CISS350: Data Structures and Advanced Algorithms
Assignment 1

Name: ____

Objectives

1. Computation of big-O of the worst case runtime performance a given algorithm.
2. Computation of big-O of the best case runtime performance a given algorithm.
3. Computation of big-O of the average case runtime performance a given algorithm.

In your Fedora virtual machine, as `student` user, execute the following in your bash shell: `pip install scipy --user`.

For this assignment, you will need to modify q02.tex, q03.tex, etc. q01.tex has a complete solution for your reference.

Given an algorithm, $T_w(n)$ denotes the worst runtime, $T_a(n)$ the average runtime, and $T_b(n)$ the best runtime. $T(n)$ denotes the worst runtime of the algorithm.

Recall that

$$An^2 + Bn + C = O(n^2)$$

and

$$Bn + C = O(n)$$

and

$$C = O(1)$$

where A, B, and C are constants. In general

$$a_d n^d + a_{d-1} n^{n-1} + \cdots + a_0 = O(n^d)$$

where a_i's are constants. Of course it's also true that

$$3n^2 - 5n + 10 = O(n^3)$$

and

$$3n^2 - 5n + 10 = O(n^{3.5})$$

and in fact

$$3n^2 - 5n + 10 = O(n^k)$$

for any real number $k \geq 2$. But the practice is to provide the best (i.e. tightest) bound.

For grading purposes, you must follow the following instructions:

1. When assigning times t_i, start with t_1 (not t_0) and use $t_1, t_2, ...$ without skipping any index.
2. When writing down exact runtimes, list the term with the highest growth rate. For instance, do not write

$$T(n) = t_1 + (t_2 + t_6)n^{100} + t_3 n^{15.25}$$

   Instead, write this:

$$T(n) = (t_2 + t_6)n^{100} + t_3 n^{15.25} + t_1$$

3. When writing the constants for each term in the runtime, arrange the constants in ascending index values. For instance, do not write this:

$$T(n) = (t_3 + t_1 + t_2)n^{100} + (t_7 + t_2 + t_1)n^{15.25}$$

   Instead, write this:

$$T(n) = (t_1 + t_2 + t_3)n^{100} + (t_1 + t_2 + t_7)n^{15.25}$$

4. Write legibly and clearly. If there's any ambiguity, then I reserve the right to interpret what you are trying to convey. And I am usually very good at picking the wrong interpretation.
5. Good writing means clear and unambiguous writing. "$x = 42$" is clearer than "42". It does not even take that much time (nor waste ink) to write the former. If I see any ambiguity like the above, you will get a 0 for the whole question.
6. There's a big difference between "$x = 42$" and "x 42". No one would say a"John tall" when it should be "John is tall". If I see something similar to "x 42", you will get a 0 for whole question.
7. In general, if I see any obviously bad/improper mathematical writing, I will give you a zero.

Q1.

Solution provided.

This is a solved problem. All the big-O computations in this assignment are somewhat similar to this question. Therefore study the solution very carefully. It will help you present your solutions in LaTeX.

The goal is to compute the big-O of the runtime performance of the following algorithm. Here's the algorithm:

```
INPUT:  x - array of doubles
        n - size of x
OUTPUT: result is stored in z
ALGORITHM:

    for i = 0, 1, 2, ..., n - 1:
        z = z + x[i] * x[i]
```

This is rewritten with timings as follows:

```
INPUT:  x - array of doubles
        n - size of x
OUTPUT: result is stored in z
ALGORITHM:
         i = 0                   time t1
LOOP:    if i >= n:              time t2
             goto ENDLOOP        time t3
         z = z + x[i] * x[i]     time t4
         i = i + 1               time t5
         goto LOOP               time t6
ENDLOOP:
```

The only relevant timing of the statements are given.

(a) Write down $T(n)$ in terms of n and the t_1, t_2, t_3, ... You should write it as a polynomial of n from the highest degree term to the lowest.

(b) Write down the big-O of $T(n)$ as $O(n^k)$ where k is the smallest possible positive integer.

SOLUTION (a) The timings with the number of times a statement is executed is as follows:

```
         i = 0                    time t1    1
LOOP:    if i >= n:               time t2    n + 1
             goto ENDLOOP         time t3    1
         z = z + x[i] * x[i]      time t4    n
         i = i + 1                time t5    n
         goto LOOP                time t6    n
ENDLOOP:
```

Therefore

$$T(n) = (t_2 + t_4 + t_5 + t_6)n + (t_1 + t_2 + t_3)$$

(b) We have

$$T(n) = O(n)$$

□

Q2. The goal is to compute the big-O of the runtime performance of the following algorithm. Here's the algorithm:

```
INPUT:  x - array of doubles
        n - size of x
        a - double
        b - int
OUTPUT: result is stored in z
ALGORITHM:
    z = a + b * b

    for i = 0, 1, 2, ..., n-1:
        a = a + 1
        x[i] = a + z + b;
        z = z + 1

    for i = 0, 1, 2, ..., (n-1)/4:
        x[i] = x[i] * z

    z = z + n
```

