Environments: Collision_v0

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1 Introduction

Collision_v0 is a simple environment for training reinforcement learning agents. It is a two-dimensional space containing colliding particles. The movement of these particles is constrained by the walls of the environment as well as the fluctuation of the other particles. Every collision in the environment (particle $\leftrightsquigarrow$ wall, particle $\leftrightsquigarrow$ particle) is elastic, meaning that the kinetic energy of the two colliding particles remains the same after the collision.

The environment also accommodates an agent that is able to move around the constrained space. The environment is fully-observable by the agent, and the agent receives the information about the environment as an image (see: Fig. 1). The goal is to train the agent to avoid colliding with other particles and the walls of the environment. If a collision involves the agent (agent $\leftrightsquigarrow$ wall, agent $\leftrightsquigarrow$ particle), the episode terminates. Figure 1 represents a sample Collision_v0 environment as the number of particles and the size of the area can be defined by the user. The green circle embodies the agent while the red circles denote the particles.

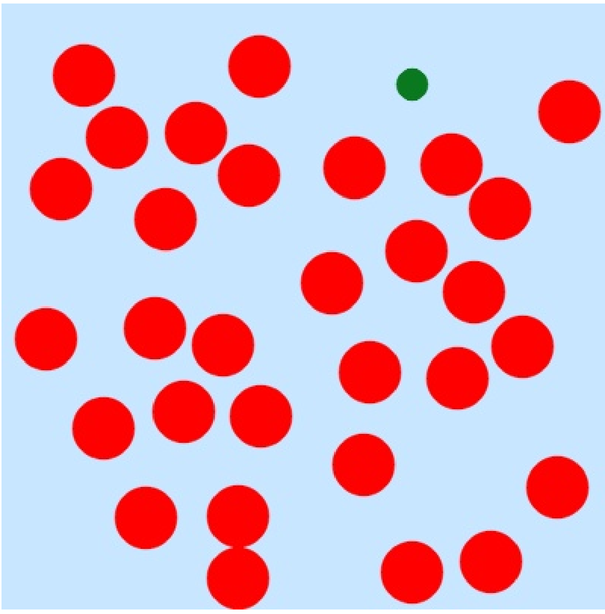

Figure 1: Collision_v0 environment is a constrained two dimensional area which holds particles and an agent. The agent should avoid any collisions with other particles and the walls of the environment.

2 Dynamics

The dynamics of the environment are encapsulated in two laws of the environment: *conservation of momentum*, and *conservation of kinetic energy*. As every collision in this environment is elastic, both momentum and kinetic energy are conserved. These laws can be expressed for the whole system

$$\sum_{i=1}^{N} m_i \|\mathbf{v}_{i,0}\| = \sum_{i=1}^{N} m_i \|\mathbf{v}_{i,t}\| \tag{1}$$

$$\sum_{i=1}^{N} m_i \|\mathbf{v}_{i,0}\|^2 = \sum_{i=1}^{N} m_i \|\mathbf{v}_{i,t}\|^2, \tag{2}$$

where the environment has N particles, and it evolves for $t = 0, ..., T$ timesteps. Further on, m_i and $\mathbf{v}_i$ denote the mass and the velocity of the i:th particle respectively. We assume that the mass of the particles is fixed. These laws provide an easy debugging tool for the purpose of analysing the dynamics of the system.

2.1 Collisions

Computationally the most time consuming part is to check for collisions within the system. Let's first look at a simple case of particle $\leftrightsquigarrow$ wall collisions.

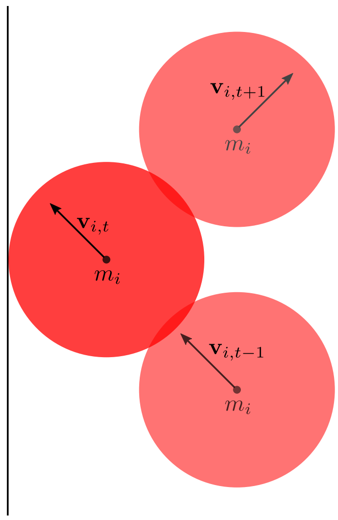

Figure 2: When a particle collides with the wall, the direction of the particle changes.

