Computing Exercise 2

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The aim of this exercise is to generate a function that obeys Simpson 1/3 rule and perform numerical integration for. Then further investigation on how the parameter relation, specifically n, number of interval and z, distance between aperture and screen. Plot of diffraction will be produced for 1 and 2 dimensional, over different shape of aperture and check if the pattern were observed with the same z for each different aperture.

PART(A)

Problem

Part(a) requires to write a layout, which will perform Simpson rule integration, specifically Simpson's $\frac{1}{3}$ rule, which will be used here. Then apply it on sine function, over the range of 0 to π. Output values will be compared with the literature value that were posted in the lecture 3's notes.

$$I = \frac{1}{3}h[[f(a)+f(b)] + \sum_{i=1,3,5}^{n=1} 4f(x_i) + \sum_{i=2,4,6}^{n=2} 2f(x_i)] \quad (1)$$

Here I is known as the Simpson's $\frac{1}{3}$ rule. Equation 1 is the general form and will be useful for generating under different condition.

To show the difference between Simpson's $\frac{3}{8}$'s rule.

$$I = \frac{3}{8}h[[f(a)+f(b)] \sum_{i=1,4,7}^{n=2} 3[f(x_i)+f(x_{i+1})] + \sum_{i=3,6,9}^{n=3} 2f(x_i)] \quad (2)$$

Results

Summation of odd and even number will be split into 2 part, in a "for" loop, as according to Equation 1.These can be differentiated by using %, modulus, where even number will produce 0 but odd number produce 1 for when "% 2".

```
h = (Upper_limit - Lower_limit) / n
S = (f(Lower_limit) + f(Upper_limit))
if i % 2 == 0:
    Sum_of_even += 2*f(Lower_limit + i*h)
elif i % 2 != 0:
    Sum_of_odd += 4*f(Lower_limit + i*h)
i += 1
```

TABLE I: Value of integral for sine function comparing to the literature value

N, number of interval	Generated integral output	% difference
2	2.0943951023931953	$4.55e^{-12}$%
4	2.0045597549844207	$1.03e^{-12}$%
6	2.0008631896735367	$1.83e^{-12}$%
8	2.0002691699483877	$4.38e^{-12}$%
10	2.0001095173150043	$2.14e^{-13}$%

Table 1 compares the literature values of sine function over the range from 0 to π from the lecture 3's notes with the generated value from the code.

TABLE II: Value of integral generated for sine function comparing to the exact literature value, 2

N, number of interval	Integral difference
2	0.0943
4	0.00455
6	8.63E-4
8	2.69E-4
10	1.09E-4
20	6.78E-6
40	4.23E-7
80	2.64E-8
160	1.65E-9
320	1.03E-10

Table 2 compares the generated value with the final literature value, which is 2. Integral difference is the generated value minus the literature value.

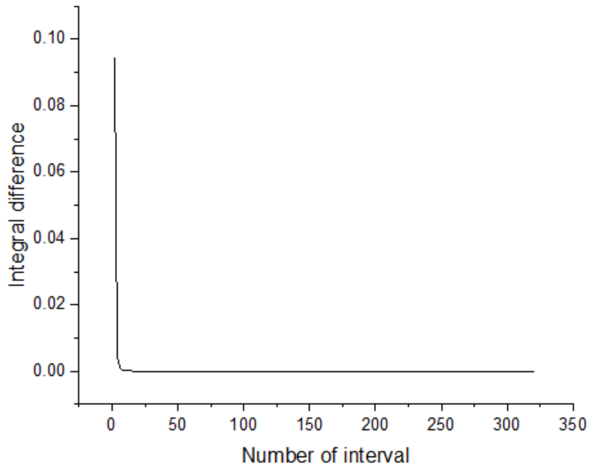

FIG. 1: Extracting datas from Table 2, where the difference between output value and literature value is plot against the number of interval, n for integration

Discussion

The integral generated is exactly the same with the literature value provided (with more accuracy, up till 16 decimal points (d.p.) , where literature value only up till 13d.p.). This was further supported by the fact that the percentage difference calculated was very small, in the power of 10^{-12}.

