
$\tilde{X}$

$E[\widetilde{m_1}\widetilde{R_1}] = Cov(\widetilde{m_1}, \widetilde{R_1}) + E[\widetilde{m_1}]E[\widetilde{R_1}]$

$$\begin{aligned} Cov(\widetilde{x}, \widetilde{y}) &= E[(\widetilde{x} - \mu_x)(\widetilde{y} - \mu_y)] \\ &= E[\tilde{x}\tilde{y}] - \mu_x\mu_y \\ &= E[\tilde{x}(\tilde{y} - \mu_y)] \\ &= E[(\tilde{x} - \mu_x)\tilde{y}] \end{aligned}$$

.

$V_0 \widetilde{X_1} \widetilde{m_1}$

$$V_0 = E[\widetilde{m_1}\widetilde{X_1}]$$

m_1

$V_0 = 1 X_1 = R_f R_f$

$$1 = E[\widetilde{m_1} R_f] = R_f E[\widetilde{m_1}]$$

$E[\widetilde{m_1}] = \dfrac{1}{R_f}$

$\widetilde{R_1} = \dfrac{\widetilde{X_1}}{V_0}$

$$1 = E[\widetilde{m_1}\widetilde{R_1}] = Cov(\widetilde{m_1}, \widetilde{R_1}) + E[\widetilde{m_1}]E[\widetilde{R_1}]$$

$E[\widetilde{m_1}] = \frac{1}{R_f}$

$$E[\widetilde{R_1}] = R_f \left(1 - Cov(\widetilde{m_1}, \widetilde{R_1})\right) = R_f - R_f Cov(\widetilde{m_1}, \widetilde{R_1})$$

$-R_f Cov(\widetilde{m_1}\widetilde{R_1})$

$\widetilde{m_1}$

$$\widetilde{m_1} = a - b\widetilde{R_m}(b > 0)$$

$\widetilde{m_1}$

$$\begin{aligned} Cov(\widetilde{m_1}, \widetilde{R_j}) &= Cov(a - b\widetilde{R_m}, \widetilde{R_j}) \\ &= -bCov(\widetilde{R_m}, \widetilde{R_j}) \end{aligned}$$

$$E[\widetilde{R_j}] = R_f - R_f Cov(\widetilde{m_1}, \widetilde{R_j})$$

$$E[\widetilde{R_m}] = R_f - R_f Cov(\widetilde{m_1}, \widetilde{R_m})$$

$$\frac{E[\widetilde{R_j}] - R_f}{E[\widetilde{R_m}] - R_f} = \frac{Cov(\widetilde{m_1}, \widetilde{R_j})}{Cov(\widetilde{m_1}, \widetilde{R_m})} = \frac{-bCov(\widetilde{R_m}, \widetilde{R_j})}{-bCov(\widetilde{R_m}, \widetilde{R_m})} = \frac{Cov(\widetilde{R_m}, \widetilde{R_j})}{Var[\widetilde{R_m}]} = \beta_j$$

$$E[\widetilde{R_j}] - R_f = \beta_j(E[\widetilde{R_m}] - R_f)$$

$W_0 C_0 W_0 - C_0 (W_0 - C_0)\widetilde{R_p} = \widetilde{C_1} U(C_0, \widetilde{C_1}) = U(C_0) + E[U(\widetilde{C_1})] max U(C_0) + E[U(\widetilde{C_1})]$

$E[U(\widetilde{C_1})] = E[U(\sum_{j=0}^{J} w_j \widetilde{R_j})]$subject to $\sum_{j=0}^{J} w_j = 1$

$\mathscr{L} \equiv u(C_0) + E[u(\displaystyle\sum_{j=0}^{J} w_j \widetilde{R_j})] + \lambda(1 - \sum_{j=0}^{J} w_j)$

$$\frac{\partial \mathscr{L}}{\partial C_0} = 0$$

$$\frac{\partial \mathscr{L}}{\partial w_j} = 0$$

$\dfrac{\partial \mathscr{L}}{\partial C_0} = u'(C_0) + E[u'(\widetilde{C_1})(-\displaystyle\sum_{j=0}^{J} w_j \widetilde{R_j})] = 0$

$u'(C_0) = E[u'(\widetilde{C_1})(\displaystyle\sum_{j=0}^{J} w_j \widetilde{R_j})]$

$\dfrac{\partial \mathscr{L}}{\partial w_j} = 0 + E[u'(\widetilde{C_1})(w_0 - C_0)\widetilde{R_j}] - \lambda = 0$

$E[u'(\widetilde{C_1})\widetilde{R_j}] = \dfrac{\lambda}{w_0 - C_0}$

$\dfrac{\partial \mathscr{L}}{\partial \lambda} = 1 - \displaystyle\sum_{j=0}^{J} w_j$

$E[u'(\widetilde{C_1}) \displaystyle\sum_{j=0}^{J} w_j \widetilde{R_j}] = \dfrac{\lambda}{w_0 - C_0} \displaystyle\sum_{j=0}^{J} w_j = \dfrac{\lambda}{w_0 - C_0}$

$$u'(C_0) = E[u'(\widetilde{C_1})(\sum_{j=0}^{J} w_j \widetilde{R_j})]$$