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Voluntary versus mandatory disclosure of liability insurance coverage limit

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ABSTRACT

This article analyzes the disclosure of the liability insurance coverage limit and the impact of mandating disclosure of the coverage limit in a setting where voluntary disclosure of a firm's cash flow information is subject to litigation risk and the firm has directors' and officers' (D&O) liability insurance. Disclosure of cash flow information is costly, but disclosure of the insurance coverage limit features no direct disclosure friction. We find that, when the litigation environment is weak, the usual unraveling argument applies, and the manager always voluntarily discloses the coverage limit in equilibrium. However, when the litigation environment is strong, either no coverage limit is disclosed or only sufficiently high coverage limits are disclosed in equilibrium. Further analysis shows that mandatory disclosure of the coverage limit increases the voluntary disclosure of cash flow information.

1. Introduction

Most public firms in the United States and Canada have directors' and officers' (D&O) liability insurance (Lin et al., 2013). By covering litigation settlements and related costs from shareholder litigation, liability insurance determines how effective shareholder litigation can promote (timely) disclosures in public equity markets (e.g. Spindler, 2011; Spindler, 2017).[1] Despite the increasing popularity of D&O insurance, U.S. firms are not required to disclose their D&O insurance information.[2] In contrast, Canadian law mandates the disclosure of this information. This difference in rules can directly affect firms' voluntary disclosure of value-relevant information. This paper studies how mandatory versus voluntary disclosure of D&O insurance information affects managerial voluntary disclosure of value relevant information.

The model considers a manager of a publicly traded firm who must decide whether to promptly inform investors about cash flow news. When the manager delays disclosure of bad news, there will be litigation against the firm to help investors secure compensation for their incurred investment losses. We assume that the firm has bought liability insurance that covers any awarded damages up to a certain coverage limit; any excess damages must be paid by the firm. In our setting, the manager's decision to disclose cash flow information is driven by the following trade-off: the benefits of nondisclosure include a higher share price from withholding bad news and savings of disclosure cost, whereas the expected cost of nondisclosure is

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[1] For instance, as the Committee on Capital Markets Regulation concluded: *"The potential deterrent function of private securities litigation is debatable because virtually all the costs fall on the corporation and its insurer, which means they are ultimately borne by the shareholders"* (Committee on Capital Market Regulation, 2006).

[2] One exception to the rule is that New York state does mandate disclosure of D&O insurance information for firms incorporated there.

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the expected damages that must be covered by the firm. The liability insurance affects both the benefits and cost of nondisclosure of cash flow information. A higher liability insurance coverage limit will increase the benefits of nondisclosure by increasing the nondisclosure price. This is because investors expect to receive higher damage payments from the insurance of a nondisclosing firm. A higher insurance coverage limit will also reduce the cost of nondisclosure, as it reduces the damages that must be covered by the firm. Hence a manager's disclosure decision of the coverage limit can influence investors' beliefs about the value of that limit, which in turn affects the manager's disclosure of cash flow information. We assume that, before privately observing the cash flow information, the manager privately knows the insurance coverage limit and decides whether to disclose it. Besides studying the manager's voluntary disclosure strategy vis-a-vis both the coverage limit and the cash flow information, we also analyse how mandating disclosure of the coverage limit affects the manager's voluntary disclosure of cash flow information.

To examine the impact of mandating disclosure of the coverage limit on the disclosure of cash flow information, we first analyse the disclosure equilibrium of that information and the coverage limit when both disclosures are voluntary. For the disclosure equilibrium of cash flow information, we find that, when the litigation environment is weak or when the litigation environment is strong but the insurance coverage limit is high, the manager only discloses good news about the cash flow and withholds bad news. This is because, in both cases, the firm's expected payments of damages are sufficiently low. Then when the firm withholds bad news about the cash flow, the benefit of an inflated share price exceeds the expected litigation cost so that the bad news is not disclosed. When the litigation environment is strong and the coverage limit is low, however, the manager discloses both good and bad news about the cash flow. Bad news is also disclosed in this case, because the expected damages paid by the firm are sufficiently high, exceeding the benefit of an inflated share price. The same trade-off applies both when the manager discloses the insurance coverage limit and when the manager does not disclose it.

For the manager's disclosure decision of the liability insurance coverage limit, we find that voluntary disclosure of the limit unravels when the litigation environment is weak. Unraveling arises because the market value of a firm that withholds information about both the coverage limit and cash flow increases with that coverage limit. Then the usual unraveling argument applies (cf. Grossman, 1981, Milgrom, 1981), as the manager observing a relatively high coverage limit always prefers to disclose it rather than pooling with low coverage limits.

When the litigation environment is strong, the manager either never discloses the coverage limit or only discloses the limit when it is sufficiently high. This nondisclosure can be sustained in our setting, even absent disclosure frictions, such as exogenous disclosure cost (Verrecchia, 1983) or uncertain information endowment (Dye, 1985; Jung and Kwon, 1988). Unraveling does not arise in this case, because the market value of a firm that withholds information about both the coverage limit and cash flow changes nonmonotonically with the coverage limit. This nonmonotonic relation results from the fact that the manager's disclosure strategy of the cash flow information differs between high and low coverage limits. When the litigation environment is strong and the coverage limit is low, the manager discloses relatively bad news about the cash flow to avoid the litigation cost. In this case, a higher coverage limit raises the net benefits of not disclosing the cash flow information, which then induces the manager to withhold more bad news about the cash flow. This in turn decreases the market value of a nondisclosing firm. In contrast, when the coverage limit is high, the manager only discloses good news about the cash flow. A higher coverage limit raises the net benefits of no disclosure, so that more good news about the cash flow is withheld, which in turn increases the market value of a nondisclosing firm. This nonmonotonic relation between the coverage limit and the market value of a firm that withholds information about both the coverage limit and cash flow prevents unraveling and sustains partial disclosure of the coverage limit in equilibrium.

Based on the above analysis, we examine the effect of mandatory disclosure of the coverage limit. We find that, when nondisclosure of the limit is supported in equilibrium, mandating its disclosure results in more voluntary disclosure of cash flow information. The explanation for this is that, when the manager decides to disclose the coverage limit, she does not have the cash flow information yet. At this stage, her expected utility is determined by the expected damages covered by the liability insurance, and these expected damages are higher when there is less disclosure of cash flow information. Hence the manager chooses not to disclose the coverage limit when it results in less disclosure of cash flow information. Consequently, mandating disclosure of the coverage limit increases voluntary disclosure of cash flow information. This result implies that the rule mandating disclosure of basic D&O insurance policies in Canada can promote prompter disclosure of value relevant information to investors. Prompt release of material information is considered essential for the proper functioning of capital markets, and many stock exchanges require firms to timely disclose value relevant information. Enforcement of this rule, however, is complicated, as it is difficult to verify when firms become aware of material information. We find that mandatory disclosure of the coverage limit also affects a firm's incentives to timely disclose value relevant information.

This article contributes to the literature in two ways. First, the results demonstrate that mandatory disclosure of the coverage limit can increase voluntary disclosure of cash flow information. This result is relevant for stock market regulators, like the US Securities and Exchange Commission, who aim to increase market transparency. Second, the article presents a setting where partial disclosure of private information is sustained without the usual exogenous disclosure frictions (cf. Verrecchia, 1983; Dye, 1985; Jung and Kwon, 1988): disclosure of the coverage limit bears no direct costs, and the market knows the firm is privately informed about the coverage limit, but nevertheless nondisclosure of the limit is sustainable in equilibrium, because of the nonmonotonic relation between the coverage limit and the market value of a firm withholding information about the coverage limit and the cash flow in our setting.

The remainder of this article is organized as follows. Section 2 reviews related literature, and Section 3 introduces the model. Section 4 presents the equilibrium analyses, and Section 5 shows how mandating disclosure of the insurance coverage limit affects the voluntary disclosure of cash flow news. Section 6 concludes.

2. Related literature

Our article contributes to the literature investigating the relation between litigation and managerial disclosures. Dye (2017) investigates a model where the seller of an asset is subject to penalties if the seller is discovered to have withheld material information. The model shows that an increased penalty may decrease the likelihood of disclosure. Trueman (1997) finds that shareholder litigation can motivate the manager to disclose both good and extremely bad news. Marinovic and Varas (2016) examines a dynamic disclosure setting and shows that litigation penalties allow firms to withhold good news and avoid choosing inefficient disclosure policies. Evans and Sridhar (2002) shows that shareholder litigation cannot always improve the credibility of voluntary disclosure. Ma (2020) studies how shareholder litigation influences the information content of managerial disclosure and finds that stricter litigation does not necessarily increase the manager's disclosure precision. Laux and Stocken (2012) and Nan and Wen (2019) focus on litigation and managerial misreporting; Laux and Stocken (2012) finds that higher litigation penalties may increase misreporting due to managerial optimism, whereas Nan and Wen (2019) finds that litigation penalties can improve investment efficiency in a setting where managers can misreport the project outcome before raising capital. We study different issues here. While the literature has mainly focused on how litigation affects managerial disclosure of value relevant information, we examine managerial disclosure of the coverage limit and how mandating its disclosure influences voluntary disclosure of value relevant information.

To our best knowledge, Caskey (2014) is the only study that examined the relation between liability insurance and managerial disclosures. Our study differs in three respects. First, the research question differs. Caskey (2014) examines the pricing implications of securities class action lawsuits and how litigation insurance can reduce the market response to shareholder litigation. This paper is about the disclosure of insurance coverage limit. Hence the coverage limit is public information in Caskey (2014) but remains uncertain to investors in our model. Second, Caskey (2014) analyzes managerial misreporting, whereas our article is about strategically withholding value relevant information. Third, the manager's incentives differ. In Caskey (2014), the manager only cares about the short-term effect of misreporting but not the long-term effect.

3. Model

Fig. 1 summarizes the sequence of events of our model. Table 1 in Appendix A summarizes all notations. We consider a firm with liability insurance that covers any future litigation damages up to a maximum amount of $Z > 0$. Any damages exceeding Z must be paid by the firm. Z thus captures the firm's liability insurance coverage limit. To maintain our focus, we choose to take the liability insurance and the level of coverage limit Z as given and study how mandatory disclosure of the coverage limit Z affects the manager's disclosure strategy.

The firm generates a liquidating cash flow x at date $t = 4$. We assume that x is uniformly distributed on $[0, x_H]$ and without loss of generality we may assume $x_H = 1$. The realization of x is unknown for all parties at $t = 0$. The sequence of events is then as follows. At date $t = 0$, the manager privately observes the coverage limit $Z \in [0, 1]$ of the insurance policy that has been bought and decides whether to publicly disclose Z. Disclosure of Z must be truthful. We denote the disclosure decision of Z by $d_Z(Z) \in \{Z, nd_Z\}$, where $d_Z(Z) = Z$ corresponds to disclosure of Z and $d_Z(Z) = nd_Z$ corresponds to no disclosure. Investors believe that the coverage limit $Z \in [0, 1]$ follows a distribution with probability density function $h(Z)$.

At $t = 1$, the manager receives perfect private information about the liquidating cash flow x and decides whether to publicly disclose or to withhold this information. Denote the cash flow disclosure decision by $d_x(x) \in \{x, nd_x\}$, where $d_x(x) = x$ corresponds to disclosure of x and $d_x(x) = nd_x$ corresponds to no disclosure. Following Verrecchia (1983), we assume that this disclosure must be truthful and, if the manager discloses x, the firm incurs a proprietary cost $c > 0$, which reduces the firm's liquidating cash flow.[3]

At $t = 2$, current investors sell shares to future investors at price $\pi(d)$, where we define the manager's disclosure strategy as a pair $d = (d_Z, d_x)$.[4] To simplify notation, we assume that there is a representative current investor and a representative future investor. At $t = 3$, the capital market receives a public disclosure about the firm's liquidating cash flow x.[5]

[3] Our results remain qualitatively the same if the disclosure cost c is a personal cost to the manager and represents the manager's time and effort in preparing the disclosure at $t = 1$.

[4] The assumption resembles Caskey (2014). In the current model, we exogenously assume complete share turnover between current and future investors. However, one can show that complete share turnover also arises endogenously when one adds a trading decision of current investors at $t = 2$ in our model. In that case, current investors decide whether to hold or to sell the shares at $t = 2$. Complete share turnover arises because, under litigation, future investors who have purchased shares at an inflated price can receive damage payments from the insurer. Current investors who hold shares at $t = 1$ are not entitled to the damage payments. Consequently future investors are willing to pay a higher price for these shares than current investors. Detailed analyses are available from the authors upon request.

[5] We assume that the firm does not incur any proprietary cost when publicly disclosing x at $t = 3$. This assumption is consistent with the argument of Verrecchia (1983) that the proprietary cost decreases over time; that is, we assume that the proprietary cost has reduced to zero at $t = 3$. Our results remain qualitatively the same if we assume that the firm still incurs a proprietary cost c at $t = 3$. In that case, the firm incurs the proprietary cost either from disclosing x at $t = 1$ or from disclosing x at $t = 3$.

$t = 0$ — The manager privately observes the insurance coverage limit Z; the manager makes disclosure decision $d_Z(Z)$.

$t = 1$ — The manager receives private information about the cash flow x; the manager makes disclosure decision $d_x(x)$; disclosure of x has a cost $c > 0$.

$t = 2$ — Future investor buys shares at price $\pi(d)$.

$t = 3$ — Firm publicly discloses cash flow x.

$t = 4$ — Firm is being investigated and if litigation succeeds, damage payments D are paid to the future investor by the insurance company (and firm if $D > Z$).
Firm pays net liquidating cash flow to the future investor.

Fig. 1. Sequence of events.

Table 1
Notation.

