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STAT 5170

Homework 6

1. $\hat{\Gamma}_p = \begin{bmatrix} \hat{\gamma}(0) & \hat{\gamma}(1) \\ \hat{\gamma}(1) & \hat{\gamma}(0) \end{bmatrix} = \begin{bmatrix} 1382.2 & 1114.4 \\ 1114.4 & 1382.2 \end{bmatrix}$

$\hat{\gamma}_p = \begin{bmatrix} \hat{\gamma}(1) \\ \gamma(2) \end{bmatrix} = \begin{bmatrix} 1114.4 \\ 591.73 \end{bmatrix}$

$\hat{\phi} = \hat{\Gamma}_p^{-1} \cdot \hat{\gamma}_p = \begin{bmatrix} 1.3175495 \\ -0.6341682 \end{bmatrix}$ $\quad \phi_1 = 1.3175495$ $\quad \phi_2 = -0.6341682$

$\sigma_w^2 = \hat{\gamma}(0) - \phi' \gamma_p = 289.1791$

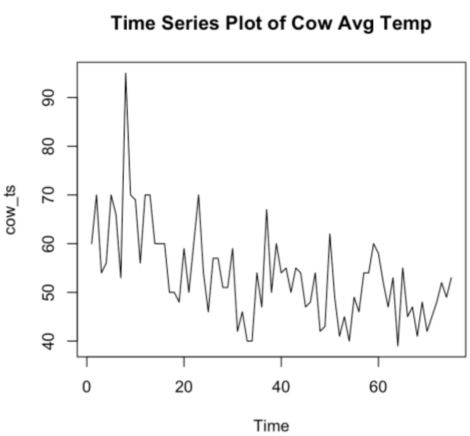
$\mathrm{Var}(\hat{\phi}) = \dfrac{\hat{\sigma}_w^2 \cdot \hat{\Gamma}_p^{-1}}{100} = \begin{bmatrix} 0.005978 & -0.00482 \\ -0.00482 & 0.005978 \end{bmatrix}$

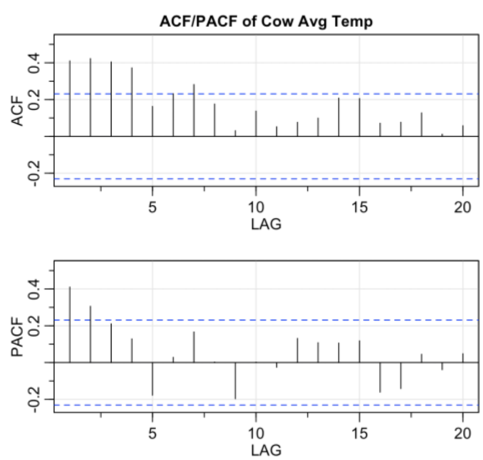
95% CI for $\phi_1 = \cancel{\phi}\ 1.3175495 \pm 1.96 \cdot \sqrt{0.005978} = (1.1660, 1.4691)$

95% CI for $\phi_2 = -0.6341682 \pm 1.96 \cdot \sqrt{\cancel{0.00482}\ 0.005978} = (-0.7857, -0.4826)$

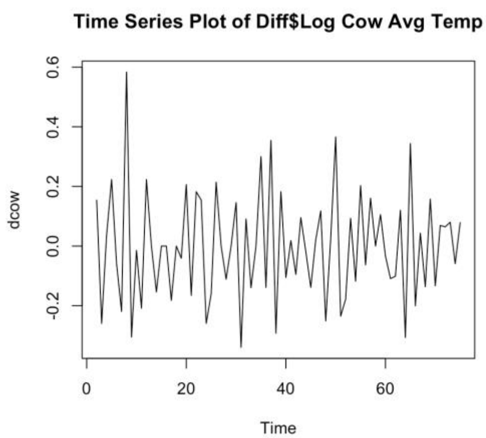
2.

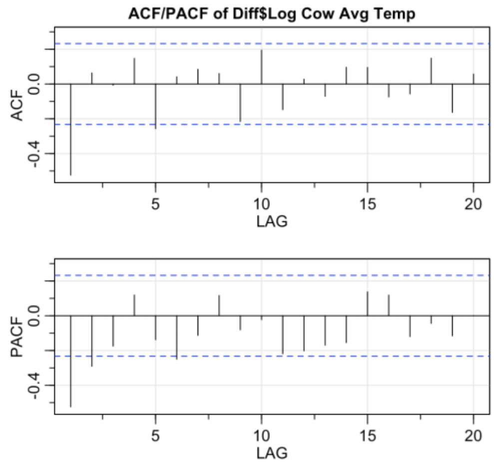
After reading in the data, we created times series plot, ACF plot and PACF plot shown below.





