Self-Service or Staffed Checkout? Service Channel Strategies in Supermarkets

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Self-service technology (SST) has become increasingly prevalent across industries, especially in supermarkets, allowing consumers to complete transactions independently without waiting for an agent. While SST is often praised for reducing waiting times and enhancing convenience, thereby stimulating demand, its broader impact on service systems remains nuanced. This paper investigates how SST affects firm performance and consumer behavior in service queueing systems with heterogeneous demand. We develop a queueing-game-theoretic model featuring both traditional human-staffed channels and self-service channels. Our analysis reveals that consumers with low demand volumes tend to prefer SST, while those with high demand volumes opt for staffed service. In contrast, consumers with moderate demand volumes may exit the system entirely. Interestingly, a larger demand volume does not necessarily translate into higher firm profits, due to the strategic behavioral responses of consumers induced by SST. We further show that the firm's optimal pricing strategy for SST follows a U-shaped pattern with respect to the inconvenience level of consumer using SST, reaching its minimum at a moderate level of inconvenience. Although greater convenience generally improves firm profitability, it can paradoxically reduce the overall consumer surplus. A win-win outcome for both the firm and consumers emerges only when the inconvenience associated with SST is kept at a moderate level. Extending the analysis to long-term capacity decisions, we find that firms may strategically reduce staffed service capacity in response to SST adoption, potentially reversing SST's short-term benefits. Taken together, our study underscores that the effectiveness of SST hinges on both average demand and the level of consumer inconvenience. While SST can improve efficiency under high demand, it may lead to a lose-lose outcome when demand is low.

Key words: self-service technology; hybrid-service channel; firm profit; consumer surplus

1. Introduction

Queues are an inherent feature of many service systems, including banks, hospitals, supermarket checkouts, and call centers, as they often emerge when service capacity falls short of meeting demand fluctuations. Long waiting times can significantly damage consumer satisfaction. In fact, the Retail Industry Executive Survey (Stack 2020) reveals that sectors such as retail and restaurants could lose up to 75% of their consumers due to long wait times caused by insufficient service capacity. This issue has become a bottleneck in the growth of the service industry, driven primarily by rising labor costs. According to reports by Paycor (2022), labor expenses can account for up to 70% of total business costs, prompting businesses to seek ways to reduce these costs. One increasingly adopted approach is the implementation of self-service technology (SST).

In supermarket retailing, delays often occur when consumers must wait to check out after collecting their items. To mitigate this, two primary types of SST are commonly implemented: (i) scan-as-you-shop SST, where consumers scan items as they add them to their carts, and (ii) self-checkout kiosk SST, where consumers scan and pay for their items at designated kiosks after completing their shopping. Figure 1 illustrates examples of both methods in practice. The scan-as-you-shop SST allows consumers to use handheld scanners or mobile apps to scan items during their shopping process, enabling them to bypass traditional checkout queues entirely (Ivanov 2019). This approach has gained significant traction in major retail businesses, including Tesco ("Scan as You Shop"), Amazon ("Amazon Go"), Walmart ("Scan & Go"), and Kroger ("Scan, Bag, Go"). By contrast, self-checkout kiosks consist of centrally managed stations, where consumers independently complete their transactions. This traditional form of SST has become widely adopted as retailers aim to enhance operational efficiency. For instance, Circle K's installation of over 10,000 self-checkout machines demonstrates the industry's growing shift toward automation, promoting consumer autonomy while reducing reliance on labor.[1] Fueled by rapid advancements in digital technology, SST is becoming increasingly prevalent across many other service sectors, including banking, hospitality, fast-food industry, transportation, and entertainment (Tan and Netessine 2020). These systems facilitate the integration of digital solutions with physical operations, streamlining service processes, improving efficiency, minimizing consumer wait times, lowering labor costs, and boosting profitability (Redyref 2022).

Despite its advantages, the adoption of SST presents several notable challenges. One significant issue is the additional effort cost required from consumers. For instance, consumers may struggle with tasks such as accurately identifying products - like distinguishing between different types of apples in a supermarket - or properly scanning barcodes. These effort costs are particularly burdensome during the early stages of SST implementation, when the systems are not yet fully

[1] https://retailwire.com/discussion/circle-ks-new-self-checkouts-are-kicking-barcode-scanners-to-the-curb/.

Figure 1 **Two SST Implementations in Practice.**

(a) Self-scanning device (b) Self-checkout kiosk

optimized for user-friendliness (O'Neill 2022). That said, consumers must weigh the trade-off between reduced waiting times and the additional effort costs associated with using SST. From the operational perspective, implementing SST entails considerable costs for firms. Mobile app-based SST solutions typically require firms to pay subscription or maintenance fees to app platforms or service providers, while kiosk-based SST necessitates substantial investments in purchasing or leasing self-checkout equipment. These expenses can offset the efficiency gains from increased system throughput (Bomee 2019).

Despite the widespread adoption of SST in supermarket settings, the existing literature has yet to comprehensively examine its dual impacts. This study aims to fill this gap by systematically investigating the implications of SST adoption for both firms and consumers, offering new insights into the underlying trade-offs. In contrast to most existing models that assume consumers have homogeneous and normalized demand, our framework explicitly incorporates heterogeneity in purchasing amount. This feature captures the inherent variability in supermarket shopping behavior, enabling a more realistic and nuanced analysis. A key finding that emerges from this framework is a novel *dual-threshold* channel selection pattern: consumers with either small or large purchase volumes tend to choose self-service or staffed checkout, respectively, while those with moderate demand volumes opt out of the system entirely. This behavioral structure underscores the critical role of demand heterogeneity in shaping equilibrium channel usage. Building on this foundation, we proceed to address the following key research questions:

Q1. How should a firm adjust its pricing strategy in response to consumer behavior when SST is available as an alternative service channel?

Q2. How do the amount of demand volume and the convenience level of SST influence the firm's profitability and consumer surplus?

Q3. Does the introduction of SST consistently benefit both the firm and its consumers? If not, when should firms offer an SST option, and what trade-offs should they consider in the adoption decision?

To address these questions, we develop a queueing-game-theoretic model that captures a supermarket's interactions with delay- and price-sensitive consumers who exhibit heterogeneous product demands. We first analyze an agent-service system, where checkout services are exclusively provided by sales agents. In this setup, consumers decide whether to use the sales-agent channel, incurring waiting costs associated with queueing. Next, we focus our analysis on a hybrid-service system, where both sales agents and SST machines are available for checkout. Consumers in this system can choose between the sales-agent channel, the SST channel, or opt not to place an order. Consumers utilizing the SST channel incur effort costs associated with operating the system, whether through SST kiosks or handheld scanners, while the supermarket bears operating costs to maintain them. In both systems, the supermarket determines the product price, influencing consumer decisions and overall system dynamics. Our analysis aims to shed light on the strategic and operational implications of SST adoption within the supermarket industry. The main findings of this paper are summarized as follows.

Equilibrium strategy. We begin by analyzing consumers' equilibrium ordering behaviors in both the agent-service system (without SST) and the hybrid-service system (with SST). In the agent-service system, only high-demand consumers choose to place orders. However, the introduction of SST enables the firm to attract lower-demand consumers, resulting in equilibrium ordering behavior characterized by a dual-threshold strategy. Anticipating these consumer behaviors, the firm in the hybrid-service system adopts a non-monotonic pricing strategy that depends on the level of inconvenience associated with SST usage. Specifically, when SST inconvenience is low, the firm employs a *volume strategy*, reducing prices to capture demand from all consumer segments. Conversely, when SST inconvenience is high, the firm shifts to a *margin strategy*, moderately increasing prices to mitigate cannibalization within the SST channel.

Firm profitability. Our analysis confirms that greater SST convenience (i.e., lower inconvenience cost) consistently enhances firm profitability. This is because a lower inconvenience cost enables the firm to adopt a margin strategy by raising prices. In contrast, when SST is highly inconvenient, the firm is forced to lower prices, even below levels observed when SST is not offered. Moreover, while higher potential demand generally increases profitability in the agent-service system, this effect does not always hold in the hybrid system with SST. Interestingly, when SST is highly convenient and the magnitude of demand is moderate, an increase in potential demand can paradoxically reduce profit, underscoring the unique dynamics induced by SST adoption.

Consumer surplus. Contrary to conventional wisdom that lower SST effort costs would uniformly benefit consumers, our findings suggest otherwise, especially when inconvenience is minimal. In these cases, reducing SST inconvenience may actually reduce consumer surplus, potentially resulting in a surplus even lower than in the agent-service system. Our study further reveals that the consumer surplus is maximized at a moderate level of inconvenience. This outcome is driven by the firm's non-monotonic pricing strategy.

When to adopt SST? Our findings indicate that SST adoption is inefficient when potential consumer demand is low, as it consistently reduces both firm profit and consumer surplus—a lose-lose outcome regardless of inconvenience level. When SST is either extremely convenient or inconvenient, it benefits only one party, resulting in win-lose or lose-win scenarios. A win-win outcome arises only when demand is high and inconvenience remains moderate. We further investigate the firm's long-term capacity adjustment decisions by endogenizing service capacity. It indicates that the introduction of SST induces the firm to cut down service capacity, reversing its short-term benefit on consumers.

The remainder of the paper is organized as follows. Section §2 reviews the relevant literature. In §3, we present the model framework. Section §4 analyzes consumers' equilibrium ordering behavior and the firm's equilibrium pricing strategy in an agent-service system without SST, followed by a similar analysis for the hybrid-service system with SST in §5. The broader implications of SST adoption are examined in §6. Section §7 conducts several model extensions for robustness checks. Finally, §8 concludes the paper and suggests avenues for future research. Technical proofs and additional results are given in the appendix.

2. Literature Review

Our paper contributes to three streams of literature: queueing economics, multichannel service in queueing systems, and self-service technology.

2.1. Queueing economics models

Our work advances the queueing economics literature, which examines consumers' strategic behaviors in queueing systems. The seminal work by Naor (1969) analyzes how consumers rationally decide whether to join a system based on observable queue lengths. Later, Edelson and Hilderbrand (1975) extend this framework to unobservable queue settings. These foundational studies have inspired extensive research in queueing economics (see Hassin and Haviv (2003) and Hassin (2016) for reviews). Generally, consumers adopt a threshold-based equilibrium strategy when queue lengths are observable and a mixed strategy when they are not. For instance, Debo et al. (2012) explore a setting where consumers receive private signals about service quality in addition to observing queue lengths, leading to the existence of ”holes” equilibrium strategies, where consumers may choose to join or abstain based on the queue length and their private signals. Similarly, Hassin and Roet-Green (2018) study equilibrium behavior in systems with two servers, revealing that consumer decisions may deviate from threshold-type strategies, particularly in intermediate queue states. In contrast to these studies, our research focuses on a supermarket checkout system offering both traditional sales-agent services and SST options. By incorporating a queueing model to describe the sales-agent service process, we investigate how SST impacts consumer behavior and firm strategies within this service system.

2.2. Multichannel service in queueing systems

Our work also aligns with the growing literature on multichannel service in queueing systems. Ghosh et al. (2020) use a queueing model to study consumer behavior in omnichannel systems, revealing that app users adopt dual-threshold strategies, such as ordering online for moderate queues and opting for offline channels otherwise. Roet-Green and Yuan (2020) examine consumer joining decisions in systems where only physical store queues are visible, demonstrating that partially observable information can reduce congestion and enhance social welfare under certain conditions. Omnichannel service systems have been extensively explored across industries such as food delivery (Baron et al. (2022), Chen et al. (2022)) and retail (Roet-Green and Yuan (2020), Sun et al. (2024)). For example, Baron et al. (2022) find that offering an online pre-ordering option increases revenue but may reduce consumer utility and social welfare when both channels are used in equilibrium. Chen et al. (2022) analyze restaurant queueing systems, showing that delivery platforms do not always boost demand and can negatively impact social welfare. In retail, Sun et al. (2024) reveal that online exchange policies can inadvertently increase congestion, making them less effective than in-store exchanges in larger markets. Distinct from these studies, we examine a specific type of multichannel service where the additional channel is SST, a self-service process driven by consumers themselves rather than traditional service providers.

2.3. Self-service technology

Finally, our research contributes to the growing body of work on SST, which has transformed consumer-firm interactions and introduced new dynamics into multichannel service systems. SST adoption has been explored in various settings, including grocery stores Bhargavi (2016), restaurants Gao and Su (2018), and retail Gao and Su (2017a,b). Sampson and Froehle (2006) and Roels (2014) highlight that SST requires significant consumer effort as it involves a co-productive process. Roels (2014) further develops an analytical framework for optimizing co-production systems by showing how task standardization should guide both the degree of interaction between customers and providers. Karmarkar and Roels (2015) extend this perspective by framing service co-production as a three-stage process—production, sharing, and consumption—emphasizing that value realization depends on all three. During the COVID-19 pandemic, SST also demonstrated its value in enhancing supply chain flexibility Heiman et al. (2022). In the banking context, Xue et al. (2007) and Buell et al. (2010) show that self-service channel usage can enhance firm performance and customer retention, particularly when customers are efficient or face high switching costs. Field (2024) discusses how self-service technologies (SSTs) shift service tasks to customers and fundamentally reshape service process design across a range of sectors, including retail, banking, healthcare, and education. Meanwhile, Campbell and Frei (2010) provide evidence that increased utilization of online banking channels positively impacts market share, highlighting the potential benefits of SST adoption. At the same time, Scherer et al. (2015) and Saldanha et al. (2021) caution against fully replacing traditional services with SST, noting that high innovation in online services may diminish the positive effects of SST. While these works offer important insights into SST adoption and design, prior studies rarely consider how SST shapes the behaviors of strategic consumers, and none of them explicitly account for congestion or strategic customer behavior, which are central to our analysis.

