Bayesian Inference and Gravitational Waves in Undergraduate Computational Physics

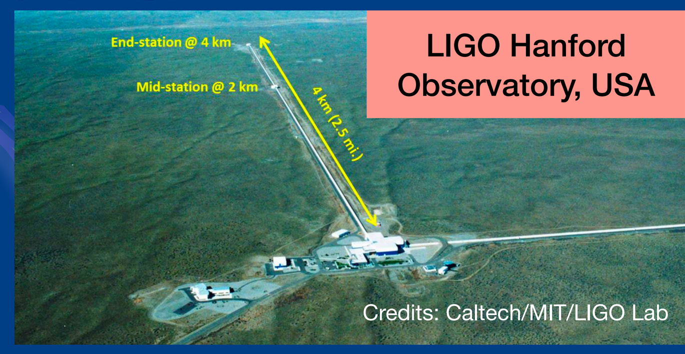
THE UNIVERSITY OF MELBOURNE

OzGrav

ARC Centre of Excellence for Gravitational Wave Discovery

Yi Shuen Christine Lee†, Kok Hong Thong, Yutong Bu

School of Physics, The University of Melbourne, Victoria 3010, Australia.



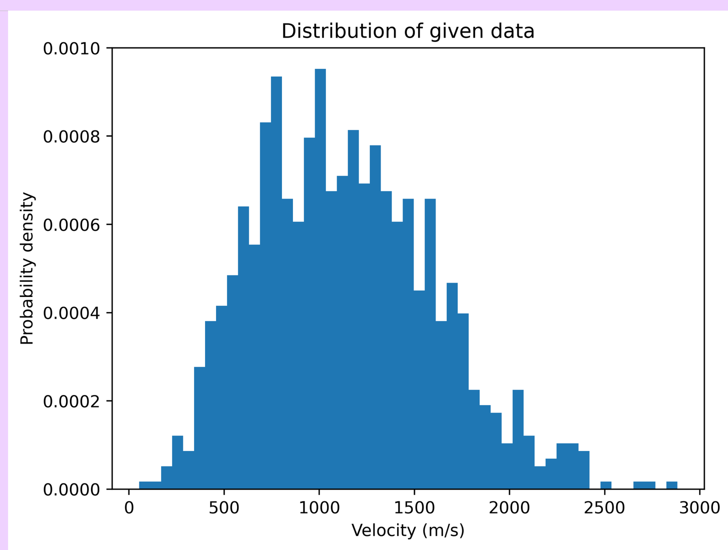
PROBLEM STATEMENT

Bayesian inference is a widely used statistical tool which prevails among diverse research areas including science, engineering, health care, social science, sports etc. Despite its broad applicability in various physics experiments, Bayesian inference is often overlooked or under-discussed in undergraduate physics coursework. Furthermore, undergraduate coursework strongly focuses on the theoretical aspect of physics with a **lack of mention about ongoing research work**.

- We present a research-informed computational physics lab activity which introduces **Bayesian inference in the context of gravitational-wave (GW) detection**, a cutting-edge research topic awarded the 2017 Nobel Prize in Physics.
- This activity targets third year undergraduate students majoring in physics, to foster interests in pursing further studies and research in physics.
- We discuss the structure and objectives of this activity, while showcasing the results produced by students who have undertaken the activity.

