Question 3a

We have

$$\boldsymbol{w} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-}1(\boldsymbol{\Phi}^T\boldsymbol{x}) \tag{1}$$

and

$$f(t=1) = \boldsymbol{w}^T\boldsymbol{\phi}(t=1) \tag{2}$$

Rearranging the factors and substituting for w, we get:

$$\begin{aligned} f(t=1) &= \boldsymbol{w}^T\boldsymbol{\phi}(t=1) \\ &= \boldsymbol{\phi}(t=1)^T\boldsymbol{w} \\ &= \boldsymbol{\phi}(t=1)^T(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}(\boldsymbol{\Phi}^T\boldsymbol{x}) \\ &= [\boldsymbol{\phi}(t=1)^T(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T)]\boldsymbol{x} \\ &= \boldsymbol{v}^T\boldsymbol{x} \end{aligned} \tag{3}$$

It follows that

$$\begin{aligned} \boldsymbol{v} &= [\boldsymbol{\phi}(t=1)^T(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T)]^T \\ &= [(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T)]^T(\boldsymbol{\phi}(t=1)^T)^T \\ &= (\boldsymbol{\Phi}^T)^T((\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1})^T\boldsymbol{\phi}(t=1) \\ &= \boldsymbol{\Phi}((\boldsymbol{\Phi}^T\boldsymbol{\Phi})^T)^{-1}\boldsymbol{\phi}(t=1) \\ &= \boldsymbol{\Phi}((\boldsymbol{\Phi}^T(\boldsymbol{\Phi})^T)^T)^{-1}\boldsymbol{\phi}(t=1) \\ &= \boldsymbol{\Phi}(\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\phi}(t=1) \end{aligned} \tag{4}$$