The error of approximating a Simpson rule can be denoted by

$$error = (\frac{1}{90})(\frac{b-a^{5}}{N})|f^{(4)}\varepsilon| \tag{3}$$

where ε is any number, but must be between upper limit, b and lower limit, a. As h was defined initially in the python codes; for equation(3), error is proportional to h^5.

This relationship was described by Figure 1 as well, which produced a graph of $y = x^{-5} + C$, where C is constant as the function didn't intersect at y = 0.

PART(B)

Problem

Part(b) require to form a simpson rule for Equation(5) of the question, which was represented below,

$$X(x, y', z) = \int_{x_2'(y')}^{x_1'(y')} \exp\left[\frac{ik}{2z}(x - x')^2\right] dx' \tag{4}$$

$$|X_{(x)}|^2 = X_{(x)} X^*_{(x)} \tag{5}$$

where $X^*_{(x)}$ is the complex conjugate of $X_{(x)}$

Then plot a graph of $|X_{(x)}|$ against x to investigate the relation between z, distance of aperture to the screen and n, number of interval for integration.

Result

Following figure included will be varying either n or z alone while the other parameter was kept constant. As for when only z is changing.

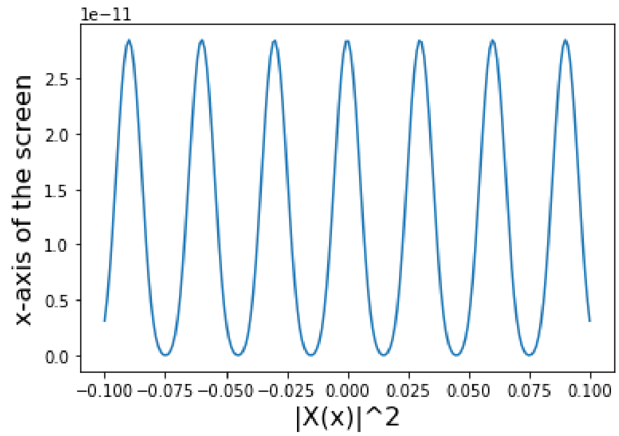

FIG. 2: when number of interval for integration, n = 2 and distance between aperture to screen, z = 10

