

2. (a) $u_t = E(x_t) = E(w_t) + kE(w_{t-1}) + \dots + kE(w_0) = 0$

$\gamma(s, t)$: if $h=0$, $\gamma(s, t) = E(x_t^2) = (1 + k^2 + k^4 + \dots + k^{2t}) \sigma_w^2 = (1 + t k^2) \sigma_w^2$

if $h=1$, $\gamma(s, t) = E(x_t x_{t-1}) = (k + k^3 + \dots + k^{2t-1}) \sigma_w^2 = (k + (t-1) k^3) \sigma_w^2$

if $h=2$, $\gamma(s, t) = E(x_t x_{t-2}) = (k^2 + k^4 + \dots + k^{2t-2}) \sigma_w^2 = (k^2 + (t-2) k^4) \sigma_w^2$

...

if $h=t$, $\gamma(s, t) = E(x_t x_0) = k \sigma_w^2$

if $h > t$, $\gamma(s, t) = 0$

As we can see, $\gamma(s, t)$ depends on h and t , so it is not stationary

(b) $\nabla x_t = x_t - x_{t-1} = w_t - 1 + k w_{t-1} - k w_{t-2} + \dots + k w_0$

$= w_t + k w_{t-1} + k w_{t-2} + \dots + k w_0 - w_{t-1} - k w_{t-2} - \dots - k w_0$

$= w_t + (k-1) w_{t-1}$

$u_t = E(\nabla x_t) = E(w_t) + (k-1) E(w_{t-1}) = 0$

$\gamma(s, t)$: if $h=0$, $\gamma(s, t) = E(\nabla x_t^2) = (1 + (k-1)^2) \sigma_w^2$

if $h=1$, $\gamma(s, t) = E(\nabla x_t \nabla x_{t-1}) = (k-1) \sigma_w^2$

if $h \geq 2$, $\gamma(s, t) = 0$

Since $u_t = 0$ and $\gamma(s, t)$ only depends on h , it is stationary.