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STAT 5170

Homework 7

1. (a) $x_t - \Phi x_{t-4} = w_t - \theta w_{t-1}$

$x_t(1-\Phi B^4) = w_t(1-\theta B)$

$p=0,\ d=0,\ q=1,\ P=1,\ D=0,\ Q=0,\ S=4$

(b) $Var(x_t) = \Phi^2 Var(x_{t-4}) + \sigma_w^2 + \theta^2\sigma_w^2$

$\gamma(0) = \Phi^2\gamma(0) + (1+\theta^2)\sigma_w^2$

$\gamma(0) = \dfrac{(1+\theta^2)}{(1-\Phi^2)}\sigma_w^2$

(c) $E(x_t x_{t-h}) = \Phi E(x_{t-4}x_{t-h}) + E(w_t x_{t-h}) - \theta E(w_{t-1}x_{t-h})$

$$\begin{cases} h=1, & \gamma(1) = \Phi\gamma(3) - \theta\sigma_w^2 \\ h=2, & \gamma(2) = \Phi\gamma(2) \\ h=3, & \gamma(3) = \Phi\gamma(1) \end{cases}$$

This gives $\gamma(3) = \Phi^2\gamma(3) - \theta\Phi\sigma_w^2 \Rightarrow \gamma(3) = \dfrac{-\theta\Phi}{(1-\Phi^2)}\sigma_w^2$

$\gamma(2) = 0$ since Φ is non-zero

$\gamma(1) = \dfrac{\gamma(3)}{\Phi} = \dfrac{-\theta}{(1-\Phi^2)}\sigma_w^2$

$\rho(1) = \dfrac{\gamma(1)}{\gamma(0)} = \dfrac{-\theta}{1+\theta^2}$

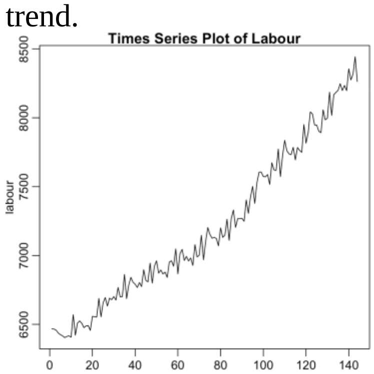
$\rho(2) = \dfrac{\gamma(2)}{\gamma(0)} = 0$

$\rho(3) = \dfrac{-\theta\Phi}{1+\theta^2}$

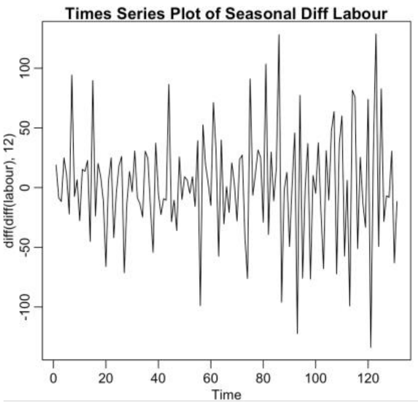
2.

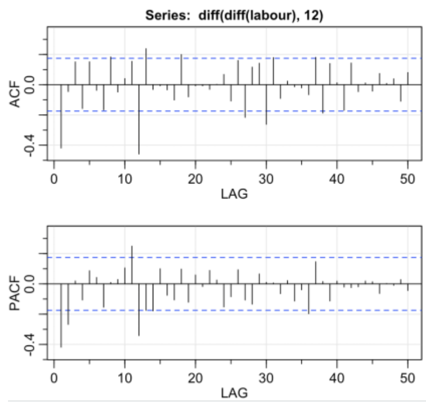
I. Exploratory Data Analysis

The time series of data shows that there appears to be an increasing trend and seasonal trend.



Then we apply differencing and seasonal differencing. The time series, ACF, PACF of the data are shown as follows.





II. Model Fitting.

Since we apply differencing and seasonal differencing, d=1 and D=1.
By looking at lags 12, 24, 36 … of ACF and PACF, we can see P=1 and Q=1.
By looking at lags 1, 2, …, 11 of ACF and PACF, we consider MA(1), AR(2), and ARMA(1,1).
Here are all possible models that we can try:

i. ARIMA(0,1,1)x(0,1,1)$_{12}$
ii. ARIMA(2,1,0)x(1,1,1)$_{12}$
iii. ARIMA(1,1,1)x(1,1,1)$_{12}$
iv. ARIMA(1,1,0)x(1,1,0)$_{12}$

III. Model Selection.

By going through all models one by one, I found that ARIMA(0,1,1)x(0,1,1)$_{12}$ passes all diagnostics and has all coefficients significant. It also gives smaller AIC, AIC$_c$ and BIC.

```
$ttable
     Estimate     SE t.value p.value
ma1   -0.4724 0.0700 -6.7502       0
sma1  -0.6939 0.0814 -8.5224       0
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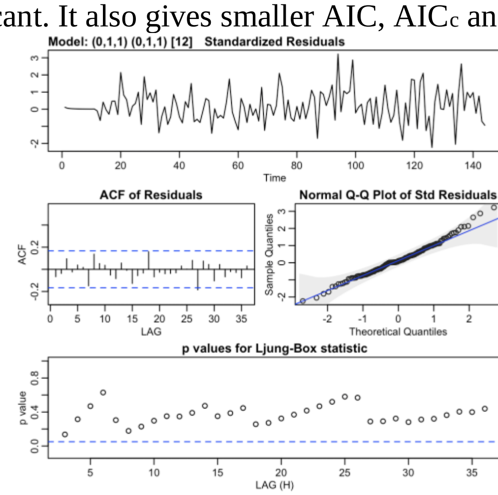

IV. Prediction.

As we can see from the table as well as the plot (true observation is blue and prediction is red), there are 9 observations outside the prediction level. This is because there was rather intense recession in 1990-1991. If we use pre-change model to predict forecast post-change values, most values will not make sense because the two parts of data has different model dynamics.

lower	table$V1[145:168]	upper
8342.431	8402.800	8475.169
8408.675	8426.400	8558.754
8373.421	8428.500	8539.035
8361.978	8466.500	8541.789
8336.685	8451.800	8529.652
8343.662	8502.300	8548.944
8294.920	8412.500	8511.819
8432.703	8553.200	8660.626
8363.255	8477.500	8601.695
8379.925	8479.400	8628.436
8526.312	8645.700	8784.502
8355.297	8263.000	8622.815
8479.689	8436.400	8769.200
8547.551	8199.393	8851.168
8513.325	8323.465	8830.421
8502.516	8606.800	8832.541
8477.580	8199.389	8820.047
8484.712	8480.800	8839.183
8435.973	8425.700	8802.056
8573.641	8630.000	8950.978
8503.987	8489.400	8892.252
8520.378	8491.600	8919.272
8666.429	8699.800	9075.676
8495.028	8530.500	8914.373