Symbol	Definition
x	Liquidating cash flow.
Z	Coverage limit of liability insurance.
$h(Z)$	Probability density function of Z.
c	Cost when the manager discloses x at date 1.
π	Share price at date 2.
d	Pair of disclosure strategies $d = (d_Z, d_x)$.
d_x	Disclosure decision of x, $d_x = x$ denotes disclosure and $d_x = nd_x$ denotes no disclosure.
d_Z	Disclosure decision of Z, $d_Z = Z$ denotes disclosure and $d_Z = nd_Z$ denotes no disclosure.
q	Probability that the litigation will be successful.
$D(x)$	Damage payments to investors when litigation succeeds.
$D_c(x)$	Damages covered by the liability insurance.
$D_u(x)$	Damages not covered by the liability insurance.
k	The weight that the manager's objective function assigns to the liquidating cash flow and damage payments covered by the firm.
α	Damage factor, fraction of economic losses recovered from a successful litigation.
$\bar{x}$	The upper disclosure threshold of x.
$\underline{x}$	The lower disclosure threshold of x.
$\underline{Z}_{nd}$	The threshold where the disclosure of x changes from two-tail to upper-tail disclosure when the coverage limit Z is not disclosed.
$\underline{Z}$	The threshold where the disclosure of x changes from lower-tail to upper-tail disclosure when the coverage limit Z is disclosed.
δ	Likelihood of disclosing x before the manager observes the coverage limit Z.

At $t = 4$, a third party will investigate the firm's disclosure practice (Dye, 2017; Trueman, 1997). We assume that with probability $q \in (0, 1)$, the third party investigation can present sufficient evidence to the court that the manager has withheld cash flow information, so that damage payments $D(x)$ are paid to the future investor.[6] The insurer covers the damage payments up to the coverage limit Z; the excess amount is paid by the firm and reduces the firm's liquidating cash flow. Finally, the remaining liquidating cash flow is distributed to the future investor.

We model litigation against the firm consistent with securities class action lawsuit. For example, under Rule 10b-5 of the US Securities Exchange Act of 1934, investors can be compensated for economic losses caused by the firm's misleading statement or omissions of releasing material information to the market. Our model focuses on the latter cause, that is, omissions. Litigation against the firm only arises when investors have purchased shares at an inflated price and have suffered monetary damages from the firm's omission of value relevant information; that is, when $x < \pi(d_Z, nd_x)$. Considering the above aspects, we define the total damage payments $D(x)$ when $x < \pi(d_Z, nd_x)$ as

$$D(x) = \alpha[\pi(d_Z, nd_x) - x], \tag{1}$$

where $\alpha \in [0, 1]$ is a damage factor that represents the fraction of the economic loss $\pi(d_Z, nd) - x$ that the future investor can recover from the litigation.[7] Let $D_c(x) = min(D(x), Z)$ denote the damages that are covered by the insurance policy and let

[6] Note that, even though investors know that no disclosure means the manager has withhold the cash flow information, the shareholder may not always succeed in suing the manager due to legal technicalities from the legal process. For instance, shareholders can loss a case if they fail to provide all required documents on time. Our results also continue to hold when $q = 1$.

[7] A damage factor $\alpha < 1$ may arise because of settlement of a class action lawsuit or estimation errors in the true value of the asset.

$D_u(x) = max(D(x) - Z, 0)$ denote the damages that are not covered. Then we can write total damages as the sum of covered and uncovered damages; that is, $D(x) = D_c(x) + D_u(x)$.

We assume that the manager chooses the disclosure strategy d to maximize her expected utility $E[U_m(\pi(d), x)]$, where

$$U_m(\pi(d), x) = \pi(d) + k(x - E[D_u(x)]). \tag{2}$$

$\pi(d)$ is the $t = 2$ share price of the firm and $D_u(x)$ is the damages not covered by the liability insurance. The manager's utility is a weighted average of the firm's share price after disclosure, the liquidating cash flow, and the expected damage payments not covered by the liability insurance. The manager cares about the short-term share price $\pi(d)$, possibly because her compensation consists of stock and stock options of which the value depends on that price. $k > 0$ is the relative weight that the manager assigns to the liquidating cash flow and the expected uncovered damages. It can represent a setting where the firm pays for the uncovered damages and the subsequent reduced firm performance lowers the manager's compensation. Alternatively, it can represent a reputational cost for the manager that is proportional to the uncovered damages. This objective function resembles Dye (2017) and Trueman (1997).

To simplify our equilibrium analysis, we implicitly assume that the cost c of disclosing x is sufficiently low to exclude the full nondisclosure equilibrium where all values of x are withheld.[8] In other words, low disclosure cost c ensures that some values of x are always disclosed in equilibrium. Intuitively, when the disclosure cost c is sufficiently high, a manager will prefer to withhold information about x, even for the highest value of x; that is, when $x = 1$.

Before defining the equilibrium of the model, we would like to clarify a few features about the disclosure of the coverage limit Z. First, the disclosure of Z differs from the disclosure of x in two ways. One is that the manager has no duty to disclose the coverage limit Z; that is, investors cannot sue the firm for withholding information about Z. The other one is that the firm incurs a proprietary cost when disclosing x but not when disclosing Z. This is plausible, given that the coverage limit Z is unlikely to be proprietary information to competitors like the financial information x.

Second, the disclosure decision of Z is taken before the manager receives private information about the cash flow x so that the disclosure decision of Z cannot reveal information about the value of x.

Third, we assume that investors cannot perfectly anticipate the firm's insurance coverage limit Z because of some unmodeled information asymmetry between the firm and investors. For instance, Core (1997) finds that the coverage limit is determined by a firm's litigation risk, distress probability, and growth opportunity. To the extent that investors are not perfectly informed about these factors, they are uncertain about the coverage limit as assumed in our setting.

Lastly, we do not include the insurance premium in our model. When the premium is known to the market and the investor cannot perfectly anticipate the value of the coverage limit based on the insurance premium, the magnitude of the premium does not affect the equilibrium disclosure strategies. This is because the premium is a sunk cost for the manager at the time she needs to decide about disclosures.[9] In this case, we can interpret the probability density function $h(Z)$ of the coverage limit Z as the investor's beliefs with respect to Z after observing the insurance premium. If the insurance premium is unknown to the market and is paid out of the liquidating cash flow before $t = 0$, it also has no impact on the manager's disclosure strategies. If the insurance premium is unknown to the market and reduces the liquidating cash flow x, the investor will try to infer the value of the insurance premium from the coverage limit Z. The investor's updating of beliefs will greatly complicate the analysis and is out of the scope of the current paper.

An equilibrium consists of the disclosure strategy $d^* = (d_Z^*, d_x^*)$ and the price function $\pi^*(d^*)$ such that:

(i) Given $\pi^*(d^*)$, the manager's disclosure strategy d^* maximizes $E[U_m(\pi^*(d^*), x)]$ for each value of $x \in [0, 1]$ and $Z \in [0, 1]$.
(ii) Given the manager's disclosure strategy d^*, the price function $\pi^*(d^*)$ is based on the future investor's rational belief regarding the insurance coverage limit, the liquidating cash flow, and possible damage payments.

4. Equilibrium analysis

We solve for the equilibrium by backward induction and present the equilibrium analysis in three steps. Section 4.1 analyzes the disclosure decision of x when Z is (not) disclosed. Section 4.2 analyzes the disclosure decision of Z by deriving the manager's best response function, given a conjectured nondisclosure price $\pi(nd_Z, nd_x)$. For a given Z, we present the manager's expected utility as a function of the nondisclosure price and show that the disclosure decision of Z only depends on a comparison of the two nondisclosure prices $\pi(Z, nd_x)$ and $\pi(nd_Z, nd_x)$. Finally, Section 4.3 combines the prior results to derive the equilibrium strategies.

[8] Specifically, when the manager discloses the coverage limit Z, this condition is equivalent to $c < \frac{1}{1+k} - \frac{1-\sqrt{1-\alpha q}}{\alpha q(1+k)}$. When the manager does not disclose the coverage limit Z, the condition is $c < \frac{1-\pi^*(nd_Z, nd_x)}{1+k}$. It is not tractable to write the condition on c based on exogenous parameters in this case because it is not tractable to derive the explicit expression of nondisclosure price $\pi^*(nd_Z, nd_x)$.

[9] This argument applies both when the insurance premium is paid out of cash flows distributed to investors before $t = 0$ and when the insurance premium is paid after $t = 0$ and reduces the liquidating cash flow x.

4.1. Disclosure decision of x

This section analyzes the disclosure decision in three steps. First, we analyse the manager's trade-off of disclosing x when taking the disclosure decision of Z and the conjectured nondisclosure price $\pi(d_Z, nd_x)$ as given. Second, we derive the disclosure strategies that can arise when Z is not disclosed. Finally, we derive disclosure strategies that are supported in equilibrium when Z is disclosed.

To analyze the disclosure decision of x for a given disclosure decision d_Z of the coverage limit Z, we first derive the manager's expected utility for disclosure and nondisclosure of x at $t = 1$ after privately observing the value of x. If the manager discloses x, i.e., $d = (d_Z, x)$, the $t = 2$ share price equals $\pi(d_Z, x) = x - c$. Because there is no litigation, the price is independent of the coverage limit Z, and thus the manager's disclosure decision d_Z. The liquidating cash flow equals $x - c$, and the manager's utility equals

$$U_m(\pi(d_Z, x), x) = \pi(d_Z, x) + k(x - c) = (1 + k)(x - c). \tag{3}$$

If the manager does not disclose x, that is, $d = (d_Z, nd_x)$, the $t = 2$ share price equals $\pi(d_Z, nd_x)$. The future investor will sue the firm when the disclosed value of x at $t = 3$ is lower than the date 2 share price, that is, when $x < \pi(d_Z, nd_x)$. There are two potential outcomes with respect to litigation: failure and success. When the litigation fails, damage payments are nonexistent, and the liquidating cash flow is not affected by the litigation; that is, the liquidating cash flow equals x. When the litigation succeeds, the future investor who has purchased shares at an inflated price will receive damage payments. The liquidating cash flow x will be reduced by uncovered damage payments $D_u(x)$. Given that the future investor sues the firm, the litigation will succeed with probability q. Hence the manager's expected utility equals

$$U_m(\pi(d_Z, nd_x), x) = \pi(d_Z, nd_x) + k[x - qD_u(x)]. \tag{4}$$

The manager discloses x if and only if the expected utility of disclosure at $t = 1$ is higher than the expected utility of no disclosure. Applying Eqs. (3) and (4), substituting $D_u(x) = \max(0, \alpha(\pi(d_Z, nd_x) - x) - Z)$, and rearranging terms yields

$$kq \max(0, \alpha(\pi(d_Z, nd_x) - x) - Z) > \pi(d_Z, nd_x) - x + (1 + k)c. \tag{5}$$

The right-hand side, that is, $\pi(d_Z, nd_x) - x + (1 + k)c$ reflects the benefit of not disclosing x; it equals the inflation in share price $\pi(d_Z, nd_x) - x$ that the manager obtains at $t = 2$ plus the savings in disutility of the disclosure cost $(1 + k)c$. The left-hand side of the inequality, that is, $kq \max(0, \alpha(\pi(d_Z, nd_x) - x) - Z)$, reflects the cost of not disclosing the cash flow x and consists of the expected disutility of uncovered damage payments. The value of this cost depends on the value of x.

If $x \geqslant \pi(d_Z, nd_x) - \frac{Z}{\alpha}$, uncovered damages equal zero either because there is no litigation (i.e., $x \geqslant \pi(d_Z, nd_x)$) or because the insurance covers all damages (i.e., $\pi(d_Z, nd_x) + \frac{Z}{\alpha} \leqslant x < \pi(d_Z, nd_x)$). As there is no cost of withholding x, disclosure of x is preferred as long as there is no benefit; that is, $x \geqslant \pi(d_Z, nd_x) + (1 + k)c$. Define $\bar{x} = \pi(d_Z, nd_x) + (1 + k)c$. Observe that, when both the share price $\pi(d_Z, nd_x)$ and the disclosure cost c are relatively high, then $\bar{x} > 1$. Intuitively, a high nondisclosure price and a high disclosure cost make the manager with high values of x also enjoy a benefit from not disclosing x. Given that the support of x is $[0, 1]$, disclosure of x is preferred for $x > \min(\bar{x}, 1)$.

If $x < \pi(d_Z, nd_x) - \frac{Z}{\alpha}$, uncovered damages equal $\alpha(\pi(d_Z, nd_x) - x) - Z$, so that inequality (5) reduces to

$$kq\alpha(\pi(d_Z, nd_x) - x) - kq\alpha\frac{Z}{\alpha} > \pi(d_Z, nd_x) - x + (1 + k)c.$$

The right-hand side again reflects the benefit of not disclosing x, and the left-hand side again represents the cost. The above inequality cannot be satisfied when $kq\alpha \leqslant 1$. We interpret $kq\alpha$ as representing the strictness of the litigation environment, with a higher value of $kq\alpha$ representing a stronger litigation environment. Intuitively, when the litigation environment is weak, the cost of no disclosure can never exceed the benefit, so that all x satisfying $x < \pi(d_Z, nd_x) - \frac{Z}{\alpha}$ are not disclosed. When $kq\alpha > 1$, the cost of no disclosure exceeds the benefit when x is sufficiently low, so that disclosure is preferred, that is, when $x < \pi(d_Z, nd_x) - \frac{kq\alpha\frac{Z}{\alpha} + (1+k)c}{kq\alpha - 1}$. This implies that, when the litigation environment is strong, the manager discloses low values of x to avoid the litigation cost. Define $\underline{x} = \pi(d_Z, nd_x) - \frac{kq\alpha\frac{Z}{\alpha} + (1+k)c}{kq\alpha - 1}$. Combined with the support of x being $[0, 1]$, disclosure of x is preferred for $x < \max(\underline{x}, 0)$.