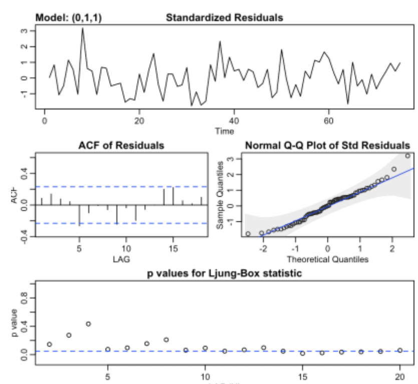
We have seen that there is decreasing trend and non-constant variance. We perform log transform and difference. The new plots are shown below. There are two candidate models: MA(1), AR(2), ARMA(1,1).





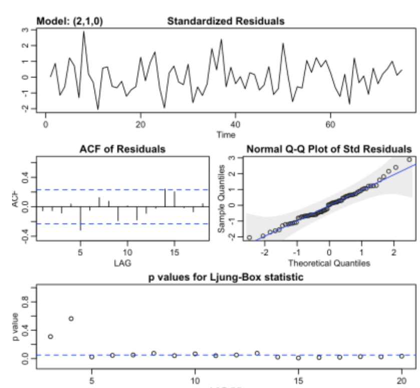
Let us fit MA(1) first. Some p-values for Ljung-Box statistics are signficant, which suggests that MA(1) may not be a right model.

```
$ttable
         Estimate     SE t.value p.value
ma1       -0.8745 0.1149 -7.6124  0.0000
constant  -0.0042 0.0023 -1.7837  0.0787

sigma^2 estimated as 0.02019:  log likelihood = 38.68,  aic = -71.36

$AIC
[1] -0.9642841

$AICc
[1] -0.9620001

$BIC
[1] -0.870876
```



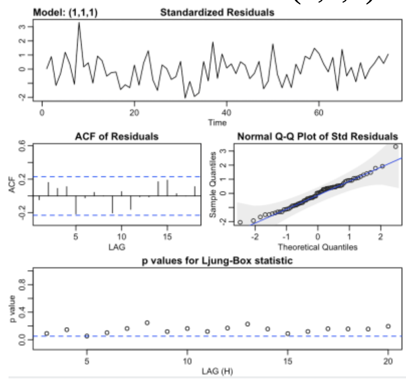
The AR(2) model does not pass p-value diagnostics, so we rule it out.

```
$ttable
         Estimate     SE t.value p.value
ar1       -0.6715 0.1107 -6.0653  0.0000
ar2       -0.2845 0.1113 -2.5553  0.0128
constant  -0.0026 0.0088 -0.2990  0.7658

sigma^2 estimated as 0.02154:  log likelihood = 36.75,  aic = -65.5

$AIC
[1] -0.8851305

$AICc
[1] -0.8804973

$BIC
[1] -0.7605865
```



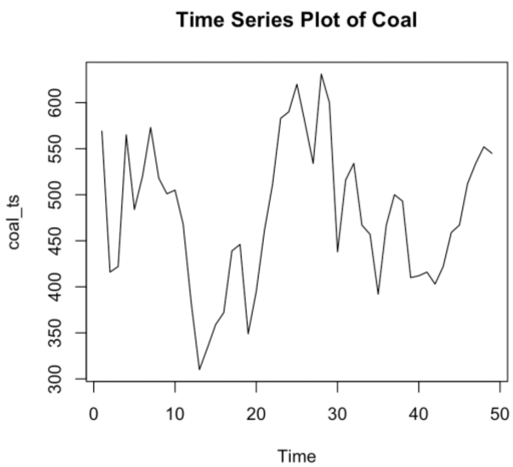
Then we fit ARMA(1,1) model. The model passes all diagnostics and it also gives samller sigma square. Though the estimated ar coefficient is insignificant, the diagnostics of ARIMA(1,1,1) is better than that of ARIMA(0,1,1). Thus we conclude ARIMA(1,1,1) is the best model.

