CISS240: Introduction to Programming
Assignment 12

Objectives

1. Write functions
2. Make function calls
3. Break down a program into functions

The purpose of this assignment is to write functions. You will also see how problems are broken down into subproblems (or subgoals). You have seen this before in the context of breaking down a programarm into different goals within the code in `main()`. The difference now is that we break down the program into pieces and then create functions for the pieces instead of keeping these pieces inside `main()`.

The first problem (Q1 and Q2) involves writing a simple tool to analyze the representation of primes using polynomials. A basic function is developed in Q1 so that it can be used in Q2. The function checks if an integer is prime.

Questions Q3–Q5 involve the rewrite of our calendar month printing program. Q3 involves writing a function that tells us if a year is a leap year. The second function (in Q4) tells us how many days there are in a given month and year. Q4 obviously must use Q3 since the days in a month for the case of February depends on whether the year is a leap year or not. Finally, Q5 uses the functions in Q3–Q4 to print the calendar itself.

Q6 involves writing several functions to analyze the famous unsolved "3x+1" problem.

There are several short functions to be implemented in Q7. The functions are then used to compute an approximation to the value of π (which we know is roughly 3.14159...) using a technique called Monte–Carlo simulation.

Q1. [NOT GRADED – SEE NOTES]

First write a function `isprime()` such that `isprime(n)` returns `true` when `n` is prime and `false` when `n` is not a prime. Here's the skeleton code which you MUST use. Several test cases are included. Note that the integers 0 and 1 are NOT primes. Negative integers are also not primes.

```
#include <iostream>


bool isprime(int n)
{
    ... your code here ...
}


int main()
{
    int i;
    std::cin >> i;

    std::cout << i << (isprime(i) ? " is prime" : " is not prime")
              << std::endl;

    return 0;
}
```

Test 1.

```
-3
-3 is not prime
```

Test 2.

```
0
0 is not prime
```

Test 3.

```
1
1 is not prime
```

Test 4.

```
2
2 is prime
```

Test 5.

```
3
3 is prime
```

Test 6.

```
12
12 is not prime
```

Test 7.

```
23
23 is prime
```

Q2. For this question, you should use the `isprime()` function from $Q1$.

This question involves polynomials that generate primes. Consider the following polynomial:

$$p(x) = x^2 + 2x + 3$$

Note that

$$\begin{aligned} p(0) &= 0 * 0 + 2 * 0 + 3 = 3, \text{which is a prime} \\ p(1) &= 1 * 1 + 2 * 1 + 3 = 6, \text{which is not a prime} \\ p(2) &= 2 * 2 + 2 * 2 + 3 = 11, \text{which is a prime} \end{aligned}$$

Therefore this $p(x)$, when x ranges from 0 to 2 (inclusive), generates 2 primes, i.e., 3 and 11.

Write a program that accepts A, B, C, N and then analyzes all the polynomials

$$p(x) = a\,x^2 + b\,x + c$$

for a ranging from 1 to A (inclusive), b ranging from 1 to B (inclusive), c ranging from 1 to C (inclusive) and reporting the number of x ranging from 0 to N (inclusive) such that $p(x)$ is prime.

Finally, print the polynomial that represents the most primes. If there's a tie, report the one that is smallest according to "dictionary order", i.e., the one that appears first.

Note that all integers are printed with `std::setw(5)`.

[An experiment for fun: Is there a polynomial $p(x)$ that represents primes for **all** $x = 0, 1, 2, 3, \ldots$? In general, suppose that the prime length of a polynomial $p(x)$ is the number n such that $p(0), p(1), p(2), \ldots, p(n-1)$ are all primes. What is the largest prime length you can find and what is the polynomial?]

Test 1

```
1 1 1 5
    1x^2 +    1x +    1:    4
largest
    1x^2 +    1x +    1:    4
```

In this case $A = 1$, $B = 1$, $C = 1$, $N = 5$.

Therefore, we're only analyzing a single polynomial, i.e., $x^2 + x + 1$ for $x = 0, 1, 2, 3, 4, 5$.