Based on the above analysis, we can now derive the manager's disclosure strategy of x when Z is not disclosed and when Z is disclosed.

4.1.1. Disclosure of x given non-disclosure of Z

We first summarize the disclosure strategy of x when Z is not disclosed in the lemma below.

Lemma 1. When Z is not disclosed at $t = 0$ and $(1 + k)c < 1 - \pi(nd_Z, nd_x)$, there exists a threshold $0 \leqslant \underline{Z}_{nd} < 1$, with $\underline{Z}_{nd} = 0$ either when $kq\alpha \leqslant 1$ or when $\pi(nd_Z, nd_x) \leqslant \frac{(1+k)c}{kq\alpha - 1}$, such that:

(A) for $Z < \underline{Z}_{nd}$, the best response disclosure strategy of x is two-tail disclosure where the manager discloses x either when $x < \underline{x}$ or when $x > \bar{x}$, and does not disclose x otherwise;
(B) for $Z \geqslant \underline{Z}_{nd}$, the best response disclosure strategy of x is upper-tail disclosure, where the manager discloses x when $x > \bar{x}$ and does not disclose x otherwise,

with $\bar{x} = \pi(nd_Z, nd_x) + (1+k)c$ and $\underline{x} = \pi(nd_Z, nd_x) - \frac{kq\alpha\frac{Z}{\alpha}+(1+k)c}{kq\alpha-1}$.

Lemma 1 shows the manager's best response disclosure strategy of x for a given conjectured nondisclosure price $\pi(nd_Z, nd_x)$. The condition $(1+k)c < 1 - \pi(nd_Z, nd_x)$ reflects our initial assumption that the disclosure cost is relatively low, such that full nondisclosure of x cannot arise as the best response disclosure strategy when Z is not disclosed.[10] Even though Z is not disclosed at $t = 0$ and does not influence the nondisclosure price $\pi(nd_Z, nd_x)$, the manager privately observes Z, and the value of Z determines the uncovered damages and the manager's expected utility. Hence the best response disclosure strategy of x does depend on Z.

When the value of Z is high (i.e., $Z \geqslant \underline{Z}_{nd}$), the manager adopts the upper-tail disclosure strategy, where good news about x is disclosed. When the coverage limit is low (i.e., $Z < \underline{Z}_{nd}$), the uncovered damages are high and can exceed the benefit of nondisclosure, so that bad news about x is also disclosed; that is, a two-tail disclosure strategy arises. For $Z = \underline{Z}_{nd}$, the lower disclosure threshold satisfies $\underline{x} = \pi(nd_Z, nd_x) - \frac{kq\alpha\frac{\underline{Z}_{nd}}{\alpha}+(1+k)c}{kq\alpha-1} = 0$. Hence, at $Z = \underline{Z}_{nd}$, the manager observing $x = 0$ is indifferent between disclosing and not disclosing x. Two-tail disclosure of x only arises when both the litigation environment is strong, that is, $kq\alpha > 1$, and the nondisclosure price is sufficiently high, such that the uncovered damages are high, that is, $\pi(nd_Z, nd_x) > \frac{(1+k)c}{kq\alpha-1}$. Otherwise, $\underline{Z}_{nd} = 0$, so that the best response disclosure strategy of x is always upper-tail disclosure; that is, only good news about x is disclosed.

The equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ when both Z and x are not disclosed is the expected value of $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ over all nondisclosed coverage limits. Intuitively, it depends on the expected liquidating cash flow $E[x \mid Z, nd_Z, nd_x]$ of a nondisclosing firm and the expected covered damages $E[D_c(x) \mid Z, nd_Z, nd_x]$, that is, the damage payments that the investor expects to receive from the insurance company. The uncovered damages do not affect the nondisclosure price because these damage payments have no net effect on the future investor's payoff: uncovered damages reduce the liquidating cash flow of the firm, so that the future investor is both paying and receiving the uncovered damages. At this stage, we cannot derive the equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ because the best response disclosure strategy for Z has not been derived yet, so that we cannot determine investors' beliefs about Z and x, given nondisclosure of Z. However, we know that, when valuing a firm that does not disclose both Z and x, the investor can anticipate for a given value of nondisclosed Z, the manager's disclosure strategy of x and thus the expected value of the cash flow $E[x \mid Z, nd_Z, nd_x]$ and the expected covered damages $E[D_c(x) \mid Z, nd_Z, nd_x]$ for this given value of coverage limit Z. Hence the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ critically depends on the coverage limit Z.

Fig. 2 shows how the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ changes with Z. For $Z \geqslant \underline{Z}_{nd}$, the manager follows an upper-tail disclosure strategy of x. In this case, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ weakly increases with Z. When the coverage limit $Z \geqslant \underline{Z}_{nd}$ is relatively low, the insurance only partially covers the damage payments for the values of x satisfying $\alpha(\pi(nd_Z, nd_x) - x) > Z$. In this case, an increase in Z increases the expected damages covered by the insurance, which in turn increases the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ and thus the nondisclosure price $\pi(nd_Z, nd_x)$. Furthermore, the increase in nondisclosure price $\pi(nd_Z, nd_x)$ results in a higher disclosure threshold $\bar{x} = \pi(nd_Z, nd_x) - (1+k)c$, which increases the expected cash flow $E[x \mid Z, nd_Z, nd_x]$, given no disclosure of x, and thus further increases the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$. When the coverage limit is sufficiently high, so that the insurance fully covers all damages for all values of x, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ is independent of Z, because the coverage limit is never binding and the expected covered damages do not depend on Z.

For $Z < \underline{Z}_{nd}$, the manager follows a two-tail disclosure strategy of x. In this case, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ decreases with Z. While a higher coverage limit increases the expected covered damages $E[D_c(x) \mid Z, nd_Z, nd_x]$ and thus the nondisclosure price, a higher coverage limit also reduces the uncovered damages and the cost of nondisclosure, so that less bad news will be disclosed; that is, the lower disclosure threshold $\underline{x} = \pi(nd_Z, nd_x) - \frac{kq\alpha\frac{Z}{\alpha}+(1+k)c}{kq\alpha-1}$ decreases when Z increases. Furthermore, observe that this lower threshold features a multiplier effect $\frac{kq\alpha}{kq\alpha-1}$ with respect to the coverage limit Z. An increase in the coverage limit Z results in less disclosure of relatively bad cash flow information, which in turn decreases the value $E[x \mid Z, nd_Z, nd_x]$ of a nondisclosing firm and thus decreases the nondisclosure price $\pi(nd_Z, nd_x)$. This decreases the damage payments $D(x) = \alpha(\pi(nd_Z, nd_x) - x)$ and the cost of nondisclosure and lowers $\underline{x}$ even further. Because of this multiplier effect, the latter effect dominates the former, so that the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ decreases with Z.

In summary, when $\underline{Z}_{nd} = 0$, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ monotonically increases with Z, but when $\underline{Z}_{nd} > 0$, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ changes nonmonotonically with Z.

[10] This condition also implies that the lower-tail disclosure strategy, where the manager discloses x when $x < \underline{x}$ and does not disclose x otherwise, cannot arise as the best response disclosure strategy of x when Z is not disclosed. This is because the nondisclosure price $\pi(nd_Z, nd_x)$ is the same for all values of Z that are not disclosed. Given the condition $(1+k)c < 1 - \pi(nd_Z, nd_x)$, the manager always prefers to disclose good news about x.

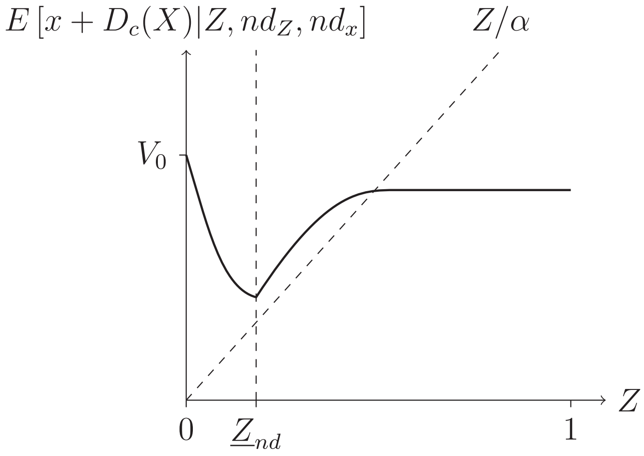

Fig. 2. Expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ of a nondisclosing firm as a function of coverage limit Z. The nondisclosure price $\pi(nd_Z, nd_x)$ is taken as given, and the corresponding disclosure strategy of x is the best response disclosure strategy as described in Lemma 1. $V_0 = \pi(nd_Z, nd_x) + \frac{1}{2}\frac{kq\alpha - 2}{kq\alpha - 1}(1+k)c$. The dashed line Z/α represents the nondisclosure price at which an upper-tail disclosure equilibrium switches from partial coverage to full coverage.

4.1.2. Disclosure of x given disclosure of Z

For the disclosure strategy of x when the coverage limit Z has been disclosed at $t = 0$, we can derive both the best response disclosure strategy of the manager and the equilibrium nondisclosure price $\pi^*(Z, nd_x)$. The equilibrium nondisclosure price $\pi^*(Z, nd_x)$ is implicitly defined by the following equation.

$$\pi^*(Z, nd_x) = E[x \mid d_Z(Z) = Z, d_x(x) = nd_x] + E[D_c(x) \mid d_Z(Z) = Z, d_x(x) = nd_x], \tag{6}$$

with $D_c(x) = \max\left((\alpha(\pi^*(Z, nd_x) - x), Z\right)$. Similar to the case when Z is not disclosed, the nondisclosure price depends on the expected liquidating cash flow of a nondisclosing firm and the expected covered damages. Using Eq. (6), we can derive the equilibrium nondisclosure price $\pi^*(Z, nd_x)$ and equilibrium disclosure strategy d_x^* when Z is disclosed at $t = 0$, as summarized below.

Lemma 2. When Z is disclosed at $t = 0$, there exists a threshold $0 \leqslant \underline{Z} < 1$, with $\underline{Z} = 0$ if and only if $kq\alpha \leqslant 2$, such that

(A) for $Z < \underline{Z}$, the equilibrium disclosure strategy d_x^* is lower-tail, where the manager discloses x when $x < \underline{x}$ and does not disclose x otherwise, with $\underline{x} = \pi^*(Z, nd_x) - \frac{kq\alpha\frac{Z}{\alpha} + (1+k)c}{kq\alpha - 1}$; the nondisclosure price in this case equals

$$\pi^*(Z, nd_x) = 1 - \sqrt{\left(\frac{kqZ + (1+k)c}{kq\alpha - 1}\right)^2 - 2q\left[\frac{Z\left(\frac{Z}{\alpha} + (1+k)c\right)}{kq\alpha - 1} + \frac{\alpha}{2}\frac{Z^2}{\alpha^2}\right]}; \tag{7}$$

(B) for $Z \geqslant \underline{Z}$, the equilibrium disclosure strategy d_x^* is upper-tail, where the manager discloses x when $x > \overline{x}$ and does not disclose x otherwise, with $\overline{x} = \pi^*(Z, nd_x) + (1+k)c$; the nondisclosure price in this case equals

$$\pi^*(Z, nd_x) = \begin{cases} qZ + \sqrt{(1+k)^2 c^2 - \alpha q(1 - \alpha q)\frac{Z^2}{\alpha^2}} & \text{if } Z < \alpha\pi^*(Z, nd_x) \\ \frac{\alpha q(1+k)c}{\sqrt{1 - \alpha q}} & \text{if } Z \geqslant \alpha\pi^*(Z, nd_x). \end{cases} \tag{8}$$

Similar to Lemma 1, the best response disclosure of x when Z is disclosed depends on the value of Z. When Z is high (i.e., $Z \geqslant \underline{Z}$), there is an upper-tail disclosure where good news about x is disclosed. For the threshold $Z = \underline{Z}$, it holds that the lower disclosure threshold satisfies $\underline{x} = \pi^*(\underline{Z}, nd_x) - \frac{kq\alpha\frac{Z}{\alpha} + (1+k)c}{kq\alpha - 1} = 0$; that is, the manager is indifferent between disclosure and no disclosure of $x = 0$.

There are two noteworthy differences from Lemma 1. First, for low values of Z (i.e., $Z < \underline{Z}$), the manager adopts a lower-tail disclosure strategy, instead of a two-tail disclosure strategy. The two-tail disclosure strategy is not supported in equilibrium when Z is disclosed, because the probability of no disclosure $\overline{x} - \underline{x} = (1+k)c + \frac{kqZ + (1+k)c}{kq\alpha - 1}$ does not depend on the nondisclosure price $\pi(Z, nd_x)$. Consequently, the expected overvaluation $\pi(Z, nd_x) - E[x \mid d_Z(Z) = Z, d_x(x) = nd_x]$ and the expected covered damages $E[D_c(x) \mid d_z(Z) = Z, d_x(x) = nd_x]$ do not depend on $\pi(Z, nd_x)$. Note that the two-tail disclosure strategy of x can be supported when Z is not disclosed, because the nondisclosure price $\pi(nd_Z, nd_x)$ depends on the the expected value of x and the expected covered damages for *all* values of Z that are not disclosed rather than a specific value of Z.

Second, when Z is disclosed, disclosure of bad news $x < \underline{x}$ occurs when $kq\alpha > 2$ rather than $kq\alpha > 1$. To support nondisclosure of $x = 1$ in the lower-tail disclosure strategy, the nondisclosure price $\pi^*(Z, nd_x)$ must be sufficiently high; that is,

$\pi^*(Z, nd_x) > 1 - (1+k)c$. For this, the disclosure threshold $\underline{x}$ must be sufficiently high, which arises when $kq\alpha > 2$. This is because a higher $kq\alpha$ increases the manager's disutility from the uncovered damages, which makes disclosure of bad news about x more attractive.