Research on SST in queueing systems remains largely underexplored. Gao and Su (2018) illustrate that introducing online order-placing self-service technology in restaurants, which allows consumers to place orders via mobile apps before arriving, benefits both tech-savvy and conventional consumers. Notably, the integration of SST in this context does not lead to workforce reductions but rather enhances kitchen capacity, accelerating food preparation. Restaurants can also implement self-service kiosks, enabling consumers to order without direct interaction with employees. Their findings suggest that firms should adopt online self-order technology when consumers are highly sensitive to wait times, whereas offline self-order options are more suitable in other scenarios. Building on the growing adoption of mobile-order-and-pay applications, Kang et al. (2024) examine the efficiency of omnichannel service systems in coffee shops using two-stage queueing models. They reveal that mobile users, who bypass the ordering stage and proceed directly to food preparation, may inadvertently reduce system throughput if prioritization policies are poorly designed. While these studies primarily segment consumers based on their SST usage preferences and emphasize reducing wait times for tech-savvy users, our research offers a distinct perspective. We focus on the co-productive dynamics of self-checkout services, where consumers actively participate in the service process alongside self-checkout machines. Our analysis considers a heterogeneous consumer base with varying product demands and accounts for the effort costs incurred when using SST.

Our findings highlight that firms should adopt SST selectively, as its benefits are not universally distributed among stakeholders. By examining the interplay of consumer behavior, demand heterogeneity, and firm strategies, we provide actionable insights into the conditions under which SST adoption can be beneficial or detrimental.

3. The Model

We consider a service system operated by a supermarket. Notice that we primarily focus on supermarkets similar to Daiso and Dollarama[2], which offer products at prices that are generally comparable, with each product priced by the supermarket at P. Delay- and price-sensitive consumers arrive according to a Poisson process with arrival rate Λ. Consumers vary in the quantity of products they intend to purchase, referred to as their *purchasing amount* (PA). Specifically, an arbitrary consumer demands x units of the product, with x uniformly distributed over $[0, \bar{n}]$, i.e., $x \sim U[0, \bar{n}]$, where $\bar{n}$ denotes the maximum demand volume. Consumers have a common valuation V for a single unit of the product, which represents the perceived reward they derive from consuming that product. Therefore, for a consumer with demand x, the total reward of ordering is $x \cdot V$. Consumers who order products require checkout services, which can be provided by either the sales agents or by consumers themselves using SST. Throughout the paper, we use the pronoun "he" to refer to the service provider (i.e., the supermarket) and "she" to refer to the consumer.

When checking out with the sales agents, the times to process each product are *independent and identically distributed* (I.I.D.), following an exponential distribution with rate μ. This rate, μ, represents the service capacity of the sales agents. Consumers who seek service from sales agents form a queue, and their orders are processed following a *first-come first-served* (FCFS) discipline. Each delay-sensitive consumer incurs a waiting cost of h per unit time when queueing for the sales agents' service. We assume that the time required for agent-side checkout is proportional to the order quantity, and that the agent service imposes no additional disutility beyond this time cost. Accordingly, when a tagged consumer has N consumers ahead of her, her expected waiting cost is given by $h \cdot \sum_{i=1}^{N} x_i$, where each $x_i \sim U[0, \bar{n}]$ denotes the purchase quantity of the i-th consumer.

[2] Daiso and Dollarama are discount chains selling various household goods at fixed or near-fixed low prices, usually under a few dollars.

In this context, V represents the consumer's per-unit net utility after accounting for the time cost of agent-side item scanning.

In contrast, we assume that consumers incur no waiting time when choosing SST service, as each consumer is assigned a dedicated self-checkout machine.[3] However, consumers face an additional cost when using SST: the effort involved in self-scanning product barcodes, especially if they are not fully familiar with the operation of the machines or scanners. The effort cost for a consumer with demand volume x is assumed to be $e(x) = kx^2/2$, where k, referred to as the inconvenience parameter, measures the inconvenience cost of using SST. This convex specification captures the increasing cognitive and physical burden associated with scanning larger baskets. Empirical support for this assumption can be found in Huang and Nusrat (2024)[4], who show that self-checkout experiences become significantly more burdensome—and reduce consumer loyalty—when the number of purchased items exceeds approximately 15. The inconvenience cost form is commonly adopted in the extant SST literature to capture a convexly increasing effort cost with service demand (see, e.g., Roels 2014). Moreover, when the supermarket decides to provide SST as a service option, it incurs an operating cost per unit of time, denoted by $\mathcal{C}$, which can be regarded as the rental or maintenance fee for the SST kiosks and scanners.

In the human-agent supermarket model, where checkout services are exclusively provided by sales agents, each consumer decides whether to shop based on their purchasing amount and the anticipated average waiting time.[5] In the hybrid-service supermarket model, which includes both sales-agent and SST checkout options, consumers who choose to shop must also decide between using the sales-agent cashier or the SST machine. We define the fractions of consumers utilizing checkout services from sales agents and SST as q_A and q_T respectively, such that $q_A + q_T \leq 1$. The remaining fraction $1 - q_A - q_T$ represents consumers who choose to balk. The parameters $V, \mathcal{C}, h, k, \bar{n}, \Lambda, \mu$, as well as the distribution of consumers' demand, are common knowledge. For clarity, all relevant notations are summarized in Table 1.

To assess the impact of SST comprehensively, we begin in Section 4 by analyzing a benchmark setting in which only staffed checkout (i.e., sales-agent service) is available. Section 5 then introduces a hybrid system that offers both staffed and self-service channels. By comparing equilibrium outcomes across these two settings, we identify how the introduction of SST alters consumers' ordering behavior and channel choices. Section 6 further examines the effects of SST on

[3] This assumption is particularly realistic for handheld scanners, where product scanning occurs during shopping. For kiosk-based SST, we assume sufficient machine availability to eliminate queueing delays.

[4] `https://www.lebow.drexel.edu/news/does-self-checkout-impact-grocery-store-loyalty`

[5] This assumption reflects the grocery retail context, where service times vary widely across customers type. In contrast, in settings like ticketing machines or airport kiosks with uniform service times, queue visibility is more informative.

key performance metrics, including optimal pricing, system throughput, average order size, firm profit, and consumer surplus. In Section 7.1, we extend the model by endogenizing service capacity as a long-term decision, allowing the firm to jointly determine optimal pricing and capacity levels. Finally, Section 7.2 considers an extension with heterogeneous products, where consumer demands are correlated across product types. Throughout the paper, we use the subscripts "A" and "H" to refer to the agent-service and hybrid-service systems, respectively, for ease of notation.[6]

Table 1 Glossary of Main Notation

Symbol	Description
P	Price of a single product
q_A/q_T	Fractions of consumers using agent/SST services
U_A/U_T	Utility of consumers using agent/SST services
Λ	Consumer arrival rate
λ	Total order arrival rate
μ	Processing rate of orders by sales agents
V	Value of an individual order to consumers
h	Waiting cost per unit time
k	Inconvenience parameter for SST users
$e(\cdot)$	Effort cost function for a consumer with demand volume
x	Consumer demand, uniformly distributed over $[0, \bar{n}]$
$\bar{n}$	Upper bound of consumer demand
C	Operating cost for the supermarket to provide SST

4. Agent-Service Model

In this section, we consider the agent-service system, where only sales-agent service is provided by the supermarket. In this case, we first analyze consumers' equilibrium ordering behavior and then derive the supermarket's optimal pricing strategy.

4.1. Equilibrium Ordering Behavior

We first study consumers' equilibrium ordering behavior. Denote the total order arrival rate (TOAR) - the number of products that need to be processed (i.e., scanned) per time unit - as λ. All consumers wait in the check-out queue which is serviced by sales agents. Given λ and μ such that $\lambda < \mu$, the expected queueing time is $\mathbb{E}[W] = \lambda/(\mu(\mu - \lambda))$, which is increasing in λ. For a given TOAR λ, a tagged consumer with demand $x > 0$ has a utility (if she places an order)

$$U_A(x) = (V - P)x - h\mathbb{E}[W] = (V - P)x - h \cdot \frac{\lambda}{\mu(\mu - \lambda)}. \tag{1}$$

[6] We also consider the SST-only scenario (denoted by "T") in Section 7.1. However, it is excluded from the main model with exogenous capacity, as it would leave the existing agents idle—an outcome that is clearly suboptimal.

Since $U_A(x)$ is strictly increases in x and $U_A(0) < 0$, there must exist a threshold $\bar{x}_A \in [0, \bar{n}]$ such that, in equilibrium, consumers will choose to place orders if and only if their demand is no less than $\bar{x}_A$, which is given by

$$\bar{x}_A \equiv \min\{x > 0 | U_A(x) \geq 0\}.$$

To compute $\bar{x}_A$, we note that TOAR can be computed as $\lambda = \Lambda \int_{\bar{x}_A}^{\bar{n}} (t/\bar{n}) dt = \Lambda(\bar{n}^2 - \bar{x}_A^2)/(2\bar{n})$. Combined with Equation (1), we know that $\bar{x}_A$ uniquely solves the equation

$$U_A(\bar{x}_A) = (V - P)\bar{x}_A - \frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)/(2\bar{n})}{\mu\,[\mu - \Lambda(\bar{n}^2 - \bar{x}_A^2)/(2\bar{n})]} = 0. \tag{2}$$

We are now ready to state consumers' equilibrium ordering behavior in the agent-service system.

LEMMA 1 **(Equilibrium Ordering Behavior in the Agent-Service System)**. *In the agent-service system, for any price P, there exists a threshold $\bar{x}_A$ such that consumers will place orders if and only if their demand is above this threshold (i.e., $x \geq \bar{x}_A$), where $\bar{x}_A$ uniquely solves Equation* (2)*. Moreover, $\bar{x}_A(P)$ is convexly increasing in P.*

Lemma 1 characterizes the equilibrium ordering behavior of consumers in the agent-service system. As shown in Lemma 1, consumers will opt in only when their demand is sufficiently large. This result is consistent with common intuition: consumers with higher product demand are more willing to visit the supermarket while those with lower product demand are less inclined to visit the supermarket. Notice that the consumers' reward gain from ordering increases with their demand x, and all consumers incur the same expected waiting cost for checkout, which can be estimated based on their previous experiences. Thus, a consumer with an extremely low demand will balk, as the reward from a low demand is insufficient to cover the expected waiting cost. Moreover, the equilibrium cutoff $\bar{x}_A(P)$ can be regarded as an ordering threshold for consumers, determined by the price set by the supermarket. Lemma 1 also shows that the equilibrium ordering threshold $\bar{x}_A(P)$ increases as the price P increases, implying that fewer consumers choose to opt in as the price increases. This is because a higher price reduces the net benefit of ordering each unit of the product ($V - P$), as a result, consumers require a higher demand x to generate enough benefit $(V - P) \cdot x$ to cover their waiting costs.

4.2. Optimal Pricing Strategy

Next, we study the supermarket's optimal pricing strategy, taking into account consumers' equilibrium ordering behavior in the agent-service system. By Lemma 1, we derive the equilibrium system throughput for the agent-service system as follows:

$$TH_A(P) = \Lambda \int_{\bar{x}_A(P)}^{\bar{n}} \frac{t}{\bar{n}} dt = \frac{\Lambda(\bar{n}^2 - \bar{x}_A(P)^2)}{2\bar{n}}. \tag{3}$$

Denote by $\Pi_A(P)$ the profit per unit of time for the supermarket when the price is P. The supermarket's optimization problem is given as follows:

$$\max_{P} \quad \Pi_A(P) = P \cdot TH_A(P), \qquad (4)$$
$$s.t. \;\; \bar{x}_A(P) \text{ uniquely solves (2).}$$

By solving Equation (4), we can determine the supermarket's optimal pricing strategy in the agent-service system, as characterized by the following proposition.

PROPOSITION 1 **(Optimal Pricing in the Agent-Service System)**. *In the agent-service system, the supermarket's optimal price, P_A^*, uniquely solves the equation*

$$F(P) \equiv \bar{n}^2 - \bar{x}_A^2(P) - 2P\bar{x}_A(P) \cdot \frac{\partial \bar{x}_A(P)}{\partial P} = 0, \qquad (5)$$

where

$$\frac{\partial \bar{x}_A(P)}{\partial P} = \frac{\mu}{h\Lambda} \cdot \frac{\bar{x}_A^2[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2(P))]^2}{[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2(P))](\bar{n}^2 + \bar{x}_A^2(P)) + 2(\bar{n}^2 - \bar{x}_A^2(P))\Lambda\bar{x}_A^2(P)}, \qquad (6)$$

and the supermarket's optimal profit is

$$\Pi_A^* = \Pi_A(P_A^*) = \Lambda(\bar{n}^2 - \bar{x}_A^2(P_A^*))P_A^*/(2\bar{n}). \qquad (7)$$

Moreover, P_A^* decreases (resp., increases) in $\bar{n}$ when $\bar{n}$ is sufficiently small (resp., large), and Π_A^* consistently increases in $\bar{n}$.

Proposition 1 identifies the optimal pricing strategy for the supermarket in the agent-service system, revealing a non-monotonic relationship between the optimal price and consumers' potential demand volume $\bar{n}$. Specifically, when potential demand is low: The supermarket lowers prices as potential demand increases. In this scenario, the overall order volume is minimal, and the service capacity of sales agents significantly exceeds the potential consumer demand. With ample capacity to accommodate all orders, the supermarket is incentivized to reduce prices to attract consumers with lower *willingness to pay* (WTP). This strategy maximizes the utilization of service capacity, ensuring that resources are used efficiently.

