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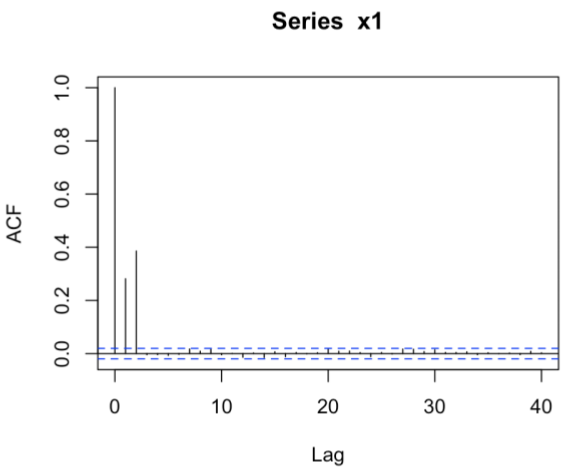
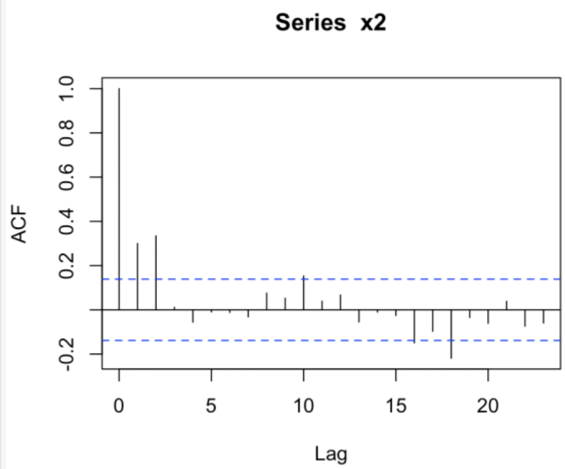
STAT 5170

Homework 2

2. The left plot is the ACF plot for 10000 observations, and the right plot is the ACF plot for 200 observations.





This is consistent with the model generated.

The bigger sample size gives narrower blue lines, whereas the smaller sample size has wider blue lines. Some correlations, which are nearly zero in the 10000-ACF, become non-zero or even outside the blue lines in the 200-ACF.

$$E(x_t x_{t-3}) = E[(w_t - w_{t-3})(w_{t-3} - w_{t-6})]$$
$$= E(w_t w_{t-3}) - E(w_{t-3}^2) + E(w_{t-3} w_{t-6})$$
$$= -Var(w_{t-3}) = -1$$

3. (a) $E(x_t) = E(w_t - w_{t-3}) = E(w_t) - E(w_{t-3}) = 0$

$\gamma(s,t) = E(x_t x_s) = E((w_t - w_{t-3})(w_s - w_{s-3}))$

$= E(w_t w_s) - E(w_t w_{s-3}) - E(w_s w_{t-3}) + E(w_{t-3} w_{s-3})$

if $|s-t| = 0$, $\gamma(s,t) = E(w_t^2) + E(w_{t-3}^2) = Var(w_t) + Var(w_{t-3}) = 2$

if $|s-t| = 3$, $\gamma(s,t) = -E(w_t^2) \cancel{- E(w_{t-3}^2) = -Var(w_t) - Var(w_{t-3}) = -2}$ -1

if $|s-t| \neq 0$ and $|s-t| \neq 3$, $\gamma(s,t) = 0$.

As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, x_t is weakly stationary.

(b) $E(x_t) = E(w_t w_{t-2}) = Cov(w_t, w_{t-2}) = 0$.

$\gamma(s,t) = E(x_t x_s) = E(w_t w_{t-2} w_s w_{s-2}) = E(w_t w_s) E(w_{t-2} w_{s-2})$

if $|s-t| = 0$, $\gamma(s,t) = Var(w_t) \cdot Var(w_{t-2}) = 1$.

if $|s-t| \neq 0$, $\gamma(s,t) = 0$.

As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, x_t is weakly stationary.

4. $E(x_t) = E(w_t) + \theta_1 E(w_{t-1}) + \theta_2 E(w_{t-2}) + \theta_3 E(w_{t-3}) = 0$.