This is rewritten with timings as follows:

```
INPUT:  x - array of doubles
        n - size of x
        a - double
        b - int
OUTPUT: z

ALGORITHM:
          z = a + b * b                     time t1

          i = 0                             time t2
LOOP1:    if i >= n:                        time t3
              goto ENDLOOP1                 time t4
          a = a + 1                         time t5
          x[i] = a + z + b;                 time t6
          z = z + 1                         time t7
          i = i + 1                         time t8
          goto LOOP1                        time t9
ENDLOOP1:
```

```
          i = 0                               time t10
LOOP2:    if i > (n - 1) / 4:                 time t11
              goto ENDLOOP2                   time t12
          x[i] = x[i] * z                     time t13
          i = i + 1                           time t14
          goto LOOP2                          time t15
ENDLOOP2:
          z = z + n                           time t16
```

(You can pretend that $n - 1$ is divisible by 4 so that $(n - 1)/4$ is an integer.)

(a) Write down $T(n)$ in terms of n and the t_1, t_2, t_3, ... You should write it as a polynomial of n from the highest degree term to the lowest.

(b) Write down the big-O of $T(n)$ as $O(n^k)$ where k is the smallest possible positive integer.

SOLUTION

(a) The timings with the number of times a statement is executed is as follows (complete it like the solution given earlier):

```
          z = a + b * b                        time t1

          i = 0                                time t2
LOOP1:    if i >= n:                           time t3
              goto ENDLOOP1                    time t4

          a = a + 1                            time t5
          x[i] = a + z + b;                    time t6
          z = z + 1                            time t7
          i = i + 1                            time t8
          goto LOOP1                           time t9
ENDLOOP1:

          i = 0                                time t10
LOOP2:    if i > (n - 1) / 4:                  time t11
              goto ENDLOOP2                    time t12
          x[i] = x[i] * z                      time t13
          i = i + 1                            time t14
          goto LOOP2                           time t15
ENDLOOP2:
          z = z + n                            time t16
```

SOLUTION. `a04q02/doc/q02s.tex`

Q3. The goal is to compute the big-O of the runtime performance of the following algorithm. (The best, worst, and average runtimes are the same.) Here's the algorithm:

```
INPUT:  x - array of doubles
        n - size of x
        a - int
OUTPUT: None
ALGORITHM:

    a = a * a

    for i = 0, 1, 2, ..., n-1:
        x[i] = x[i] + a

        for j = 0, 1, 2, ..., i:
            x[j] = x[i] * a
```

(a) Write down $T(n)$ in terms of n and the $t_1, t_2, \ldots$. You should write it as a polynomial of n from the highest degree term to the lowest.

(b) Write down the big-O of $T(n)$ as $O(n^k)$ where k is the smallest possible positive integer.

SOLUTION

(a) The timings with the number of times a statement is executed is as follows (complete it like the solution given earlier):

```
           a = a * a                time t1

           i = 0                    time t2
LOOP1:     if i >= n:               time t3
               goto ENDLOOP1        time t4
           x[i] = x[i] + a          time t5

           j = 0                    time t6
LOOP2:     if j > i:                time t7
               goto ENDLOOP2        time t8
           x[j] = x[i] * a          time t9
           j = j + 1                time t10
           goto LOOP2               time t11
ENDLOOP2:
           i = i + 1                time t12
           goto LOOP1               time t13
ENDLOOP1:
```

SOLUTION. `a04q03/doc/q03s.tex`

Q4. The goal is to compute the big-O of the best and worst runtime performance of the following algorithm. Here's the algorithm:

```
INPUT:  x - array of doubles
        n - size of x
        a - int
OUTPUT: None
ALGORITHM:

    a = x[0] / 2

    for i = 0, 1, 2, ..., 100000:

        for j = 0, 1, 2, ..., n-1:
            x[j] = x[j] + a

        for j = 0, 1, 2, ..., (n - 1)/2:
            if x[j] < 5:
                x[j] = x[j] * x[0]
```

```
INPUT:  x - array of doubles
        n - size of x
        a - int
OUTPUT: None
ALGORITHM:

          a = x[0] / 2              time t1

          i = 0                     time t2
LOOP1:    if i > 100000:            time t3
              goto ENDLOOP          time t4

          j = 0                     time t5
LOOP2:    if j > n - 1:             time t6
              goto ENDLOOP2         time t7
          x[j] = x[j] + a           time t8
          j = j + 1                 time t9
          goto LOOP2                time t10
ENDLOOP2:
          j = 0                     time t11
LOOP3:    if j > (n - 1)/2:         time t12
```

```
                goto ENDLOOP3       time t13

            if x[j] >= 5:           time t14
                goto ELSE           time t15
            x[j] = x[j] * x[0]      time t16
ELSE:       j = j + 1               time t17
            goto LOOP3              time t18
ENDLOOP3:
            i = i + 1               time t19
            goto LOOP1              time t20
ENDLOOP1:
```

(a) Write down $T_w(n)$ in terms of n and the t_1, t_2,.... You should write it as a polynomial of n from the highest degree term to the lowest.