Before moving on to analyze the disclosure decision of Z, we first analyze how the equilibrium nondisclosure price $\pi^*(Z, nd_x)$ changes with Z.

Lemma 3. When $kq\alpha \leqslant 2$, $\pi^*(Z, nd_x)$ weakly increases with Z; when $kq\alpha > 2$, $\pi^*(Z, nd_x)$ decreases with Z when $Z < \underline{Z}$ and weakly increases with Z when $Z \geqslant \underline{Z}$.

Fig. 3 shows how the equilibrium nondisclosure price $\pi^*(Z, nd_x)$ changes with the coverage limit Z. The intuition resembles the case when Z is not disclosed. When the litigation environment is relatively weak, that is, $kq\alpha \leqslant 2$, $\underline{Z} = 0$, only the upper-tail disclosure of x arises. In this case, $\pi^*(Z, nd_x)$weakly increases in $Z \geqslant \underline{Z}$. The expression of $\pi^*(Z, nd_x)$ is shown in Eq. (8). When the value of Z is relatively low, that is, $\underline{Z} \leqslant Z < \alpha\pi^*(Z, nd_x)$, the insurance only provides partial coverage of litigation damages for low values of x (i.e., for $x < \pi^*(Z, nd_x) - \frac{Z}{\alpha}$). In this case, the nondisclosure price $\pi^*(Z, nd_x)$ increases when Z increases. This is again because an increase in Z increases the expected damages and the nondisclosure price $\pi^*(Z, nd_x)$, which in turn results in a higher disclosure threshold $\bar{x} = \pi^*(Z, nd_x) - (1+k)c$ and the expected cash flow $E[x \mid d_Z(Z) = Z, d_x(x) = nd_x]$ of a nondisclosing firm. This further increases the nondisclosure price $\pi^*(Z, nd_x)$. When the value of Z is relatively high, that is, $Z \geqslant \alpha\pi^*(Z, nd_x)$, the insurance fully covers all damages for all values of x, making the nondisclosure price $\pi^*(Z, nd_x)$ independent of Z.

When the litigation environment is relatively strong, that is, $kq\alpha > 2$, then $\underline{Z} > 0$ and the lower-tail disclosure of x arises when $Z < \underline{Z}$, and the upper-tail disclosure of x arises when $Z \geqslant \underline{Z}$. $\pi^*(Z, nd_x)$ nonmonotonically changes with Z in this case. Specifically, $\pi^*(Z, nd_x)$ again weakly increases with Z in the upper-tail disclosure of x. For the lower-tail disclosure of x, the nondisclosure price $\pi^*(Z, nd_x)$ decreases when Z increases. An increase in Z increases the expected covered damages, which then increase the nondisclosure price. However, a higher coverage limit Z also decreases the cost of nondisclosure for the manager, so that less bad news will be disclosed and thus decreases the nondisclosure price $\pi^*(Z, nd_x)$. Similar to the case when Z is not disclosed, the latter effect dominates the former, so that the nondisclosure price decreases when coverage limit Z increases.

Finally, observe that the nondisclosure price $\pi^*(Z, nd_x)$ is minimal at $Z = \underline{Z}$, with $\underline{Z} = 0$ when $kq\alpha \leqslant 2$.

4.2. Disclosure decision of Z

This section analyzes the manager's disclosure decision of Z in two steps. First, we analyze the manager's expected utility at $t = 0$, anticipating the equilibrium disclosure behavior at $t = 1$ and all subsequent events. Second, we compare the manager's expected utility for disclosure and nondisclosure of Z, to derive the manager's best response disclosure decision of Z for a conjectured nondisclosure price $\pi(nd_Z, nd_x)$.

4.2.1. Manager's expected utility

When the manager decides whether to disclose Z at $t = 0$, her expected utility depends on which disclosure equilibrium of x arises. For a given value of Z and a given disclosure decision d_Z, the manager correctly anticipates the corresponding best response disclosure strategy of x, as summarized in Lemma 1 and Lemma 2. If the manager discloses x at $t = 1$, her utility equals $(1+k)(x-c)$, which is independent of Z and her disclosure decision d_Z. If the manager does not disclose x, her utility equals $\pi(d_Z, nd_x) + k(x - qD_u(x))$. Hence the manager's expected utility for a conjectured nondisclosure price $\pi(d_Z, nd_x)$ equals

$$E[U_m(\pi(d_Z, nd_x), x)] = Pr(d_x(x) = x \mid d_Z) \cdot E[(1+k)(x-c) \mid d_x(x) = x, d_Z]$$
$$+ Pr(d_x(x) = nd_x \mid d_Z)E[\pi(d_Z, nd_x) + k(x - qD_u(x)) \mid d_x(x) = nd_x, d_Z].$$

We can rewrite the above expected utility expression as

$$E[U_m(\pi(d_Z, nd_x), x)] = (1+k)(E[x] - c) + Pr(d_x(x) = nd_x|d_Z) \cdot E[\pi(d_Z, nd_x) - x + (1+k)c - kqD_u(x) \mid d_x(x) = nd_x, d_Z]. \quad (9)$$

The first component of Eq. (9) is the expected utility of a full disclosure strategy of x. The second component represents the manager's expected net benefit of no disclosure. Recall that $\pi(d_Z, nd_x) - x + (1+k)c$ is the benefit of no disclosure, which consists of the overvaluation in share price and the avoided disutility of disclosure cost $(1+k)c$. $kqD_u(x)$ is the expected cost of no disclosure, which consists of the uncovered damages. Furthermore, the term $\pi(d_Z, nd_x) - x + (1+k)c - kqD_u(x)$ is always positive, because the manager chooses not to disclose x if and only if this leads to a positive net benefit.

Eq. (9) shows that the manager's expected utility $E[U_m(\pi(d_Z, nd_x), x)]$ increases with both the probability of no disclosure $Pr(d_x(x) = nd_x|d_Z)$ and the net benefit of no disclosure $E[\pi(d_Z, nd_x) - x + (1+k)c - kqD_u(x) \mid d_x(x) = nd_x, d_Z]$. As both components depend on the nondisclosure price $\pi(d_Z, nd_x)$, we first analyse how the manager's expected utility changes with the nondisclosure price $\pi(d_Z, nd_x)$, with results summarized below.

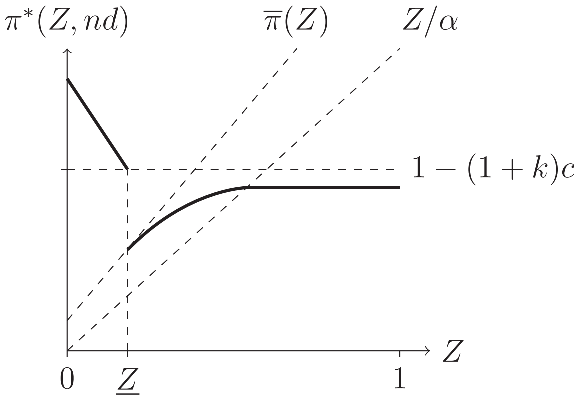

Fig. 3. Equilibrium nondisclosure price $\pi^*(Z, nd_x)$ as a function of the coverage limit Z. The upper-tail disclosure of x arises when $Z \geqslant \underline{Z}$, with $\underline{Z} = 0$ when $kq\alpha \leqslant 2$. The lower-tail disclosure of x arises when $Z < \underline{Z}$. The dashed line $\overline{\pi}(Z) = \frac{kqZ+(1+k)c}{kq\alpha-1}$ represents the nondisclosure price for which a manager with coverage limit Z is indifferent between disclosing and not disclosing cash flow $x = 0$; that is, an upper-tail disclosure equilibrium exists only if $\pi^*(Z, nd_x) \leqslant \overline{\pi}(Z)$. The dashed line Z/α represents the nondisclosure price at which an upper-tail disclosure equilibrium switches from partial coverage to full coverage.

Lemma 4. Given coverage limit Z and disclosure decision d_Z, the manager's expected utility $E[U_m(\pi(d_Z, nd_x), x)]$ is

(A) independent of the nondisclosure price $\pi(d_Z, nd_x)$ when the best response disclosure of x is a two-tail disclosure strategy;
(B) increasing with the nondisclosure price $\pi(d_Z, nd_x)$ when the best response disclosure of x is an upper-tail disclosure strategy;
(C) decreasing with the nondisclosure price $\pi(d_Z, nd_x)$ when the best response disclosure of x is a lower-tail disclosure strategy.

Furthermore, the manager's expected utility $E[U_m(\pi(d_Z, nd_x), x)]$ is highest in case (A).

To explain Lemma 4, note that how the manger's expected utility $E[U_m(\pi(d_Z, nd_x), x)]$ changes with the nondisclosure price $\pi(d_Z, nd_x)$ depends on how the probability of no disclosure $Pr(d_x(x) = nd_x|d_Z)$ changes with $\pi(d_Z, nd_x)$. This is because the net benefit of no disclosure $E[\pi(d_Z, nd_x) - x + (1+k)c - kqD_u(x) \mid d_x(x) = nd_x, d_Z]$ increases when the probability of no disclosure increases. In the best response disclosure strategy of x, the manager withholds information about x if and only if it results in a net benefit; that is, the more information is withheld, the higher the expected utility of no disclosure is. Hence all that matters for the manager's expected utility is how the probability of no disclosure changes with the nondisclosure price $\pi(d_Z, nd_x)$.

Lemma 4A then follows from the observation that the probability of no disclosure equals $\bar{x} - \underline{x} = (1+k)c + \frac{kqZ+(1+k)c}{kq\alpha-1}$ in a two-tail disclosure strategy of x and is independent of the nondisclosure price. Lemma 4B holds because in an upper-tail disclosure strategy of x, the probability of no disclosure equals $\bar{x} = \pi(d_Z, nd_x) + (1+k)c$ and is increasing in $\pi(d_Z, nd_x)$. Intuitively, around the upper disclosure threshold $\bar{x}$, the value of x is higher than the nondisclosure price $\pi(d_Z, nd_x)$, so that there is no litigation cost when withholding information about x. Hence, when deciding whether to disclose x, the manager trades off the undervaluation $\bar{x} - \pi(d_Z, nd_x)$ from no disclosure against the disclosure cost $(1+k)c$. A higher nondisclosure price decreases the undervaluation from not disclosing x and thus increases the probability of no disclosure. The reverse argument applies for 4C: the probability of no disclosure in a lower-tail disclosure strategy of x equals $1 - \underline{x} = 1 - \pi(d_Z, nd_x) + \frac{kqZ+(1+k)c}{kq\alpha-1}$ and decreases with $\pi(d_z, nd_x)$. The intuition is as follows. Around the lower disclosure threshold $\underline{x}$, a higher nondisclosure price increases both the benefit (i.e., overvaluation) and the cost of nondisclosure (i.e., litigation). The increased cost exceeds the increased benefit in this case, because the litigation environment is strong and $kq\alpha > 1$. Hence more values of x are disclosed when $\pi(d_Z, nd_x)$ increases. The manager earns the highest expected utility when a two-tail disclosure of x arises because this leads to the highest probability of nondisclosure.

4.2.2. Best response disclosure decision of Z

For a given coverage limit Z and a conjectured nondisclosure price $\pi(nd_Z, nd_x)$, the manager discloses Z if and only if her expected utility at $t = 0$ from disclosing Z is higher than not disclosing Z; that is,

$$E[U_m(\pi(Z, nd_x), x)] \geqslant E[U_m(\pi(nd_Z, nd_x), x)].$$

Hence, to derive the manager's best response disclosure decision of Z, we need to compare the manager's expected utilities when Z is disclosed and when it is not disclosed. The analysis in Section 4.2.1 shows that the expected utility comparison depends on the probability of no disclosure of x, which in turn depends on the nondisclosure price $\pi(d_Z, nd_x)$. In general,

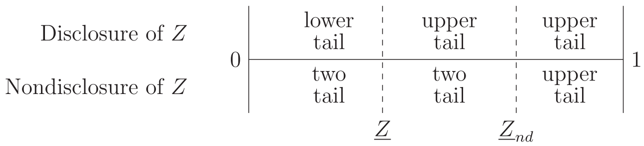

Fig. 4. Comparison of best response disclosure strategy of x when the manager discloses Z and when the manager does not disclose Z, with $\underline{Z} > 0$ if and only if $kq\alpha > 2$ and $\underline{Z}_{nd} > 0$ if and only if $kq\alpha > 1$ and $\pi(nd_Z, nd_x) > \frac{(1+k)c}{kq\alpha-1}$.

the manager chooses the disclosure strategy of Z that leads to a higher probability of nondisclosure of x. We summarize the detailed best response disclosure strategy of Z below.

Lemma 5. When $\pi(nd_Z, nd_x)$ satisfies $\pi^*(\underline{Z}, nd_x) < \pi(nd_Z, nd_x) < 1 - (1+k)c$, there exists a threshold $Z^* \in (\underline{Z}_{nd}, 1]$, such that the manager prefers disclosure of Z if and only if $Z > Z^*$. In particular,

(A) for $Z < \underline{Z}_{nd}$, the manager always prefers nondisclosure of Z;
(B) for $Z \geqslant \underline{Z}_{nd}$, the manager prefers disclosure of Z if and only if $\pi(nd_Z, nd_x) > \pi^*(Z^*, nd_x)$.

The condition $\pi^*(\underline{Z}, nd_x) < \pi(nd_Z, nd_x)$ in Lemma 5 suggests that the nondisclosure price $\pi(nd_Z, nd_x)$ when both Z and x are not disclosed is higher than the lowest nondisclosure price $\pi^*(\underline{Z}, nd_x)$ when Z is disclosed.[11] This condition implies that $\underline{Z}_{nd} > \underline{Z}$, that is, the threshold $\underline{Z}_{nd}$, where disclosure of x switches from two-tail to upper-tail disclosure when Z is not disclosed, is higher than the threshold $\underline{Z}$, where disclosure of x switches from lower-tail to upper-tail disclosure when Z is disclosed.