$\gamma(s,t) = E(x_t x_s) = E[(w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \theta_3 w_{t-3})(w_s + \theta_1 w_{s-1} + \theta_2 w_{s-2} + \theta_3 w_{s-3})]$

if $|s-t| = 0$, $\gamma(s,t) = E(w_t^2) + \theta_1^2 E(w_{t-1}^2) + \theta_2^2 E(w_{t-2}^2) + \theta_3^2 E(w_{t-3}^2) = (1 + \theta_1^2 + \theta_2^2 + \theta_3^2)\sigma_w^2$

if $|s-t| = 1$, $\gamma(s,t) = \theta_1 E(w_t w_{s-1}) \cancel{+ \theta_1 E(w_{t-1} w_s)} + \theta_1\theta_2 E(w_{t-1} w_{s-2}) \cancel{+ \theta_2\theta_1 E(w_{t-2} w_{s-1})}$
$+ \theta_2\theta_3 E(w_{t-2} w_{s-3}) \cancel{+ \theta_3\theta_2 E(w_{t-3} w_{s-2})}$

$= \cancel{2}(\theta_1 + \theta_1\theta_2 + \theta_2\theta_3)\sigma_w^2$

if $|s-t| = 2$, $\gamma(s,t) = \theta_2 E(w_t w_{s-2}) + \theta_1\theta_3 E(w_{t-1} w_{s-3}) \cancel{+ \theta_2 E(w_{t-2} w_s) + \theta_3\theta_1 E(w_{t-3} w_{s-1})}$

$= \cancel{2}(\theta_2 + \theta_1\theta_3)\sigma_w^2$

if $|s-t| = 3$, $\gamma(s,t) = \theta_3 E(w_t w_{s-3}) \cancel{+ \theta_3 E(w_{t-3} w_s) = 2\theta_3\sigma_w^2} = \theta_3 \sigma_w^2$

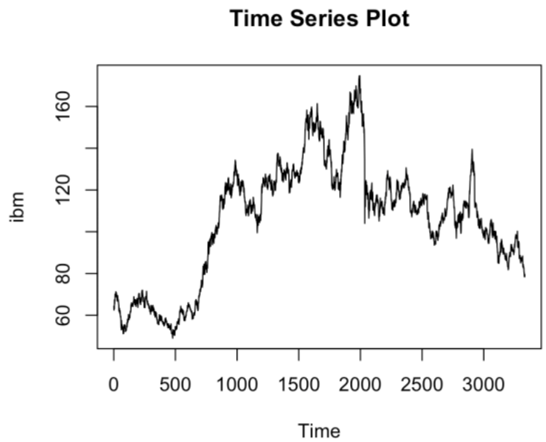
if $|s-t| \geq 4$, $\gamma(s,t) = 0$

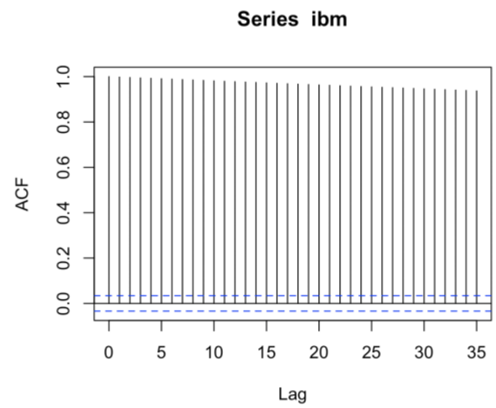
As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, MA(3) is weakly stationary

$$\gamma(s,t) = \begin{cases} (1 + \theta_1^2 + \theta_2^2 + \theta_3^2)\sigma_w^2, & h = 0 \\ (\theta_1 + \theta_1\theta_2 + \theta_2\theta_3)\sigma_w^2, & h = 1 \\ (\theta_2 + \theta_1\theta_3)\sigma_w^2, & h = 2 \\ \theta_3 \sigma_w^2, & h = 3 \\ 0, & h \geq 4 \end{cases}$$

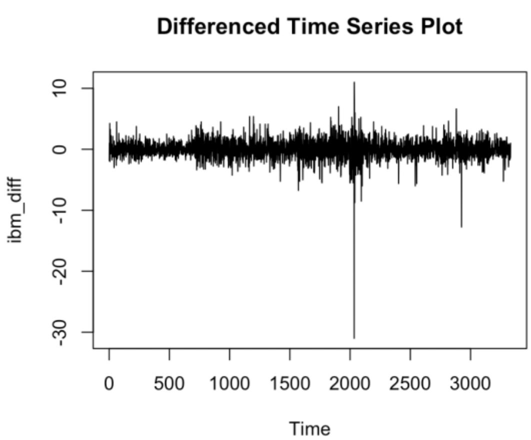
5.