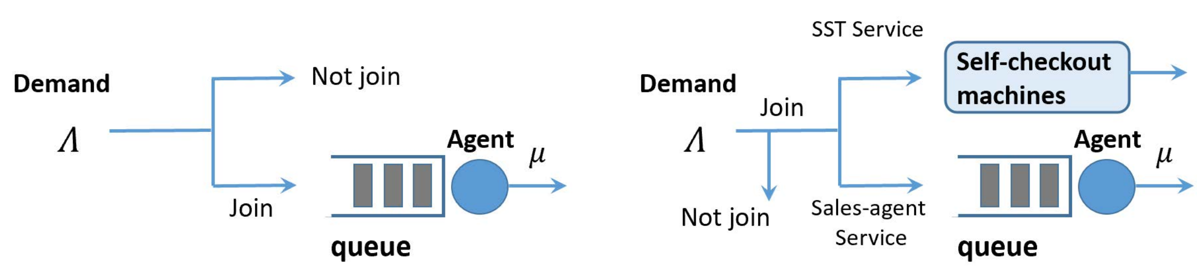
When potential demand is high, the supermarket raises prices as potential demand increases. Here, the service capacity of sales agents becomes relatively constrained, leading to longer waiting times for consumers. The supermarket, unable to accommodate a surge in consumer arrivals, strategically increases prices to manage demand levels. By doing so, it not only reduces the strain on service capacity but also improves profit margins by capturing higher revenue from consumers who are willing to pay the elevated price. This pricing adjustment balances demand management with profitability, prioritizing margins over system throughput. This non-monotonic pricing strategy highlights the supermarket's dual objectives of optimizing resource utilization and maximizing profitability under varying demand conditions.

Furthermore, our analysis reveals that the supermarket consistently benefits from an increase in consumers' potential demand. This aligns with common intuition, as higher potential demand typically translates into a greater volume of orders, thereby enhancing revenue opportunities. Additionally, the supermarket can further optimize its profit by strategically adjusting prices to align with the changing demand dynamics, ensuring that both throughput and profitability are maximized.

5. Hybrid-Service System

In this section, we analyze the hybrid-service system, where the supermarket offers both sales-agent and SST services. Refer to Figure 2 for an illustration of the two-process flow. Parallel to our examination of the agent-service system, we first characterize consumers' equilibrium ordering behavior in this dual-service setup. We then investigate the supermarket's optimal pricing strategy to maximize profitability in the hybrid-service environment.

Figure 2 Agent-Service vs. Hybrid-Service.



(a) Agent-service system

(b) Hybrid-service system

5.1. Equilibrium Ordering Behavior

We first study consumers' equilibrium ordering behavior. Consider a tagged consumer with demand x. If she chooses the sales-agent service, her expected utility $U_A(x)$ is given by (1), and if she chooses the SST service, her expected utility is

$$U_T(x) = (V - P)x - \frac{1}{2}kx^2. \tag{8}$$

Recall that if a consumer opts for the sales-agent service, her expected utility, $U_A(x)$, is influenced by the overall congestion resulting from the ordering behavior of other consumers who also choose the sales-agent service. Differently, if a consumer chooses the SST service, her expected utility is independent of the ordering behavior of other consumers. Moreover, in equilibrium, a consumer with demand x chooses the SST service if and only if $U_T(x) \geq \max\{U_A(x), 0\}$. Likewise, she chooses the sales-agent service if and only if $U_A(x) \geq \max\{U_T(x), 0\}$. One can verify that $U_T(x)$ is concave in x and $U_T(0) = 0$. Specifically, $U_T(x)$ increasing in x when $x \in \left[0, \frac{V-P}{k}\right]$ and is decreasing in x when $x \in \left(\frac{V-P}{k}, \bar{n}\right]$. Thus, we have $U_T(x) \geq 0$ if and only if $x \in \left[0, \min\{\frac{2(V-P)}{k}, \bar{n}\}\right]$.

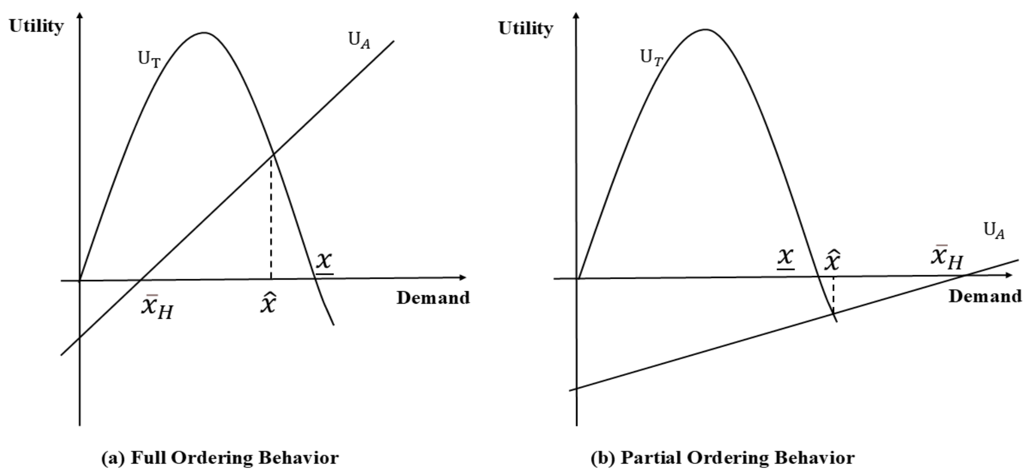
Given TOAR λ, we plot consumers' utility functions in the two service channels (i.e., $U_A(x)$ and $U_T(x)$) with respect to different consumer demand x in Figure 3. In Figure 3, we denote the intersection points of the two functions with the x-axis as $\bar{x}_H(P)$ and $\underline{x}$, respectively. In particular, $\bar{x}_H(P)$ uniquely solves (2) thus $\bar{x}_H(P) = \bar{x}_A(P)$ and $\underline{x}(P) = 2(V-P)/k$. Moreover, the two utility functions intersect at $\hat{x}$. As Figure 3 indicates, there are two cases depending on the relationship between $\bar{x}_H$ (the demand threshold for consumers willing to choose sales-agent service) and $\underline{x}(P)$ (the demand threshold for consumers willing to choose SST service): in case (a), $\bar{x}_H(P) < \underline{x}(P)$, and all consumers either place orders through sales-agent service ($x \leq \hat{x}$) or SST service ($x > \hat{x}$), leading a full market coverage; in case (b), $\bar{x}_H(P) > \underline{x}(P)$, and consumers whose demand falls between the two thresholds ($\underline{x}(P) < x < \bar{x}_H(P)$) balk, leading a partial market coverage.

Since $U_A(x)$ is increasing in x and $U_T(x)$ is concave in x, it follows that $U_T(x) - U_A(x)$ is concave in x and $U_T(0) - U_A(0) > 0$. Thus, the two utility functions will cross at most once over $x \geq 0$, and we denote the unique intersection point as $\hat{x}$. One can verify that $U_A(x) < U_T(x)$ if and only if $x \in [0, \hat{x})$. That is, $\hat{x}$ uniquely solves $U_A(x) = U_T(x)$, which is equivalent to

$$h \cdot \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1}{2}kx^2, \tag{9}$$

where $\lambda = \Lambda \cdot \frac{\bar{n}+x}{2} \cdot \frac{\bar{n}-x}{\bar{n}}$. The expression of λ is derived since consumers prefer to choose the sales-agent service when $U_A(x) \geq \max\{U_T(x), 0\}$ for $x \geq \hat{x}$. In addition, the left (resp., right) hand side of Equation (9) is decreasing (resp., increasing) with x, which guarantees the uniqueness of $\hat{x}$.

Figure 3 Consumers' Utility Functions in the Hybrid-Service Model.



The following lemma characterizes consumers' equilibrium ordering behavior in the hybrid-service system.

LEMMA 2 **(Equilibrium Ordering Behavior in the Hybrid-Service System)**. *In the hybrid-service system, there exists a threshold for the inconvenience parameter, $\bar{k} = 2(V-P)/\bar{x}_H(P)$, such that consumers' equilibrium ordering behavior is given as follows:*

(1) **High Convenience.** *When $k \le \bar{k}$, consumers prefer SST if their demand $x \in [0, \hat{x})$, and prefer the sales-agent service if their demand $x \in [\hat{x}, \bar{n}]$, where*

$$\hat{x} = \sqrt{[\sqrt{8hk\Lambda^2\mu\bar{n}^2 + (2h\Lambda + k\mu\bar{n}(2\mu - \Lambda\bar{n}))^2} - 2h\Lambda + k\mu\bar{n}(\Lambda\bar{n} - 2\mu)]/2k\Lambda\mu}.$$

(2) **Low Convenience.** *When $k > \bar{k}$, consumers prefer SST if their demand $x \in [0, \underline{x}(P)]$, opt out if their demand $x \in (\underline{x}(P), \bar{x}_H(P)]$, and choose the sales-agent service if their demand $x \in (\bar{x}_H(P), \bar{n}]$, where $\underline{x}(P) = 2(V-P)/k$ and $\bar{x}_H(P)$ uniquely solves Equation (2).*

Lemma 2 characterizes the equilibrium ordering behavior of consumers in the hybrid-service system. A comparison with Lemma 1 reveals that the introduction of SST attracts consumers with relatively low service demand who would otherwise opt out of placing orders. These consumers are drawn to the SST option because it significantly reduces waiting time, which is a key factor in their decision-making process. Although SST usage involves an effort cost for consumers to self-checkout, this cost remains relatively minor for those with low service demand. Consequently, the convenience of reduced waiting time outweighs the effort cost, making SST an appealing choice for these consumers. However, as Lemma 2 highlights, SST is less attractive to consumers with high service demand. For these consumers, the effort cost $e(x)$, which increases with the number of products x, becomes a significant deterrent. The cumulative effort cost associated with using SST machines surpasses the waiting cost of the sales-agent service, leading these consumers to prefer checking out through sales agents. Thus, the choice between SST and sales-agent service is determined by a trade-off between waiting costs and effort costs, depending on the consumer's demand level.

In addition, it is worth noticing that the inconvenience parameter k also plays a critical role in determining consumers' equilibrium ordering behavior as well as the market coverage. We have verified that consumers with sufficiently high demand (with $x \geq \bar{x}_A(P) = \bar{x}_H(P)$) place orders in the agent-service system, while low-demand consumers (with $x < \bar{x}_A(P) = \bar{x}_H(P)$) never use the sales-agent service, regardless of whether the SST channel is available. Specifically, as shown in Lemma 2(1), when the SST inconvenience parameter k is small (i.e., SST is less inconvenient), the effort cost for using SST machines becomes negligible, making the SST service highly attractive to consumers, especially those with low demand.

The maximum demand of consumers willing to place orders and choose the SST (i.e., $\underline{x}(P)$) currently exceeds the minimal demand of consumers who choose sales-agent service $\bar{x}_H(P)$. Consequently, all consumers who would opt out in the agent-service system choose to use the SST service in the hybrid-service system. Additionally, some consumers who would have chosen the sales-agent service now prefer to use SST, resulting in full market coverage.

As established in Lemma 2, when the use of SST is highly inconvenient (i.e., when k is large), not all consumers place orders, resulting in partial market coverage. In this scenario, consumers with moderate service demand face a dilemma: the effort costs of using SST machines become significant, while their moderate demand makes enduring long queues for the sales-agent service unjustifiable. Consequently, these consumers opt out, leaving the market underserved. When k is large, the SST channel primarily attracts low-demand consumers, but the maximum demand of consumers willing to use the SST service (i.e., $\underline{x}(P)$) falls short of the overall ordering threshold $\bar{x}_H(P)$. As a result, only consumers with either very low or very high service demand choose to participate in the system. Specifically, low-demand consumers prefer the SST service to avoid lengthy waiting times, while high-demand consumers opt for the sales-agent service to mitigate the substantial effort costs associated with SST usage. This segmentation underscores the critical role of effort cost in shaping consumers' channel preferences and market participation.

5.2. Optimal Pricing Strategy

Next, we study the supermarket's optimal pricing strategy, in the hybrid-service model. We first characterize the equilibrium system throughput.

LEMMA 3 **(Equilibrium System Throughput in the Hybrid-Service Case)**. *In the hybrid-service system, the equilibrium system throughput, $TH_H(P)$, is given as follows:*

(1) *When $k \le \bar{k}$, $TH_H(P) = \Lambda \bar{n}/2$.*

(2) *When $k > \bar{k}$, $TH_H(P) = \Lambda(\bar{n}^2 - \bar{x}_H^2(P) + \underline{x}^2(P))/(2\bar{n})$.*

Denote by $\Pi_H(P)$ the profit per unit time for the supermarket when the price is P. The supermarket's optimization problem is given as follows:

$$\begin{aligned} \max_{P} \quad & \Pi_H(P) = P \cdot TH_H(P) - \mathcal{C}, \\ s.t. \quad & \bar{x}_H(P) \text{ uniquely solves (2)}, \end{aligned} \tag{10}$$

where $TH_H(P)$ is given by Lemma 3. By solving Equation (EC.26), we can determine the supermarket's optimal pricing strategy in the hybrid-service system, as characterized by the following proposition.

In the following, we use $\bar{x}_H$ and $\underline{x}$ to denote the ordering threshold under endogenous pricing.

PROPOSITION 2 **(Optimal Pricing Strategy in the Hybrid-Service Case)**. *In the hybrid-service system, there exists a threshold on inconvenience parameter, $\hat{k}$, such that the supermarket's optimal pricing strategy is given as follows:*

(1) **High Convenience.** *When $k \le \hat{k}$, the consumer market is fully covered. The supermarket's optimal price is $P_H^* = \bar{P}$, which is decreasing in k, and the supermarket's optimal profit is $\Pi_H^* = \Lambda \bar{n} \bar{P}/2 - \mathcal{C}$, which is decreasing in k, where $\bar{P}$ uniquely solves*

$$k\bar{x}_H(P) = 2(V - P). \tag{11}$$

(2) **Low Convenience.** *When $k > \hat{k}$, the consumer market is partially covered. The supermarket's optimal price is $P_H^* = \hat{P}$, which is increasing in k, and the supermarket's optimal profit is $\Pi_H^* = \Lambda \hat{P}(\bar{n}^2 - \bar{x}_H^2 + \underline{x}^2)/(2\bar{n}) - \mathcal{C}$, which is decreasing in k, where $\hat{P}$ uniquely solves*

$$G(P) \equiv \bar{n}^2 + \frac{4(V-P)(V-3P)}{k^2} - \bar{x}_H^2(P) - 2P\bar{x}_H(P) \cdot \frac{\partial \bar{x}_H(P)}{\partial P} = 0. \tag{12}$$

Moreover, $\hat{k}$ uniquely solves $G(\bar{P}(\hat{k})) = 0$.