Here is the analysis by hand:

x	$x^2 + x + 1$

0	$0^2 + 0 + 1 = 1$	not prime
1	$1^2 + 1 + 1 = 3$	prime
2	$2^2 + 2 + 1 = 7$	prime
3	$3^2 + 3 + 1 = 13$	prime
4	$4^2 + 4 + 1 = 21$	not prime
5	$5^2 + 5 + 1 = 31$	prime

Hence, there are 4 primes represented by this polynomial for x running from 0 to 5.

Test 1.

```
2 2 2 10
    1x^2 +     1x +     1:     6
    1x^2 +     1x +     2:     1
    1x^2 +     2x +     1:     0
    1x^2 +     2x +     2:     5
    2x^2 +     1x +     1:     5
    2x^2 +     1x +     2:     5
    2x^2 +     2x +     1:     6
    2x^2 +     2x +     2:     1
largest
    1x^2 +     1x +     1:     6
```

In this case, note that there is a tie: there are two polynomials that achieve 6 primes for x running from 0 to 10. The earlier one is reported as the "winner".

Test 2.

```
10 10 10 100
    1x^2 +     1x +     1:    32
    1x^2 +     1x +     2:     1
    1x^2 +     1x +     3:    15
    1x^2 +     1x +     4:     0
    1x^2 +     1x +     5:    30
    1x^2 +     1x +     6:     0
    1x^2 +     1x +     7:    32
    1x^2 +     1x +     8:     0
    1x^2 +     1x +     9:    13
    1x^2 +     1x +    10:     0
    1x^2 +     2x +     1:     0
    1x^2 +     2x +     2:    19
    1x^2 +     2x +     3:    11
    1x^2 +     2x +     4:    17
    1x^2 +     2x +     5:    21
    1x^2 +     2x +     6:     7
... output not shown ...
   10x^2 +    10x +     5:     1
   10x^2 +    10x +     6:     0
   10x^2 +    10x +     7:    27
   10x^2 +    10x +     8:     0
   10x^2 +    10x +     9:    11
   10x^2 +    10x +    10:     0
largest
    8x^2 +    10x +     1:    53
```

The output of this test case is too long to be shown in its entirety. Part of it is snipped. Note that, in this case, the maximum density of primes represented is 53/101, about 52.4%. Can we find a better polynomial to represent primes for $x = 0, \ldots, 100$ if we expand our search?

Test 3.

```
20 20 20 100
... output not shown ...
largest
    7x^2 +     7x +    17:    70
```

Now we have achieved a density of 70/101, almost 70%.

Test 4.

```
30 30 30 100
... output not shown ...
largest
    1x^2 +    23x +    23:    75
```

The maximum density is about 75%.

Test 5.

```
50 50 50 100
... output not shown ...
largest
    1x^2 +     1x +    41:    87
```

Still improving!

Test 6.

```
60 60 60 100
... output not shown ...
largest
    1x^2 +     1x +    41:    87
```

Oops . . . we still have the same polynomial.

Test 7.

```
70 70 70 100
... output not shown ...
largest
    1x^2 +     1x +    41:    87
```

Wait . . . what's happening??? Expanding our brute force search doesn't come up with a better polynomial . . . hmmm . . .

Test 8.

```
80 80 80 100
... output not shown ...
largest
    1x^2 +     1x +    41:    87
```

Is this a coincidence???

Test 9.

```
100 100 100 100
... output not shown ...
largest
    1x^2 +     1x +    41:    87
```

What!!! Is there something special about $x^2 + x + 41$?!?

Q3. Write a function `is_leap_year()` such that `is_leap_year(year)` returns `true` exactly when the `year` (an `int`) is a leap year. You must use the skeleton code given below.

```
#include <iostream>


//------------------------------------------------------------------------
// This function returns true if year is a leap year. Otherwise false is
// returned.
// (This function has no output.)
//------------------------------------------------------------------------
bool is_leap_year(int year)
{
    ... your code here ...
}


int main()
{
    int y;
    std::cin >> y;
    std::cout << is_leap_year(y) << std::endl;

    return 0;
}
```

For your reference, here's a relevant program from a previous assignment:

```
#include <iostream>

int main()
{
    int year = 0;
    std::cin >> year;

    bool is_leap_year = ((year % 4 == 0)
                         && (year % 100 != 0
                             || (year % 100 == 0 && year % 400 == 0)
                            )
                        );

    if (is_leap_year)
    {
        std::cout << "leap year" << std::endl;
    }
    else
```

```
    {
        std::cout << "not leap year" << std::endl;
    }

    return 0;
}
```

Make sure you test your program thoroughly.

