Appendix I
Proof of Lemma 1

Proof: According to (1), for an FDI attack $\mathbf{a} = \mathbf{H}_0\mathbf{c}$, $\mathsf{a} \in \mathrm{col}(\mathsf{A}^{\mathsf{T}})$ holds. Denote the system's cut matrix as $\mathbf{B}$, where $\mathbf{B} \in \mathbb{R}^{n\times m}$. Each row of $\mathbf{B}$ corresponds to a fundamental cut of the system, and $\mathrm{col}(\mathbf{A}^{\mathsf{T}}) = \mathrm{col}(\mathbf{B}^{\mathsf{T}})$ [23]. Therefore, $\mathbf{a} \in \mathrm{col}(\mathbf{B}^{\mathsf{T}})$ holds. Thus, the set of nonzero elements of $\mathbf{a}$ must be the union of the system cuts. ■

Appendix II
Proof of Lemma 2

Proof: Since $\mathbf{F}_0 = \mathbf{C}\mathbf{X}_0$, we have $\mathbf{F}_0\mathbf{H}_0 = \mathbf{X}_0\mathbf{X}_0^{-1}\mathbf{C}\mathbf{A}_{-r}^{\mathsf{T}} = \mathbf{0}$. Then for any FDI attack $\mathbf{a} = \mathbf{H}_0\mathbf{c}$, $\mathbf{F}_0\mathbf{a} = \mathbf{0}$ holds. ■

Appendix III
Proof of Lemma 3

Proof: Regardless of stealthiness, the complete system can detect all attacks after MTD, where DoA = 0. Suppose the initial power flow vector $\mathbf{z} = \mathbf{H}_0\boldsymbol{\theta}_0$, since the HMTD strategies will not change the power flow, there must be $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_k \in \mathbb{R}^n$ that satisfy $\mathbf{H}_0\boldsymbol{\theta}_0 = \mathbf{H}_1\boldsymbol{\theta}_1 = \cdots = \mathbf{H}_k\boldsymbol{\theta}_k = \mathbf{z}$. For an attack as $\mathbf{a} = k\mathbf{z}, k \in \mathbb{R}$, we have $\mathbf{a} \in \mathrm{col}(\mathbf{H}_0) \bigcap \cdots \bigcap \mathrm{col}(\mathbf{H}_k)$, which is undetectable. Thus, the DoA in any system after HMTD is no less than 1. ■

Appendix IV
Proof of Theorem 1

Proof: Suppose $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is the topology of a power system, and the loop matrix $\mathbf{C}$ of $\mathcal{G}$ can be transformed into a block diagonal matrix

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}^{c_1} & & & \\ & \mathbf{C}^{c_2} & & \\ & & \ddots & \\ & & & \mathbf{C}^{c_t} \end{bmatrix}$$

where $c_1, c_2, \dots, c_t$ are t subblocks in $\mathcal{G}$. Since the loop matrix does not contain any bridge, each subblock is an independent complete system. According to Lemma 3, the minimum DoA of each subblock is 1. Then, the minimum DoA of the whole system is $s + t$. ■

Appendix V
Proof of Theorem 3

Proof: (Sufficiency): The system has feasible HMTD scheme is equivalent to $rank\left(\begin{bmatrix}\mathbf{J}\\ \tilde{\mathbf{Z}}\end{bmatrix}\right) < m$, where $\mathbf{J} = \mathbf{C}\mathbf{Z}$ and each row of $\tilde{\mathbf{Z}}$ has only one non-zero element. Each row of the loop matrix $\mathbf{C}$ corresponds to a fundamental loop of the system, which is formed by adding an edge to a spanning tree $\mathbb{T}$ [23]. The loop corresponding to each row has a unique edge, which belongs to the cotree $\bar{\mathbb{T}}$ corresponding to the tree $\mathbb{T}$ [23]. The *Hermite Normal form* of the loop matrix $\mathbf{C}$ is $\begin{bmatrix}\mathbf{C}_{\bar{\mathbb{T}}} & \mathbf{C}_{\mathbb{T}}\end{bmatrix}$, where $\mathbf{C}_{\bar{\mathbb{T}}} \in \mathbb{R}^{(m-n)\times(m-n)}$ is a diagonal matrix representing the cotree branch part and $\mathbf{C}_{\mathbb{T}} \in \mathbb{R}^{(m-n)\times n}$ is the part of the tree branches [24]. Obviously, $rank(\mathbf{C}) = m - n$. If $\bar{\mathbb{D}}$ contains a spanning tree, the Hermite Normal form of $\tilde{\mathbf{Z}}$ must contain an n-dimensional diagonal matrix corresponding to this spanning tree, and $rank\left(\begin{bmatrix}\mathbf{J}\\ \tilde{\mathbf{Z}}\end{bmatrix}\right) = m$ always holds. Conversely, if $\bar{\mathbb{D}}$ does not contain any spanning tree, $rank\left(\begin{bmatrix}\mathbf{J}\\ \tilde{\mathbf{Z}}\end{bmatrix}\right) < m$ always holds.