Proposition 2 identifies the optimal pricing strategy for the supermarket in the hybrid-service system and highlights its critical dependence on k. When the SST inconvenience parameter k is small (i.e., SST is less inconvenient), the minimal effort cost associated with using SST machines makes them highly appealing to low-demand consumers who would otherwise opt out in an agent-service system. These consumers adopt SST in the hybrid-service model, resulting in full market coverage. A smaller k reflects a stronger consumer preference for SST, facilitating higher order volumes and improved efficiency.

As k increases, indicating higher effort costs for using SST, consumers with higher demand levels tend to switch to sales-agent services. This shift increases the load on the sales-agent channel, creating a bottleneck and limiting the system's overall efficiency and profitability. In contrast, SST users are less affected by congestion but are more price-sensitive. By strategically lowering prices, the supermarket can incentivize consumers to shift from sales agents back to SST, relieving congestion in the sales-agent channel and optimizing system performance. To maintain full market coverage as k rises, the supermarket adopts a volume strategy, setting lower prices to counterbalance the increased effort cost associated with SST usage. This approach ensures sustained consumer engagement, maximizes service efficiency, and preserves overall profitability despite rising SST effort costs.

On the other hand, when the use of SST is highly inconvenient (i.e., when k is large), the supermarket primarily attracts consumers with extremely low demand. Consumers with higher demand prefer the sales-agent service, while those with moderate demand may opt out entirely, as neither the waiting cost associated with the sales-agent channel nor the effort cost of SST is sufficiently outweighed by the benefits of ordering. As a result, achieving full market coverage becomes challenging, leading to only partial coverage. Moreover, as k continues to increase and SST becomes less attractive, price reductions lose their effectiveness in drawing consumers toward SST. In this scenario, the supermarket may pivot to a strategy focused on raising prices to encourage consumers to rely more heavily on the sales-agent service rather than attempting to stimulate demand through SST. This approach aligns with a margin strategy, which prioritizes maximizing profitability through higher profit margins instead of volume expansion. As illustrated in Figure 4, the optimal price initially decreases with k and then begins to rise for any fixed level of potential demand. Notably, the supermarket charges the lowest price when k is moderate (i.e., when $k = \hat{k}$). In other words, the most conservative pricing strategy is adopted when the inconvenience of SST is neither excessively high nor negligibly low.

Furthermore, Proposition 2 reveals that the supermarket can consistently achieve higher profits as the convenience of using SST increases (i.e., as k decreases). This result aligns with intuition: as k decreases, the effort cost associated with using SST becomes less significant, making SST more appealing to a broader range of consumers. As more consumers opt for SST at checkout, the system experiences a higher volume of transactions, leading to increased throughput. This improved operational efficiency allows the supermarket to capitalize on the expanded demand, ultimately enhancing overall profitability.

Figure 4(b) reveals a surprising insight: in the hybrid-service system, the supermarket may not always benefit from higher potential consumer demand ($\bar{n}$). This contrasts with the results from the agent-service system, where the supermarket's optimal profit consistently increases with $\bar{n}$ (see Proposition 1). Specifically, when k is moderate (e.g., $k = 2$), we observe that $\Pi_H^*|_{\bar{n}=5} < \Pi_H^*|_{\bar{n}=4} < \Pi_H^*|_{\bar{n}=3}$, indicating that an increase in potential demand can actually reduce the supermarket's profit. We provide explanations for this counterintuitive result below.

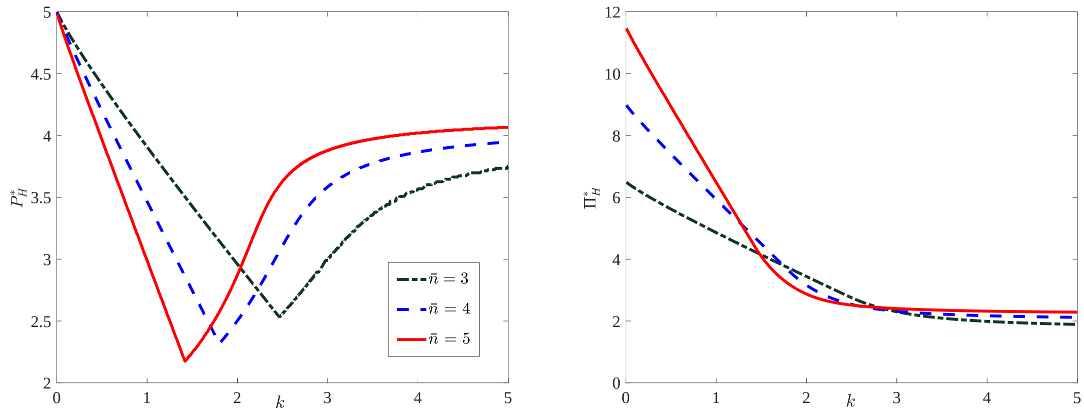
Similar to the agent-service scenario, when the potential demand is low, the supermarket has an incentive to lower prices to attract more consumers and thus maximize the utilization of service capacity. In this case, higher demand scenarios lead to more extensive use of service capacity, thereby generating greater profits. Conversely, when demand is high, actual demand is limited by the sales-agent capacity, leading to similar order volumes across different potential demand levels. Recalling the agent-service scenario, as $\bar{n}$ increases, the supermarket raises prices to alleviate service pressure, resulting in higher profit margins and increased profits. When demand is moderate, the supermarket charges the lowest price, achieving the highest throughput for the SST channel. As $\bar{n}$ continues to rise, the supermarket opts to raise prices. It is important to note that when the SST inconvenience is relatively low, consumer demand for SST becomes more sensitive to price changes. In such cases, as $\bar{n}$ increases, the throughput of the SST channel decreases, leading to a reduction in profit through this channel. Although an increase in $\bar{n}$ also enhances the throughput of the sales-agent channel, this is somewhat constrained by its limited service capacity. The divergent impacts of the two service channels on profit growth *may* result in an overall decrease in profit. This observation prompts a formal investigation into the impact of $\bar{n}$ on the supermarket's optimal profit. Theoretical findings are presented in Proposition 3.

PROPOSITION 3 **(The Impact of $\bar{n}$)**. *The impact of potential demand $\bar{n}$ depends on the inconvenience parameter k:*

(1) When k is large, the optimal profit Π_H^* increases with $\bar{n}$;

(2) When k is small, there exist two thresholds $\bar{n}_a$ and $\bar{n}_b$ with $\bar{n}_a \leq \bar{n}_b$ such that the optimal profit Π_H^* decreases with $\bar{n}$ for $\bar{n} \in [\bar{n}_a, \bar{n}_b]$ and increases with $\bar{n}$ otherwise.

Figure 4 **The Supermarket's Optimal Price and Profit as k Varies.**



(a) Optimal Price (b) Optimal Profit

Note. $V=5, \Lambda=1, \mu=1, h=1, \mathcal{C}=1$

Proposition 3 confirms that the optimal profit does not always increase with $\bar{n}$. This result suggests that higher potential demand improves the supermarket's profit only when it is extremely low or high, consistent with the agent-service scenario. However, when potential demand is at a non-extreme level, the supermarket may be worse off as $\bar{n}$ increases.

6. Agent Service vs. Hybrid Service

In this section, we study the impact of SST by comparing the outcomes of the profit-maximization problem in the agent-service system and the hybrid-service system. Specifically, we examine how the provision of the SST service influences the supermarket's optimal price, system throughput, average ordering volume, optimal profit, and consumer surplus. Throughout this analysis, we use $\bar{x}_A$, $\bar{x}_H$, and $\underline{x}$ to represent the ordering thresholds in the agent-service system, the hybrid-service system, and for the sales-agent service under the optimal pricing strategy, respectively.

6.1. Optimal Price

We first study how the adoption of SST affect the supermarket's optimal pricing strategy.

THEOREM 1 **(Optimal Price)**. *There exists a threshold $\underline{k}$ such that adopting SST improves the supermarket's optimal price (i.e., $P_H^* > P_A^*$) if and only if $k < \underline{k}$, where $\underline{k}$ uniquely solves equation $P_A^* = \bar{P}(k)$.*

Theorem 1 demonstrates that when the SST inconvenience parameter k is below the threshold $\underline{k}$, the provision of the SST service enables the supermarket to charge a higher price. In this scenario, the consumer market is fully covered when SST is provided, whereas it is only partially covered without SST (see Propositions 1 and 2). The supermarket raises prices compared to the agent-service system for the following two key reasons. On the one hand, providing SST improves the system throughput by capturing low-demand consumers who opt in for the SST service. On the other hand, introducing SST attracts moderate-demand consumers who would have chosen the sales-agent service to switch to SST, thereby raising the ordering threshold for selecting the sales-agent service (see Figure 3 (a), where this threshold raises from $\bar{x}_H$ to $\hat{x}$). In the case of a low inconvenience parameter, even with a higher price, providing SST can still attract all consumers who would have opted out in the agent-service system, enabling the SST channel to offset the throughput reduction from the sales-agent channel. This allows the market to achieve full coverage and further enhance profits. Consequently, when SST is easy to use, the profit-maximizing supermarket could charge a higher price to extract more surplus from consumers. Specifically, when the inconvenience parameter is close to zero, almost all consumers choose SST due to the negligible effort cost, prompting the supermarket to set prices that closely approach V to maximize their extraction of consumer rewards.

However, when SST use is inconvenient (i.e., when k is large), the supermarket charges a lower price with SST than without it. In this scenario, the effort cost of using SST is high, leading the increase in k to shift consumers toward the sales-agent service. This shift puts upward pressure on the sales-agent channel's arrivals, and its limited service capacity becomes a bottleneck in handling the growing demand. Consequently, continuing to charge a higher price than in the agent-service system results in demand shifting from SST to the sales-agent channel, ultimately leading to lost sales. To mitigate the cannibalization effect on the SST channel's throughput, the supermarket compensates by offering consumers a lower price with SST than without it.

6.2. System Throughput and Average Ordering Volume

We now study the impact of SST on the system throughput and the average ordering volume. First, we examine the impact of SST on the system throughput, which measures the total number of products ordered by consumers. In the agent-service system, the system throughput under the optimal price is given by

$$TH_A^* = \frac{\Lambda(\bar{n}^2 - \bar{x}_A^2)}{2\bar{n}}.$$

In the hybrid-service system, the system throughput under the optimal price is given by

$$TH_H^* = \begin{cases} \frac{\Lambda\bar{n}}{2} & \text{if} \quad k \leq \hat{k} \\ \frac{\Lambda(\bar{n}^2 - \bar{x}_H^2 + \underline{x}^2)}{2\bar{n}} & \text{if} \quad k > \hat{k}. \end{cases}$$

We next summarize how the adoption of SST affects the system throughput.

PROPOSITION 4 **(System Throughput)**. *SST always improves the system throughput (i.e., $TH_H^* > TH_A^*$).*

Proposition 4 suggests that the provision of the SST service always improves system throughput. Recall that in the agent-service system, consumers with low service demand always choose to opt out. In contrast, in the hybrid-service system, consumers with extremely low service demand always opt in and use the SST service, regardless of the value of k. Moreover, as shown in Figure 3, those who originally choose the sales-agent service in the agent-service system continue to use it when k is large, or partially switch to the SST service when k is small. Thus, the provision of the SST service consistently attracts more consumers to place orders in the supermarket.

Next, we study the impact of SST on the average ordering volume, which is defined as the ratio of system throughput to the number of consumers who place orders. Note that in our model, consumers are heterogeneous in their service demand. Thus, while the provision of the SST service always improves system throughput, it does not necessarily mean that consumers' average ordering volume will always improve. In the agent-service system, the number of consumers who place order is $\Lambda(\bar{n} - \bar{x}_H)/\bar{n}$, and consumers' average ordering volume under optimal price is given by

$$\bar{N}_A^* = \frac{TH_A^*}{\frac{\Lambda(\bar{n}-\bar{x}_H)}{\bar{n}}} = \frac{\bar{n} + \bar{x}_H}{2}.$$

By contrast, in the hybrid-service system, consumers' average ordering volume under the optimal price are divided into the following two cases based on market coverage:

$$\bar{N}_H^* = \begin{cases} \frac{TH_H^*}{\Lambda} = \frac{\bar{n}}{2}, & \text{if} \quad k \leq \hat{k} \\ \frac{TH_H^*}{\frac{\Lambda(\bar{n}-\bar{x}_H+\underline{x})}{\bar{n}}} = \frac{\bar{n}^2-\bar{x}_H^2+\underline{x}^2}{2(\bar{n}-\bar{x}_H+\underline{x})}, & \text{if} \quad k > \hat{k}. \end{cases}$$

Hence, we next summarize how the adoption affects the consumers' average demand volume.

PROPOSITION 5 **(Average Demand Volume)**. *SST always reduces the average demand volume (i.e., $\bar{N}_H^* < \bar{N}_A^*$).*

Proposition 5 shows that the provision of the SST service always reduces consumers' potential ordering volume. In the absence of SST, only consumers with high service demand place orders, resulting in a higher average ordering volume. In contrast, when SST is available, many consumers with low service demand opt to place orders, which significantly reduces the average ordering volume. Thus, while the introduction of the SST service always attracts more consumers to place orders, it inevitably leads to a decrease in the average ordering volume.

6.3. Supermarket Profit

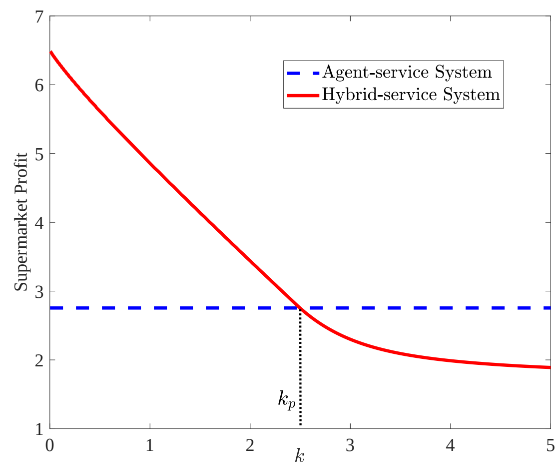
In this subsection, we further study the impact of SST on the supermarket's optimal profit.