(b) Write down the big-O of $T_w(n)$ as $O(n^k)$ where k is the smallest possible positive integer.

(c) Write down the big-O of $T_b(n)$ as $O(n^k)$ where k is the smallest possible positive integer. (You are strongly advised to compute the precise $T_b(n)$ on your own of course. For grading purposes, you just have to write down the big-O.)

SOLUTION
(a) The timings with the number of times a statement is executed is as follows (complete it like the solution given earlier):

```
          a = x[0] / 2            time t1

          i = 0                   time t2
LOOP1:    if i > 100000:          time t3
              goto ENDLOOP        time t4

          j = 0                   time t5
LOOP2:    if j > n - 1:           time t6
              goto ENDLOOP2       time t7
          x[j] = x[j] + a         time t8
          j = j + 1               time t9
          goto LOOP2              time t10
ENDLOOP2:
          j = 0                   time t11
LOOP3:    if j > (n - 1)/2:       time t12
              goto ENDLOOP3       time t13

          if x[j] >= 5:           time t14
              goto ELSE           time t15
          x[j] = x[j] * x[0]      time t16
ELSE:     j = j + 1               time t17
          goto LOOP3              time t18
ENDLOOP3:
          i = i + 1               time t19
          goto LOOP1              time t20
ENDLOOP1:
```

SOLUTION. `a04q04/doc/q04s.tex`

Q5. The goal is to compute the big-O of the best and worst runtime performance of the following algorithm. Here's the algorithm:

```
INPUT:  x - array of doubles
        n - size of x
        a - int
OUTPUT: None
ALGORITHM:

    a = x[0] / 2

    for i = 0, 1, 2, ..., 100000:

        if a < 100:
            for j = 0, 1, 2, ..., n-1:
                x[i] = x[i] + a

            for j = 0, 1, 2, ..., (n-1)/2:
                if x[j] < 5:
                    x[j] = x[j] * x[0]
```

(During your analysis, just treat $n-1$ as though it's even so that $(n-1)/2$ is an integer.)

(a) Write down the big-O of $T_b(n)$ as $O(n^k)$ where k is the smallest possible positive integer. (You are strongly advised to write down the precise expression for $T_b(n)$ on your own. For grading purposes, you just have to write down the big-O of $T_b(n)$.)

(b) Write down the big-O of $T_w(n)$ as $O(n^k)$ where k is the smallest possible positive integer. (You are strongly advised to write down the precise expression for $T_w(n)$ on your own. For grading purposes, you just have to write down the big-O of $T_w(n)$.)

SOLUTION. `a04q05/doc/q05s.tex`

Q6. State the big-O of the runtime of the following sorting algorithm

```
INPUT:   x - array
         n - size of array x

ALGORITHM:

for i = 0, 1, 2, ..., n - 2:
    for j = 0, 1, 2, ..., n - 2:
        if x[j] < x[j + 1]:
            t = x[j]
            x[j] = x[j + 1]
            x[j + 1] = t
```

(You are strongly advised to translate the above to code that allows you compute the runtime and then write down the precise $T(n)$. For grading purposes you just have to write down the big-O of $T(n)$.) SOLUTION. `a04q06/doc/q06s.tex`

Q7. The purpose of this question is to compute the big-O runtime of an algorithm using experimental and brute force curve-fitting.

First implement the bubblesorting algorithm. Use it collect runtimes for arrays of size $n = 2000, 4000, 6000$, etc. until you have enough points to plot a graph of the runtime. You should have at least 10 data points. (For each n, you should do about 10 experiments and then take the average.) Specifically, when you run your program, it prompts the user for n and k. It then performs bubblesort on `x[0]`, ..., `x[n-1]` k times and reports on the average time. Here's a test run:

```
10000 10
8.5
```

This gives you an experimental runtime

$$T(10000)$$

of the bubblesort.

Next, plot the data points on a graph. See next page on how to do it in LaTeX.