Test 1.

```
1991
0
```

Test 2.

```
2004
1
```

Test 3.

```
1200
1
```

Test 4.

```
1900
0
```

Q4. Write a function `days_in_month()` such that `days_in_month(month, year)` returns the number of days in the given `month` (an `int`) of the given `year` (an `int`). (See *a*06.)

You must use the skeleton code given below. Note that the test cases are included in the code.

You will need the function from Q3. Therefore you should copy-and-paste the function from Q3 to the code for this question.

```
#include <iostream>


//--------------------------------------------------------------------------
// This function returns true if year is a leap year. Otherwise false is
// returned.
// (This function has no output.)
//--------------------------------------------------------------------------
bool is_leap_year(int year)
{
    ... your code here ...
}


//--------------------------------------------------------------------------
// This function returns the number of days in given month and year.
// (This function has no output.)
//--------------------------------------------------------------------------
int days_in_month(int month, int year)
{
    ... your code here ...
}


int main()
{
    int m, y;
    std::cin >> m >> y;
    std::cout << days_in_month(m, y) << std::endl;

    return 0;
}
```

For your reference, here's a relevant program from a previous assignment:

```
#include <iostream>

int main()
{
```

```
int yyyymmdd = 0;
std::cin >> yyyymmdd;

int yyyy = yyyymmdd / 10000;     // year
int mm = yyyymmdd / 100 % 100;   // month
int dd = yyyymmdd % 100;         // day-of-month

bool valid = false; // true exactly when yyyymmdd is a valid date

if (1 <= mm && mm <= 12)
{
    if (mm == 2)
    {
        //-------------------------------------------------------------
        // CASE: February.
        // There are 29 or 28 days depending on whether the year
        // is a leap year or not.
        //-------------------------------------------------------------
        if (yyyy % 4 == 0
            && ((yyyy % 100 != 0) || (yyyy % 100 == 0)
                && (yyyy % 400 == 0)))
        {
            valid = (1 <= dd && dd <= 29);
        }
        else
        {
            valid = (1 <= dd && dd <= 28);
        }
    }
    else if ((mm % 2 == 1 && mm <= 7) || (mm % 2 == 0 && mm >= 8))
    {
        //-------------------------------------------------------------
        // CASE: 31 days in months 1,3,5,7,8,10,12
        //-------------------------------------------------------------
        valid = (1 <= dd && dd <= 31);
    }
    else
    {
        //-------------------------------------------------------------
        // CASE: 30 days in months 4,6,9,11
        //-------------------------------------------------------------
        valid = (1 <= dd && dd <= 30);
```

```
        }
    }

    if (valid)
    {
        std::cout << "correct" << std::endl;
    }
    else
    {
        std::cout << "incorrect" << std::endl;
    }

    return 0;
}
```

Make sure you test your program thoroughly.

Test 1.

```
2 2004
29
```

Test 2.

```
2 2003
28
```

Test 3.

```
5 2003
31
```

Test 4.

```
2 1900
28
```

Test 5.

```
9 2008
30
```

Test 6.

```
2 1200
29
```

Q5. Recall that we had a previous problem that involved printing a calendar month. Here's the problem:

Write the following program that prints a calendar month. The program prompts the user for three integers: the month, the year, and the day–of–week for the first day of the month (with 0 representing Sunday, 1 representing Monday, etc.) For instance, if the user entered

```
3 2008 6
```

It means that he/she wants the calendar for March 2008 and the first day of the month is a Saturday. The output is

```
March 2008
--------------------
Su Mo Tu We Th Fr Sa
                    1
 2  3  4  5  6  7  8
 9 10 11 12 13 14 15
16 17 18 19 20 21 22
23 24 25 26 27 28 29
30 31
```

Rewrite the calendar month printing program so that it uses the two functions `is_leap_year()` and `days_in_month()` from the previous questions. You must use the skeleton code given below.

