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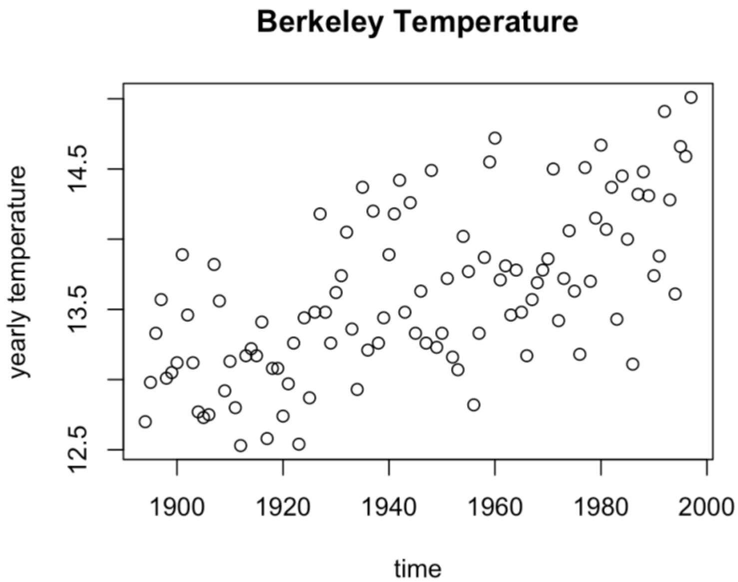
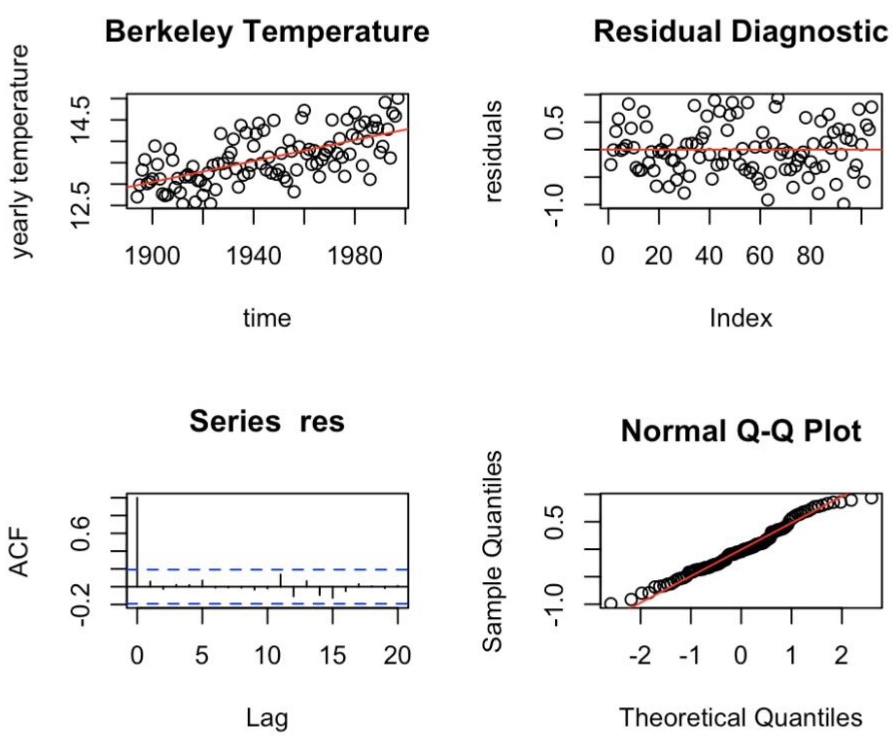
STAT 5170
Homework 3

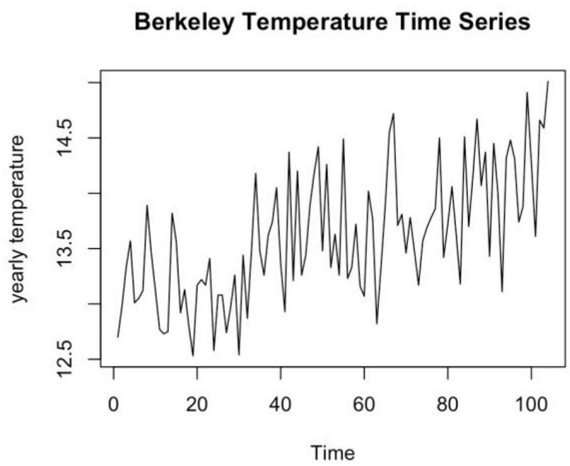
Question 1
(a) There is a growing trend with yearly temperature.

