Out[1]: Toggle show/hide

Q 8.1

a

The mass defect fractions of each reaction is defined as $\frac{m_{reactants} - m_{products}}{m_{products}}$

The masses of each of the elements in the equations are given below in amu:

In [2]:
```
mH = 1.007825031898 # * amu
mHe = 4.002603 # * amu
mC = 12. # * amu
mO = 15.994914  # * amu
mSi = 27.9769265 # * amu
mFe = 55.9349375 # * amu
```

Calculating $\frac{m_{reactants} - m_{products}}{m_{products}}$, we get the following mass defect fractions

```
4 H-1 -> He-4              : 0.00712
3 He-4 -> C-12             : 0.00065
C-12 + He-4 -> O-16        : 0.00048
2 O-16 -> Si-28 + He-4     : 0.00032
2 Si-28 -> Fe-56           : 0.00034
```

Out[3]: Toggle show/hide

b.

In general, the mass fractions of each reaction decreases as the masses of the reactants increases. This means that the nuclear time scale of each reaction becomes shorter and shorter for a star burning heavier and heavier elements to maintain hydrostatic equilibrium. As such, the star spends less and less time in each subsequent phase of stellar burning.

The slight increase in mass defect fraction for the final reaction is a bit curious. I think this might due to He-4 in the product. Since He-4 has a relatively high average binding energy per nucleon, some of the energy that might have otherwise been released goes into the He-4 nucleus.

Q 8.4

From the book we have that the core mass needed to fuse a particular element is propotional to the temperature needed to ignite fusion of that element to the 3/4 power. Since we know that the critical core mass for helium fusion is $0.3M_{\odot}$, we can use this proportionality, and T_{ign} for the other fusion processes to find the minimum masses needed to ignite those reactions:

$M_{crit} \propto T_{ign}^{3/4}$ So $\frac{M_{crit,X}}{M_{crit,He}} = \left(\frac{T_{ign,X}}{T_{ign,He}}\right)^{3/4}$

$M_{crit,X} = \left(\frac{T_{ign,X}}{T_{ign,He}}\right)^{3/4} M_{crit,He}$

From table 8.4 in the book, we have T_{thresh} for a variety of reactions, where 3-α fusion is helium fusion.

Table 8.4. Summary of the Most Important Reaction Rates in Stars

Fuel (1)	Process (2)	T_{thresh} 10^6 K (3)	Product (4)	E_{net} MeV/nucl (5)	T_c 10^6 K (6)	L_{net}/L (7)	Duration yr (8)
H	p-p chain	4	He	6.55	–	–	–
H	CNO cycle	15	He	6.25	35	0.94	1.1×10^7
He	3-α fusion	100	C,O	0.61	180	0.96	2.0×10^6
C	C-fusion	600	Ne,Mg,Na,O	0.54	810	0.16	2.0×10^3
Ne	Ne photdis	900	O,Mg,Si		1600	5.3×10^{-4}	0.7
O	O-fusion	1000	S,Si,P,Mg	0.30	1900	8.2×10^{-5}	2.6
Si	Si nucl equil.	3000	Fe,Ni,Cr,Ti	<0.18	3300	5.8×10^{-7}	0.05

Calculate the minimum core mass for each fusion phase:

```
Core Masses for fusion phases
------------------------------
C-fusion      : 1.15 M_sun
Ne-photodis   : 1.56 M_sun
O-fusion      : 1.69 M_sun
Si nucl       : 3.85 M_sun
```

Out[4]: Toggle show/hide

Q 9.1

Using the following equations for various timescales, which are dependent on the Mass, Radius, and Luminosity of the stars:

$$\tau_{dyn} = \frac{1}{\sqrt{G\bar{\rho}}}$$

$$\tau_{KH} = \frac{GM^2}{RL}$$

$\tau_{nucl} = \frac{f_M M \epsilon_n}{L} \approx \frac{(M/M_\odot)}{(L/L_\odot)} \times 10^{10} years$ for H-fusion.