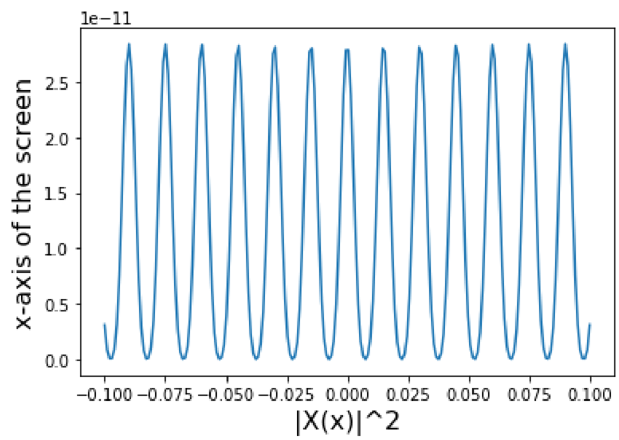

FIG. 3: when n = 2 and z = 20

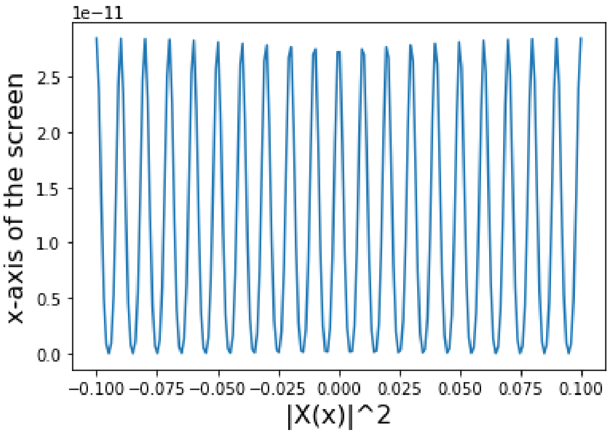

FIG. 4: when n = 2 and z = 30

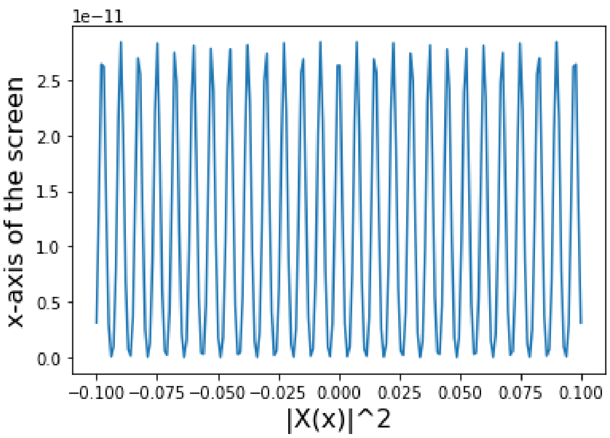

FIG. 5: when n = 2 and z = 40

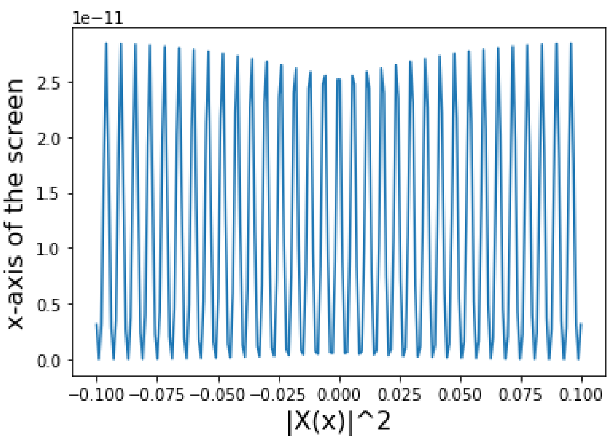

FIG. 6: when n = 2 and z = 50

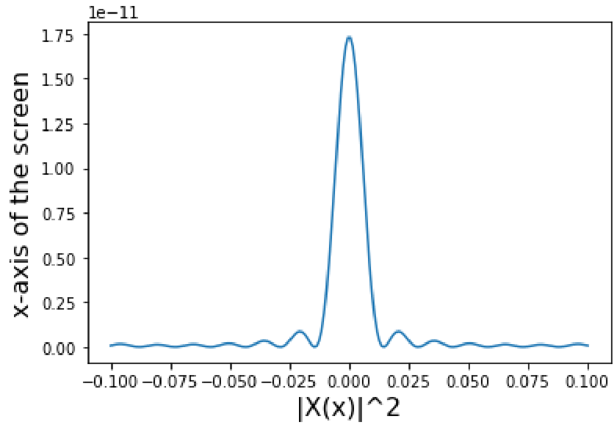

FIG. 7: when n = 16, z = 10

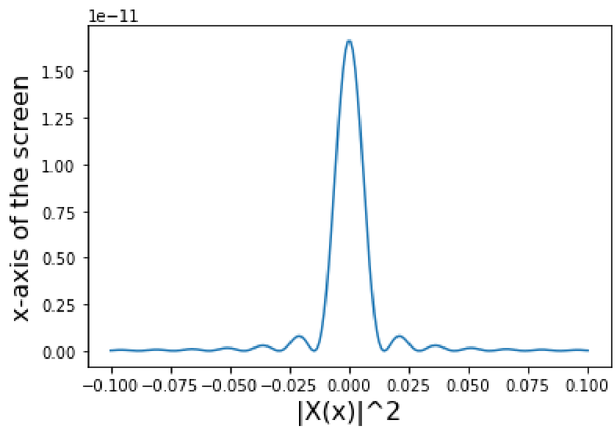

FIG. 8: when n = 32, z = 10

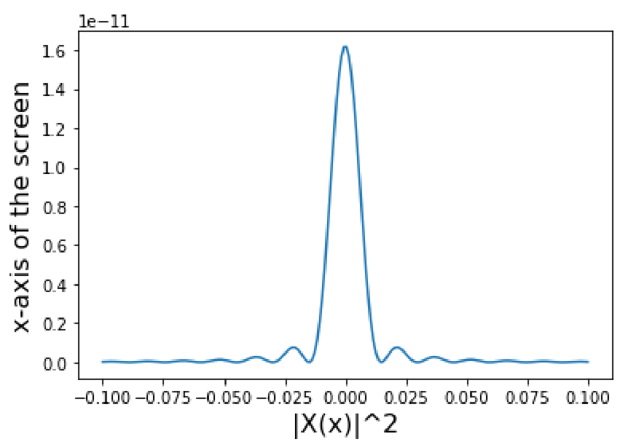

FIG. 9: n = 100, z = 10

DISCUSSION

As shown by Figure 2-6, when n is kept at 2 throughout whil increasing the z, the number of oscillation increased over the same range of x-axis, which also imply the period of 1 complete oscillation decreases as z increases. When z is equal to 50, there's an distinguishable envelope shape formed which was suppose to corresponds to carrier wave.

Then for when n increases as z was kept constant (from figure 7-9), the waveform of the figures seems to remain the same. It seems the final state of the waveform looks like a single slit diffraction pattern.

As for when the ratio of n and z were the same and both were increased in such that the ratio of both were kept the same as the initial ratio, the graph still have the same general shape, however it will be sharper as compared to before at lower value of both. This can be shown by figure , which were in appendix.