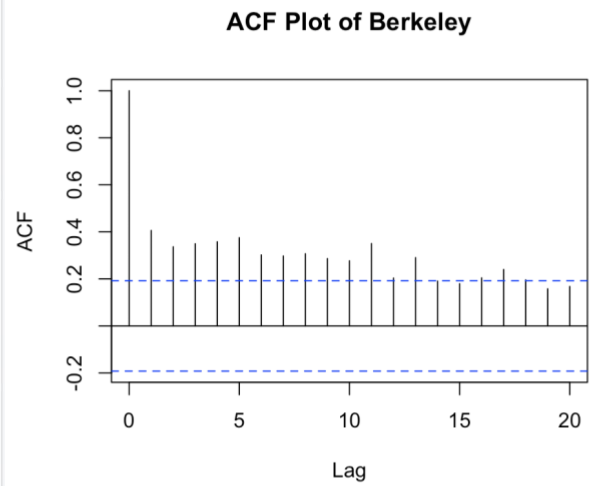


(b) The fit is reasonable. The F-statistic given by summary indicates a strong linear relationship, and diagnostic plots do not show any violation.

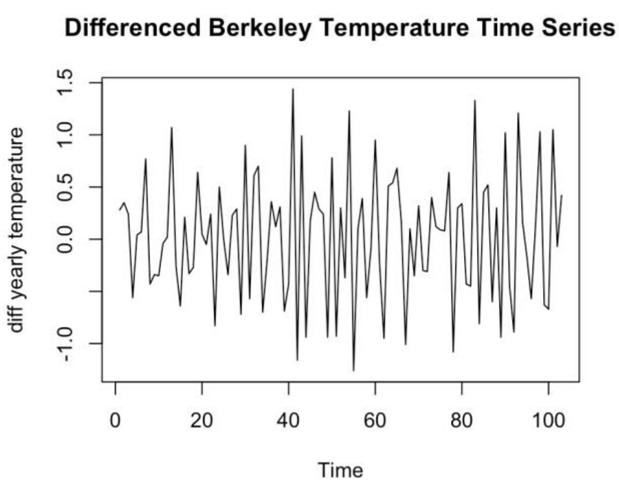


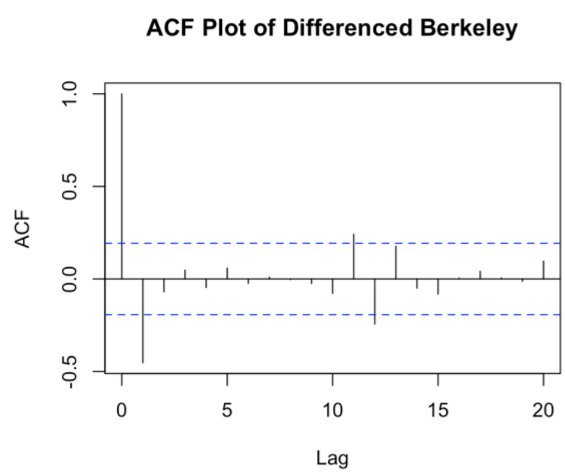
(c) The time series seems not to be stationary because it has an increasing trend. We cannot interpret the ACF plot because the time series is not stationary.





(d) The data seems to be stationary after differencing.





(e) $\nabla x_t = \nabla\beta_1 + \nabla\beta_2 t + \nabla w_t) = \beta_2 + w_t - w_{t-1}$. The corresponding model is MA(1) with β_2 as constant mean and $\theta_1 = -1$. The θ_1 explains why ACF at lag one is negative and significantly outside the confidence intervals, but the other lags show weak dependency.

2. (a) $u_t = E(x_t) = E(w_t) + kE(w_{t-1}) + \cdots + kE(w_0) = 0$

$\gamma(s,t)$: if $h=0$, $\gamma(s,t) = E(x_t^2) = (1+k^2+k^2+\cdots+k^2)\sigma_w^2 = (1+tk^2)\sigma_w^2$

if $h=1$, $\gamma(s,t) = E(x_t x_{t-1}) = (k+k^2+\cdots+k^2)\sigma_w^2 = (k+(t-1)k^2)\sigma_w^2$

if $h=2$, $\gamma(s,t) = E(x_t x_{t-2}) = (k+k^2+\cdots+k^2)\sigma_w^2 = (k+(t-2)k^2)\sigma_w^2$

...

if $h=t$, $\gamma(s,t) = E(x_t x_0) = k\sigma_w^2$

if $h>t$, $\gamma(s,t) = 0$

As we can see, $\gamma(s,t)$ depends on h and t, so it is not stationar

(b) $\nabla x_t = x_t - x_{t-1}$

$= w_t + kw_{t-1} + kw_{t-2} + \cdots + kw_0 - w_{t-1} - kw_{t-2} - \cdots - kw_0$