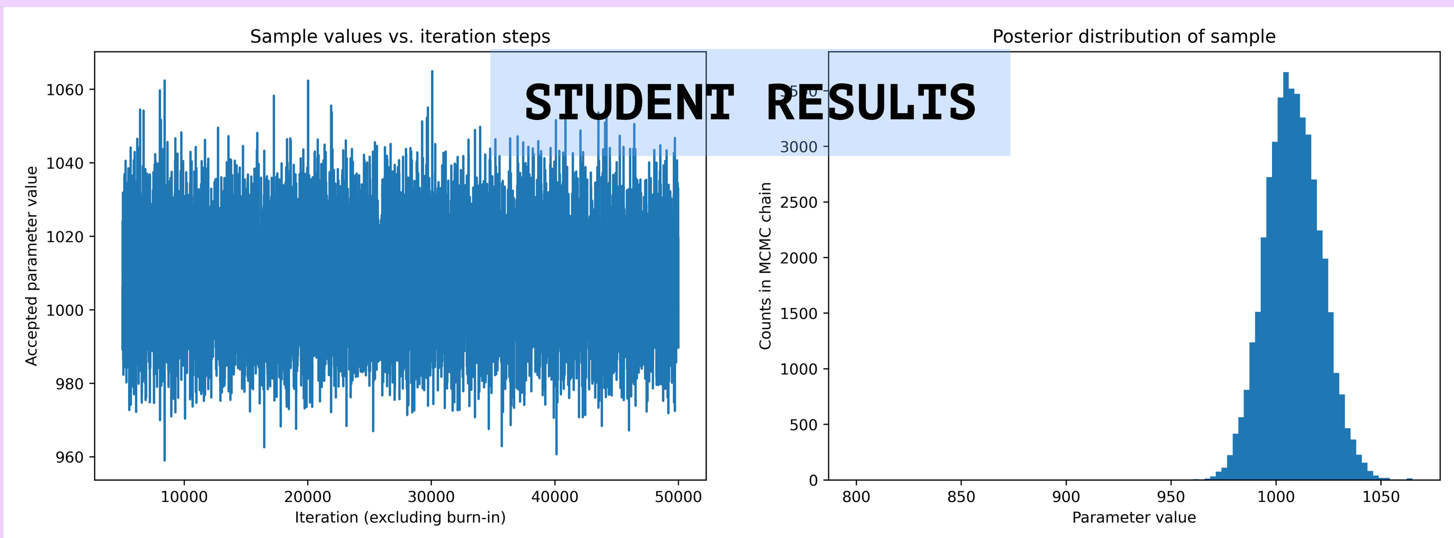
BAYESIAN INFERENCE

KEY CONCEPT	ACTIVITY
Bayes' Theorem as a tool to update probability distributions of hypotheses using existing knowledge and new observations.	**A medical test example** Using Bayes' theorem to calculate the probability of obtaining a *true* positive COVID RAT result when tested on a randomly selected individual
Bayes factor as a tool for comparing two competing statistical models for a given dataset	**A model selection example** Using the Bayes factor to determine which of two velocity probability distributions, modelled by the Maxwell-Boltzmann (MB) equation, is more strongly supported by a given dataset
Markov Chain Monte Carlo (MCMC) methods as a numerical sampling tool	Students write a **Metropolis-Hastings sampling algorithm** in the Python programming language to estimate the parameter of a model for a given dataset



(Left) Students are provided with an arbitrary dataset containing a range of velocities, v.

(Right) Assuming dataset can be modelled by the MB equation, parameterised by the peak velocity (v_p). Students write their own Metropolis-Hastings algorithm in Python to sample the posterior distribution of v_p.



