Optimal Bidding Strategy for Auto Bidding Campaigns

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Problem Statement

- Auto bidding requires advertisers to set a campaign budget.
- Bidding strategies are automated by an algorithm.
- Objective: Design a bidding algorithm to maximize ad engagement within budget.
- Importance: Poor performance could lead to advertiser churn and revenue loss.

Solution: Notations

- B: Campaign budget
- T: Total auction opportunities
- t: t^{th} auction opportunity
- x_t: Indicator if the campaign wins the t^{th} auction
- c_t: Engagement cost of the t^{th} auction
- b_t: Bid price at t
- r_t: Conversion rate(pctr) at t

Optimization Problem: CPC Campaign Example

- **Objective**: Maximize expected engagement under Second Price Auction(SPA)

$$\max_{x_t \in \{0,1\}} \sum_{t=1}^{T} x_t r_t$$

- **Constraint**: Stay within budget

$$\sum_{t=1}^{T} x_t c_t \leq B \quad \text{where } x_t = \mathbb{1}_{\{b_t > c_t\}}$$

Lagrangian Formulation

- Lagrangian:

$$\mathrm{L} = \sum_{t=1}^{T} x_t r_t - \lambda \left(\sum_{t=1}^{T} x_t c_t - B \right)$$

- Dual Problem:

$$\min_{\lambda \geq 0} \sum_{t=1}^{T} (r_t - \lambda c_t)_+ + \lambda B$$

Optimality Condition and Solution

- **Optimality Condition**:

$$b_t^* = \frac{r_t}{\lambda^*} \quad \text{where } \lambda^* \text{ is the optimal dual param}$$

- **Bid Update via Dual Online Gradient Descent (DOGD)**:

$$\lambda_{t+1} = \lambda_t - \epsilon_t \cdot \left(\frac{B}{T} - c_t \cdot \mathbb{1}_{\{r_t \geq \lambda c_t\}} \right)$$

Extension to Cost Cap

- **Problem Formulation**

$$\max_{x_t \in \{0,1\}} \sum_{t=1}^{T} x_t r_t$$

s.t.

$$\sum_{t=1}^{T} x_t c_t \leq B, \quad \sum_{t=1}^{T} x_t c_t \leq C \sum_{t=1}^{T} x_t r_t$$

- **Derive Bid Update Rule via DOGD as welll**

Alternative Solution: Model Predictive Control (MPC)

- **Model Predictive Control**
- **Dependent on Bid Landscape Forecasting**
- **Sensitive to forecasting accuracy**

A/B Testing: Budget Split Test

- Baseline: MPC controller
- Treatment: OGD
- **Result**: more than 10% improvement in ROI metrics.