$= w_t + (k-1)w_{t-1}$

$u_t = E(\nabla x_t) = E(w_t) + (k-1)E(w_{t-1}) = 0$

$\gamma(s,t)$: if $h=0$, $\gamma(s,t) = E(\nabla x_t^2) = (1+(k-1)^2)\sigma_w^2$

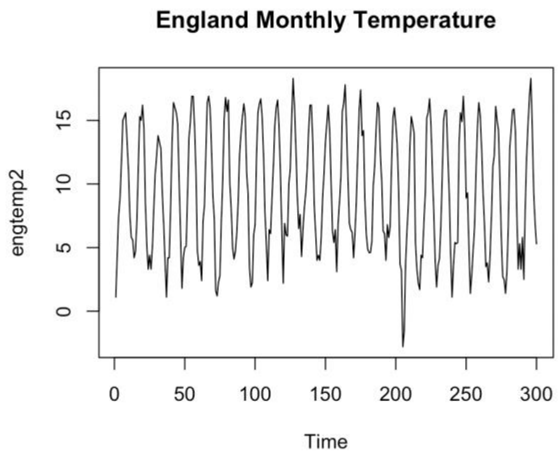
if $h=1$, $\gamma(s,t) = E(\nabla x_t \nabla x_{t-1}) = (k-1)\sigma_w^2$

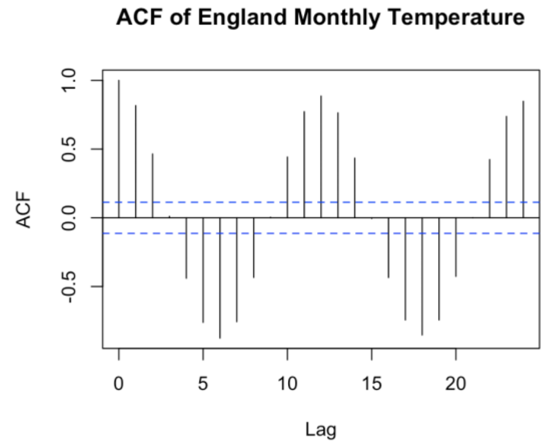
if $h \geq 2$, $\gamma(s,t) = 0$.

Since $u_t = 0$ and $\gamma(s,t)$ only depends on h, it is stationary.

Question 3

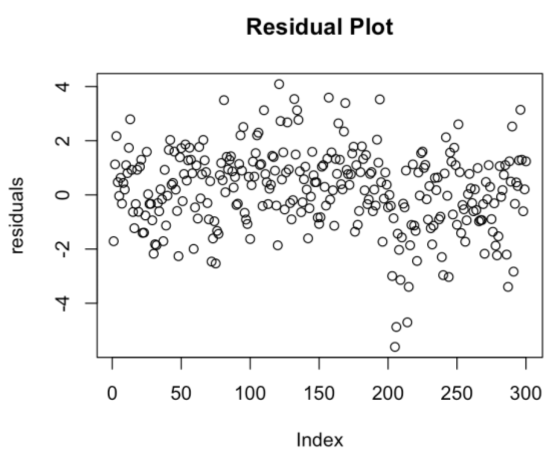
(a) There is a clear seasonal pattern: the temperature in plot rises and falls periodically over the course of the year.

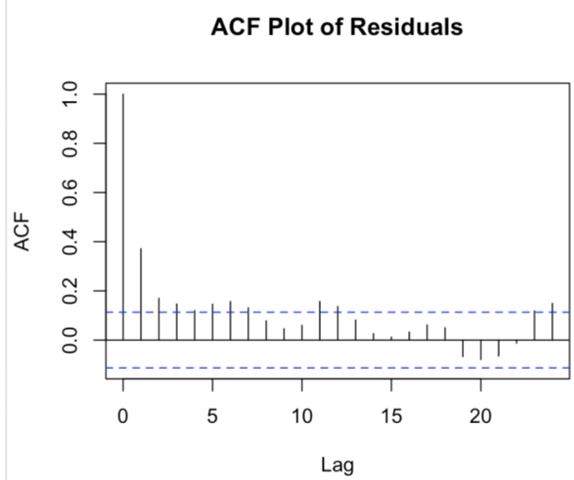




(b) $\beta_1 = 9.21731$, $\beta_2 = -5.15517$, $\beta_3 = -3.88514$

(c) There exists dependency to some extend for residuals, because several lags show correlation higher than the range. The residuals are fairly stationary, although there are some indications that this may not be the case. Overall, the residuals are more stationary than the original series.





(d) We successfully removed a large part of periodical trend, but the ACF plot indicates that there may be some other trends left.

Question 4

(a) Since we are using weekly data to evaluate yearly data, the weights are as follows

$$a_0 = a_{\pm 1} = \cdots = a_{\pm 25} = \frac{1}{52}, a_{\pm 26} = \frac{1}{104}$$

(b) From 2000 to 2008, the oil price increased continuously but experienced a drop in 2009. Beginning 2010, the oil price went up again.