THEOREM 2 **(Supermarket Profit)**. *There exists a threshold k_p such that SST improves the supermarket's optimal profit (i.e., $\Pi_H^* > \Pi_A^*$) if and only if $k < k_p$, where k_p is solved by $\Pi_A^*(P_A^*) = \Pi_H^*(P_H^*(k))$.*

As shown in Theorem 2, the provision of the SST service improves the supermarket's optimal profit when the SST inconvenience parameter k is sufficiently small. As previously discussed, in this scenario, the consumer market is partially covered in the case without SST (see Lemma 1), while it is fully covered in the case with SST (see Proposition 2). Specifically, on the one hand, offering the SST service allows the supermarket to attract low-demand consumers who might otherwise opt out if SST were unavailable. This results in the supermarket benefiting from a larger consumer market being served. On the other hand, when k is small, the effort cost of using the SST service is lower compared to the waiting cost of checkout through the sales-agent. As noted in Theorem 1, this allows the supermarket to convert the cost savings from consumers using SST, relative to those using the sales-agent service, into profit by charging a higher price for SST than for the sales-agent service, thereby benefiting from a higher profit margin. Consequently, when k is sufficiently small, the supermarket can achieve a higher profit by offering the SST service.

However, when the use of SST is sufficiently inconvenient (i.e., when k is large), the SST service becomes significantly less appealing to consumers. As a result, the additional throughput gains from attracting low-demand consumers gradually diminish as k continues to increase. Consequently, the consumer market becomes partially covered, as it fails to attract consumers with moderate demand (see Proposition 2). As Theorem 1 indicates, the supermarket cannot benefit from charging a higher profit margin and instead must lower the price (compared to the agent-service system) to ensure low-demand consumers place orders. Particularly when k is sufficiently large, the extraordinarily high effort cost of using SST deters virtually all consumers from using it, leaving only those with extremely low demand opting for the SST service. In this scenario, the hybrid-service system effectively reverts to an agent-service system, with optimal pricing resembling that of the agent-service system when k approaches infinity. Consequently, when k is large, the lower pricing results in the supermarket extracting a smaller profit margin compared to the agent-service system. Moreover, the provision of SST machines incurs additional operating costs for the supermarket. These dual factors lead to the supermarket's profit being lower in the hybrid-service system than in the agent-service system when k is sufficiently large. We present our numerical findings in Figure 5.

Figure 5 Impact of SST on Supermarket Profit



Note. $V=5, \Lambda=1, \mu=1, h=1, \bar{n}=3, \mathcal{C}=1$.

6.4. Consumer Surplus

We next proceed to study the impact of SST on the consumer surplus. In the agent-service system, the consumer surplus under the optimal price is given by

$$CS_A^* = \frac{\Lambda}{\bar{n}} \int_{\tilde{x}_H}^{\bar{n}} \left[(V - P_A^*)t - \frac{h\Lambda(\bar{n}^2 - t^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - t^2)]} \right] dt. \tag{13}$$

In the hybrid-service system,

(1) if $k \le \hat{k}$, the optimal price is $P_H^* = \bar{P}$, and the consumer surplus is

$$CS_H^* = \frac{\Lambda}{\bar{n}} \left(\int_0^{\hat{x}} \left((V - \bar{P})t - \frac{1}{2}kt^2 \right) dt + \int_{\hat{x}}^{\bar{n}} \left((V - \bar{P})t - \frac{h\Lambda(\bar{n}^2 - t^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - t^2)]} \right) dt \right); \tag{14}$$

(2) if $k > \hat{k}$, the optimal price is $P_H^* = \hat{P}$, and the consumer surplus is

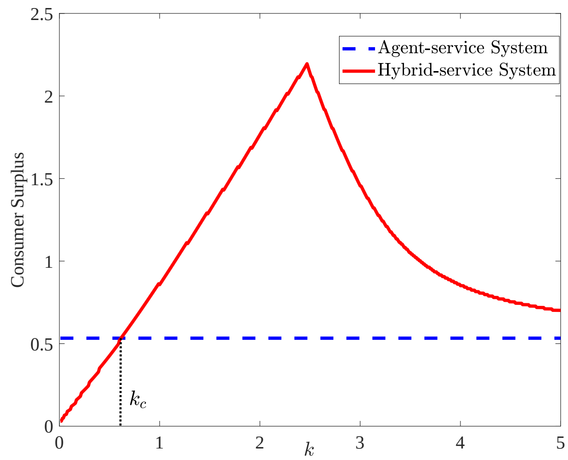
$$CS_H^* = \frac{\Lambda}{\bar{n}} \left(\int_0^{\underline{x}} \left((V - \hat{P})t - \frac{1}{2}kt^2 \right) dt + \int_{\tilde{x}_H}^{\bar{n}} \left((V - \hat{P})t - \frac{h\Lambda(\bar{n}^2 - t^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - t^2)]} \right) dt \right). \tag{15}$$

By comparing CS_A^* and CS_H^*, we can obtain the impact of SST on the consumer surplus in the following theorem.

THEOREM 3 **(Consumer Surplus)**. *There exists a threshold k_c such that SST improves the consumer surplus (i.e., $CS_A^* > CS_H^*$) if and only if $k > k_c$, where k_c is solved by $CS_S^*(P_A^*) = CS_H^*(\bar{P}(k))$.*

Common intuition might suggest that consumers would always benefit from the option of the SST service, as a new technology is provided for checkout and eliminates the waiting cost. However, Theorem 3 demonstrates that offering the SST service can actually reduce consumer surplus, particularly when the SST inconvenience parameter k is sufficiently small.

Figure 6 Impact of SST on Consumer Surplus



Note. $V = 5, \Lambda = 1, \mu = 1, h = 1, \bar{n} = 3, \mathcal{C} = 1$.

As noted in Theorem 1, when k is sufficiently small, the extremely low effort cost of using SST captures more throughput and leaves more surplus, which enables the supermarket to charge a higher price with SST than without it. In this scenario, although low-demand consumers benefit from the SST service, high-demand consumers are worse off due to the increased price, leading to a reduction in consumer surplus. As illustrated in Figure 6, when the inconvenience parameter approaches zero, consumer surplus also approaches zero, as the supermarket can charge a price that extracts all consumer surplus. This occurs because nearly all consumers opt for the SST service due to the negligible effort cost, allowing the supermarket to set a price close to the service reward, leaving no surplus for consumers. As k increases, the effort cost of using SST rises, and consumers with sufficiently high demand tend to switch to the sales-agent service. As highlighted in Proposition 2, the supermarket adopts the *volume strategy* to boost demand and achieve full market coverage, which leads to an increase in consumer surplus as prices decrease.

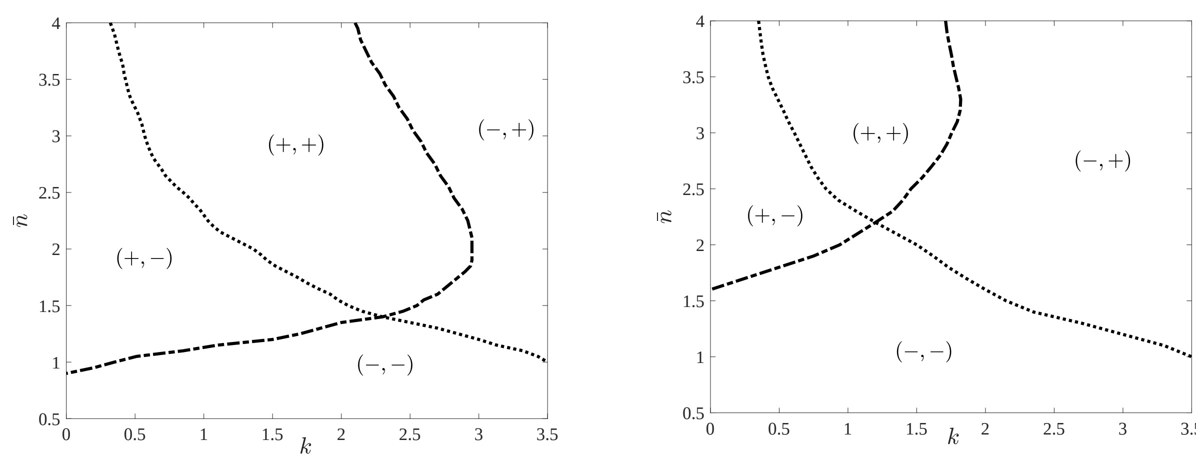
In contrast, when the use of SST becomes sufficiently inconvenient (i.e., when k is large), providing the SST service improves consumer surplus compared to not having it available. As noted in Theorem 1, when k is large, the supermarket mitigates the cannibalization effect on the SST channel by compensating consumers with a lower price for using SST compared to without it. Thus, some consumers who would have opted out of the agent-service system can benefit from a lower price by placing orders through the SST service, resulting in higher throughput with SST than without it. Consequently, consumer surplus is improved with the introduction of SST when the inconvenience cost is large.

Specifically, when the SST inconvenience parameter k is sufficiently small, note that the supermarket adopts the *volume strategy* (see Proposition 2) to maintain full market coverage by lowering prices, leading to an increase in consumer surplus as k increases. However, when the use of SST is sufficiently inconvenient, the supermarket adopts the *margin strategy*, resulting in a decrease in each consumer's surplus as k increases. Additionally, the market shifts to partial coverage, i.e., the system throughput is lower with a large k. These two factors combined lead to a downward trend in consumer surplus. Consequently, the highest consumer surplus occurs at a moderate level of inconvenience.

6.5. Incentives for SST Adoption

Based on the previous results regarding the impact of SST on the supermarket profit and the consumer surplus, we are now able to explore the supermarket's and consumers' incentives to adopt the SST service. Figure 7 illustrates the impact of SST on the supermarket profit and the consumer surplus by dividing the parameter space into two dimensions: the inconvenience parameter of using SST machines (i.e., k) and the potential demand of consumers (i.e., $\bar{n}$).

Figure 7 Incentives for SST Adoption



(a) $\mathcal{C} = 1$ (b) $\mathcal{C} = 2$

Note. Illustration of the sign of $\left(\Pi_H^* - \Pi_A^*, CS_H^* - CS_A^*\right)$ *when* $V = 5, \Lambda = 1, \mu = 1, h = 1.$

As Figure 7 indicates, there are four different regions in which the provision of SST leads to "win-win", "win-lose", "lose-win", and "lose-lose" situations for the supermarket and consumers. The first item in parentheses indicates a "win" or "lose" in the supermarket profit for the hybrid-service system compared to the agent-service system, while the second item indicates a "win" or "lose" in consumer surplus in the hybrid-service system relative to the agent-service system. From this, we are able to obtain the following key insights.

When the potential demand $\bar{n}$ is low, both the supermarket and consumers are worse off with the provision of the SST service, resulting in a *lose-lose* situation. In this scenario, the congestion in the sales-agent channel is not severe due to the small $\bar{n}$. As a result, the elimination of congestion cost through SST is not significant, and the supermarket can charge a higher price compared to without SST, as the surplus of consumers using SST is higher. Hence, consumers are worse off with SST. Moreover, adopting SST yields only minimal profit improvement for the supermarket due to the low potential demand. Additionally, the SST service incurs an extra operating cost that outweighs the increased profit from offering it. Therefore, neither the supermarket nor consumers have an incentive to adopt the SST service when the potential demand is low.

When the potential demand $\bar{n}$ is high, the supermarket and consumers may have opposing incentives for SST adoption, which critically depends on the value of the inconvenience parameter k. Specifically, when k is sufficiently small, the provision of the SST service results in a *win-lose* situation where the supermarket benefits from SST while consumers are worse off, mainly due to the higher price charged by the supermarket. When k is sufficiently large, offering the SST service leads to a *lose-win* situation where the supermarket is worse off with SST, while consumers benefit. The adoption of SST fails to achieve full market coverage due to the high effort cost, prompting the supermarket to lower prices to ensure throughput. When k is moderate, the provision of the SST service leads to a *win-win* situation, in which both the supermarket and consumers are better off with SST. This is because the supermarket charges the lowest price to secure full market coverage and consequently, consumers benefit from the low price. Also, the supermarket benefits from the high throughput despite the price concession. Consequently, only when the potential demand is large and the effort cost of using SST is moderate can both the supermarket and consumers have the incentive to adopt the SST service simultaneously.

The *win-win* area shrinks as $\mathcal{C}$ increases (as shown in Figure 7 (b) compared to Figure 7(a)), since the higher operating cost motivates the supermarket to raise prices to offset it, thereby reducing consumer surplus. Moreover, a large $\mathcal{C}$ is inherently disadvantageous for the supermarket himself.

7. Extensions

This section extends the base model along two dimensions to to confirm the robustness of our main insights. First, we adopt a long-term perspective by endogenizing service capacity, allowing us to assess how the introduction of SST influences capacity decisions. Second, we incorporate product variety by analyzing how SST affects consumer behavior when two differentiated products are available.

7.1. Endogenous Service Capacity

In our main model, the capacity of the sales-agent service is assumed to be fixed across both service scenarios. To incorporate the firm's long-term staffing decisions and better capture the operational trade-offs involved in capacity planning, we extend the model by endogenizing service capacity—treating it as a decision variable jointly optimized with price rather than a fixed parameter. We also include an SST-only system to capture scenarios where staffed service is fully replaced by self-service technology, allowing a full comparison across possible service configurations. Specifically, we assume that the cost of maintaining a service capacity level μ is given by $c(\mu) = c\mu^2$. This quadratic form captures the convex nature of capacity costs and is a standard assumption in the queueing economics literature (Cachon and Zhang 2007, Yang et al. 2021).

To comprehensively examine the impact of endogenous service capacity decisions, we extend our analysis in Appendix EC.1.1. Specifically, we compare the optimal service capacities across three systems: the Agent-only system (μ^A), the Hybrid system (μ^H), and the SST-only system (μ^T).

