Instructions:
Be verbose. Explain clearly your reasoning, methods, and results in your written work. Write clear code that is well documented. With 99% certainty, you cannot write too many code comments.
Written answers are worth 18 points. Code is worth 2 points. 10 points total.
1. When finished, respond to the question in Canvas as "done." We will record your grade there.
2. In your code repository, create a folder called "Project02."
3. In that folder, include
a. a document (PDF) with your responses.
b. All code
c. A README file with instructions for us to run your code
Everything must be checked into your repository by 8am Saturday 3/1. A pull will be done at that time. Documents and code checked in after the instructors pull will not be graded.
Data for problems can be found in CSV files with this document in the class repository.

Problem 1
Given the dataset in DailyPrices.csv, for the stocks SPY, AAPL, and EQIX
A. Calculate the Arithmetic Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

```
Last 5 rows of de-meaned arithmetic returns:
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011492 -0.014678 -0.006966
2024-12-30 -0.012377 -0.014699 -0.008064
2024-12-31 -0.004603 -0.008493  0.006512
2025-01-02 -0.003422 -0.027671  0.000497
2025-01-03  0.011538 -0.003445  0.015745
Total standard deviation: 0.012679754664908083
```

B. Calculate the Log Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

```
Last 5 rows of de-meaned log returns:
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011515 -0.014675 -0.006867
2024-12-30 -0.012410 -0.014696 -0.007972
2024-12-31 -0.004577 -0.008427  0.006602
2025-01-02 -0.003392 -0.027930  0.000613
2025-01-03  0.011494 -0.003356  0.015725
Total standard deviation (across all stocks and days): 0.012630765615542203
```

Problem 2

Given the dataset in DailyPrices.csv, you have a portfolio of

- 100 shares of SPY
- 200 shares of AAPL
- 150 shares of EQIX

A. Calculate the current value of the portfolio given today is 1/3/2025

```
Prices on 2025-01-03 for SPY, AAPL, EQIX:
SPY     591.95
AAPL    243.36
EQIX    959.97
Name: 2025-01-03 00:00:00, dtype: float64
Portfolio value on 2025-01-03 : 251862.5
```

B. Calculate the VaR and ES of each stock and the entire portfolio at the 5% alpha level assuming arithmetic returns and 0 mean return, for the following methods:

a. Normally distributed with exponentially weighted covariance with lambda=0.97

```
--- Normal EW ---
SPY: VaR = 8.27, ES = 10.37
AAPL: VaR = 4.74, ES = 5.94
EQIX: VaR = 19.67, ES = 24.66
Portfolio: VaR = 3880.54, ES = 4866.36
```

b. T distribution using a Gaussian Copula

```
--- T Copula (nu=4) ---
SPY: VaR = 7.58, ES = 11.38
AAPL: VaR = 4.34, ES = 6.52
EQIX: VaR = 18.02, ES = 27.08
Portfolio: VaR = 3556.36, ES = 5343.05
```

c. Historic simulation using the full history.

```
--- Historical ---
SPY: VaR = 8.15, ES = 10.23
AAPL: VaR = 4.99, ES = 6.84
EQIX: VaR = 23.35, ES = 30.55
Portfolio: VaR = 4314.93, ES = 5799.29
```

C. Discuss the differences between the methods.

The normal model of exponentially weighted covariance

Returns are assumed to be normally distributed with a zero mean. Exponentially weighted covariance ($\lambda = 0.97$) gives more weight to recent observations, which is useful when market conditions change quickly. Both VaR and ES have closed-form expressions. Because the method relies on parameter distributions, it produces smooth estimates. However, the normal distribution tends to underestimate extreme events because it fails to capture fat tails.

T distribution using a Gaussian Copula

This method replaces the normal assumption of t with a distribution (assuming degrees of freedom, e.g. v = 4). The t distribution has heavier tails than the normal distribution, which means that it accounts for higher probability of extreme events. A Gaussian copula is used to model the dependence structure between assets. However, the degree of freedom parameter (v) can significantly affect the results. If v is not carefully chosen or estimated from the data, it may lead to an incorrect estimation of tail risk.

Historic simulation using the full history.

This method does not assume any particular parametric shape of the revenue distribution. Instead, the actual historical return (or dollar change) distribution is used to calculate the fifth percentile and the average loss outside that percentile. This can be useful if the historical data is representative of future risks. The quality of risk estimates depends on the length and relevance of historical data. Historical data may not be representative if market conditions change. Empirical estimates may be more volatile and sensitive to outliers, especially if the data set is small or not up-to-date.

Problem 3

You have a European Call option with the following parameters

- Time to maturity: 3 months (0.25 years)
- Call Price: $3.00
- Stock Price: $31
- Strike Price: $30
- Risk Free Rate: 10%
- No dividends are paid.

A. Calculate the implied volatility

```
Implied Volatility: 33.51%
```

B. Calculate the Delta, Vega, and Theta. Using this information, by approximately how much would the price of the option change is the implied volatility increased by 1%. Prove it.

```
Delta: 0.6674
Vega: 5.6309
Theta (per year): -5.4882
If implied volatility increases by 1% (0.01), the option price increases by about $0.0563
```

C. Calculate the price of the put using Generalized Black Scholes Merton. Does Put-Call Parity Hold?

```
Put price using Black-Scholes (GBSM): $1.23
Put price using Put-Call Parity:   $1.26
```

The Black–Scholes put price and the price computed via put-call parity are approximately consistent. Put-call parity holds for this European option.

D. Given a portfolio of
a. 1 call
b. 1 put
c. 1 share of stock
Assuming the stock's return is normally distributed with an annual volatility of 25%, the expected annual return of the stock is 0%, there are 255 trading days in a year, and the implied volatility is constant. Calculate VaR and ES for a 20 trading day holding period, at alpha=5% using:
d. Delta Normal Approximation
e. Monte Carlo Simulation
Hint: Don't forget to include the option value decay in your calculations

```
     Delta-Normal Approximation
Portfolio initial value: $35.26
VaR (5%): $5.61
ES (5%): $6.82
      Monte Carlo Simulation
VaR (5%): $4.21
ES (5%): $4.65
```

E. Discuss the differences between the 2 methods. Hint: graph the portfolio value vs the stock value and compare the assumptions between the 2 methods.

Delta-Normal Approximation

Assuming the return on the underlying stock is normally distributed, it works fine for small changes around the current stock price, but ignores curvature (gamma) and other higher-level effects. However, it may underestimate risk (especially tail risk) for large price changes because it ignores nonlinear behavior.

Monte Carlo Simulation

This method fully captures the non-linear (convex) nature of option prices, including gamma and other higher-order effects. It better represents the distribution of outcomes, including extreme moves. However, it requires many simulations and is more computationally intensive.