Z's value, which was found over a range for author's code, was unusually large compare to the value provided in the question. (By a factor of 10^2.)

PART(C)

Problem

Part(c) require to use a similar equation as to equation(4), with addition of y-axis . The following equation can be represented as below,

$$E(x,y,z) = \frac{kE_0}{2z\pi}\int_{y_2'}^{y_1'} X(x,y',z)\exp\left[\frac{ik}{2z}(y-y')^2\right]dy' \tag{6}$$

$$I_{(x,y,z)} = \epsilon_o c E_{(x,y,z)} E^*_{(x,y,z)} \tag{7}$$

where E is electric field of the diffracted light with different coordinates system (x,y), away from aperture coordinates system by z, I is the intensity, ϵ_o is the permittivity of free space and c is the speed of light. Value for constant wouldn't play a big part of the role, so it was assinged at 1 for all constant involved. Equation(6) will be used in equation(7).

Results

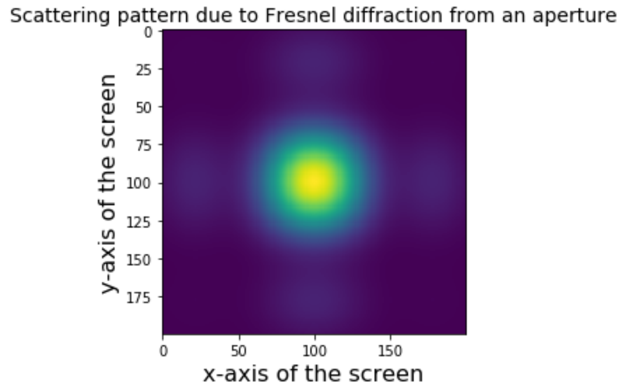

FIG. 10: when n = 50 and z = 1

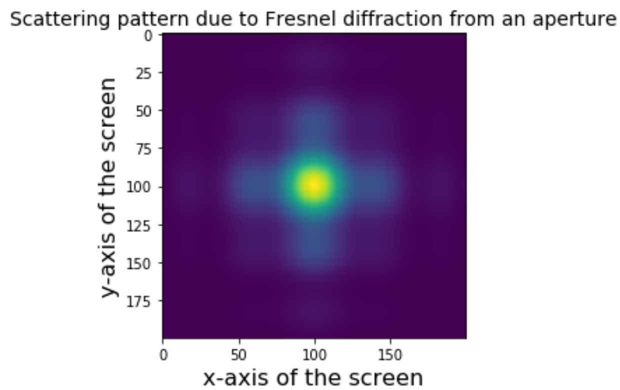

FIG. 11: when n = 50 and z = 1.7. This can be compared to question paper and had similar pattern to when z = 30mm