Our results show that as the system transitions from Agent-only to Hybrid, and eventually to SST-only, the firm progressively reduces its staffed service capacity, i.e., $\mu^T \leq \mu^H \leq \mu^A$. Moreover, the effect of the inconvenience parameter k on firm profit follows a similar directional pattern as in the baseline model with exogenous capacity: the firm benefits only when SST is sufficiently convenient (i.e., when k is low). In contrast, the effect on consumer surplus diverges. Unlike the exogenous case, the introduction of SST does not always enhance consumer welfare when k is large. In such settings, the firm strategically reduces its staffed capacity, offsetting the short-term benefits of SST and ultimately diminishing overall consumer surplus.

7.2. Heterogeneous Products

In our main model, we consider a scenario where products are homogeneous in both their base value and price. However, in reality, supermarket products are often diverse. To validate the robustness of our previous conclusions, we extended our model to include two types of products, each associated with corresponding rewards, V_1 and V_2, and prices, P_1 and P_2, where, without loss of generality, $V_1 < V_2$ and $P_1 < P_2$. The demand for these products is interrelated, reflecting the typical positive correlation observed among different product demands. This extension helps address the limitations associated with modeling only a single product type.

We denote the demand of each consumer for the two products as x and y, where $y = \alpha x$ with $\alpha \geq 0$ indicating the correlation between demands. Suppose that the demand for product 1 follows a uniform distribution over $[0, \bar{n}_1]$, then the demand for product 2 is uniformly distributed over $[0, \alpha\bar{n}_1]$. Here, $\bar{n}_1$ and $\alpha\bar{n}_1$ represent the upper bounds of consumer demand for each product type.

Our single-product model shows that providing SST attracts low-demand consumers and leads to a double-threshold equilibrium when using SST machines is highly inconvenient. In Appendix EC.1.2, we formally model consumer strategies under two product types. Our findings indicate that the most interesting result—the optimal price is non-monotonic with respect to the inconvenience parameter associated with using SST—remains robust, as introducing two product types has minimal impact on consumer decision-making. This is because when SST machines are inconvenient, only consumers with extremely low demand are captured to use it, while consumers with moderate demand tend to balk.

8. Conclusions

With advances in information technology, self-service technology (SST) has been increasingly adopted across service sectors such as supermarkets, banks, and call centers, aiming to alleviate the growing burden of rising labor costs. In the retail context, SST is expected to reduce consumer waiting times, increase system throughput, and lower operational expenses—offering presumed benefits for both firms and consumers. However, these benefits are not guaranteed; they are critically shaped by the degree of SST inconvenience and the potential magnitude of demand volume.

This paper offers a first attempt to examine the operational and economic implications of adopting self-service technology in supermarkets, where consumers differ in demand volumes and respond strategically to both pricing and service channel availability. Motivated by the trade-off between reduced waiting time and the hassle cost associated with using SST, we develop a queueing-game-theoretic model comparing a traditional agent-only system with a hybrid system offering both staffed checkout and SST.

Our analysis yields several key insights. First, SST adoption fundamentally alters consumer behavior, giving rise to a *dual-threshold* equilibrium: low-demand consumers prefer SST, high-demand consumers choose staffed checkout, while those with moderate demand often exit the system. Second, the firm's optimal pricing strategy in the hybrid system is non-monotonic in SST inconvenience—initially lowering prices to attract volume, then raising them to preserve margins as SST becomes more convenient. Third, the effects of SST on profit and consumer surplus are nuanced. Although SST always increases throughput, it does not guarantee improved outcomes for either party. When SST is too convenient or too inconvenient, only one side benefits. A win-win outcome—where both profit and consumer surplus increase—emerges only when SST inconvenience is moderate and potential demand is sufficiently high.

These findings yield important managerial implications. SST should not be adopted indiscriminately; firms must carefully assess its convenience level, operational costs, and the potential magnitude of demand volume. Notably, and contrary to intuition, maintaining a moderate level of inconvenience may be beneficial for both the firm and consumers, as it allows for market coverage expansion without triggering excessive price increases. That said, IT managers should also recognize that SST can reduce consumer surplus—particularly when aggressive pricing undermines the benefits of shorter wait times. In the long run, we further caution that SST adoption may incentivize strategic reductions in staffed service capacity, potentially exacerbating the erosion of consumer welfare. Table 2 summarizes the key insights.

Table 2 Summary of Key Findings

Dimension	Key Insight
Consumer Behavior	SST induces a dual-threshold pattern: low-demand (high-demand) consumers use SST (agent), and moderate-demand consumers exit.
Pricing Strategy	The optimal SST price exhibits a U-shaped pattern with respect to the inconvenience level.
Profit and Welfare	SST boosts throughput, but only yields a win-win outcome under high average demand and moderate inconvenience; low average demand results in a lose-lose situation.
Capacity Adjustment	SST adoption leads the firm to cut staffed capacity, further worsening the consumer surplus in the long run.

From a theoretical standpoint, our work contributes to the literature on queueing economics, multichannel service systems, and SST design by explicitly modeling endogenous consumer channel choice under demand heterogeneity and effort costs. Our dual-threshold equilibrium structure, together with the non-monotonic price and welfare implications, challenges conventional assumptions that SST is universally beneficial.

Several promising directions remain for future research. First, our model assumes no queueing delay for SST, which may not hold under capacity constraints. Incorporating congestion in SST queues would introduce new strategic trade-offs. Second, extending the model to account for heterogeneous delay sensitivity could yield richer equilibrium patterns. Third, it would be valuable to explore behavioral factors—such as risk aversion or habit persistence—in shaping consumers responses to SST. Finally, future work could analyze dynamic pricing or incentive mechanisms to guide consumers across service channels more efficiently.

References

Baron O, Chen X, Li Y (2022) Omnichannel services: The false premise and operational remedies. *Management Sci.* 62(9):865–884.

Bhargavi S (2016) Self-service in stores: What have we learnt from Amazon Go? (Dec), URL `https://www.aspiresys.com/articles/amazon-go-in-store-self-service`.

Bomee L (2019) Advantages and disadvantages of self-service kiosks. (Dec 10), URL `https://www.intuiface.com/blog/pros-and-cons-of-self-ordering-kiosk`.

Buell RW, Campbell D, Frei FX (2010) Are self-service customers satisfied or stuck? *Prod. Oper. Manag.* 19(6):679–697.

Cachon GP, Zhang F (2007) Obtaining fast service in a queueing system via performance-based allocation of demand. *Management Sci.* 53(3):408–420.

Campbell D, Frei F (2010) Cost structure, customer profitability, and retention implications of self-service distribution channels: Evidence from customer behavior in an online banking channel. *Management Sci.* 56(1):4–24.

Chen M, Hu M, Wang J (2022) Food delivery service and restaurant: Friend or foe? *Management Sci.* 68(9):6539–6551.

Debo LG, Parlour C, Rajan U (2012) Signaling quality via queues. *Management Sci.* 58(5):876–891.

Edelson NM, Hilderbrand DK (1975) Congestion tolls for poisson queuing processes. *Econometrica* 43(1):81–92.

Field JM (2024) *Designing Service Processes to Unlock Value* (New York: Business Expert Press).

Gao F, Su X (2017a) Omnichannel retail operations with buy-online-and-pickup-in-store. *Management Sci.* 63(8):2478–2492.

Gao F, Su X (2017b) Online and offline information for omnichannel retailing. *Manuf. Serv. Oper. Manag.* 19(1):84–98.

Gao F, Su X (2018) Omnichannel service operations with online and offline self-order technologies. *Management Sci.* 64(8):3595–3608.

Ghosh A, Bassamboo A, Lariviere M (2020) The queue behind the curtain: Information disclosure in omnichannel services. *Available at SSRN 3730482* .

Hassin R (2016) *Rational Queueing* (Boca Raton: CRC Press, Taylor and Francis Group).

Hassin R, Haviv M (2003) *To queue or not to queue: Equilibrium behavior in queueing systems* (New York: Springer Science & Business Media).

Hassin R, Roet-Green R (2018) Cascade equilibrium strategies in a two-server queueing system with inspection cost. *Eur. J. Oper. Res.* 267(3):1014–1026.

Heiman A, Reardon T, Zilberman D (2022) The effects of COVID-19 on the adoption of "on-the-shelf technologies": Virtual dressing room software and the expected rise of third-party reverse-logistics. *Serv. Sci.* 14(2):179–194.

Ivanov SH (2019) Ultimate transformation: How will automation technologies disrupt the travel, tourism and hospitality industries? *Zeitschrift für Tourismuswissenschaft* 11(1):25–43.

Kang K, Doroudi S, Delasay M (2024) Prioritization in the presence of self-ordering opportunities in omnichannel services. *Prod. Oper. Manag.* 33(3):737–756.

Karmarkar US, Roels G (2015) An analytical framework for value co-production in services. *Serv. Sci.* 7(3):163–180.

Naor P (1969) The regulation of queue size by levying tolls. *Econometrica* 37(1):15–24.

O'Neill R (2022) Why some people say they will never use self-service checkouts. (May 20), URL `https://www.walesonline.co.uk/news/wales-news/shoppers-tesco-self-service-tills-24009169`.

Paycor (2022) A closer look at labor costs. (Dec 8), URL `https://www.paycor.com/resource-center/articles/closer-look-at-labor-costs`.

Redyref (2022) The benefits of self-checkout kiosks in stores. URL `https://redyref.com/the-benefits-of-self-checkout-kiosks`.

Roels G (2014) Optimal design of coproductive services: Interaction and work allocation. *Manuf. Serv. Oper. Manag.* 16(4):578–594.

Roet-Green R, Yuan Y (2020) Information visibility in omnichannel queues. *Available at SSRN 3485810* .

Saldanha T, Kathuria A, Khuntia J, Konsynski BR (2021) Ghosts in the machine: How marketing and human capital investments enhance customer growth when innovative services leverage self-service technologies. *Inf. Syst. Res.* 33(1):76–109.

Sampson S, Froehle C (2006) Foundations and implications of a proposed unified services theory. *Prod. Oper. Manag.* 15(2):329–343.

Scherer A, Wunderlich N, Florian VW (2015) The value of self-service: The long-term effects of technology-based self-service usage on consumer retention. *MIS Q.* 39(1):177–200.

Stack O (2020) Why queuing is very bad for your business? (Jun 3), URL `https://orderingstack.com/blog/why-queuing-is-bad-for-your-business-and-how-you-can-get-eliminate-it`.

Sun K, Liu Y, Li X (2024) Mail back or in-store dropoff? Optimal design of product-exchange policies in omnichannel retailing systems. *Omega* 125:103024.

Tan TF, Netessine S (2020) At your service on the table: Impact of tabletop technology on restaurant performance. *Management Sci.* 66(10):4496–4515.

Xue M, Hitt LM, Harker PT (2007) Customer efficiency, channel usage, and firm performance in retail banking. *Manuf. Serv. Oper. Manag.* 9(4):535–558.

Yang L, Wang Z, Cui S (2021) A model of queue scalping. *Management Sci.* 67(11):6803–6821.

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Appendix EC.1 More Details on the Extensions in §7

EC.1.1. Endogenous Service Capacity

We begin by analyzing how the introduction of SST influences optimal capacity when both price and capacity are endogenously chosen, and compare the results with that under the Agent-service system.

PROPOSITION EC.1. **(Agent-service Requires Higher Capacity than Hybrid-Service).** When both price and capacity are endogenous, the Agent-service system requires a (weakly) higher optimal capacity than the Hybrid-service system, i.e., $\mu^A \geq \mu^H$.

This result is quite intuitive, introducing SST reduces the firm's reliance on human-operated service, leading to a lower requirement in capacity. From a design perspective, SST not only reshapes consumer behavior but also shifts the frim's optimal resource allocation.

To complete the comparison across common service configurations, we also consider an SST-service system alongside the Agent-service and Hybrid-service systems.

EC.1.1.1. SST-service System While our main analysis focuses on settings where consumers choose between agent- and self-service options, many real-world systems—such as modern gas stations and airline check-in kiosks—rely solely on SST. To assess the applicability of our model in such environments, we now examine a special case in which only self-service is available.

Consumers with demand x derive utility from SST as $U_T(x) = (V - P)x - \frac{1}{2}kx^2$. A consumer joins if and only if $U_T(x) \geq 0$, implying that all consumers with $x \leq \underline{x}(P) = 2(V-P)/k$ participate. Moreover, the service capacity μ is dedicated to the agent channel. As such, the firm does not perform any capacity allocation and only needs to make pricing decisions. The resulting system throughput is

$$TH_T(P) = \Lambda \int_0^{\underline{x}} \frac{t}{\bar{n}}\, dt = \frac{\Lambda \underline{x}^2}{2\bar{n}}. \tag{EC.1}$$

The supermarket's optimization problem is then given by:

$$\max_P \quad \Pi_H(P) = P \cdot TH_T(P) - \mathcal{C}. \tag{EC.2}$$

The consumer surplus under the optimal price is:

$$CS_T^* = \frac{\Lambda}{\bar{n}} \int_0^{\underline{x}} \left[(V - P_T^*)t - \frac{1}{2}kt^2 \right] dt. \tag{EC.3}$$

Due to the insufficient tractability of the general game, we gain qualitative insights via numerical experiments. Figure EC.1 illustrates the optimal service capacities μ^A, μ^H, and μ^T, as well as the corresponding profit and consumer surplus across the Agent-, Hybrid-, and SST- systems, under varying levels of the inconvenience parameter k. We have the following observations.

(1) First, in line with expectations, a strictly lower capacity μ^H compared to μ^A is adopted under the hybrid-service system. This is because offering SST helps alleviate system congestion and boosts consumer access. The SST-service system, in contrast, does not require human-operated capacity at all, resulting in $\mu^T = 0$ across all values of k.