To explain Lemma 5, we present the best response disclosure strategies of x when Z is (not) disclosed in Fig. 4. For both $Z < \underline{Z}$ and $\underline{Z} \leqslant Z < \underline{Z}_{nd}$, nondisclosure of Z results in a two-tail disclosure strategy of x, whereas disclosure of Z results in a lower-tail and upper-tail disclosure strategy of x, respectively. From Lemma 4, we know that two-tail disclosure of x yields the highest expected utility for the manager, because it leads to the highest probability of no disclosure of x. Therefore the manager always prefers nondisclosure of Z.

For $Z \geqslant \underline{Z}_{nd}$, both disclosure and nondisclosure of Z result in upper-tail disclosure of x. From Lemma 4, we know that, for an upper-tail disclosure strategy of x, the probability of no disclosure of x, and thus the manager's expected utility increases with the nondisclosure price, so that the manager prefers disclosure of Z if and only if its nondisclosure price is higher; that is, $\pi^*(Z, nd_x) > \pi(nd_Z, nd_x)$. Because the nondisclosure price $\pi^*(Z, nd_x)$ when Z is disclosed is (weakly) increasing in Z and the nondisclosure price $\pi(nd_Z, nd_x)$ when Z is not disclosed is independent of Z, the manager prefers to disclose high values of Z. We denote the threshold as Z^*; that is, disclosure of Z is preferred for $Z > Z^*$.

The best response disclosure strategy of Z is illustrated in Fig. 5. For $Z < \underline{Z}_{nd}$, the two-tail disclosure of x when Z is not disclosed gives the manager the highest expected utility, so that the manager always prefers not to disclose Z. For $Z \geqslant \underline{Z}_{nd}$, the manager discloses Z when the nondisclosure price is higher; that is, $\pi^*(Z, nd_x) > \pi(nd_Z, nd_x)$. Hence the threshold Z^* is where the functions $\pi^*(Z, nd_x)$ and $\pi(nd_Z, nd_x)$ intersect for $Z \geqslant \underline{Z}_{nd}$.

4.3. Disclosure equilibrium

When the manager makes the disclosure decision of Z, she takes the conjectured nondisclosure price $\pi(nd_Z, nd_x)$ that arises when Z and x are not disclosed as given. This conjectured nondisclosure price gives rise to a best response disclosure strategy of Z, where $Z > Z^*$ are disclosed. The future investor rationally anticipates this disclosure strategy of Z. When observing nondisclosure of Z and x, the future investor rationally prices the firm at $\pi^*(nd_Z, nd_x)$. In equilibrium, the nondisclosure price $\pi^*(nd_Z, nd_x)$ equals the conjectured nondisclosure price $\pi(nd_Z, nd_x)$. The equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ when both Z and x are not disclosed is the expected value of $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ over all nondisclosed coverage limits $Z \leqslant Z^*$; that is,

$$\pi^*(nd_Z, nd_x) = \int_0^{Z^*} E[x + D_c(x) \mid Z, nd_Z, nd_x] h(Z \mid Z \leqslant Z^*) dZ. \tag{10}$$

Recall from prior analyses and Fig. 2 that, when $\underline{Z}_{nd} = 0$, there is only upper-tail disclosure of x, so that the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ monotonically increases with Z. In contrast, when $\underline{Z}_{nd} > 0$, there can be either two-tail or upper-tail disclosure of x, so that the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ changes nonmonotonically with Z. This relation critically determines whether there is full or partial disclosure of Z in equilibrium. We now analyse the disclosure equilibria, with results summarized below.

[11] When $\pi^*(nd_Z, nd_x) \leqslant \pi^*(\underline{Z}, nd_x)$, we can show that there also exists a partial disclosure of the coverage limit Z. However, it is not tractable to analyze how mandatory disclosure of Z affects the likelihood of disclosing x. Hence we did not include this case in our analysis. While we consider $\pi^*(nd_Z, nd_x) \leqslant \pi^*(\underline{Z}, nd_x)$ unlikely to be supported in equilibrium, it is not tractable to prove that this is indeed the case. We also cannot find a numerical example to show that $\pi^*(nd_Z, nd_x) < \pi^*(\underline{Z}, nd_x)$ can arise in equilibrium.

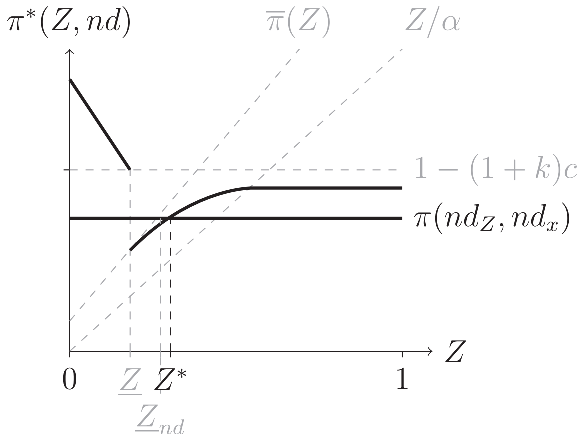

Fig. 5. Best response disclosure strategy of Z given conjectured nondisclosure price $\pi(nd_Z, nd_x)$ that satisfies $\pi^*(\underline{Z}, nd_x) \leqslant \pi(nd_Z, nd_x) < 1-(1+k)c$. The dashed line $\overline{\pi}(Z) = \frac{kqZ+(1+k)c}{kq\alpha-1}$ represents the nondisclosure price for which a manager with coverage limit Z is indifferent between disclosing and not disclosing cash flow $x = 0$; that is, an upper-tail disclosure equilibrium exists only if $\pi^*(Z, nd_x) \leqslant \overline{\pi}(Z)$. The dashed line Z/α represents the nondisclosure price at which an upper-tail disclosure equilibrium switches from partial coverage to full coverage.

Proposition 1. When $kq\alpha \leqslant 2$, there exists an equilibrium that features full disclosure of Z; that is, $d_Z^*(Z) = Z$ for all $Z \in [0, 1]$. The equilibrium disclosure strategy d_x^* is described in Lemma 2.

When the litigation environment is weak, that is, $kq\alpha \leqslant 2$, skeptical beliefs by the investor when observing nondisclosure of Z support full disclosure of Z.[12] Skeptical beliefs imply that, given a full disclosure equilibrium of Z, the investor values a firm that does not disclose both Z and x at the worst possible price, that is, $\pi^*(\underline{Z}, nd_Z)$. This implies that there is only upper-tail disclosure of x both when Z is disclosed and when Z is not disclosed; that is $\underline{Z} = \underline{Z}_{nd} = 0$. We know from Lemma 4 that the manager prefers disclosure of Z to nondisclosure of Z if and only if $\pi^*(Z, nd_x) > \pi^*(nd_Z, nd_x)$. In this setting, the usual unravelling argument applies.

To explain why unraveling arises in this case, suppose there exists partial disclosure of Z where the manager would not disclose $Z \leqslant Z^*$. Then in equilibrium, the manager should be indifferent between disclosing and not disclosing Z^*; that is, $\pi^*(nd_Z, nd_x) = \pi^*(Z^*, nd_x)$. This equilibrium condition also implies that the expected value $E[x + D_c(x) \mid Z^*, d_Z, nd_x]$ at Z^* is the same when Z is disclosed and when Z is not disclosed. However, as illustrated in Fig. 2, when $\underline{Z}_{nd} = 0$, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$, given nondisclosure of Z, monotonically increases with Z. This indicates that the expected value for lower values of Z is always lower than the expected value at $Z = Z^*$. Therefore the manager is always better off disclosing Z^* rather than withholding her information and pooling with lower values of Z. Then disclosure of Z unravels, and full disclosure arises.

Proposition 2. When $kq\alpha > 2$:

(A) if $\pi^*(\underline{Z}, nd_x) < \pi^*(nd_Z, nd_x) < \pi^*(1, nd_x)$, the equilibrium features upper-tail disclosure of Z; that is, there exists $Z^* < 1$ such that $d_Z^*(Z) = Z$ for all $Z \in (Z^*, 1]$ and $d_Z^*(Z) = nd_Z$ otherwise.
(B) if $\pi^*(1, nd_x) \leqslant \pi^*(nd_Z, nd_x) < 1-(1+k)c$, the equilibrium features full nondisclosure of Z; that is, $d_Z^*(Z) = nd_Z$ for all $Z \in [0, 1]$.

Proposition 2 shows that the equilibrium can be either partial disclosure or full nondisclosure of Z. When the equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ is not too high (i.e., $\pi^*(nd_Z, nd_x) < \pi^*(1, nd_x)$), there exists a disclosure threshold Z^* where high values of Z are disclosed. This equilibrium is illustrated in Fig. 6A. When the equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ is higher than the nondisclosure price $\pi^*(Z, nd_x)$ for all $Z \geqslant Z^*$ (i.e., $\pi^*(nd_Z, nd_x) \geqslant \pi^*(1, nd_x)$), a full nondisclosure of Z arises, as shown in Fig. 6B. Unraveling does not occur in both cases because, when the litigation environment is strong, the expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ changes nonmonotonically with Z. For low values of the coverage limit Z, the two-tail disclosure of x increases the expected value and thus can sustain a high nondisclosure price $\pi^*(nd_Z, nd_x)$ to meet the equilibrium condition $\pi^*(nd_Z, nd_x) = \pi^*(Z^*, nd_x)$.

We also perform a numerical analysis to examine both types of equilibria. An example of a setting that features partial disclosure of Z is $\alpha = 0.9$, $q = 0.7$, $k = 3.492$, and $c = 0.067$. Under this partial disclosure equilibrium, it holds that

[12] When $1 < kq\alpha < 2$, besides the full disclosure equilibrium, there may also exist a partial disclosure equilibrium of Z where $Z > Z'$ is disclosed. This is because, if the nondisclosure price $\pi^*(nd_Z, nd_x)$ is sufficiently high ($\pi^*(nd_Z, nd_x) > \frac{(1+k)c}{kq\alpha-1}$) such that $\underline{Z}_{nd} > 0$, the manager follows the two-tail disclosure strategy of x when Z is not disclosed and $Z < \underline{Z}_{nd}$. The high expected value $E[x + D_c(x) \mid Z, nd_Z, nd_x]$ under the two-tail disclosure of x may sustain a sufficiently high equilibrium nondisclosure price $\pi^*(nd_Z, nd_x)$ that meets the equilibrium condition $\pi^*(nd_Z, nd_x) = \pi^*(Z', nd_x)$. While we cannot prove that this equilibrium cannot arise, we also cannot find a numerical example to support the existence of this equilibrium.

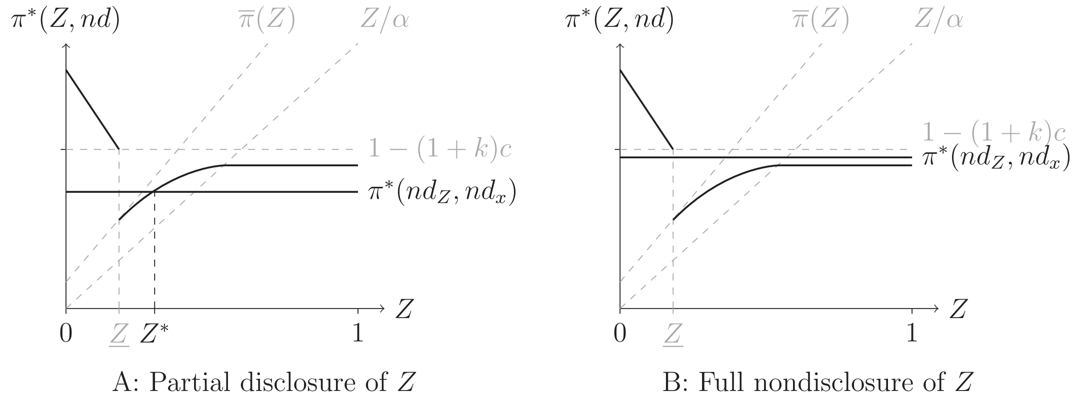

Fig. 6. Equilibrium disclosure of Z. The dashed line $\overline{\pi}(Z) = \frac{kqZ+(1+k)c}{kq\alpha-1}$ represents the nondisclosure price for which a manager with coverage limit Z is indifferent between disclosing and not disclosing cash flow $x = 0$; that is, an upper-tail disclosure equilibrium exists only if $\pi^*(Z, nd_x) \leqslant \overline{\pi}(Z)$. The dashed line Z/α represents the nondisclosure price at which an upper-tail disclosure equilibrium switches from partial coverage to full coverage.

$Z^* = 0.273$ and $\pi^*(nd_Z, nd_x) = 0.434$. For the full nondisclosure equilibrium, an example of such a setting is $\alpha = 0.9$, $q = 0.7$, $k = 5.714$ and $c = 0.045$. A full nondisclosure equilibrium of Z arises with $\pi^*(nd_Z, nd_x) = 0.530$. Observe that, in both examples, $(1+k)c = 0.303$; that is, the manager's disutility of disclosing x is the same across both settings. The reason why full nondisclosure arises for $k = 5.731$ is because a higher value of k makes the uncovered damages more costly to the manager. As a result, the manager discloses more bad cash flow information in the two-tail disclosure strategy of x, which in turn results in a higher nondisclosure price $\pi^*(nd_Z, nd_x)$. This nondisclosure price is high enough to induce nondisclosure of all coverage limits Z.