Note that we are not changing how the original program works. The point is to "factor" out useful code from our original program. This makes the program more readable – it will be very obvious once you're done with this question. Also, the functions factored out can be used for other programs.

```
#include <iostream>


//------------------------------------------------------------------------
// This function returns true if year is a leap year. Otherwise false is
// returned.
// (This function has no output.)
//------------------------------------------------------------------------
bool is_leap_year(int year)
{
    ... your code here ...
}
```

```
//------------------------------------------------------------------------
// This function returns the number of days in given month and year.
// (This function has no output.)
//------------------------------------------------------------------------
int days_in_month(int month, int year)
{
    ... your code here ...
}


//------------------------------------------------------------------------
// This function prints the calendar of the given month and year.
// The parameter day_of_first is the day of the 1st of the given month
// and year.
//------------------------------------------------------------------------
void print_calendar_month(int month, int year, int day_of_first)
{
    ... your code here ...
}


int main()
{
    int month, year, day_of_first;
    std::cin >> month >> year >> day_of_first;

    print_calendar_month(month, year, day_of_first);

    return 0;
}
```

For your reference, here's the relevant program from a previous assignment:

```
#include <iostream>
#include <iomanip>
#include <cmath>

int main()
{
    int month, year, day_of_first;
    std::cin >> month >> year >> day_of_first;
```

```
    switch (month)
    {
        case 1: std::cout << "January "; break;
        case 2: std::cout << "February "; break;
        case 3: std::cout << "March "; break;
        case 4: std::cout << "April "; break;
        case 5: std::cout << "May "; break;
        case 6: std::cout << "June "; break;
        case 7: std::cout << "July "; break;
        case 8: std::cout << "August "; break;
        case 9: std::cout << "September "; break;
        case 10: std::cout << "October "; break;
        case 11: std::cout << "November "; break;
        case 12: std::cout << "December "; break;
    }

    std::cout << year << '\n'
              << "--------------------\n"
              << "Su Mo Tu We Th Fr Sa\n";


    // Compute the number of days in the month and store in days_in_month
    int days_in_month = 0;
    if (1 <= month and month <= 12)
    {
        if (month == 2) // 28 or 29 days
        {
            if (year % 4 == 0
                && ((year % 100 != 0) || (year % 100 == 0)
                    && (year % 400 == 0)
                    )
                )
            {
                days_in_month = 29;
            }
            else
            {
                days_in_month = 28;
            }
        }
        else if ((month % 2 == 1 && month <= 7) ||
                 (month % 2 == 0 && month >= 8)) // 31 days
        {
            days_in_month = 31;
        }
```

```
        else // 30 days
        {
            days_in_month = 30;
        }
    }

    // Print spaces before the first day of the month
    for (int i = 0; i < day_of_first; i++)
    {
        std::cout << "   ";
    }

    int day_of_week = day_of_first;
    for (int day_in_month = 1; day_in_month <= days_in_month; day_in_month++)
    {
        std::cout << std::setw(2) << day_in_month << ' ';

        day_of_week++;
        if (day_of_week == 7)
        {
            std::cout << std::endl;
            day_of_week = 0;
        }
    }

    std::cout << std::endl;

    return 0;
}
```

Make sure you test your program thoroughly.

Test 1.

```
3 2008 6
March 2008
--------------------
Su Mo Tu We Th Fr Sa
                    1
 2  3  4  5  6  7  8
 9 10 11 12 13 14 15
16 17 18 19 20 21 22
23 24 25 26 27 28 29
30 31
```

Test 2.

```
3 2008 4
March 2008
--------------------
Su Mo Tu We Th Fr Sa
                1  2  3
 4  5  6  7  8  9 10
11 12 13 14 15 16 17
18 19 20 21 22 23 24
25 26 27 28 29 30 31
```

Test 3.

```
2 2008 5
February 2008
--------------------
Su Mo Tu We Th Fr Sa
                   1  2
 3  4  5  6  7  8  9
10 11 12 13 14 15 16
17 18 19 20 21 22 23
24 25 26 27 28 29
```

Test 4.

```
2 2009 0
February 2009
--------------------
Su Mo Tu We Th Fr Sa
 1  2  3  4  5  6  7
 8  9 10 11 12 13 14
15 16 17 18 19 20 21
22 23 24 25 26 27 28
```

Q6. The following problem is called the "3x+1" problem. Consider the following function T:

$$\text{if } n \text{ is odd, } T(n) = 3n + 1$$
$$\text{if } n \text{ is even, } T(n) = n/2$$