(2) Second, consistent with Theorem 1, when capacity is endogenously determined, the optimal price under the hybrid system decreases with k and eventually falls below that of the agent system. For small k, however, it may be slightly higher as consumers exhibit a stronger preference for using SST, leading to higher surplus extraction through pricing. In SST-service system, as k increases, the firm lowers the price to offset rising effort costs and retain full participation. However, when k is large enough, maintaining throughput would require steep price cuts that hurt profitability. The firm thus stabilizes the price to preserve margin from the remaining participating consumers. Moreover, in each service system, higher k consistently leads to a reduction in profit.

(3) Third, consistent with Theorem 3, when capacity is endogenously determined, systems that include SST tend to achieve the highest consumer surplus at intermediate levels of k. However, in the SST-only system, high effort costs at large k reduce participation and hurt consumer surplus.

These observations confirm that the main qualitative insights hold even when the capacity is endogenized. That is, the long-run capacity adjustment does not change our main results, highlighting the robustness of our model.

EC.1.2. Heterogeneous Products

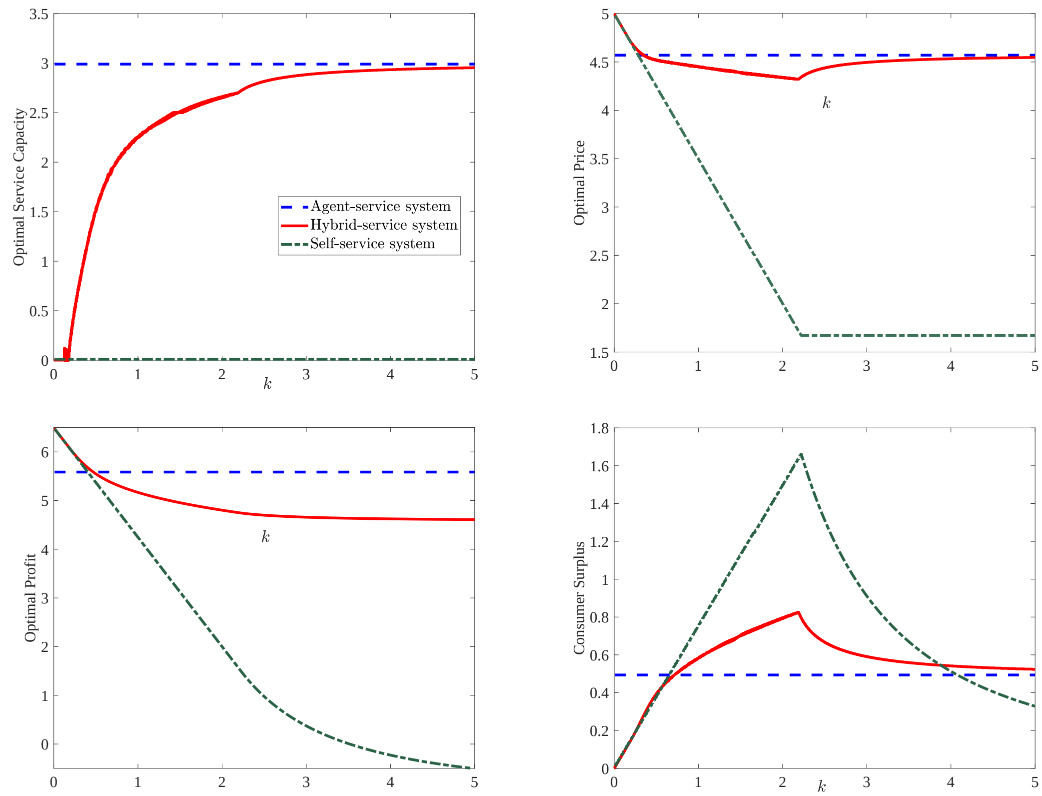
In this section, we use the superscript "(2)" to indicate the heterogeneous case.

EC.1.2.1. Agent-Service System For a given aggregated arrival rate λ for all products, considering a tagged consumer with demand x for product 1 and αx for product 2, her expected utility of placing an order is

$$U_A^{(2)}(x) = [(V_1 - P_1) + (V_2 - P_2)\alpha]x - h \cdot \frac{\lambda}{\mu(\mu - \lambda)} = (V_1 + \alpha V_2 - \widetilde{P})x - h \cdot \frac{\lambda}{\mu(\mu - \lambda)}, \quad \text{(EC.4)}$$

where we denote $\widetilde{P} = (P_1 + \alpha P_2)$ as the *full price* per unit demand of product 1 (and it also requires α unit of product 2). Since $U_A^{(2)}(x)$ strictly increases with x and $U_A^{(2)}(0) < 0$, there must exist a

Figure EC.1 Hybrid-Service System vs. Agent-Service System with Endogenous Capacity



Note. $\mu = 1$, $h = 1$, $V = 5$, $c = 0.1$, $\mathcal{C} = 1$, $\bar{n} = 3$, $\mu = 3$.

threshold $\bar{x}_A^{(2)} \in [0, \bar{n}_L]$ such that in equilibrium, consumers choose to place orders if and only if their demand for product 1 is above that threshold (i.e., $x > \bar{x}_A^{(2)}$), where $\bar{x}_A^{(2)}$ is a function of the full price $\widetilde{P}$. Thus, the aggregated effective arrival rate of orders is given by

$$\lambda = (1+\alpha)\Lambda \int_{\bar{x}_A^{(2)}}^{\bar{n}_L} \frac{t}{\bar{n}_L} dt = \frac{(1+\alpha)\Lambda(\bar{n}_L^2 - (\bar{x}_A^{(2)})^2)}{2\bar{n}_L}.$$

Combined with Equation (EC.4), the minimal demand of consumers who would like to place orders for product 1, $\bar{x}_A^{(2)}$, uniquely solves

$$U_A^{(2)}(\bar{x}_A^{(2)}) = (V_1 + \alpha V_2 - \widetilde{P})\bar{x}_A^{(2)} - \frac{(1+\alpha)h\Lambda(\bar{n}_1^2 - (\bar{x}_A^{(2)})^2)}{\mu\left[2\bar{n}_L\mu - (1+\alpha)\Lambda(\bar{n}_L^2 - (\bar{x}_A^{(2)})^2)\right]} = 0. \tag{EC.5}$$

By solving Equation (EC.5), we obtain consumers' equilibrium ordering behavior in the agent-service system, which is characterized by the following lemma.

LEMMA EC.1. **(Equilibrium Ordering Behavior in The Agent-Service System).** In the agent-service system, for any full price $\widetilde{P}$, there exists a threshold $\bar{x}_A^{(2)}(\widetilde{P})$ such that consumers place orders if and only if their demand for product 1 exceeds that threshold (i.e., $x > \bar{x}_A^{(2)}(\widetilde{P})$), where $\bar{x}_A^{(2)}(\widetilde{P})$ uniquely solves

$$(V_1 + \alpha V_2 - \widetilde{P})x = \frac{(1+\alpha)h\Lambda(\bar{n}_1^2 - x^2)}{\mu[2\mu\bar{n}_1 - (1+\alpha)\Lambda(\bar{n}_1^2 - x^2)]}. \tag{EC.6}$$

Moreover, $\bar{x}_A^{(2)}(\widetilde{P})$ is increasing in $\widetilde{P}$.

Lemma EC.1 shows that consumers opt in only when their demand for both products is large enough. This is consistent with common intuition and similar to the result for a single product type. Since the service reward acquired by a consumer with low demand cannot compensate for the waiting cost incurred by queueing, the consumer chooses to balk.

By Lemma EC.1, the equilibrium system throughput of product 1 in the agent-service system is given by

$$TH_A^{(2)}(\widetilde{P}) = \Lambda \int_{\bar{x}_A^{(2)}(\widetilde{P})}^{\bar{n}_1} \frac{t}{\bar{n}_1} dt = \frac{\Lambda(\bar{n}_1^2 - \bar{x}_A^{(2)}(\widetilde{P})^2)}{2\bar{n}_1}. \tag{EC.7}$$

Denote by $\Pi_A^{(2)}(\widetilde{P})$ the profit per unit time for the supermarket when the full price is $\widetilde{P}$. The supermarket's optimization problem is as follows

$$\begin{aligned} \max \quad & \Pi_A^{(2)}(\widetilde{P}) = TH_A^{(2)}(\widetilde{P}) \cdot \widetilde{P}, \\ s.t. \quad & \bar{x}_A^{(2)}(\widetilde{P}) \text{ uniquely solves (EC.6).} \end{aligned} \tag{EC.8}$$

Solving the above optimization problem, we derive the optimal full price $\widetilde{P}_A^*$. The optimal price of product 1 is given as $P_1^* = \min\left(\max\{0, \widetilde{P}_A^* - \alpha V_2\}, V_1\right)$, and the optimal price of product 2 is given as $P_2^* = (\widetilde{P}_A^* - P_1^*)/\alpha$.

EC.1.2.2. Hybrid-Service System In this subsection, we consider a hybrid-service setting, where an additional SST service channel is introduced by the supermarket. We consider a tagged consumer with demand x for product L and y for product H, where $y = \alpha x$, i.e., her demand for product H is proportional to her demand for product L. If she chooses the sales-agent service, her expected utility is $U_A^{(2)}(x)$, as given by Equation (EC.4), while if she uses the SST service, her expected utility is

$$U_T^{(2)}(x) = (V_1 + \alpha V_2 - \widetilde{P})x - \frac{1}{2}k(1+\alpha)^2 x^2. \tag{EC.9}$$

In equilibrium, consumers with demand x for product L prefer to use SST if and only if $U_T^{(2)}(x) \geq \max\{U_A^{(2)}(x), 0\}$. Likewise, she would choose sales-agent service if and only if $U_A^{(2)}(x) \geq \max\{U_T^{(2)}(x), 0\}$.

It can be verified that $U_T^{(2)}(x)$ is concave in x and $U_T^{(2)}(0)=0$. Specifically, $U_T^{(2)}(x)$ increases in $x \in \left[0, \frac{V_1+\alpha V_2-\widetilde{P}}{k(1+\alpha)^2}\right]$ and decreases in $x \in \left(\frac{V_1+\alpha V_2-\widetilde{P}}{k(1+\alpha)^2}, \bar{n}_1\right]$. Thus, $U_T^{(2)}(x) \geq 0 \Leftrightarrow x \in \left[0, \min\{\frac{2(V_1+\alpha V_2-\widetilde{P})}{k(1+\alpha)^2}, \bar{n}_1\}\right]$. Similar to the case of a single product type, we denote the intersection points of the two functions, $U_A^{(2)}(x)$ and $U_T^{(2)}(x)$, with the x-axis as $\bar{x}_D^{(2)}$ and $\underline{x}^H$, respectively, where $\bar{x}_H^{(2)}$ is uniquely solved by (EC.6); thus, $\bar{x}_H^{(2)} = \bar{x}_A^H$ and $\underline{x}^{(2)} = \frac{2(V_1+\alpha V_2-\widetilde{P})}{k(1+\alpha)^2}$. In particular, two cases are specified: $\bar{x}_H^{(2)} \leq \underline{x}^{(2)}$ and $\bar{x}_H^{(2)} > \underline{x}^{(2)}$.

The two utility functions intersect at most once for $x \geq 0$, and we denote this unique intersection as $\hat{x}^{(2)}$. It can be verified that $U_A^{(2)}(x) < U_T^{(2)}(x)$ if and only if $x \in [0, \hat{x}^{(2)})$. In other words, $\hat{x}^H$ is the unique solution to $U_A^{(2)}(x) = U_T^{(2)}(x)$, which is equivalent to

$$h \cdot \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1}{2}k(1+\alpha)^2 x^2, \tag{EC.10}$$

where $\lambda = (1+\alpha)\Lambda(\bar{n}_1+x)(\bar{n}_1-x)/2\bar{n}_1$. The expression of λ is derived since consumers prefer to choose sales-agent service when $U_A^{(2)}(x) \geq \max\{U_T^{(2)}(x), 0\}$ for $x \geq \hat{x}^{(2)}$. In addition, the left (right) hand side of Equation (EC.10) is decreasing (increasing) with x, which guarantees the uniqueness of $\hat{x}^{(2)}$.

The following lemma characterizes the equilibrium order-placing strategy of consumers.

LEMMA EC.2. **(Equilibrium Strategy in The Hybrid-Service System).** In the hybrid-service system, there exists a threshold $\check{k} = 2(V_1+\alpha V_2-\widetilde{P})/((1+\alpha)^2\bar{x}_H^{(2)}(\widetilde{P}))$ such that the consumers' equilibrium ordering strategy is given as follows

(1) **(High Convenience)** If $k \leq \check{k}$, consumers choose SST service if their demand of product 1 $x \in [0, \hat{x}^{(2)})$, and choose sales-agent service if their demand of product 1 $x \in [\hat{x}^{(2)}, \bar{n}_1]$, where $\hat{x}^{(2)} = \sqrt{\frac{\sqrt{[2h\Lambda+k(1+\alpha)\mu\bar{n}_1(2\mu-(1+\alpha)\Lambda\bar{n}_1)]^2-8h\Lambda^2\bar{n}_1^2k(1+\alpha)^2\mu}-[2h\Lambda+k(1+\alpha)\mu\bar{n}_1(2\mu-(1+\alpha)\Lambda\bar{n}_1)]}{2k(1+\alpha)^2\mu\Lambda}}$.

(2) **(High Inconvenience)** If $k > \check{k}$, consumers choose SST service if their demand $x \in [0, \underline{x}^{(2)}]$, and opt out if their demand $x \in (\underline{x}^{(2)}, \bar{x}_D^{(2)}]$, and choose sales-agent service if their demand $x \in (\bar{x}_H^{(2)}, \bar{n}_1]$, respectively, where $\underline{x}^{(2)} = \frac{2(V_1+\alpha V_2-\widetilde{P})}{k(1+\alpha)^2}$ and $\bar{x}_H^{(2)}$ uniquely solves (EC.6).