Before proceeding to analyse the effect of mandating disclosure of coverage limit Z, we would like to discuss several features of the voluntary disclosure equilibrium of Z. First, we assume that the manager decides on the disclosure of the coverage limit Z before she observes the cash flow x. In our setting, the manager's voluntary disclosure decision of the coverage limit will not change after observing the cash flow x; that is, a manager who chooses not to disclose Z at $t = 0$ will make the same disclosure decision about Z after she observes the value of x at $t = 1$. This is because the manager's disclosure decision of the coverage limit is determined by the comparison of the nondisclosure price $\pi^*(Z, nd_x)$ when Z is disclosed and the nondisclosure price $\pi^*(nd_Z, nd_x)$ when Z is not disclosed. This decision is independent of the realized value of the cash flow. Hence the manager's disclosure decision of Z will not change after observing the cash flow.

Second, we find that partial disclosure of the coverage limit can arise without any direct disclosure frictions of Z. This result is driven by the fact that the model features two consecutive disclosure decisions: the disclosure decision of Z depends on the subsequent disclosure strategy of x, which in turn depends on the disclosure decision of Z. In particular, the manager's disclosure decision of Z depends on the market value of the firm when the cash flow information about x is withheld from the market. This value changes nonmonotonically with the coverage limit Z because the disclosure strategy of x differs between low and high coverage limits. When the coverage limit is low, the manager discloses relatively bad news about the cash flow to avoid litigation. In this case, a higher coverage limit raises the net benefits of not disclosing the cash flow information, which then induces the manager to withhold more bad news about the cash flow. This in turn decreases the market value of a nondisclosing firm. In contrast, when the coverage limit is high, the manager only discloses good news about the cash flow. A higher coverage limit raises the net benefits of no disclosure, so that more good news about the cash flow is withheld, which in turn increases the market value of a nondisclosing firm. Such nonmonotonicity sustains the partial disclosure equilibrium of the coverage limit and does not arise in standard disclosure games in the literature.

Lastly, the existence of partial disclosure without any direct disclosure friction critically relies on our assumption of using disclosure cost rather than an uncertain managerial information endowment to get partial disclosure of the cash flow information x. This is because, with an uncertain managerial information endowment, an uninformed manager who is not subject to litigation will always prefer a higher nondisclosure price. This makes the manager always prefer a higher nondisclosure price when deciding whether to disclose the coverage limit. The usual unraveling argument again applies in this case, such that there is full disclosure of the coverage limit without any direct disclosure frictions.

5. Voluntary versus mandatory disclosure of *Z*

Our main result studies the effect of mandating disclosure of the coverage limit Z on the likelihood of disclosing x. The result provides insight on whether mandating disclosure of the coverage limit Z improves the timely disclosure of cash flow information. For this, we measure the likelihood of disclosure δ at date $t = 0$ before the manager learns the coverage limit Z; that is, $\delta = 1 - \int_0^1 Pr(d_x(x) = nd_x \mid Z)h(Z)dZ$.

Proposition 3. If the nondisclosure price $\pi^*(nd_Z, nd_x)$ when observing nondisclosure of both Z and x satisfies $\pi^*(\underline{Z}, nd_x) < \pi^*(nd_Z, nd_x) < 1-(1+k)c$, then the probability of disclosing x is higher when the disclosure of the coverage limit Z is mandatory than when it is voluntary.

The above result indicates that mandating the disclosure of the firm's liability insurance coverage limit can also promote more timely disclosure of the firm's financial performance. The reason is that, in equilibrium, the manager only withholds information about Z when the corresponding disclosure strategy d_x^* features less disclosure of cash flow information x than when Z would be disclosed. The manager prefers a disclosure strategy of Z with less disclosure of x because less disclosure of x leads to higher expected damages covered by the insurance, which in turn increases the $t=0$ expected net benefit of the disclosure strategy d_x and the manager's expected utility. This result is consistent with the argument in the literature that firms should disclose more information about their liability insurance, especially the coverage limit, as a way to discipline the manager (Baker and Griffith, 2010).

The results are relevant to stock market regulators, like the US Securities and Exchange Commission, who aim at increasing market transparency. Many stock exchanges require firms to promptly disclose value relevant information. Enforcement of this rule, however, is complicated, as it is difficult to verify when firms become aware of material information. We find that mandating the disclosure of the liability insurance coverage limit can increase incentives to disclose value relevant information.

6. Conclusions

This article studies voluntary disclosure of the liability insurance coverage limit as well as how mandatory disclosure of the coverage limit affects managers' incentives to disclose cash flow information in the presence of litigation risk. It finds that a manager fully discloses the coverage limit when the litigation environment is weak. However, when the litigation environment is strong, partial or full nondisclosure of the coverage limit can arise without any direct disclosure frictions. The result also shows that firms are more likely to disclose cash flow information when disclosure of the coverage limit is mandatory rather than voluntary.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs

Proof of Lemma 1. Take $\pi(nd_Z, nd_x) < 1-(1+k)c$ as given. This upper bound implies that the manager strictly prefers disclosure of $x=1$ when Z is not disclosed, that is, $\bar{x} < 1$. Hence, the resulting best response disclosure strategy of x is either two-tail $(\underline{x} > 0)$ or upper-tail disclosure $(\underline{x} = 0)$. The two-tail disclosure of x arises when $kq\alpha > 1$ and $\pi(nd_Z, nd_x) - \frac{kqZ+(1+k)c}{kq\alpha-1} > 0$. Because $\pi(nd_Z, nd_x)$ is independent of Z and $\frac{kqZ+(1+k)c}{kq\alpha-1}$ increases with Z when $kq\alpha > 1$, there exists a threshold $\underline{Z}_{nd}$ such that $\pi(nd_Z, nd_x) - \frac{kq\underline{Z}_{nd}+(1+k)c}{kq\alpha-1} = 0$. Consequently, the two-tail disclosure of x arises for all $Z < \underline{Z}_{nd}$ and the upper-tail disclosure of x arises for all $Z \geqslant \underline{Z}_{nd}$.

Proof of Lemma 2. To derive the disclosure equilibrium of x when Z is disclosed, we take Z as given and we use shorthand notation π for the $t=2$ nondisclosure price $\pi(Z, nd_x)$. We proceed the proof in three steps. First, we derive how the future investor price a non-disclosing firm given the conjectured nondisclosure price π and the corresponding best response disclosure strategy of x. Second, we derive the equilibrium nondisclosure price expression. Third, to show the uniqueness of the equilibrium, we analyse the conditions under which each equilibrium arises. Note that we also consider full nondisclosure strategy of x in the proof as it allows to derive an explicit condition under which the full nondisclosure equilibrium of x does not arise.

Step 1. Denote the event that litigation is successful by $\phi \in \{0,1\}$, where $\phi = 0$ corresponds to failure and $\phi = 1$ corresponds to success. The nondisclosure price π is determined by the expected utility $E[W(Z, nd_x, \phi)|d_Z = Z, d_x = nd_x]$ of the future investor, which equals

$$E[W(Z, nd_x, \phi)|Z, nd_x] = Pr(\phi = 0|Z, nd_x)E[W(Z, nd_x, 0) \mid Z, nd_x, \phi = 0] + Pr(\phi = 1|Z, nd_x)E[W(Z, nd_x, 1) \mid Z, nd_x, \phi = 1]. \quad (11)$$

When litigation fails, that is, $\phi = 0$, the future investor's utility equals

$$W(Z, nd_x, 0) = x - \pi.$$

When litigation is successful, that is, $\phi = 1$, the future investor's utility equals

$$W(Z, nd_x, 1) = x - \max(0, D(x) - Z) - \pi + D(x). \quad (13)$$

Substituting Eqs. (12) and (13) into (11) gives

$$E[W(Z, nd_x, \phi) \mid Z, nd_x] = [E[x \mid Z, nd_x] - \pi] + Pr(\phi = 1|Z, nd_x)E[D(x) - \max(0, D(x) - Z) \mid Z, nd_x, \phi = 1].$$

Rewriting yields

$$\pi = E[x \mid Z, nd_x] + Pr(\phi = 1|Z, nd_x)E[D(x) - \max(0, D(x) - Z) \mid Z, nd_x, \phi = 1]. \tag{14}$$

Using that $D(x) - \max(0, D(x) - Z) = \min(D(x), Z)$, we can rewrite Eq. (14) as

$$\pi = E[x \mid Z, nd_x] + Pr(\phi = 1|Z, nd_x)E[\min(D(x), Z) \mid Z, nd_x, \phi = 1]. \tag{15}$$

Step 2. Based on the above nondisclosure price expression, we can derive that

$$E[x \mid Z, nd_x] = \begin{cases} \frac{1}{2} & \text{if } d_x \text{ is full nondisclosure} \\ \frac{1}{2}[\pi + (1+k)c] & \text{if } d_x \text{ is upper-tail disclosure} \\ \frac{1}{2}\left(\pi - \frac{kqZ+(1+k)c}{kq\alpha-1} + 1\right) & \text{if } d_x \text{ is lower-tail disclosure} \\ \frac{1}{2}\left(\pi - \frac{kqZ+(1+k)c}{kq\alpha-1} + \pi + (1+k)c\right) & \text{if } d_x \text{ is two-tail disclosure} \end{cases}$$

$$Pr(\phi = 1|Z, nd_x) = \begin{cases} q\pi & \text{if } d_x \text{ is full nondisclosure} \\ q\frac{\pi}{\pi+(1+k)c} & \text{if } d_x \text{ is upper-tail disclosure} \\ q\frac{\frac{kqZ+(1+k)c}{kq\alpha-1}}{1-\pi+\frac{kqZ+(1+k)c}{kq\alpha-1}} & \text{if } d_x \text{ is lower-tail disclosure} \\ q\frac{\frac{kqZ+(1+k)c}{kq\alpha-1}}{(1+k)c+\frac{kqZ+(1+k)c}{kq\alpha-1}} & \text{if } d_x \text{ is two-tail disclosure} \end{cases}$$

$$E[min(D(x), Z)|Z, nd_x, \phi = 1] = \begin{cases} \frac{1}{2}\alpha\pi & \text{if } d_x \text{ is full nondisclosure or upper-taildisclosure and } \pi \leqslant \frac{Z}{\alpha} \\ \frac{1}{\pi}\left[\left(\pi - \frac{Z}{\alpha}\right)Z + \frac{1}{2}\frac{Z^2}{\alpha}\right] & \text{if } d_x \text{ is full nondisclosure or upper-tail disclosure and } \pi > \frac{Z}{\alpha} \\ \frac{kq\alpha-1}{kqZ+(1+k)c}\left[\left(\frac{kqZ+(1+k)c}{kq\alpha-1} - \frac{Z}{\alpha}\right)Z + \frac{1}{2}\frac{Z^2}{\alpha}\right] & \text{if } d_x \text{ is lower-tail or two-tail disclosure} \end{cases}$$

In equilibrium, Eq. (15) holds. For a full nondisclosure strategy and $\pi \leqslant \frac{Z}{\alpha}$, the nondisclosure price π solves

$$f_1(\pi) = \frac{1}{2} + q\frac{1}{2}\alpha\pi^2 - \pi = 0.$$

Rewriting yields

$$\pi = \frac{1}{\alpha q}\left(1 \pm \sqrt{1 - \alpha q}\right).$$

We focus our analysis on the negative root of the nondisclosure price, because with a positive root, $\pi > 1$. This implies that a non-disclosing firm is valued higher than the highest value of the liquidating cash flow x, which is unlikely to happen. Hence, we consider

$$\pi = \frac{1}{\alpha q}\left(1 - \sqrt{1 - \alpha q}\right). \tag{16}$$

For a full nondisclosure strategy and $\pi > \frac{Z}{\alpha}$, the nondisclosure price π solves

$$f_2(\pi) = \frac{1}{2} + q\left[\left(\pi - \frac{Z}{\alpha}\right)Z + \frac{1}{2}\frac{Z^2}{\alpha}\right] - \pi = 0.$$

Rewriting yields

$$\pi = \frac{1}{2(1 - qZ)} - \frac{1}{2(1 - qZ)}\alpha q\frac{Z^2}{\alpha^2}. \tag{17}$$

For an upper-tail disclosure strategy and $\pi \leqslant \frac{Z}{\alpha}$, the nondisclosure price π solves

$$f_3(\pi) = \frac{1}{2}(\pi + (1+k)c) + q\frac{\pi}{\pi + (1+k)c}\frac{1}{2}\alpha\pi - \pi = 0.$$

Rewriting yields

$$\pi = \frac{(1+k)c}{\sqrt{1 - \alpha q}}. \tag{18}$$

For an upper-tail disclosure strategy and $\pi > \frac{Z}{\alpha}$, the nondisclosure price π solves

$$f_4(\pi) = \frac{1}{2}(\pi + (1+k)c) + q\frac{1}{\pi + (1+k)c}\left[\frac{1}{2}\frac{Z^2}{\alpha} + Z\left(\pi - \frac{Z}{\alpha}\right)\right] - \pi = 0.$$

Rewriting yields

$$\pi = qZ \pm \sqrt{(1+k)^2c^2 - \alpha q(1-\alpha q)\frac{Z^2}{\alpha^2}}.$$

As $\pi > \frac{Z}{\alpha}$ holds in the upper-tail disclosure equilibrium, only the positive root is feasible, that is,

$$\pi = qZ + \sqrt{(1+k)^2c^2 - \alpha q(1-\alpha q)\frac{Z^2}{\alpha^2}}. \tag{19}$$

For a lower-tail disclosure strategy, the nondisclosure price π solves

$$f_5(\pi) = \frac{1}{2}\left(1 + \pi - \frac{kqZ + (1+k)c}{kq\alpha - 1}\right) + \frac{q}{1 - \pi + \frac{kqZ+(1+k)c}{kq\alpha-1}}\left[\left(\frac{kqZ + (1+k)c}{kq\alpha - 1} - \frac{Z}{\alpha}\right)Z + \frac{1}{2}\frac{Z^2}{\alpha}\right] - \pi = 0.$$

Rewriting yields

$$\pi = 1 \pm \sqrt{\left(\frac{kqZ + (1+k)c}{kq\alpha - 1}\right)^2 - 2q\left[\frac{Z(\frac{Z}{\alpha} + (1+k)c)}{kq\alpha - 1} + \frac{\alpha}{2}\frac{Z^2}{\alpha^2}\right]}.$$

We again focus our analysis on the negative root of the nondisclosure price. One reason is as explained before, with a positive root, a non-disclosing firm is valued higher than the highest value of the liquidating cash flow x, which is unlikely to happen. The second reason is that the insurance company is likely to price protect itself by assuming the firm will adopt the disclosure strategy with a higher probability of no disclosure and charging the corresponding insurance premium. In the lower-tail disclosure equilibrium, the firm does not disclose $x \in \left[\pi - \frac{kqZ+(1+k)c}{kq\alpha-1}, 1\right]$, and the nondisclosure price with a negative root generates a higher probability of no disclosure. Hence, we consider

$$\pi = 1 - \sqrt{\left(\frac{kqZ + (1+k)c}{kq\alpha - 1}\right)^2 - 2q\left[\frac{Z(\frac{Z}{\alpha} + (1+k)c)}{kq\alpha - 1} + \frac{\alpha}{2}\frac{Z^2}{\alpha^2}\right]}. \tag{20}$$

For a two-tail disclosure strategy, the nondisclosure price π solves

$$f_6(\pi) = \frac{1}{2}\left(\pi - \frac{kqZ + (1+k)c}{kq\alpha - 1} + \pi + (1+k)c\right) + \frac{q\left[\left(\frac{kqZ+(1+k)c}{kq\alpha-1} - \frac{Z}{\alpha}\right)Z + \frac{1}{2}\frac{Z^2}{\alpha}\right]}{(1+k)c + \frac{kqZ+(1+k)c}{kq\alpha-1}} - \pi = 0,$$

which is independent of π. Hence, the two-tail disclosure strategy cannot be supported as an equilibrium when Z is disclosed.

