1 Question 1

The first removed edge breaks the single cycle graph in 2 connected components. The second removed edge breaks one of the 2 connected components into 2 connected components.
At the end of the day, there are **3 connected components**.

2 Question 2

Two graphs having identical degree distributions **are not always isomorphic** to each other. Here is an example. See graph 1 and graph 2. Both have the same identical degree distribution (3 nodes with 2 degrees, and 2 nodes with 1 degree) but are not isomorphic, since some connections are lost from graph 1 to graph 2 (see for example nodes 1 and 0).

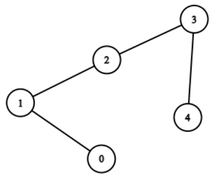

Figure 1: Graph 1.

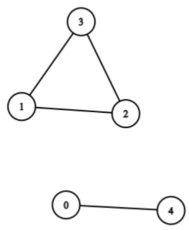

Figure 2: Graph 2.

3 Question 3

Let's supposed $n \geq 3$. Since a fully connected graph with n nodes have $\frac{n(n-1)}{2}$ edges, a n-nodes graph with $\frac{n(n-1)}{2} - 1$ edges is a graph where every nodes are connected except two of them (let's say node 0 and node 1). It means that every triplet is a closed triplet, except the triplets composed of the node 0, the node 1 and any other node.
We have $\binom{n}{3}$ possible triplets and $n-2$ triplets with at least nodes 0 and 1. At the end of the day, the global clustering coefficient is:

$$1 - \frac{n-2}{\binom{n}{3}} = 1 - \frac{6}{n(n-1)}$$

4 Question 4

According to [1], the smallest eigenvalue of $D - W$ is 0, which means that the smallest eigenvalue of $L_{rw} = D^{-1}(D - W)$ is 0 as well. On Prop.2 of [1], the author proves that the multiplicity k of the eigenvalue 0 is the number of components of the graph and that the corresponding eigenvectors are the indicator vectors of the connected components.

5 Question 5

The output is clearly stochastic since the K-Means algorithm chooses randomly the initial cluster centers. However, you can make it deterministic by imposing the initial cluster centers.

6 Question 6

Computing the modularity means computing first the values l_c, d_c of each cluster and the global value m.
Graph 1: the blue community has $l_c = 4$ and $d_c = 9$. The green community has $l_c = 3$ and $d_c = 7$. $m = 8$.
So $Q = \frac{47}{128} \approx 0.367$.
Graph 2: the blue community has $l_c = 1$ and $d_c = 4$. The orange community has $l_c = 2$ and $d_c = 8$. The green community has $l_c = 1$ and $d_c = 4$.
$m = 8$.
So $Q = \frac{1}{8} = 0.125$.

7 Question 7

$\Phi(P_4) = [3, 2, 1]$ and $\Phi(K_4) = [6, 0, 0]$.
So $K(P_4, P_4) = 9 + 4 + 1 = 14$,
$K(P_4, K_4) = 6 * 3 = 18$,
$K(K_4, K_4) = 6 * 6 = 36$.

References

[1] Ulrike von Luxburg. A tutorial on spectral clustering. *CoRR*, abs/0711.0189, 2007.