Necessity: Assume there are feasible HMTD schemes in the system, and $rank\left(\begin{bmatrix}\mathbf{J}\\ \tilde{\mathbf{Z}}\end{bmatrix}\right) < m$ holds. According to the sufficiency proof, if $\bar{\mathbb{D}}$ contains a spanning tree, $rank\left(\begin{bmatrix}\mathbf{J}\\ \tilde{\mathbf{Z}}\end{bmatrix}\right) = m$ always holds, which is inconsistent with the assumption. ■

Appendix VI
Proof of Theorem 4

Proof: Suppose the loop matrix $\mathbf{C}$ can be transformed into a block diagonal matrix with t elements $\mathbf{C}^{c_1}, \dots, \mathbf{C}^{c_t}$, where $c_1, c_2, \dots, c_t$ are t subblocks in $\mathcal{G}$. We can devide $\mathbf{L}_k$ into $\mathbf{L}_k^{c_1}, \dots, \mathbf{L}_k^{c_t}$ as well. According to Theorem 1, we need to prove that for any c_i, there $\exists k$ such that $rank(\mathbf{L}_k^{c_i}) = m^{c_i} - 1$, where m^{c_i} is the column number of $\mathbf{C}^{c_i}$.

In the k-th MTD, we have $\mathbf{F}_k^{c_i} = \mathbf{C}^{c_i}\mathbf{X}_k^{c_i}$, where $\mathbf{X}_k^{c_i} = diag(x_{k,1}, x_{k,2}, \dots x_{k,m^{c_i}})$ is the diagonal matrix composed of all line reactance in c_i at the k-th MTD. We have $\mathbf{F}_k^{c_i} = \begin{bmatrix}\mathbf{f}_{k,\cdot 1}^{c_i} & \mathbf{f}_{k,\cdot 2}^{c_i} & \dots & \mathbf{f}_{k,\cdot m^{c_i}}^{c_i}\end{bmatrix}$, $\mathbf{C}^{c_i} = \begin{bmatrix}\mathbf{C}_{\cdot 1}^{c_i} & \mathbf{C}_{\cdot 2}^{c_i} & \dots & \mathbf{C}_{\cdot m^{c_i}}^{c_i}\end{bmatrix}$, where $\mathbf{f}_{k,\cdot l}^{c_i}$ and $\mathbf{C}_{\cdot l}^{c_i}$ are the l-th column vectors of $\mathbf{F}_k^{c_i}$ and $\mathbf{C}^{c_i}$, respectively. Then $\mathbf{f}_{k,\cdot l}^{c_i} = x_{k,l}\mathbf{C}_{\cdot l}^{c_i}$. Consider the case where only the parameter of the line k is different from the element in $\mathbf{x}_0$ at the k-th MTD, i.e.,

$$x_{k,l} = \begin{cases}(1+\delta_k)x_{0,l}, & l = k\\ x_{0,l}, & l \neq k\end{cases}$$

where $\delta_k \neq 0$. Then the composite matrix after m^{c_i} MTD is

$$\mathbf{L}_{m^{c_i}}^{c_i} = \begin{bmatrix}\mathbf{F}_0^{c_i}\\ \mathbf{F}_1^{c_i}\\ \vdots\\ \mathbf{F}_{m^{c_i}}^{c_i}\end{bmatrix} = \begin{bmatrix}\mathbf{C}^{c_i} & & & \\ & \mathbf{C}^{c_i} & & \\ & & \ddots & \\ & & & \mathbf{C}^{c_i}\end{bmatrix}\begin{bmatrix}\mathbf{X}_0^{c_i}\\ \mathbf{X}_1^{c_i}\\ \vdots\\ \mathbf{X}_{m^{c_i}}^{c_i}\end{bmatrix}.$$