Step 3. Now we analyse the conditions under which each equilibrium can arise.

The full nondisclosure strategy of x with $\pi \leqslant \frac{Z}{\alpha}$ arises when $\pi \geqslant 1 - (1+k)c$ and $\pi \leqslant \frac{Z}{\alpha}$. Replacing the expression of π in Eq. (16) yields

$$c \geqslant \frac{1}{1+k} - \frac{1 - \sqrt{1-\alpha q}}{\alpha q(1+k)} = \bar{c}, \tag{21}$$

$$\frac{Z}{\alpha} \geqslant \frac{1}{\alpha q}\left(1 - \sqrt{1-\alpha q}\right). \tag{22}$$

The full nondisclosure strategy of x with $\pi > \frac{Z}{\alpha}$ arises when $\pi \geqslant 1 - (1+k)c$ and $\pi > \frac{Z}{\alpha}$. Replacing the expression of π in Eq. (17) yields

$$\frac{Z}{\alpha} > \frac{1}{\alpha q}\left(1 + \sqrt{1-\alpha q}\right), \tag{23}$$

$$\frac{Z}{\alpha} < \frac{1}{\alpha q}\left(1 - \sqrt{1-\alpha q}\right), \tag{24}$$

$$c \geqslant \frac{1}{1+k} - \frac{1}{2(1+k)(1-qZ)} + \frac{\alpha q}{2(1+k)(1-qZ)}\frac{Z^2}{\alpha^2}. \tag{25}$$

Because the right hand side of Eq. (25) decreases with Z and equals $\frac{1}{1+k} - \frac{1-\sqrt{1-\alpha q}}{\alpha q(1+k)}$ when $Z = \frac{1}{q}\left(1 - \sqrt{1-\alpha q}\right)$, condition (25) implies $c \geqslant \bar{c}$ is not binding. Conditions (21) and (25) implies that when $c < \bar{c} = \frac{1}{1+k} - \frac{1-\sqrt{1-\alpha q}}{\alpha q(1+k)}$, a full nondisclosure strategy of x cannot be supported in equilibrium when Z is disclosed.

The upper-tail disclosure strategy of x with $\pi \leqslant \frac{Z}{\alpha}$ arises when $\pi < 1-(1+k)c$ and $\pi \leqslant \frac{Z}{\alpha}$. Replacing the expression of π in Eq. (18) yields

$$c < \frac{1}{1+k} - \frac{1-\sqrt{1-\alpha q}}{\alpha q(1+k)} = \bar{c}, \tag{26}$$

$$Z \geqslant \frac{\alpha(1+k)c}{\sqrt{1-\alpha q}}. \tag{27}$$

The upper-tail disclosure strategy of x with $\pi > \frac{Z}{\alpha}$ arises when $kq\alpha \leqslant 1,\ \pi < 1-(1+k)c,\ \pi > \frac{Z}{\alpha}$, and $\pi - \frac{kqZ+(1+k)c}{kq\alpha-1} \geqslant 0$. In this case, the insurance only provides partial coverage of litigation damages for some values of x. Lemma 3 shows that the nondisclosure price in this case is always lower than the upper-tail disclosure strategy with $\pi \leqslant \frac{Z}{\alpha}$, where the insurance provides full coverage of litigation damages for all values of x. Given our assumption that $c < \bar{c}$ so that the full nondisclosure of x does not arise, the condition $\pi < 1-(1+k)c$ is always satisfied for the upper-tail disclosure with $\pi \leqslant \frac{Z}{\alpha}$, as shown by Eq. (26). When $kq\alpha \leqslant 1$, the third condition $\pi - \frac{kqZ+(1+k)c}{kq\alpha-1} \geqslant 0$ is always satisfied. Replacing the expression of π in Eq. (19), $\pi > \frac{Z}{\alpha}$ can be rewritten as

$$Z < \frac{\alpha(1+k)c}{\sqrt{1-\alpha q}}. \tag{28}$$

When $kq\alpha > 1$, the third condition $\pi - \frac{kqZ+(1+k)c}{kq\alpha-1} \geqslant 0$ on π yields

$$\frac{Z}{\alpha} \geqslant (1+k)c\frac{-kq\alpha(1-\alpha q) - \alpha q + (kq\alpha-1)\sqrt{k^2q^2\alpha^2(1-\alpha q)+\alpha^2q^2}}{k^2q^2\alpha^2(1-\alpha q)+\alpha q}. \tag{29}$$

The lower-tail disclosure strategy of x arises when $kq\alpha < 1,\ \pi > \frac{Z}{\alpha},\ \pi > 1-(1+k)c$, and $\pi > \frac{kqZ+(1+k)c}{kq\alpha-1}$. Lemma 3 shows that the nondisclosure price decreases with Z in this case, while both $\frac{Z}{\alpha}$ and $\frac{kqZ+(1+k)c}{kq\alpha-1}$ increase with Z. One can show that at $Z = \underline{Z},\ \pi^*(\underline{Z}, nd_x) > \frac{kqZ+(1+k)c}{kq\alpha-1} > \frac{Z}{\alpha}$. Hence, $\pi > \frac{Z}{\alpha}$ and $\pi > \frac{kqZ+(1+k)c}{kq\alpha-1}$ always hold for the lower-tail disclosure strategy of x.

Replacing the expression of π in Eq. (20) into condition $\pi > 1-(1+k)c$ yields

$$\frac{Z}{\alpha} < (1+k)c\frac{-kq\alpha(1-\alpha q) - \alpha q + (kq\alpha-1)\sqrt{k^2q^2\alpha^2(1-\alpha q)+\alpha^2q^2}}{k^2q^2\alpha^2(1-\alpha q)+\alpha q}. \tag{30}$$

One can show that the right-hand side of (30) is negative if and only if $kq\alpha - 2 < 0$. Define $\underline{Z}$ by

$$\underline{Z} = \max\left(0, (1+k)c\frac{-kq\alpha(1-\alpha q) - \alpha q + (kq\alpha-1)\sqrt{k^2q^2\alpha^2(1-\alpha q)+\alpha^2q^2}}{k^2q^2\alpha^2(1-\alpha q)+\alpha q}\right). \tag{31}$$

Then a lower-tail disclosure strategy of x only arises when $Z < \underline{Z}$.

To complete the proof, observe that the existence conditions (27), (28), (29), and (30) for the upper-tail and lower-tail disclosure strategies of x are mutually exclusive, so that the nondisclosure price and corresponding disclosure strategy are uniquely determined.

Proof of Lemma 3. When $kq\alpha \leqslant 2$, the manager follows the upper-tail disclosure strategy of x. When $Z < \alpha\pi^*(Z, nd_x)$, taking the partial derivative of $\pi^*(Z, nd_x)$ in Eq. (8) with respect to $\frac{Z}{\alpha}$ yields

$$\frac{\partial \pi^*(Z, nd_x)}{\partial \frac{Z}{\alpha}} = \alpha q - \frac{\alpha q(1-\alpha q)\frac{Z}{\alpha}}{\sqrt{(1+k)^2c^2 - \alpha q(1-\alpha q)\frac{Z^2}{\alpha^2}}},$$

which is positive if and only if

$$\sqrt{(1+k)^2c^2 - \alpha q(1-\alpha q)\frac{Z^2}{\alpha^2}} > (1-\alpha q)\frac{Z}{\alpha}.$$

Rewriting yields $\frac{Z^2}{\alpha^2} < \frac{(1+k)^2c^2}{1-\alpha q}$, which is always satisfied in the upper-tail disclosure strategy because of (28). Hence, $\pi^*(Z, nd_x)$ increases with Z in this case. When $Z \geqslant \alpha\pi^*(Z, nd_x)$, based on the expression of $\pi^*(Z, nd_x)$ in Eq. (8), it is straightforward to see that it is independent of Z. Combining these two cases we know that $\pi^*(Z, nd_x)$ weakly increases with Z when $kq\alpha \leqslant 2$.

When $kq\alpha > 2$, the manager follows the lower-tail disclosure strategy of x for $Z < \underline{Z}$. In this case, taking the partial derivative of $\pi^*(Z, nd_x)$ in Eq. (7) with respect to Z yields

$$\frac{\partial \pi^*(Z, nd_x)}{\partial Z} = -\frac{\frac{kq}{kq\alpha-1}\frac{kqZ+(1+k)c}{kq\alpha-1} - q\left[\frac{2\frac{Z}{\alpha}+(1+k)c}{kq\alpha-1} + \frac{Z}{\alpha}\right]}{\sqrt{\left(\frac{kqZ+(1+k)c}{kq\alpha-1}\right)^2 - 2q\left[\frac{Z\left(\frac{Z}{\alpha}+(1+k)c\right)}{kq\alpha-1} + \frac{1}{2}\frac{Z^2}{\alpha}\right]}},$$

which is negative if and only if

$$\frac{k}{kq\alpha-1}\frac{kqZ+(1+k)c}{kq\alpha-1} - \left[\frac{2\frac{Z}{\alpha}+(1+k)c}{kq\alpha-1} + \frac{Z}{\alpha}\right] > 0.$$

Rewriting the above inequality yields

$$\left(\frac{k^2q\alpha}{(kq\alpha-1)^2} - \frac{2}{kq\alpha-1} - 1\right)\frac{Z}{\alpha} + \frac{(1+k)c}{kq\alpha-1}\left(\frac{k}{kq\alpha-1} - 1\right) = \frac{k^2\alpha q(1-\alpha q)+1}{(kq\alpha-1)^2}\frac{Z}{\alpha} + \frac{(1+k)c}{kq\alpha-1}\frac{k(1-\alpha q)+1}{kq\alpha-1} > 0,$$

which always holds because $1-\alpha q \geqslant 0$ and $kq\alpha - 1 > 0$. Hence, $\pi^*(Z, nd_x)$ decreases with Z when $Z < \underline{Z}$. When $kq\alpha > 2$ and $Z \geqslant \underline{Z}$, the manager again follows the upper-tail disclosure strategy of x. As shown before, $\pi^*(Z, nd_x)$ weakly increases with Z in this case.

Proof of Lemma 4. We analyse how the manager's expected utility changes with the non-disclosure price for a given disclosure decision d_Z of Z. Let π be shorthand notation for the non-disclosure price $\pi(d_Z, nd_x)$. Define $x' = \pi + (1+k)c$. When the upper-tail disclosure of x with $\pi \leqslant \frac{Z}{\alpha}$ arises, the manager's expected utility at $t = 0$ equals

$$\begin{aligned} &x'\int_0^{x'}(\pi + kx)\tfrac{1}{x'}dx + (1-x')\int_{x'}^1(1+k)(x-c)\tfrac{1}{1-x'}dx \\ &= kE[x] + \pi(\pi + (1+k)c) + \tfrac{1}{2} - \tfrac{1}{2}(\pi + (1+k)c)^2 - (1+k)c(1-\pi-(1+k)c) \\ &= \tfrac{1}{2}k + \tfrac{1}{2}(\pi + (1+k)c)^2 + \tfrac{1}{2} - (1+k)c. \end{aligned} \tag{32}$$

It is straightforward to see that the above expected utility increases with π.