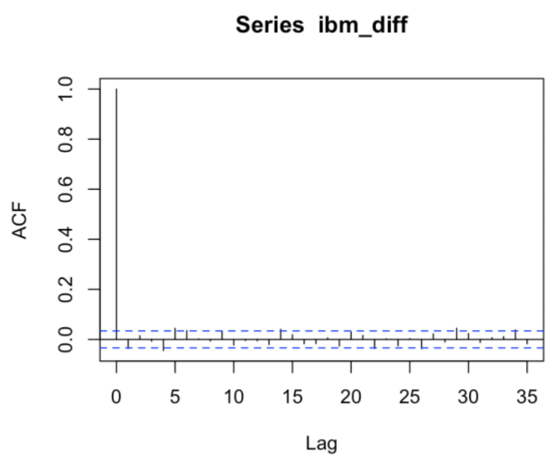
(a) It is not stationary. The ACF plot is decaying very slowly and remains well above the dotted blue lines, and it is an indication of a non-stationary time series.



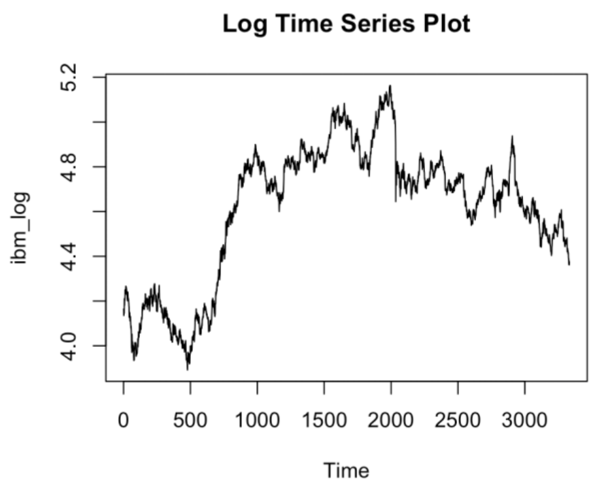


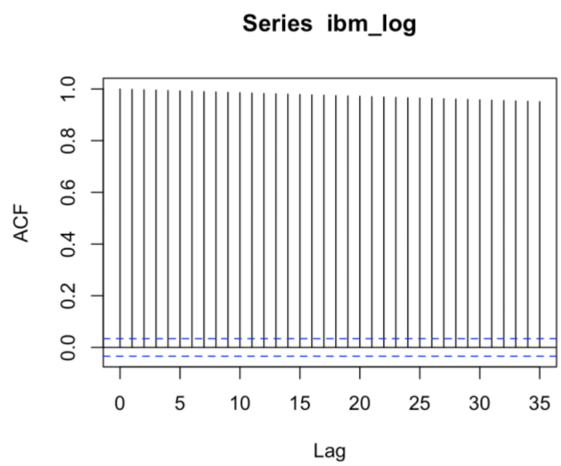
(b) The differenced time series is not stationary, because the variance is not stable.



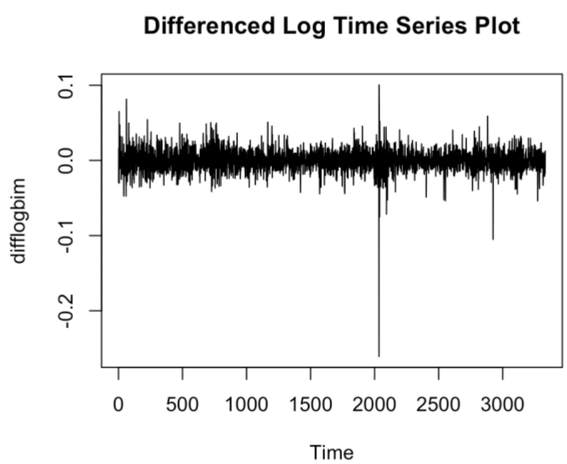


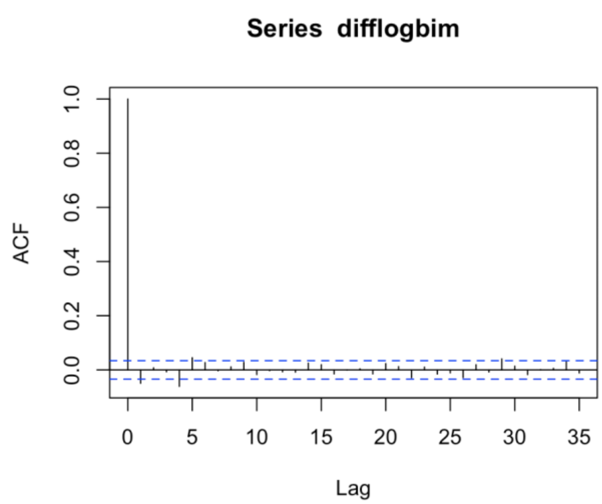
(c) The transformed data does not appear stationary. The ACF plot is still decaying very slowly and remains well above the dotted blue lines, so it is a non-stationary time series.