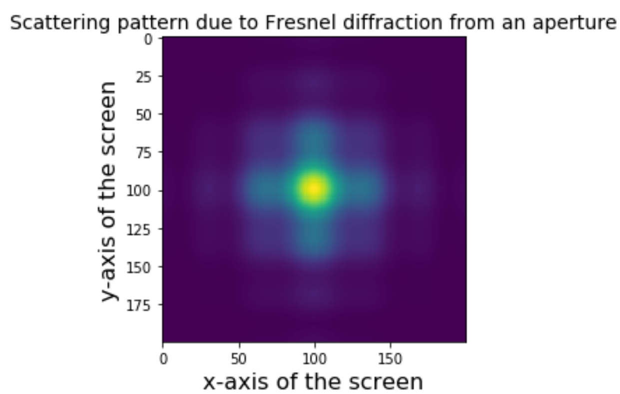

FIG. 12: when n = 50 and z = 2; similar pattern to when z = 20mm

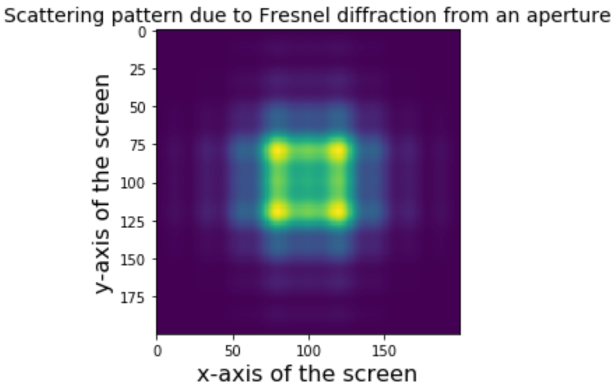

FIG. 13: when = 50 and z = 3similar pattern to when z = 15mm

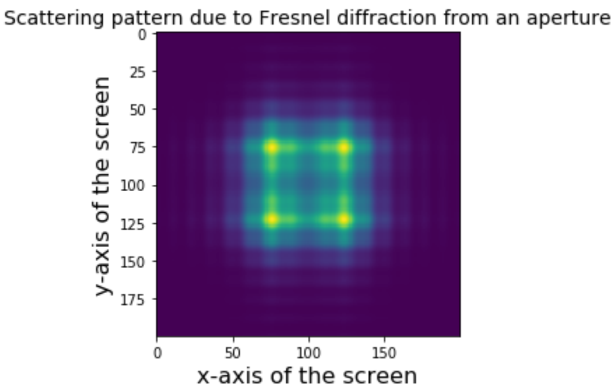

FIG. 14: when n = 50 and z = 5

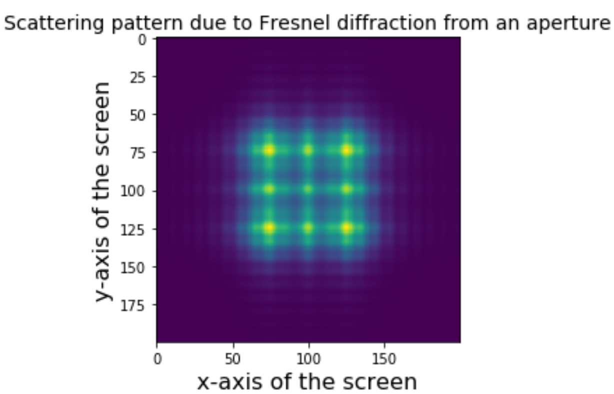

FIG. 15: when n = 50 and z = 7

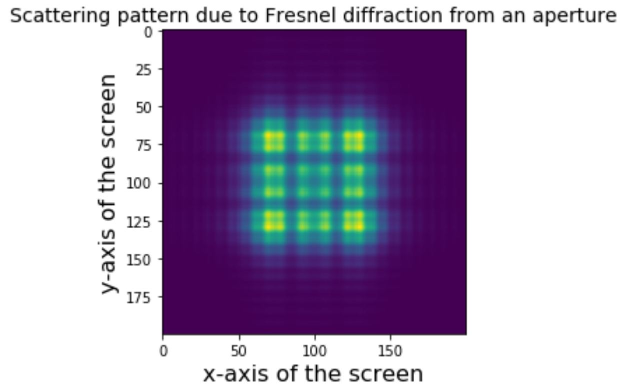

FIG. 16: when n = 50 and z = 8, similar pattern to when z = 10mm

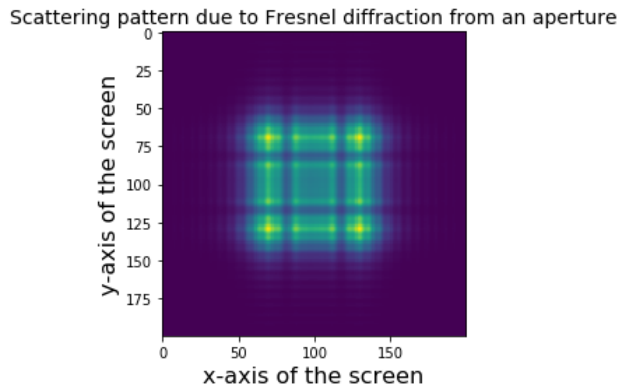

FIG. 17: when n = 50 and z = 10

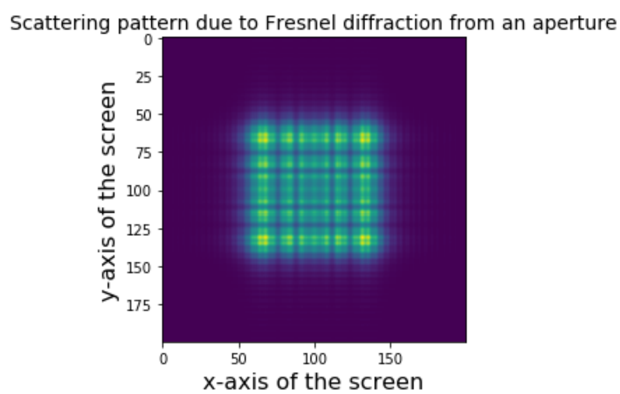

FIG. 18: when n = 50 and z = 15, similar pattern to when z = 5mm

Discussion

Input of n must be at least 20 so as the pattern created won't be too blur and unclear or even shifted slightly. So if n was increased, the image of the pattern will be sharper and more focused , where the external pattern that was not in the center, will disappear.

What was predicted was when z decreases (from 30mm to 5mm), pattern observed will be divided into many smaller square. However, for these case, when z increases (from 1 till 15), the pattern observed divided into many smaller square, which was supposed to be the opposite. Perhaps this could be due to the limit of value taken to be too small or results in a opposite sign in the intermediate stage.