$\mathbf{X}_k^{c_i}$ can be written as $\begin{bmatrix}x_{k,1}\mathbf{e}_1 & x_{k,2}\mathbf{e}_2 & \cdots & x_{k,m^{c_i}}\mathbf{e}_{m^{c_i}}\end{bmatrix}$, where $\mathbf{e}_1 = \begin{bmatrix}1 & 0 & \dots & 0\end{bmatrix}^{\mathsf{T}}, \dots, \mathbf{e}_{m^{c_i}} = \begin{bmatrix}0 & 0 & \dots & 1\end{bmatrix}^{\mathsf{T}}$ are the unit basis vectors of m^{c_i}-dimensional space. Then $\mathbf{X}_k^{c_i} = \mathbf{X}_0^{c_i} + \begin{bmatrix}\mathbf{0} & \dots & \delta_k x_{0,k}\mathbf{e}_k & \dots & \mathbf{0}\end{bmatrix}$. Construct $\mathbf{X}^*$ by

$$\begin{bmatrix}\mathbf{X}_0^{c_i}\\ \mathbf{X}_1^{c_i}\\ \vdots\\ \mathbf{X}_{m^{c_i}}^{c_i}\end{bmatrix} - \begin{bmatrix}\mathbf{0}^{c_i}\\ \mathbf{X}_0^{c_i}\\ \vdots\\ \mathbf{X}_0^{c_i}\end{bmatrix} = \begin{bmatrix}x_{0,1}\mathbf{e}_1 & x_{0,2}\mathbf{e}_2 & \dots & x_{0,m^{c_i}}\mathbf{e}_{m^{c_i}}\\ \delta_1 x_{0,1}\mathbf{e}_1 & & & \\ & \delta_2 x_{0,2}\mathbf{e}_2 & & \\ & & \ddots & \\ & & & \delta_{m^{c_i}} x_{0,m^{c_i}}\mathbf{e}_{m^{c_i}}\end{bmatrix}.$$

Denote the diagonal matrix expanded by m^{c_i} system loop matrices by $\hat{\mathbf{C}}^{c_i} = diag(\mathbf{C}^{c_i}, \mathbf{C}^{c_i}, \dots, \mathbf{C}^{c_i})$, then

$$\hat{\mathbf{C}}^{c_i}\mathbf{X}^* = \begin{bmatrix}\mathbf{F}_{0,\cdot 1}^{c_i} & \mathbf{F}_{0,\cdot 2}^{c_i} & \dots & \mathbf{F}_{0,\cdot m^{c_i}}^{c_i}\\ \delta_1\mathbf{F}_{0,\cdot 1}^{c_i} & & & \\ & \delta_2\mathbf{F}_{0,\cdot 2}^{c_i} & & \\ & & \ddots & \\ & & & \delta_{m^{c_i}}\mathbf{F}_{0,\cdot m^{c_i}}^{c_i}\end{bmatrix}.$$

Since every column of $\mathbf{C}^{c_i}$ has non-zero elements, then $rank(\hat{\mathbf{C}}^{c_i}\mathbf{X}^*) = m^{c_i}$, i.e., $rank(\mathbf{L}^{c_i}_{m^{c_i}}) = m^{c_i}$ without the stealth requirements.

Denote the part of $\mathbf{z}$ corresponding to c_i as $\mathbf{z}^{c_i}$, $\mathbf{L}^{c_i}_{m^{c_i}}\mathbf{z}^{c_i} = \mathbf{0}$ always holds when considering the stealthiness requirements. Thus, $rank(\mathbf{L}^{c_i}_{m^{c_i}}) = m^{c_i} - 1$. For the whole system, we have $rank(\mathbf{L}_{m^{c_i}}) = m - s - t$. So there must $\exists k < m, k \in \mathbb{N}^+$ that satisfies $rank(\mathbf{L}_k) = m - s - t$. $\blacksquare$