Since the collision is elastic, the magnitude of the velocity of the particle does not change. Depending on the wall the particle collides with, either the speed in x or y-direction changes. If the particle collides with right or left side of the space, then $\mathbf{v}_{i,t+1} = (v^x_{i,t+1}, v^y_{i,t+1}) = (-v^x_{i,t}, v^y_{i,t})$, where $\mathbf{v}_{i,t+1}$ denotes the velocity after the collision, and $\mathbf{v}_{i,t}$ velocity before the collision. Collision with the up or down side of the area work in similar fashion $\mathbf{v}_{i,t+1} = (v^x_{i,t+1}, v^y_{i,t+1}) = (v^x_{i,t}, -v^y_{i,t})$. As one can see, particle $\leftrightsquigarrow$ wall collisions are simple to calculate, but let's move on to particle $\leftrightsquigarrow$ particle collisions.

Particle $\leftrightsquigarrow$ particle collisions are more complicated. This arises from the fact that both the magnitude and direction of the velocity might change.

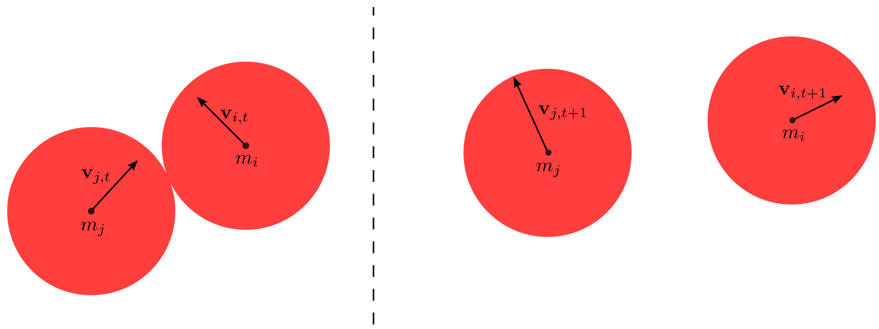

Figure 3: When two particles collide, the magnitude and the directions of the particles changes so that the momentum and kinetic energy of the system is conserved.

The full derivation of the particle $\leftrightsquigarrow$ particle collisions is in A. The velocities of the two particles in Figure 3 are

$$\mathbf{v}_{i,t+1} = \mathbf{v}_{i,t} - \frac{2m_j}{m_j + m_i} \frac{\langle \mathbf{v}_{i,t} - \mathbf{v}_{j,t}, \mathbf{x}_{i,t} - \mathbf{x}_{j,t} \rangle}{\|\mathbf{x}_{i,t} - \mathbf{x}_{j,t}\|^2} (\mathbf{x}_{i,t} - \mathbf{x}_{j,t}) \tag{3}$$

$$\mathbf{v}_{j,t+1} = \mathbf{v}_{j,t} - \frac{2m_i}{m_i + m_j} \frac{\langle \mathbf{v}_{j,t} - \mathbf{v}_{i,t}, \mathbf{x}_{j,t} - \mathbf{x}_{i,t} \rangle}{\|\mathbf{x}_{j,t} - \mathbf{x}_{i,t}\|^2} (\mathbf{x}_{j,t} - \mathbf{x}_{i,t}), \tag{4}$$

where $\mathbf{x}_{i,t} = (x_{i,t}, y_{i,t})$ and $\mathbf{x}_{j,t} = (x_{j,t}, y_{j,t})$ denote the positions of particles i and j respectively. Moreover, as the number of particles increases, further optimizations regarding collision checks needs to be considered.

2.2 Collision detection

Checking for every possible collision in the environment is time consuming. Every particle might collide with one of the walls of the environment or any of the other particles, meaning that without any optimization the time complexity for collision checks is $\mathcal{O}(n^2)$ where n is the number of particles in the environment.

3 Agents

4 Results

A Particle collisions