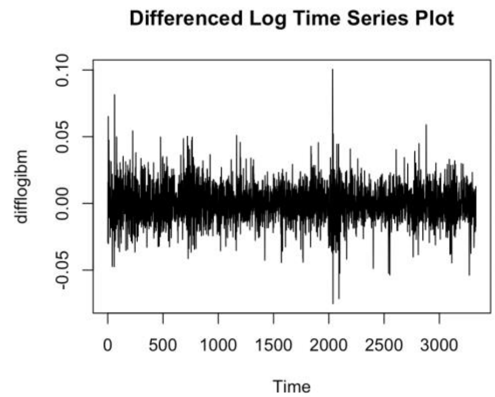


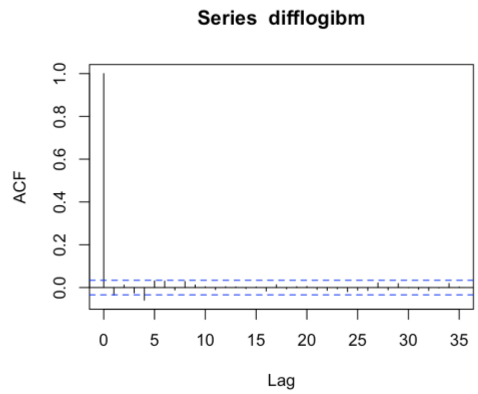
(d) log(diff(ibm)) is not a good idea because if we take difference first, we get some negative data and then log() does not work.
According to the ACF plot, this transformation succeed in creating stationary data.

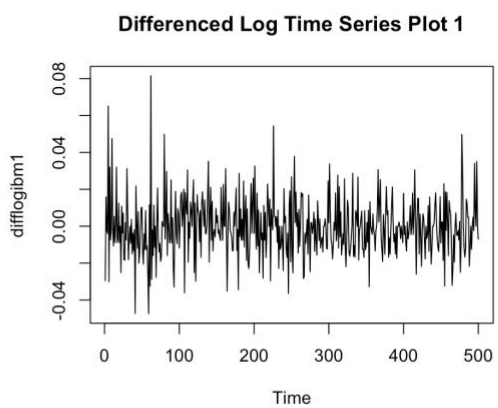


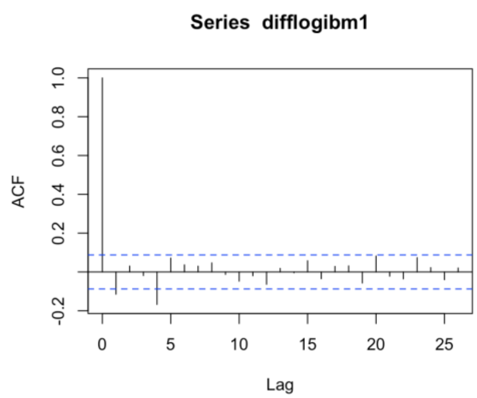


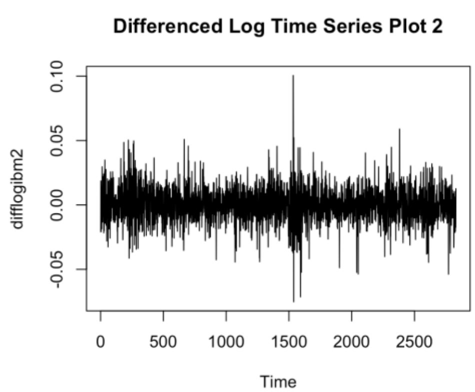
(e) The first section contains fewer observations than the second section. Smaller sample size lead to a wider area of dotted blue lines and more lags outside the area. Bigger sample size lead to a narrower area of dotted blue lines.

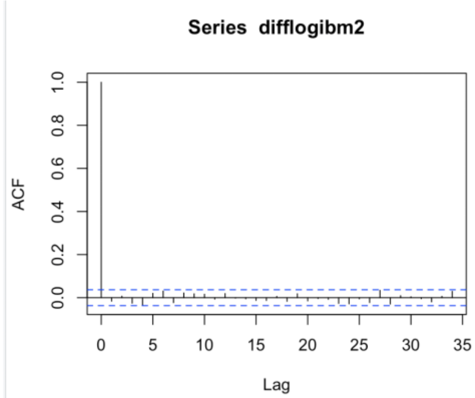












(f) It is reasonable, because the ACF plot for difflogibm2 is similar to the Sample ACF plot for Gaussian White Noise. We take the mean and variance of difflogibm2 to estimate δ and σ_ω.

$E(d_t) = \delta + E(w_t) = 0.0002646076$, so $\delta = 0.0002646076$

$Var(d_t) = Var(w_t) = \sigma_w^2 = 0.0001759611$, so $\sigma_\omega = \sqrt{0.0001759611} = 0.01326503$