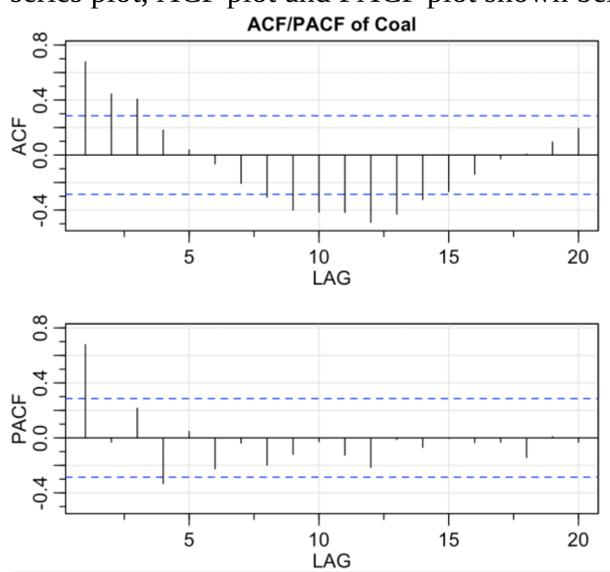
```
$ttable
         Estimate     SE  t.value p.value
ar1        0.1815 0.1161   1.5635  0.1224
ma1       -1.0000 0.0760 -13.1560  0.0000
constant  -0.0043 0.0009  -4.7231  0.0000

sigma^2 estimated as 0.01916:  log likelihood = 39.36,  aic = -70.71

 $AIC
 [1] -0.9555972

 $AICc
 [1] -0.950964

 $BIC
 [1] -0.8310532
```



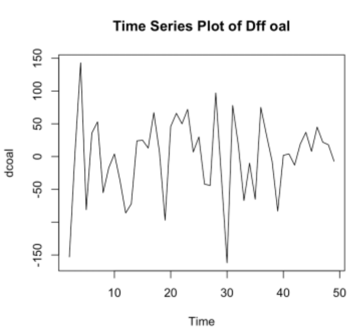
3.

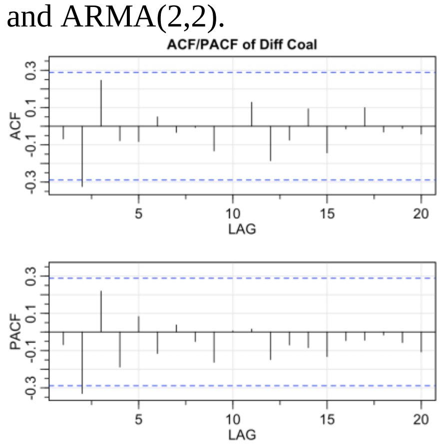
After reading in the data, we created times series plot, ACF plot and PACF plot shown below.





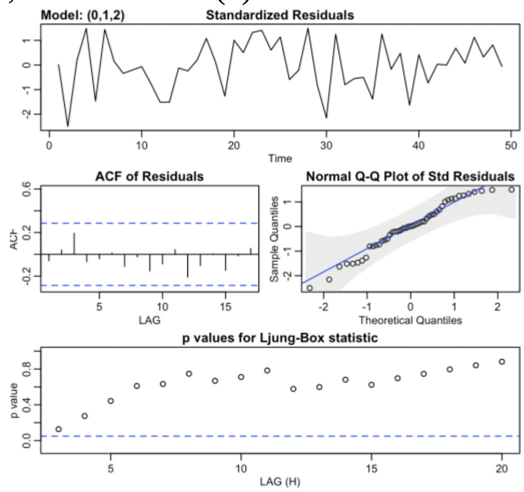
The plot indicates non-staionarity, so we perform differencing. The plots for differenced data are shown below. The differenced data shows stationarity and there are four candidate models: AR(2), MA(2), ARMA(1,1) and ARMA(2,2).





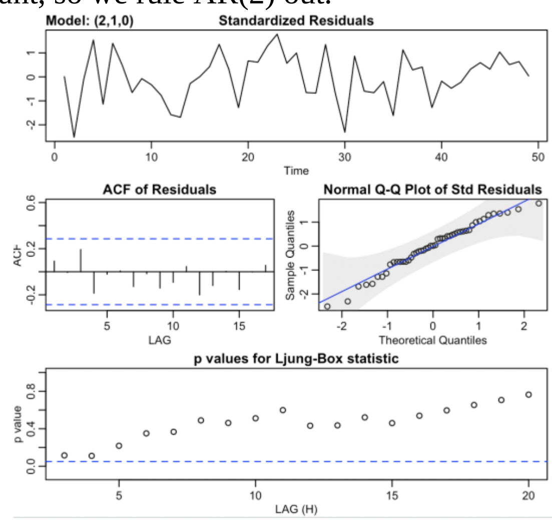
Let us fit MA(2). The ma1 coefficient is insignificant, so we rule MA(2) out.

```
$ttable
         Estimate     SE t.value p.value
ma1        0.0985 0.1312  0.7512  0.4564
ma2       -0.4666 0.1355 -3.4441  0.0013
constant   0.0631 5.1830  0.0122  0.9903

sigma^2 estimated as 3051:  log likelihood = -260.93,  aic = 529.86

$AIC
[1] 11.03875

$AICc
[1] 11.05012

$BIC
[1] 11.19469
```



