$$E(x_t x_{t-3}) = E[(w_t - w_{t-3})(w_{t-3} - w_{t-6})]$$
$$= E(w_t w_{t-3}) - E(w_{t-3}^2) + E(w_{t-3} w_{t-6})$$
$$= -Var(w_{t-3}) = -1$$

3. (a) $E(x_t) = E(w_t - w_{t-3}) = E(w_t) - E(w_{t-3}) = 0$

$\gamma(s,t) = E(x_t x_s) = E((w_t - w_{t-3})(w_s - w_{s-3}))$

$= E(w_t w_s) - E(w_t w_{s-3}) - E(w_s w_{t-3}) + E(w_{t-3} w_{s-3})$

if $|s-t| = 0$, $\gamma(s,t) = E(w_t^2) + E(w_{t-3}^2) = Var(w_t) + Var(w_{t-3}) = 2$

if $|s-t| = 3$, $\gamma(s,t) = E(w_t^2) - E(w_{t-3}^2) = Var(w_t) - Var(w_{t-3}) = 2 - 1$

if $|s-t| \neq 0$ and $|s-t| \neq 3$, $\gamma(s,t) = 0$.

As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, x_t is weakly stationary.

(b) $E(x_t) = E(w_t w_{t-2}) = Cov(w_t, w_{t-2}) = 0$.

$\gamma(s,t) = E(x_t x_s) = E(w_t w_{t-2} w_s w_{s-2}) = E(w_t w_s) E(w_{t-2} w_{s-2})$

if $|s-t| = 0$, $\gamma(s,t) = Var(w_t) \cdot Var(w_{t-2}) = 1$.

if $|s-t| \neq 0$, $\gamma(s,t) = 0$.

As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, x_t is weakly stationary.

4. $E(x_t) = E(w_t) + \theta_1 E(w_{t-1}) + \theta_2 E(w_{t-2}) + \theta_3 E(w_{t-3}) = 0$.

$\gamma(s,t) = E(x_t x_s) = E[(w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \theta_3 w_{t-3})(w_s + \theta_1 w_{s-1} + \theta_2 w_{s-2} + \theta_3 w_{s-3})]$

if $|s-t| = 0$, $\gamma(s,t) = E(w_t^2) + \theta_1^2 E(w_{t-1}^2) + \theta_2^2 E(w_{t-2}^2) + \theta_3^2 E(w_{t-3}^2) = (1 + \theta_1^2 + \theta_2^2 + \theta_3^2)\sigma_w^2$

if $|s-t| = 1$, $\gamma(s,t) = \theta_1 E(w_t w_{s-1}) + \theta_1 E(w_{t+1} w_s) + \theta_1 \theta_2 E(w_{t-1} w_{s-2}) + \theta_2 \theta_1 E(w_{t-2} w_{s-1})$

$+ \theta_2 \theta_3 E(w_{t-2} w_{s-3}) + \theta_3 \theta_2 E(w_{t-3} w_{s-2})$

$= (\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3)\sigma_w^2$

if $|s-t| = 2$, $\gamma(s,t) = \theta_2 E(w_t w_{s-2}) + \theta_1 \theta_3 E(w_{t-1} w_{s-3}) + \theta_2 E(w_{t-2} w_s) + \theta_3 \theta_1 E(w_{t-3} w_{s-1})$

$= (\theta_2 + \theta_1 \theta_3)\sigma_w^2$

if $|s-t| = 3$, $\gamma(s,t) = \theta_3 E(w_t w_{s-3}) + \theta_3 E(w_{t-3} w_s) = 2\theta_3 \sigma_w^2 = \theta_3 \sigma_w^2$

if $|s-t| \geq 4$, $\gamma(s,t) = 0$

As $u_t = 0$ and $\gamma(s,t)$ only depends on $|s-t|$, MA(3) is weakly stationary

$$\gamma(s,t) = \begin{cases} (1 + \theta_1^2 + \theta_2^2 + \theta_3^2)\sigma_w^2 & , h = 0 \\ (\theta_1 + \theta_1\theta_2 + \theta_2\theta_3)\sigma_w^2 & , h = 1 \\ (\theta_2 + \theta_1\theta_3)\sigma_w^2 & , h = 2 \\ \theta_3 \sigma_w^2 & , h = 3 \\ 0 & , h \geq 4 \end{cases}$$