Another way to write this (mathematically) is:

$$T(n) = \begin{cases} 3n + 1, & \text{if } n \text{ is odd} \\ n/2, & \text{if } n \text{ is even} \end{cases}$$

Note that there $T(n)$ is made up of two formulas; you need to choose the right one. Here are some examples:

$T(42) = 42/2 = 21$	(you choose the $n/2$ formula since $n = 42$ is even)
$T(5) = 3(5) + 1 = 16$	(you choose the $3n + 1$ formula since $n = 5$ is odd)

Note that you can use the "output" of $T(42)$ which is 21 and apply T to it again: $T(21)$.

$$T(42) = 21$$
$$T(21) = 3(21) + 1 = 64$$

and yet again:

$$T(64) = 64/2 = 32$$

Of course you can do this as many times as you like, using outputs as inputs:

$T(42) = 42/2 = 21$	because 42 is even
$T(21) = 3(21) + 1 = 64$	because 21 is odd
$T(64) = 64/2 = 32$	because 64 is even
$T(32) = 32/2 = 16$	because 32 is even
$T(16) = 8$	because 16 is even
$T(8) = 4$	because 8 is even
$T(4) = 2$	because 4 is even
$T(2) = 1$	because 2 is even

OK . . . that's enough . . . let's stop at 1. Altogether we applied T eight times and in doing

that we got a sequence of numbers:

$$42, 21, 64, 32, 16, 8, 4, 2, 1$$

The "3x+1" problem/conjecture states that if you apply enough T to a given strictly positive integer n, you will always get 1.

Well, this is definitely the case for 42. But is this true in general for all strictly positive n?

The "3x+1" problem is still an open problem: No one has proven it yet. (By the way, there is a £1000 reward for proving it ... in case you need some money.) This problem is an example of a dynamical system.

Write a program that prompts the user for two integers and verifies the conjecture for all integers in the range described by the two integers by printing the sequences for each integer until it reaches 1. (See explanation in Test 1.)

The following skeleton must be used.

```
#include <iostream>

//------------------------------------------------------------------------
// The function T returns T(n) where:
//   if n is odd, T(n) = 3 * n + 1
//   if n is even, T(n) = n / 2 (integer division)
// (This function has no output.)
//------------------------------------------------------------------------
int T(int n)
{
    ... your code here ...
}


//------------------------------------------------------------------------
// The function print_3x_plus_1(n) prints the sequence of integers on
// applying T until 1 is reached.
//
// Example: On calling print_3x_plus_1(3),
// 3 10 5 16 8 4 2 1
// is printed.
//------------------------------------------------------------------------
void print_3x_plus_1(int n)
```

```
{
    ... your code here ...
}


int main()
{
    int a, b;
    std::cin >> a >> b;

    for (int n = a; n <= b; ++n)
    {
        print_3x_plus_1(n);
    }

    return 0;
}
```

Test 1.

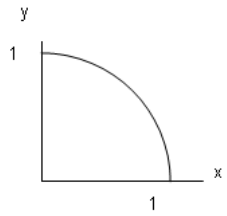
```
1 10
1
2 1
3 10 5 16 8 4 2 1
4 2 1
5 16 8 4 2 1
6 3 10 5 16 8 4 2 1
7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
8 4 2 1
9 28 14 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
10 5 16 8 4 2 1
```

As you can see, the user enters 1 and 10 and the program runs n from 1 to 10 and for each n, continually applies T until 1 is reached. For instance, when n is 6, continually applying T gives the sequence $6, 3, 10, 5, 16, 8, 4, 2, 1$. In every case, the sequence does ultimately reach 1.

Test 2.

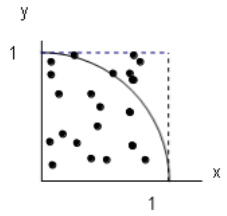
```
11 20
11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
12 6 3 10 5 16 8 4 2 1
13 40 20 10 5 16 8 4 2 1
14 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
15 46 23 70 35 106 53 160 80 40 20 10 5 16 8 4 2 1
16 8 4 2 1
17 52 26 13 40 20 10 5 16 8 4 2 1
18 9 28 14 7 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
19 58 29 88 44 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
20 10 5 16 8 4 2 1
```

Q7. The goal here is to compute an approximation of the value of π using the "Monte–Carlo" method. The Monte–Carlo method is a very famous approximation method for computing numbers and has many important applications in simulation, engineering sciences, game theory, etc. The idea is very simple. Look at the following:



This is the quarter circle of radius 1. The area of a full circle of radius r is πr^2. For the circle of radius 1, the area would be $\pi 1^2$ which is π. So our quarter circle would have an area of $\pi/4$. (Here, the / refers to mathematical division and not integer division.)

