CISS358: Algorithm Analysis
Assignment 3

Objectives

- Review of proof using induction and WOP.
- Prove the correctness of an algorithm using loop invariants.

Q1. Recall from class, I mentioned that there are some proofs that you must know because these proofs almost form a template for many other proofs. The goal of this question is not to design a proof, but rather to re-study a given proof very carefully and write an absolutely correct proof in your own words and possible filling gaps if necessary.

Prove the following using WOP.

Euclidean property of $\mathbb{N}$. If $a, b \in \mathbb{N} = \{0, 1, 2, ...\}$ with $b \leq 0$, then there exist q and r in $\mathbb{N}$ such that

$$a = bq + r, \;\; 0 \leq r < b$$

(More is true: q, r are unique. The proof is not difficult and if probably proven in discrete 1. I definitely talk about it in 451.)

SOLUTION.

Q2. Prove using invariants and induction that the following algorithm (bottom-up recursion for exponentiation) is correct.

```
ALGORITHM: power
INPUTS: x - base
        e - exponent
OUTPUT: x^e, i.e., x raised to power of e

ret = 1
while e > 0:
    if e % 2 == 1:
        ret = ret * x
    e = e / 2
    x = x * x

return ret
```

(Note: `/` is integer division.)

SOLUTION.

Q3. Prove that the selection sort algorithm is correct using invariants and induction.

SOLUTION.