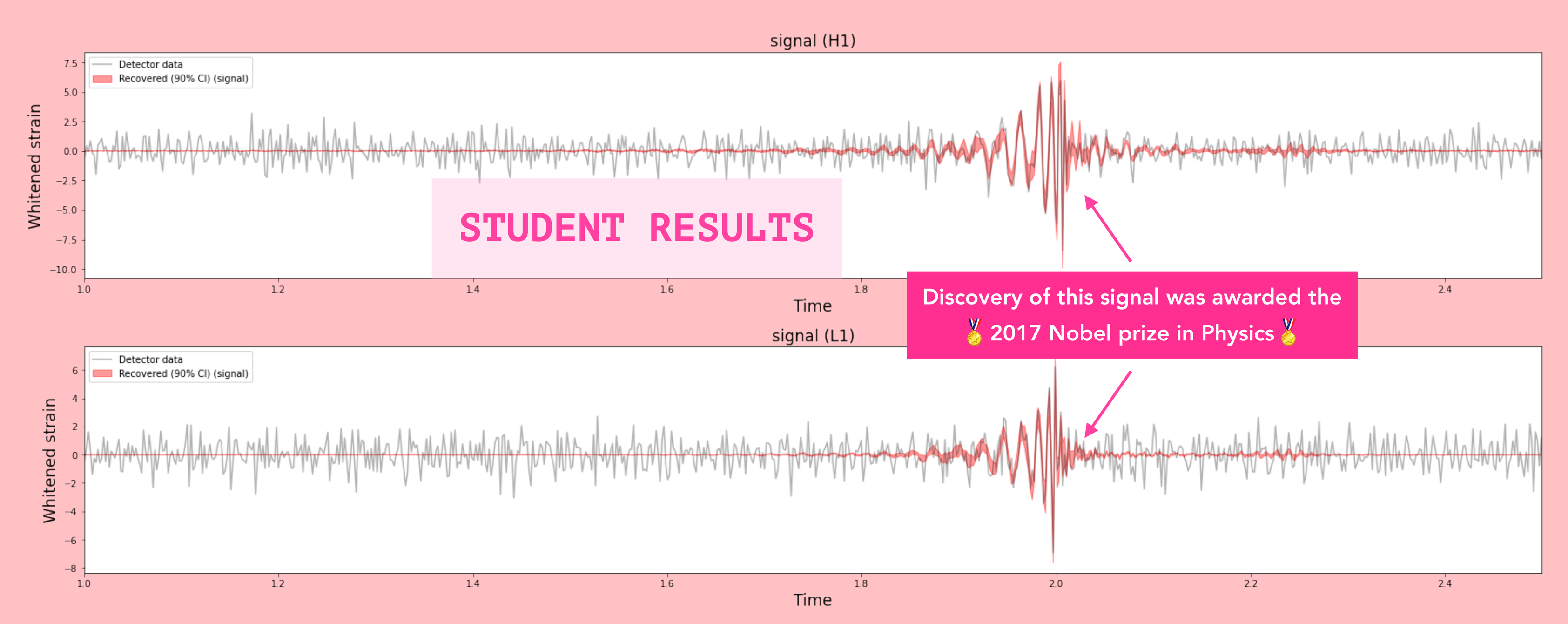
GRAVITATIONAL-WAVE DATA ANALYSIS

With the foundations of Bayesian Inference in hand, students now work with the data of a Bayesian GW detection algorithm called *BayesWave* [1]. *BayesWave* is one of the leading transient GW search algorithm in the LIGO-Virgo-KAGRA collaboration. The aim of this exercise is to give students the experience of managing, analysing and presenting large data sets. Students choose between one of the two following activities.

WAVEFORM RECOVERY

This exercise simulates the experience of detecting a GW while introducing the concept of credible intervals (CI) in Bayesian statistics.

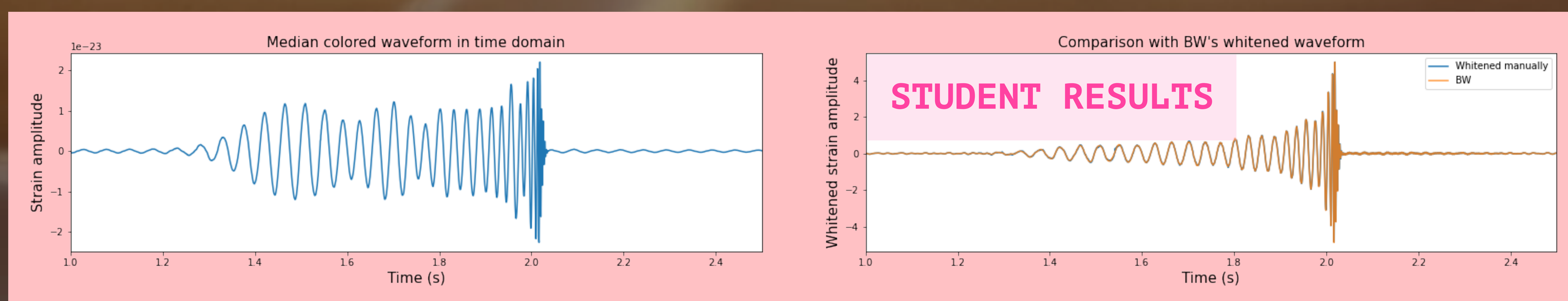
Students extract relevant output from the *BayesWave* analysis to visualise GW signals. The plot below shows: (i) the detector data (grey) containing the signal of GW150914, the first ever GW detection in 2015 and (ii) the 90% credible interval of *BayesWave*'s representation of the signal (red).



SIGNAL PROCESSING

This exercise demonstrates noise whitening, a signal processing procedure applicable beyond GW searches.

Noise power in GW detectors fluctuates across the frequency spectrum i.e. noise is coloured. The whitening process equalises the noise power, so that excess signal power at any frequency becomes more obvious. The left panel below shows a coloured GW signal. The right panel shows the same signal whitened by a student (blue), compared to the signal whitened by *BayesWave* (orange).



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[1] N. Cornish and T. Littenberg, Classical Quantum Gravity 32, 135012 (2015).

E-mail†: ylee9@student.unimelb.edu.au Twitter: @AstroOrca