex/latex/tools/verbatim.sty
Package: verbatim 2003/08/22 v1.5q LaTeX2e package for verbatim enhancements
\every@verbatim=\toks39
\verbatim@line=\toks40
\verbatim@in@stream=\read2
)
No file main.aux.
\openout1 = `main.aux'.

LaTeX Font Info:    Checking defaults for OML/cmm/m/it on input line 22.
LaTeX Font Info:    ... okay on input line 22.
LaTeX Font Info:    Checking defaults for T1/cmr/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
```

```
LaTeX Font Info:    Checking defaults for OT1/cmr/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
LaTeX Font Info:    Checking defaults for OMS/cmsy/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
LaTeX Font Info:    Checking defaults for OMX/cmex/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
LaTeX Font Info:    Checking defaults for U/cmr/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
LaTeX Font Info:    Checking defaults for PD1/pdf/m/n on input line 22.
LaTeX Font Info:    ... okay on input line 22.
(/usr/share/texlive/texmf-dist/tex/latex/ucs/ucsencs.def
File: ucsencs.def 2011/01/21 Fixes to fontencodings LGR, T3
)
*geometry* driver: auto-detecting
*geometry* detected driver: pdftex
*geometry* verbose mode - [ preamble ] result:
* driver: pdftex
* paper: a4paper
* layout: <same size as paper>
* layoutoffset:(h,v)=(0.0pt,0.0pt)
* modes:
* h-part:(L,W,R)=(85.35826pt, 426.79135pt, 85.35826pt)
* v-part:(T,H,B)=(85.35826pt, 702.78308pt, 56.9055pt)
* \paperwidth=597.50787pt
* \paperheight=845.04684pt
* \textwidth=426.79135pt
* \textheight=702.78308pt
* \oddsidemargin=13.08827pt
* \evensidemargin=13.08827pt
* \topmargin=-23.91173pt
* \headheight=12.0pt
* \headsep=25.0pt
* \topskip=10.0pt
* \footskip=30.0pt
* \marginparwidth=49.79231pt
* \marginparsep=11.0pt
* \columnsep=10.0pt
* \skip\footins=9.0pt plus 4.0pt minus 2.0pt
* \hoffset=0.0pt
* \voffset=0.0pt
* \mag=1000
* \@twocolumnfalse
* \@twosidefalse
* \@mparswitchfalse
* \@reversemarginfalse
* (1in=72.27pt=25.4mm, 1cm=28.453pt)

(/usr/share/texlive/texmf-dist/tex/context/base/supp-pdf.mkii
[Loading MPS to PDF converter (version 2006.09.02).]
\scratchcounter=\count140
\scratchdimen=\dimen172
\scratchbox=\box39
\nofMPsegments=\count141
\nofMParguments=\count142
\everyMPshowfont=\toks41
\MPscratchCnt=\count143
\MPscratchDim=\dimen173
\MPnumerator=\count144
```

```
\makeMPintoPDFobject=\count145
\everyMPtoPDFconversion=\toks42
) (/usr/share/texlive/texmf-dist/tex/latex/oberdiek/epstopdf-base.sty
Package: epstopdf-base 2010/02/09 v2.5 Base part for package epstopdf

(/usr/share/texlive/texmf-dist/tex/latex/oberdiek/grfext.sty
Package: grfext 2010/08/19 v1.1 Manage graphics extensions (HO)
)
Package grfext Info: Graphics extension search list:
(grfext)              [.png,.pdf,.jpg,.mps,.jpeg,.jbig2,.jb2,.PNG,.PDF,.JPG,.JPE
G,.JBIG2,.JB2,.eps]
(grfext)              \AppendGraphicsExtensions on input line 452.

(/usr/share/texlive/texmf-dist/tex/latex/latexconfig/epstopdf-sys.cfg
File: epstopdf-sys.cfg 2010/07/13 v1.3 Configuration of (r)epstopdf for TeX Liv
e
))
ABD: EveryShipout initializing macros
\AtBeginShipoutBox=\box40
Package hyperref Info: Link coloring ON on input line 22.