For the upper-tail disclosure of x with $\pi > \frac{Z}{\alpha}$, the manager's expected utility equals

$$\begin{aligned} &\left(\pi - \frac{Z}{\alpha}\right)\int_0^{\pi-\frac{Z}{\alpha}}[\pi + kx - kq(\alpha(\pi-x)-Z)]\frac{1}{\pi-\frac{Z}{\alpha}}dx + \left((1+k)c + \frac{Z}{\alpha}\right)\int_{\pi-\frac{Z}{\alpha}}^{x'}(\pi+kx)\frac{1}{(1+k)c+\frac{Z}{\alpha}}dx \\ &\quad + (1-x')\int_{x'}^1(1+k)(x-c)\frac{1}{1-x'}dx = \frac{1}{2}k + \frac{1}{2}(\pi+(1+k)c)^2 - \frac{1}{2}kq\alpha\left(\pi - \frac{Z}{\alpha}\right)^2 + \frac{1}{2} - (1+k)c. \end{aligned} \tag{33}$$

To see how the manager's expected utility changes with π, one can derive that

$$\frac{\partial EU_m}{\partial \pi} = \pi + (1+k)c - kq\alpha\left(\pi - \frac{Z}{\alpha}\right) = (1-kq\alpha)\pi + (1+k)c + kqZ.$$

When $kq\alpha \leqslant 1$, the above condition is always positive. When $kq\alpha > 1$, the above condition remains to be positive because $\pi < \frac{kqZ+(1+k)c}{kq\alpha-1}$ always holds in this equilibrium. Hence, the manager's expected utility also increases with π in this case.

For the lower-tail disclosure of x, the manager's expected utility equals

$$\begin{aligned} &\left(\pi - \tfrac{kqZ+(1+k)c}{kq\alpha-1}\right)\int_0^{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}(1+k)(x-c)\tfrac{1}{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}dx + \left(\tfrac{kqZ+(1+k)c}{kq\alpha-1} - \tfrac{Z}{\alpha}\right)\int_{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}^{\pi-\frac{Z}{\alpha}}[\pi + kx - kq(\alpha(\pi-x)-Z)]\tfrac{1}{\frac{kqZ+(1+k)c}{kq\alpha-1}-\frac{Z}{\alpha}}dx \\ &\qquad + \left(1-\pi+\tfrac{Z}{\alpha}\right)\int_{\pi-\frac{Z}{\alpha}}^1(\pi+kx)\tfrac{1}{1-\pi+\frac{Z}{\alpha}}dx \\ &\qquad = \tfrac{1}{2}k + \tfrac{1}{2}\left[\tfrac{(kqZ+(1+k)c)^2}{kq\alpha-1} - kq\alpha\tfrac{Z^2}{\alpha^2}\right] + \pi(1-(1+k)c) - \tfrac{1}{2}\pi^2. \end{aligned} \tag{34}$$

Taking partial derivative with π, we have

$$\frac{\partial EU_m}{\partial \pi} = 1 - (1+k)c - \pi,$$

which is always negative because $\pi > 1-(1+k)c$, i.e, the manager prefers not to disclose $x = 1$ in a lower-tail disclosure strategy. Thus, the manager's expected utility decreases with π.

For the two-tail disclosure strategy, the manager's expected utility equals

$$\begin{aligned}&\left(\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}\right)\int_0^{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}(1+k)(x-c)\frac{1}{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}dx\\&+\left(\frac{kqZ+(1+k)c}{kq\alpha-1}-\frac{Z}{\alpha}\right)\int_{\pi-\frac{kqZ+(1+k)c}{kq\alpha-1}}^{\pi-\frac{Z}{\alpha}}[\pi+kx-kq(\alpha(\pi-x)-Z)]\frac{1}{\frac{kqZ+(1+k)c}{kq\alpha-1}-\frac{Z}{\alpha}}dx\\&+\left((1+k)c+\frac{Z}{\alpha}\right)\int_{\pi-\frac{Z}{\alpha}}^{x'}(\pi+kx)\frac{1}{(1+k)c+\frac{Z}{\alpha}}dx+(1-x')\int_{x'}^{1}(1+k)(x-c)\frac{1}{1-x'}dx\\&=\frac{1}{2}k+\frac{1}{2}\left[\frac{(kqZ+(1+k)c)^2}{kq\alpha-1}-kq\alpha\frac{Z^2}{\alpha^2}\right]+\frac{1}{2}(1-(1+k)c)^2.\end{aligned}\tag{35}$$

It is straightforward to see that the manager's expected utility is independent of π.

The expected utility from the two-tail disclosure of x is higher than the upper-tail disclosure of x with $\pi\leqslant\frac{Z}{\alpha}$ if and only if

$$\frac{1}{2}\left[\frac{(kqZ+(1+k)c)^2}{kq\alpha-1}-kq\alpha\frac{Z^2}{\alpha^2}\right]+\frac{1}{2}(1-(1+k)c)^2>\frac{1}{2}(\pi+(1+k)c)^2+\frac{1}{2}-(1+k)c,$$

which is equivalent to $(\pi+(1+k)c)^2<\frac{kq\alpha}{kq\alpha-1}\left((1+k)c+\frac{Z}{\alpha}\right)^2$. This condition always holds because $\pi\leqslant\frac{Z}{\alpha}$ and $\frac{kq\alpha}{kq\alpha-1}>1$. Hence, expected utility from the two-tail disclosure of x is higher than the upper-tail disclosure of x with $\pi\leqslant\frac{Z}{\alpha}$.

The expected utility from the two-tail disclosure of x is higher than the upper-tail disclosure of x with $\pi>\frac{Z}{\alpha}$ if and only if

$$\frac{1}{2}\left[\frac{(kqZ+(1+k)c)^2}{kq\alpha-1}-kq\alpha\frac{Z^2}{\alpha^2}\right]+\frac{1}{2}(1-(1+k)c)^2>\frac{1}{2}(\pi+(1+k)c)^2-\frac{1}{2}kq\alpha\left(\pi-\frac{Z}{\alpha}\right)^2+\frac{1}{2}-(1+k)c,$$

which is equivalent to $\frac{1}{2}((kq\alpha-1)\pi-kqZ-(1+k)c)^2>0$ and always holds. Hence, expected utility from the two-tail disclosure of x is higher than the upper-tail disclosure of x with $\pi>\frac{Z}{\alpha}$.

The expected utility from the two-tail disclosure of x is higher than the lower-tail disclosure of x if and only if

$$\frac{1}{2}(1-(1+k)c)^2>\pi(1-(1+k)c)-\frac{1}{2}\pi^2,$$

which is equivalent to $\frac{1}{2}(\pi-1+(1+k)c)^2>0$ and always holds. Hence, expected utility from the two-tail disclosure of x is higher than the lower-tail disclosure of x.

In summary, the above comparison shows that the two-tail disclosure of x yields the highest expected utility of the manager at $t=0$.

Proof of Lemma 5. We compare the expected utility of the manager to derive her optimal disclosure strategy d_Z. First, we show that $\underline{Z}\leqslant\underline{Z}_{nd}$. Recall that $\underline{Z}$ is the coverage limit where the best response disclosure strategies changes from lower-tail to upper-tail when Z is disclosed, i.e., $\pi^*(\underline{Z},nd_x)-\frac{kq\underline{Z}+(1+k)c}{kq\alpha-1}=0$. Similarly, $\underline{Z}_{nd}$ is the coverage limit where the best response disclosure strategy of x changes from lower-tail to upper-tail when Z is not disclosed, i.e., $\pi(nd_Z,nd_x)-\frac{kq\underline{Z}_{nd}+(1+k)c}{kq\alpha-1}=0$. Because $\pi(nd_Z,nd_x)>\pi^*(\underline{Z},nd_x)$, it follows that $\underline{Z}<\underline{Z}_{nd}$.

Consider $Z<\underline{Z}<\underline{Z}_{nd}$. We know that disclosure of Z results in a lower-tail disclosure strategy of x and that non-disclosure of Z results in a two-tail disclosure strategy of x. Lemma 4 implies that the manager's expected utility is the highest in the two-tail disclosure strategy of x. Hence, the manager chooses not to disclose Z.

Next, consider $\underline{Z}\leqslant Z<\underline{Z}_{nd}$. Disclosure of Z results in an upper-tail disclosure strategy of x and that non-disclosure of Z results in a two-tail disclosure strategy of x. By the same argument as above, Lemma 4 implies that the manager chooses non-disclosure of Z.

Finally, consider $Z\geqslant\underline{Z}_{nd}>\underline{Z}$. Both disclosure of Z and non-disclosure of Z results in an upper-tail disclosure strategy of x. Lemma 4 implies that the manager prefers disclosure of Z if and only if $\pi^*(Z,nd_x)>\pi(nd_Z,nd_x)$. While $\pi(nd_Z,nd_x)$ is independent of Z, Lemma 3 implies that $\pi^*(Z,nd_x)$ is (weakly) increasing in Z. Hence, there exist $Z^*\geqslant\underline{Z}_{nd}$ such that $\pi^*(Z,nd_x)>\pi(nd_Z,nd_x)$ for all $Z>Z^*$. Note that $Z^*=1$ if $\pi(nd_Z,nd_x)>\pi^*(1,nd_x)$ holds.

Proof of Proposition 1. To show that a full disclosure equilibrium of Z exists, assume that the out-of-equilibrium beliefs are skeptical, i.e., when the market observes non-disclosure of Z, the non-disclosure price $\pi^*(nd_Z,nd_x)=\min_Z\pi^*(Z,nd_x)$. When $kq\alpha\leqslant 2$, only an upper-tail disclosure strategy of x arises when Z is disclosed. Hence, $\min_Z\pi^*(Z,nd_x)=\pi^*(0,nd_x)=(1+k)c$. Then it holds that $\pi^*(nd_Z,nd_x)=(1+k)c\leqslant\frac{(1+k)c}{kq\alpha-1}$ so that there is only upper-tail disclosure of x when Z is not disclosed. In this case, Lemma 5 shows that the manager prefers disclosure of Z if and only if $\pi^*(Z,nd_x)>\pi^*(nd_Z,nd_x)$.

Lemma 3 shows that $\pi^*(Z,nd_x)$ weakly increases with Z in the upper-tail disclosure of x. Then for each $Z\in(0,1]$, the manager strictly prefers disclosure of Z because $\pi^*(Z,nd_x)>\pi^*(nd_Z,nd_x)$. For $Z=0$, the manager is indifferent between disclosing and not disclosing Z.

Proof of Proposition 2. To prove part (A), let $\pi^*(nd_Z, nd_x)$ be such that $\pi^*(\underline{Z}, nd_x) < \pi^*(nd_Z, nd_x) < \pi^*(1, nd_x)$. Together with $kq\alpha > 2$, the condition implies that $0 < \underline{Z} < \underline{Z}_{nd}$. Lemma 3 shows that $\pi^*(Z, nd_x)$ is weakly increasing in Z for $Z \geqslant \underline{Z}$. As $\pi^*(nd_Z, nd_x)$ is independent of Z, there exists $Z^* < 1$ such that $\pi^*(Z^*, nd_x) = \pi^*(nd_Z, nd_x)$. Lemma 5(B) then implies that nondisclosure of Z is preferred for all Z satisfying $\underline{Z} \leqslant Z \leqslant Z^*$. Lemma 5(A) implies that all $Z < \underline{Z}_{nd}$ are not disclosed. Hence, disclosure of Z is preferred for all $Z > Z^*$.

To prove part (B), let $\pi^*(nd_Z, nd_x)$ be such that $\pi^*(1, nd_x) \leqslant \pi^*(nd_Z, nd_x) < 1 - (1+k)c$. Lemma 5(A) shows that all $Z < \underline{Z}_{nd}$ are not disclosed. Because for $Z > \underline{Z}_{nd}$ the manager strictly prefers disclosure of Z over nondisclosure of Z if and only if $\pi^*(Z, nd_x) > \pi^*(nd_Z, nd_x)$, the condition $\pi^*(nd_Z, nd_x) \geqslant \pi^*(1, nd_x)$ implies $\pi^*(nd_Z, nd_x) \geqslant \pi^*(Z, nd_x)$ for all $Z \geqslant \underline{Z}_{nd}$. Hence, the manager chooses not to disclose Z for all $Z \in [0, 1]$.

Proof of Proposition 3. The proof shows that for each value of $Z \leqslant Z^*$ and $kq\alpha > 2$, the likelihood of disclosure of x is (weakly) higher with disclosure of Z than with nondisclosure of Z. Observe that a full nondisclosure of Z features $Z^* = 1$. Furthermore, as proved in Lemma 5, $0 < \underline{Z} \leqslant \underline{Z}_{nd}$.

Consider $Z \leqslant \underline{Z}$. When Z is disclosed, the disclosure of x is lower-tail and the likelihood of disclosing x equals $\pi^*(Z, nd_x) - \frac{kqZ+(1+k)c}{kq\alpha-1}$. When Z is not disclosed, the disclosure of x is two-tail and the likelihood of disclosing x equals $1 - (1+k)c - \frac{kqZ+(1+k)c}{kq\alpha-1}$. As $\pi^*(Z, nd_x) > 1 - (1+k)c$ when $Z \leqslant \underline{Z}$, disclosing Z increases the likelihood of disclosure of x. Next, consider $\underline{Z} < Z \leqslant \underline{Z}_{nd}$. When Z is disclosed, the disclosure of x is upper-tail and the likelihood of disclosing x equals $1 - \pi^*(Z, nd_x) - (1+k)c$. When Z is not disclosed, the disclosure of x is again two-tail and the likelihood of disclosing x is $1 - (1+k)c - \frac{kqZ+(1+k)c}{kq\alpha-1}$. Using that $Z > \underline{Z}$ implies $\pi^*(Z, nd_x) < \frac{kqZ+(1+k)c}{kq\alpha-1}$, it follows that the likelihood of disclosing x is higher with disclosure of Z than with nondisclosure of Z. Finally, consider $\underline{Z}_{nd} < Z \leqslant Z^*$. Then the disclosure of x is upper-tail both when Z is disclosed and not disclosed. The likelihood of disclosing x equals $1 - \pi^*(Z, nd_x) - (1+k)c$ when Z is disclosed and $1 - \pi^*(nd_Z, nd_x) - (1+k)c$ when Z is not disclosed. Because in equilibrium $\pi^*(nd_Z, nd_x) \geqslant \pi^*(Z, nd_x)$ for all $Z \leqslant Z^*$, it follows that the likelihood of disclosing x is lower when Z is not disclosed.

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