Finally, find an n^d (i.e., find d with d as small as possible) such that

$$T(n) = O(n^d)$$

You do that by finding an N and a C such that the curve of Cn^d is above $T(n)$ for all $n \geq N$, i.e., such that for $n \geq N$,

$$T(n) \leq Cn^d$$

Include the following for this question:

(a) C++ source code.
(b) Bash shell session with collection of runtimes.
(c) Graph showing the plot of $T(n)$ and Cn^d. State clearly the C and the d. Also, indicate clearly N on the n–axis.

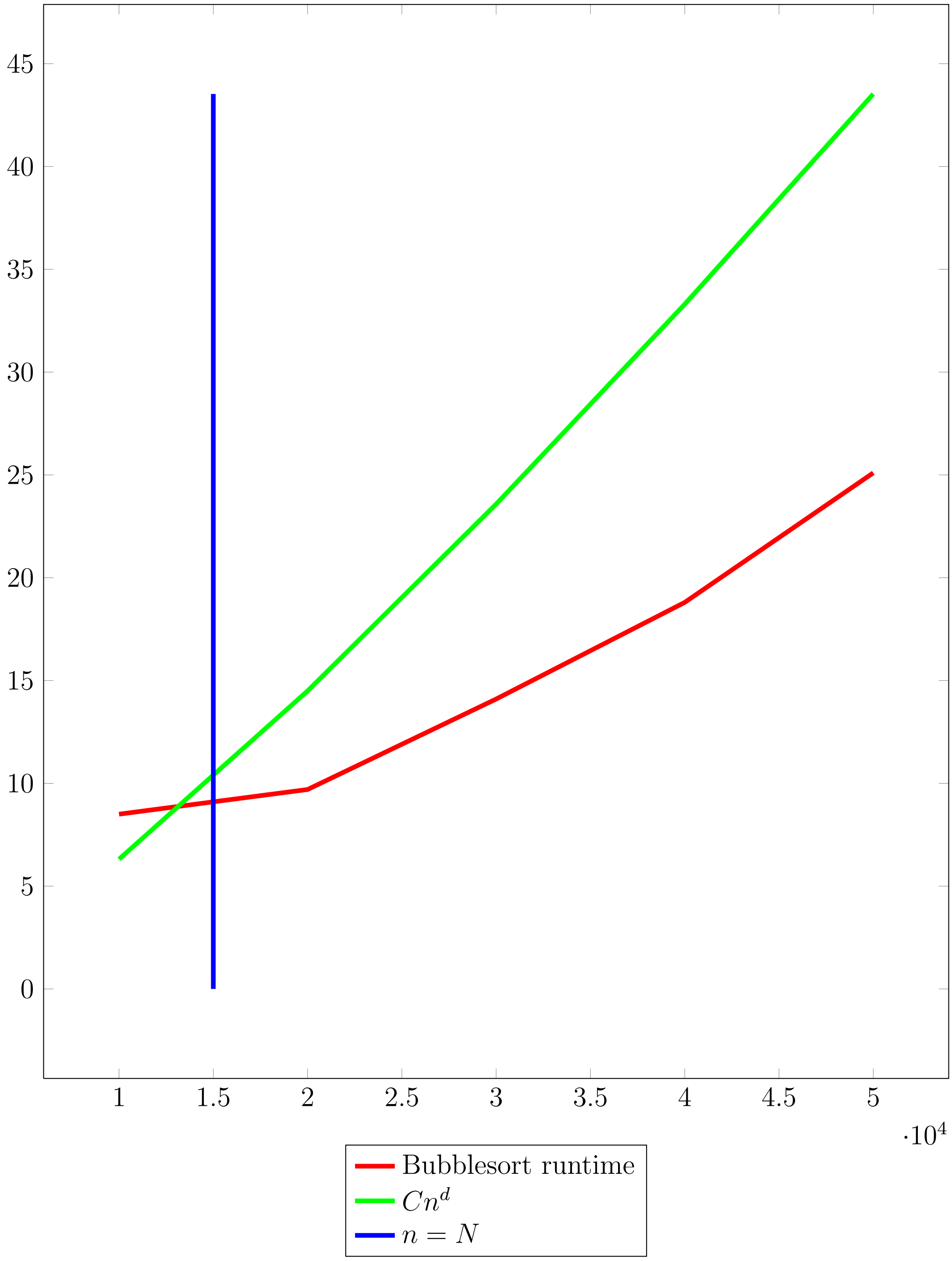
(a)

```
REPLACE THIS WITH YOUR C++ SOURCE FILE
```

(b)

```
REPLACE THIS WITH YOUR BASH SHELL SESSION WITH RUNTIMES COLLECTION
[student@localhost bubblesort]$ g++ *.cpp
[student@localhost bubblesort]$ ./a.out
10000 10
8.5
[student@localhost answer]$ ./a.out
20000 10
9.7
```

(c) In the follow change, modify C, N, d and the data in the python code for drawing the graph.



$$N = 15000, \quad C = 0.0001, \quad d = 1.2$$