(/usr/share/texlive/texmf-dist/tex/latex/hyperref/nameref.sty
Package: nameref 2012/10/27 v2.43 Cross-referencing by name of section

(/usr/share/texlive/texmf-dist/tex/generic/oberdiek/gettitlestring.sty
Package: gettitlestring 2010/12/03 v1.4 Cleanup title references (HO)
)
\c@section@level=\count146
)
LaTeX Info: Redefining \ref on input line 22.
LaTeX Info: Redefining \pageref on input line 22.
LaTeX Info: Redefining \nameref on input line 22.
\@outlinefile=\write4
\openout4 = 'main.out'.


<expl.png, id=7, 578.16pt x 433.62pt>
File: expl.png Graphic file (type png)
 <use expl.png>
Package pdftex.def Info: expl.png used on input line 33.
(pdftex.def)              Requested size: 298.75264pt x 224.06114pt.

<t_err.png, id=9, 578.16pt x 433.62pt>
File: t_err.png Graphic file (type png)
 <use t_err.png>
Package pdftex.def Info: t_err.png used on input line 42.
(pdftex.def)              Requested size: 298.75264pt x 224.06114pt.
 [1

{/var/lib/texmf/fonts/map/pdftex/updmap/pdftex.map}pdfTeX warning (ext4): desti
nation with the same identifier (name{figure.1}) has been already used, duplica
te ignored

\@EveryShipout@Output ...@Org@Shipout \box \@cclv

l.45
      <./expl.png>] <impl.png, id=28, 578.16pt x 433.62pt>
File: impl.png Graphic file (type png)
```

```
 <use impl.png>
Package pdftex.def Info: impl.png used on input line 51.
(pdftex.def)            Requested size: 298.75264pt x 224.06114pt.

[2pdfTeX warning (ext4): destination with the same identifier (name{figure.2})
has been already used, duplicate ignored

\@EveryShipout@Output ...@Org@Shipout \box \@cclv

l.58
     pdfTeX warning (ext4): destination with the same identifier (name{figure.3
}) has been already used, duplicate ignored

\@EveryShipout@Output ...@Org@Shipout \box \@cclv

l.58
     <./t_err.png> <./impl.png>] <ph_e.png, id=38, 578.16pt x 433.62pt>
File: ph_e.png Graphic file (type png)

<use ph_e.png>
Package pdftex.def Info: ph_e.png used on input line 61.
(pdftex.def)            Requested size: 298.75264pt x 224.06114pt.
 <ph_i.png, id=39, 578.16pt x 433.62pt>
File: ph_i.png Graphic file (type png)
 <use ph_i.png>
Package pdftex.def Info: ph_i.png used on input line 67.
(pdftex.def)            Requested size: 298.75264pt x 224.06114pt.
 [3pdfTeX warning (ext4): destination with the same identifier (name{figure.4})
 has been already used, duplicate ignored

\@EveryShipout@Output ...@Org@Shipout \box \@cclv

l.73
     pdfTeX warning (ext4): destination with the same identifier (name{figure.5
}) has been already used, duplicate ignored

\@EveryShipout@Output ...@Org@Shipout \box \@cclv

l.73
     <./ph_e.png> <./ph_i.png>] <lag.png, id=50, 578.16pt x 433.62pt>
File: lag.png Graphic file (type png)

<use lag.png>
Package pdftex.def Info: lag.png used on input line 76.
(pdftex.def)            Requested size: 298.75264pt x 224.06114pt.
File: ph_i.png Graphic file (type png)
 <use ph_i.png>
Package pdftex.def Info: ph_i.png used on input line 87.
(pdftex.def)            Requested size: 298.75264pt x 224.06114pt.
LaTeX Font Info:    Try loading font information for T1+cmtt on input line 93.

(/usr/share/texlive/texmf-dist/tex/latex/base/t1cmtt.fd
File: t1cmtt.fd 1999/05/25 v2.5h Standard LaTeX font definitions
)
File: functions.py  (verbatim)
 [4pdfTeX warning (ext4)
```