For He-fusion (e.g. in the case of the red supergiant, $\tau_{nucl,He} \approx 0.1\tau_{nucl,H}$

We can calculate the various time scales for each of the 4 stars:

In [5]:
```python
stars = {
    '1MS': {'M': 1, 'R': 1, 'L': 1},
    '60MS': {'M': 60, 'R': 15, 'L': 8e5},
    '15RSG': {'M': 15, 'R': 3.3e3, 'L': 4.5e5},
    '0.6WD': {'M': .6, 'R': .012, 'L': 1e-3}
}
```

```
1MS
-----------------------------------
Dynamical:              3.26e+03 s
Kelvin-Helmholtz:       9.91e+14 s
Nuclear:                3.16e+17 s

Timescale Ratios:       1 : 3.0e+11 : 9.7e+13

60MS
-----------------------------------
Dynamical:              2.45e+04 s
Kelvin-Helmholtz:       2.97e+11 s
Nuclear:                2.37e+13 s

Timescale Ratios:       1 : 1.2e+07 : 9.7e+08

15RSG
-----------------------------------
Dynamical:              1.60e+08 s
Kelvin-Helmholtz:       1.50e+08 s
Nuclear:                1.05e+12 s

Timescale Ratios:       1 : 9.4e-01 : 6.6e+03

0.6WD
-----------------------------------
Dynamical:              5.53e+00 s
Kelvin-Helmholtz:       2.97e+19 s
Nuclear:                nan s

Timescale Ratios:       1 : 5.4e+18 : nan
```

Out[6]: Toggle show/hide

Since the white dwarf is no longer undergoing fusion, the nuclear timescale is undefined. Other trends of interest:

1. The Dynamical time scale tends to be by far the shortest, followed by the thermal timescale, and finally the nuclear time scale. (dynamical time scale and thermal time scale are flipped for the 15 $M_\odot$ red super giant due to its huge radius (which leads to very low $\bar{\rho}$) and high luminosity (which speeds up the thermal time scale).
2. The nuclear time scale for stars that are fusing are much longer than either the thermal or dynamical time scales. This implies that the stars are in both hydrostatic equilibrium and thermal equilibrium, any perturbation out of these disequilibria would easily have time to settle into a new equilibrium over the (nuclear) lifetime of the stars. An exception might be the super

massive star (60\M$_\odot$ star), whose thermal timescale is about 10% of the nuclear time, so we would have a good chance of observing this star out of thermal equilibrium if it ever happened.
3. Since the white dwarf is no longer undergoing fusion, its lifetime is determined by the thermal timescale, i.e. the time it will take to radiate it's thermal energy and cool.

Q 9.2

If the sun were to suddenly stop fusing, the gravity would no longer be balanced by pressure and the sun would collapse on itself on the order of the **dynamical time scale** (~30 minutes, see Q9.1), until it could be supported by electron degeneracy. This change would likely be noticeable by many people, because while the sun would remain the same luminosity, its angular size on the sky would decrease by a factor of about 100, meaning the surface brightness of the sun would increase by 10000x! Thus the sun would be much more difficult to look at, and would be able to shine through a variety of atmospheric layers (e.g. atmosphere at sunset, smog, or thin clouds) much more strongly.

However if those with access to a neutrino or gravitational wave detector might be able to tell on order of the photon travel time to the Earth, since those effects would occur immediately.

Q 11.1

A star (star 1) with a polytrope index $\gamma = 5/3$ has a stiffer equation of state than a star with $\gamma = 4/3$. This means that as a force is exerted compressing the star (e.g. gravity), the gas pressure required to match that compressive force can be accomplished by a less dense gass for $\gamma = 5/3$ than for a star with $\gamma = 4/3$, since $P \propto \rho^{\gamma}$. This means that for two stars of the same mass, and thus central gravitational force, the star with $\gamma = 4/3$ will have a more concentrated density structure and a higher density at the center.

Another way to think about this using the lagrangian formulation. Suppose| we build up a star by adding dm of mass in a spherical shells until we reach M_{star}. Each dm for the star with $\gamma = 4/3$ will cause the material to compress more than for the star with $\gamma = 5/3$, thus resulting in a more concentrated density structure.