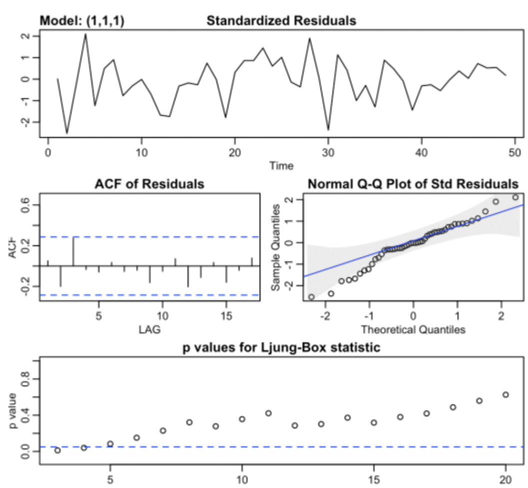
Then we fit AR(2). The ar1 coefficient is insignificant, so we rule AR(2) out.

```
$ttable
         Estimate     SE t.value p.value
ar1       -0.1053 0.1443 -0.7300  0.4692
ar2       -0.3656 0.1409 -2.5949  0.0127
constant   0.4548 5.6191  0.0809  0.9359

sigma^2 estimated as 3191:  log likelihood = -261.89,  aic = 531.78

$AIC
[1] 11.07872

$AICc
[1] 11.09009

$BIC
[1] 11.23466
```



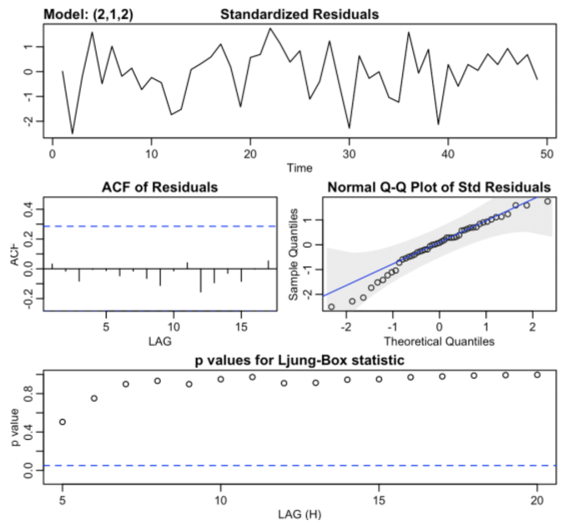
Then we fit ARMA(1,1). The p-value diasnostics plot has significant p-values, so we rule ARMA(1,1) out.

```
$ttable
         Estimate     SE  t.value p.value
ar1        0.7311 0.1096   6.6687  0.0000
ma1       -1.0000 0.0562 -17.8070  0.0000
constant   0.1117 1.8122   0.0616  0.9511

sigma^2 estimated as 3175:  log likelihood = -262.69,  aic = 533.38

$AIC
[1] 11.11216

$AICc
[1] 11.12352

$BIC
[1] 11.26809
```



Lastly, we try out ARMA(2,2). It passes all diagnostics. The ma2 coefficient is insignificant, so we drop ma2 coefficient. Then we obtain the new ARMA(2,1) model.

```
$ttable
         Estimate     SE t.value p.value
ar1       -0.8783 0.4038 -2.1750  0.0352
ar2       -0.6132 0.2997 -2.0464  0.0469
ma1        0.9170 0.4795  1.9126  0.0625
ma2        0.2558 0.4459  0.5736  0.5692
constant   0.5627 6.7366  0.0835  0.9338

sigma^2 estimated as 2833:  log likelihood = -259.27,  aic = 530.55

$AIC
[1] 11.05303

$AICc
[1] 11.0828

$BIC
[1] 11.28693
```



The ARMA(2,1) model passes all diagnostics and have each significant coefficent. Thus, we conclude ARIMA(2,1,1) is the best model.

```
$ttable
         Estimate     SE t.value p.value
ar1       -0.6651 0.1946 -3.4179  0.0014
ar2       -0.4345 0.1441 -3.0140  0.0043
ma1        0.6711 0.1729  3.8805  0.0003
constant   0.5977 6.1948  0.0965  0.9236

sigma^2 estimated as 2859:  log likelihood = -259.45,  aic = 528.9

$AIC
[1] 11.01878

$AICc
[1] 11.03816

$BIC
[1] 11.21369
```