F - test ⟷ t-test

	df	SS	MS	F	p-val.
Reg	k	$RSS_1 - RSS_2$ (H_0)	$\frac{RSS_1 - RSS_2}{k}$	$\boxed{\frac{MS}{MS}}$ ⫫	p_0.
Res	$n-p-k$	RSS_2 (H_1)	$\frac{RSS_2}{n-p-k}$	$F_{k,n-p-k}$.	$\boxed{\alpha}$.
Tot	$\boxed{n-p.}$	RSS_1.			

H_0: p columns in X. ⟷ H_1: $(p+k)$ columns in X.

$p-1$ EVs + 1 int.

$(n-p)$.

$Y = X\beta + e$.

$H_0: Y = X_1\beta_1 + e$. ⟷ $H_1: Y = X_2\beta_2 + e$.

$$C(X_1) \subseteq C(X_2).$$

$H_1 = X_1(X_1'X_1)^{-1}X_1'$ $\quad H_2 = X_2(X_2'X_2)^{-1}X_2'$. ✓.

$\frac{RSS_1 - RSS_2}{RSS_2}$. ⟵

$$RSS_1 = Y'(I-H_1)Y.$$

$$RSS_2 = Y'(I-H_2)Y.$$

$$RSS_1 - RSS_2 = Y'(I-H_1)Y - Y'(I-H_2)Y$$

$$= Y'(H_2-H_1)Y. \quad \text{large?}$$

$$RSS_2 = Y'(I-H_2)Y.$$

$$RSS_1 \overset{(>)}{\geq} RSS_2. \qquad C(X_1) = C(X_2).$$

$H_2 - H_1 > 0$ positive definite.

projection operator

$$\begin{cases} H^2 = H. \\ H^T = H. \end{cases}$$

$$Y'(H_2-H_1)Y$$

$$= Y'(H_2-H_1)'(H_2-H_1)Y.$$

$$\geq 0.$$

$(H_2-H_1)^T = H_2 - H_1.$ $\quad C(H_1) \subseteq C(H_2).$ $\quad H_1 = H_2 A.$

$$(H_2-H_1)^2 = H_2^2 + H_1^2 - \underbrace{H_2H_1} - H_1H_2.$$
$$= H_2 + H_1 - \underbrace{H_2H_2}_{H_2}A - \underbrace{H_1H_2}.$$
$$= H_2 + H_1 - H_1 - H_1^T$$
$$= H_2 - H_1.$$

$$\boxed{(H_1H_2)^T = H_2H_1 = H_1.}$$

$$\boxed{H_2H_1 = H_1H_2 = H_1.}$$

$C(H_1) \subseteq C(H_2).$

$\Rightarrow$ $H_2 - H_1.$ perpendicular projection matrix.

$\Leftrightarrow$ ① $(H_2-H_1)^2 = H_2 - H_1$ idempotent

② $(H_2-H_1)^T = H_2 - H_1$ symmetric.

Under normality assumption.

$Y'AY \perp\!\!\!\perp Y'BY$ $\quad AB = 0.$

$C(H_1) \subseteq C(H_2)$

$\underline{RSS_1 - RSS_2 \perp\!\!\!\perp RSS_2.}$

$$\underbrace{(I-H_2)}\underbrace{(H_2-H_1)} = 0.$$
$$= H_2 - H_1 - H_2^2 + H_2H_1$$
$$= H_2 - H_1 - H_2 + H_1$$
$$= 0.$$

$$\underline{R^2} = \frac{SS_{reg}}{SYY} = \frac{Y'(H-\frac{1}{n}J)Y}{\Sigma(y_i-\bar{y})^2}.$$

measure how good a model is.

H_1: existing model $\Leftrightarrow$ $\underline{H_0: Y = lm(y \sim x - 1).}$

baseline.

$\uparrow$

R^2

$$\frac{Y'(H-\frac{1}{n}J)Y}{Y'(I-\frac{1}{n}J)Y} \leftarrow Y'(\tfrac{1}{n}J)Y \geq 0 \quad \longrightarrow \quad \frac{Y'(H)Y}{Y'IY.}$$

$\leq$

$R^2 \xleftrightarrow{\quad} R^2$

$\sum_{i=1}^{n} \hat{e}_i = 0.$ $\qquad J_n'(y_i - \hat{y}_i).$ $\qquad J_n = (1, 1, \cdots, 1).$

$$\vec{y} - \hat{y} = y - Hy = (I-H)y.$$

$$J_n'(y-\hat{y}) = \underbrace{J_n'(I-H)}y \overset{?}{=} 0. \checkmark.$$

$J_n' I = J_n'.$ $\qquad J_n' H \overset{?}{=} J_n'. \checkmark.$

$H J_n \overset{?}{=} J_n \quad \checkmark.$

$H = X(X'X)^{-1}X'.$ $\qquad X = \begin{pmatrix} \boxed{J_n} & \cdots \end{pmatrix}$

$H J_n = J_n.$

$$\underbrace{SYY = RSS + SS_{reg}}.$$

$$SYY = \sum_{i=1}^{n} (y_i - \bar{y})^2 \overset{\downarrow}{=} Y^T(I - \tfrac{1}{n}J_{n\times n})Y.$$

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \overset{\downarrow}{=} Y^T(I-H)Y.$$

$$SS_{reg} = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \overset{\downarrow}{=} Y^T(H - \tfrac{1}{n}\underbrace{J_{n\times n}})Y.$$

$$J = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & & 1 \\ \vdots & & & & \vdots \\ 1 & & \cdots & & 1 \end{pmatrix}_{n\times n}.$$

$J_n = (\underbrace{1, \cdots, 1}_{n})'$ $\qquad J_n = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}_{n\times n}.$

$J_{n\times n}.$