Suppose we generate a lot of random points in the square with corners $(0,0), (0,1), (1,0), (1,1)$:



and consider the total number of random points and the number of points that hit the interior of the quarter circle. Then clearly

$$\frac{\text{Number of points in quarter circle}}{\text{Total number of points}} \approx \frac{\text{Area of quarter circle}}{\text{Area of square}}$$

($\approx$ means "approximately".) But this is just

$$\frac{\text{Number of points in quarter circle}}{\text{Total number of points}} \approx \frac{\pi/4}{1}$$

which gives

$$\pi \approx \frac{4 * (\text{Number of points in quarter circle})}{\text{Total number of points}}$$

[ASIDE: In general, the Monte–Carlo method can be used to approximate areas. This is not the only method used to approximate values. In fact, the Monte–Carlo method is usually very slow in the sense that you need to generate many random events – which in this case is the generation of random points in the square – before you get a good approximation. Using

the power series method one can get an extremely good approximation of π with only 100 iterations.]

Complete the following skeleton code. You should not add any code to `main()`.

```
//-------------------------------------------------------------------------
// Prompts and returns the number of points entered by the user
//-------------------------------------------------------------------------
int getNumPoints()
{
}


//-------------------------------------------------------------------------
// randunit() returns a random double from 0.0 to 1.0
//-------------------------------------------------------------------------
double randunit()
{
}


//-------------------------------------------------------------------------
// pointIsInCircle(x, y) returns true if the point (x,y) is in the quarter
// circle
//-------------------------------------------------------------------------
bool pointIsInCircle(double x, double y)
{
}


//-------------------------------------------------------------------------
// printRunningApproximation(i, inCircle) prints for instance:
//
// 1000000         785219          3.1408760000
//
// (See Test 1)
// if i is 1000000 and inCircle is 78219. The third column is the
// approximation of pi with the number of points generated at this point
// (i.e., i) and the number of points in the quarter circle (i.e.
// inCircle)
//-------------------------------------------------------------------------
void printRunningApproximation(int i, int inCircle)
```

```
{
}


//------------------------------------------------------------------------
// printSummary(n, inCircle) prints for instance:
//
// The Pi-minator strikes again ...
// Final approximation of pi: 3.1415482200
// Number of points generated: 1000000000
//
// (See Test 1) if n is 1000000000 and inCircle is 785387055.
//------------------------------------------------------------------------
void printSummary(int n, int inCircle)
{
}


int main()
{
    srand((unsigned int) 0);
    std::cout << std::fixed << std::setprecision(10);

    int n = getNumPoints();

    int inCircle = 0;  // Counts the number of random points in
                       // the quarter circle
    for (int i = 1; i <= n; ++i)
    {
        // Generate random point (x,y) in the square
        double x = randunit();
        double y = randunit();

        // Increment inCircle if (x,y) is in the quarter circle
        if (pointIsInCircle(x, y))
        {
            ++inCircle;
        }

        printRunningApproximation(i, inCircle);
    }
```

```
    printSummary(n, inCircle);

    return 0;
}
```

Refer to **Test 1** for the output. Some of the output is not shown since the output is very long. Note that the first two columns have width of 16. The numbers in the third column have 10 decimal places after the decimal point. Furthermore, note that the running approximation is printed only after every 1000000 points are generated. Note also that the `rand()` function in your compiler might work differently from mine. However, the approximations should still move toward the value of π.

Test 1. (Only part of the output is shown.)

```
Number of points to generate: 1000000000
1000000          785219           3.1408760000
2000000          1571143          3.1422860000
3000000          2356627          3.1421693333
4000000          3141318          3.1413180000
... output not shown ...
998000000        785230080        3.1415486297
999000000        785308538        3.1415483068
1000000000       785387055        3.1415482200
The Pi-minator strikes again ...
Final approximation of pi: 3.1415482200
Number of points generated: 1000000000
```