Aside from that, pattern of diffraction was able to be generated with the program's codes and stay fixed at the centre throughout.

PART(D)

Problem

Part(d) requires to produce over different shape of aperture , mainly circle and triangle and etc if possible as originally, part(c) was done over a square or rectangle shape. One of the way to do it is by parameterise the equation of circle into one variable (either y or x), then represent it as the upper and lower limits of the chosen variable.

$$y^2 + x^2 = r^2 \tag{8}$$

Rearranging,

$$y = \sqrt{r^2 - x^2} \tag{9}$$

where r is the radius of the circle, which is also the upper limit of the x' aperture and x will be the varying variable.

Result

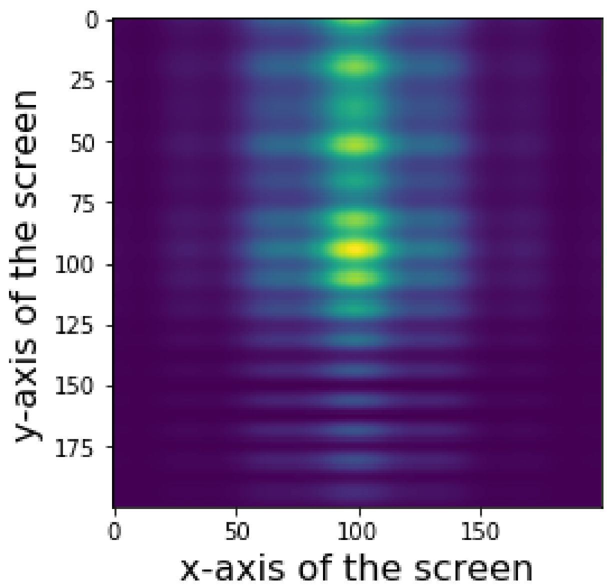

FIG. 19: For circular aperture, when n = 50 and z = 2

Discussion

Figure 19 produced didn't looks like a circular aperture, but more like a cylinder-shaped. Possible error could be dued to error in equation which was carried forward from part(c) to (d).