By Lemma EC.2, the equilibrium system throughput of product 1, $TH_D^H(\widetilde{P})$, is given by

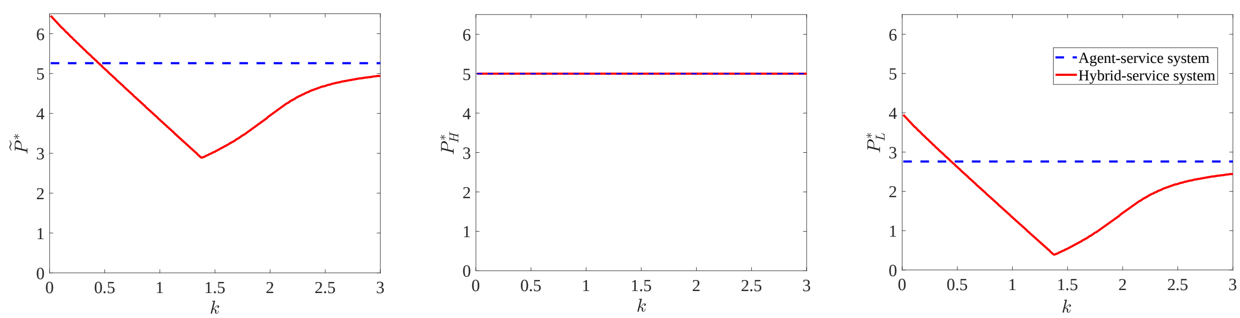
$$TH_H^{(2)}(\widetilde{P}) = \begin{cases} \Lambda\bar{n}_1/2 & \text{if} \quad k \leq \underline{x}^{(2)} \\ \Lambda\left(\bar{n}_1^2 - (\bar{x}_H^{(2)}(\widetilde{P}))^2 + \left(\frac{2(V_1+\alpha V_2-\widetilde{P})}{k(1+\alpha)^2}\right)^2\right)/(2\bar{n}_1) & \text{if} \quad k > \underline{x}^{(2)}. \end{cases}$$

Denote by $\Pi_H^{(2)}(\widetilde{P})$ the profit per unit time for the supermarket when the *full price* is $\widetilde{P}$. The supermarket's optimization problem is as follows:

$$\begin{aligned} \max \quad & \Pi_H^{(2)}(\widetilde{P}) = \widetilde{P} \cdot TH_H^{(2)}(\widetilde{P}) - \mathcal{C}, \\ s.t. \quad & \bar{x}_H^{(2)}(\widetilde{P}) \ \text{uniquely solves (EC.6).} \end{aligned} \tag{EC.11}$$

We conduct a numerical example in Figure EC.2 to compare the optimal prices in the two systems for the two types of products as k varies. It is clear that the optimal price in the agent-service system is independent of k. The supermarket adopts *volume strategy* and then *margin strategy* as k increases for the *full price*. Moreover, the SST improves the price if and only if the inconvenience parameter is small. Specifically, the optimal price of product 1 shows a similar tendency to the full price. However, the supermarket consistently charges product 2 the highest price, which approaches the value of the product 2.

Figure EC.2 Comparisons of Price under Two Scenarios.



(a) Optimal Full Price (b) Optimal Price of Product 2 (c) Optimal Price of Product 1

Note. $V_2 = 5, V_1 = 4, \alpha = 0.5, \Lambda = 1, \mathcal{C} = 1, \mu = 1, h = 1, \bar{n} = 3.$

Figure EC.3 shows the profit comparison between two systems. The SST service improves profit if and only if the inconvenience parameter k is small. When k is sufficiently large, SST fails to enhance profit as it is rarely used by consumers but incurs operating costs for offering it, resulting in profit falling short of that generated by the agent-service system, which is consistent with the single-product type case.

EC.2. Proofs

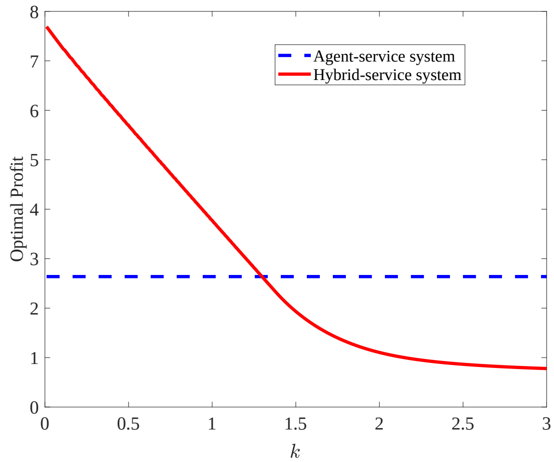
EC.2.1. Proof of Lemma 1

Given the products' aggregated arrival rate λ, consumers' utility of placing orders increases with demand x. Note that $U_A(0) = -h\lambda/(\mu(\mu-\lambda)) < 0$ and $U_A(\bar{n}) = (V-P)\bar{n} > 0$, there exists a unique solution $\bar{x}_A$ that satisfies

$$U_A(\bar{x}_A) = (V-P)\bar{x}_A - h\frac{\lambda}{\mu(\mu-\lambda)} = 0 \quad \text{and} \quad \lambda = \Lambda \cdot \frac{\bar{n}+\bar{x}_A}{2} \cdot \frac{\bar{n}-\bar{x}_A}{\bar{n}},$$

and $\bar{x}_A$ is a function of price P. When consumers' demand exceeds $\bar{x}_A$, the utility is nonnegative, which also means that consumers place orders if demand $x \in [\bar{x}_A, \bar{n}]$ and balk if $x \in [0, \bar{x}_A)$.

Figure EC.3 Comparisons of Profit under Two Scenarios



Note. $V_H = 5, V_L = 4, \alpha = 0.5, \Lambda = 1, \mathcal{C} = 1, \mu = 1, h = 1, \bar{n} = 3.$

Therefore, the aggregated arrival rate of products under equilibrium is determined to be $\lambda = \Lambda(\bar{n} + \bar{x}_A) \cdot (\bar{n} - \bar{x}_A)/(2\bar{n})$, given that x follows a uniform distribution.

To verify the monotonicity of $\bar{x}_A$ in P, note that, under equilibrium, we have $U_A(\bar{x}_A) = 0$, which is equivalent to

$$V - P = \frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)}{\mu\bar{x}_A[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]}. \tag{EC.12}$$

We denote the right hand of Equation (EC.12) as $g(\bar{x}_A) \equiv h\Lambda(\bar{n}^2 - \bar{x}_A^2)/(\mu\bar{x}_A[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)])$, and the first-order derivative of $g(\bar{x}_A)$ with respect to $\bar{x}_A$ is

$$\frac{\partial g(\bar{x}_A)}{\partial \bar{x}_A} = -\frac{h\Lambda}{\mu} \cdot \frac{[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)](\bar{n}^2 + \bar{x}_A^2) + 2(\bar{n}^2 - \bar{x}_A^2)\Lambda\bar{x}_A^2}{\bar{x}_A^2[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]^2} < 0,$$

which means that $g(\bar{x}_A)$ decreases in $\bar{x}_A$. While the left hand side of (EC.12) decreases in P, this causes the right-hand side of Equation (EC.12) to decrease with P. By using the chain rule, we have that the solution $\bar{x}_A(P)$ increases with P and $\partial\bar{x}_A(P)/\partial P > 0$.

To further verify the concavity or convexity of $\bar{x}_A$ with respect to P, we define $m(\bar{x}_A) \equiv (\bar{n}^2 - \bar{x}_A^2)/[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]$ and then $g(\bar{x}_A) \equiv h\Lambda m(\bar{x}_A)/(\mu\bar{x}_A)$. The second-order derivative of $g(\bar{x}_A)$ with respect to $\bar{x}_A$ is

$$\frac{\partial^2 g(\bar{x}_A)}{\partial \bar{x}_A^2} = \frac{h\Lambda}{\mu} \cdot \frac{1}{\bar{x}_A^3} \cdot \left(\frac{\partial^2 m(\bar{x}_A)}{\partial \bar{x}_A^2}\bar{x}_A^2 - 2\frac{\partial m(\bar{x}_A)}{\partial \bar{x}_A}\bar{x}_A + 2m(\bar{x}_A) \right).$$

Since

$$\frac{\partial^2 m(\bar{x}_A)}{\partial \bar{x}_A^2} = -\frac{4\mu\bar{n}[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2) - 4\Lambda\bar{x}_A^2]}{[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]^3} \cdot \frac{\partial m(\bar{x}_A)}{\partial \bar{x}_A} = -\frac{4\mu\bar{n}\bar{x}_A}{[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]^2},$$

then we have that

$$\frac{\partial^2 g(\bar{x}_A)}{\partial \bar{x}_A^2} = \frac{4\mu\bar{n}\bar{x}_A^2[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)] + 16\mu\bar{n}\Lambda\bar{x}_A^4 + 2(\bar{n}^2 - \bar{x}_A^2)[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]^2}{[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]^3\bar{x}_A^3} > 0.$$

Taking the second-order derivative of $\partial g(\bar{x}_A)/\partial \bar{x}_A$ with respect to P, we have that

$$\frac{\partial g(\bar{x}_A)}{\partial \bar{x}_A} \cdot \frac{\partial^2 \bar{x}_A}{\partial P^2} = -\frac{\partial^2 g(\bar{x}_A)}{\partial \bar{x}_A^2} \cdot \frac{\partial \bar{x}_A}{\partial P} < 0,$$

where the right hand of the above equation is negative since both $\partial^2 g(\bar{x}_A)/\partial \bar{x}_A^2$ and $\partial \bar{x}_A(P)/\partial P$ are positive, and then $\partial^2 \bar{x}_A/\partial P^2 > 0$. Therefore, $\bar{x}_A$ is convexly increasing in P. □

EC.2.2. Proof of Proposition 1

The second-order derivative of the profit function with respect to P is

$$\frac{\partial^2 \Pi_A(P)}{\partial P^2} = -2\left[\bar{x}_A(P) \cdot \frac{\partial \bar{x}_A(P)}{\partial P} + P\left(\left(\frac{\partial \bar{x}_A(P)}{\partial P}\right)^2 + \frac{\partial^2 \bar{x}_A(P)}{\partial P^2} \cdot \bar{x}_A(P)\right)\right] < 0.$$

The above inequality holds since we have proved that $\bar{x}_A(P)$ is convexly increasing in P, as $\partial^2 \bar{x}_A(P)/\partial P^2 > 0$ and $\partial \bar{x}_A(P)/\partial P > 0$, and this indicates that the profit function is concave in price P. Then the optimal price P_A^* is uniquely solved by

$$\frac{\partial \Pi_A(P)}{dP} = 0 \Leftrightarrow \bar{n}^2 - \bar{x}_A^2(P) - 2P\bar{x}_A(P) \cdot \frac{\partial \bar{x}_A(P)}{\partial P} = 0.$$

To explore the impact of the upper bound of consumers' demand $\bar{n}$, recall that the profit function is given by $\Pi_A(P) = \Lambda(\bar{n}^2 - \bar{x}_A(P)^2)/(2\bar{n}) \cdot P$, and for any fixed price P, the profit improves as $\bar{n}$ increases since we have

$$\frac{\partial \Pi_A}{\partial \bar{n}} = \frac{\Lambda P}{2\bar{n}^2}\left(\bar{n}^2 + \bar{x}_A^2 - 2\bar{n}\bar{x}_A \cdot \frac{\partial \bar{x}_A}{\partial \bar{n}}\right) > 0.$$

The above inequality holds since, by taking the first-order derivative of Equation (EC.12) with respect to $\bar{n}$, we have

$$\frac{\partial \bar{x}_A}{\partial \bar{n}} = \frac{2h\Lambda\bar{n} - 2\mu\bar{x}_A(V-P)(\mu - \Lambda\bar{n})}{\mu(V-P)[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2) + 2\Lambda\bar{x}_A^2] + 2h\Lambda\bar{x}_A}. \quad \text{(EC.13)}$$

Recall that the equation $(V-P)\bar{x}_A - \frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]} = 0$ holds under equilibrium and there is a straightforward result that $\frac{h\Lambda\bar{n}}{\mu(\mu - \Lambda\bar{n})} > \frac{h\Lambda\bar{n}}{\mu(2\mu - \Lambda\bar{n})} > \frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)}{\mu(2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2))}$. The last inequality holds since $\frac{h\Lambda\bar{n}}{\mu(2\mu - \Lambda\bar{n})}$ is a special case when $\bar{x}_A = 0$ in $\frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]}$, which is the average waiting cost of consumers. Hence, we have

$$(V-P)\bar{x}_A - \frac{h\Lambda\bar{n}}{\mu(\mu - \Lambda\bar{n})} < (V-P)\bar{x}_A - \frac{h\Lambda(\bar{n}^2 - \bar{x}_A^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_A^2)]} = 0,$$

and $(V-P)\bar{x}_A - h\Lambda\bar{n}/[\mu(\mu-\Lambda\bar{n})] < 0$ is equivalent to $2h\Lambda\bar{n} - 2\mu\bar{x}_A(V-P)(\mu-\Lambda\bar{n}) > 0$, which indicates that the denominator of (EC.13) is positive. Therefore, $\bar{x}_A$ increases with respect to $\bar{n}$.

We then aim to prove that $\partial\bar{x}_A/\partial\bar{n} < 1$, which is equivalent to

$$2h\Lambda\bar{n} - 2\mu\bar{x}_A(V-P)(\mu-\Lambda\bar{n}) < \mu(V-P)[2\mu\bar{n} - \Lambda(\bar{n}^2-\bar{x}_A^2) + 2\Lambda\bar{x}_A^2] + 2h\Lambda\bar{x}_A,$$

combined with $(V-P)\bar{x}_A - h\Lambda(\bar{n}^2-\bar{x}_A^2)/(\mu[2\mu\bar{n}-\Lambda(\bar{n}^2-\bar{x}_A^2)]) = 0$, we have

$$2\mu(V-P)\bar{x}_A^2[\mu - \Lambda(\bar{n}-\bar{x}_A)] + h\Lambda(\bar{n}-\bar{x}_A)^2 > 0,$$

Therefore, we have $\partial\bar{x}_A/\partial\bar{n} \in (0,1)$ and then

$$\frac{\partial\Pi_A}{\partial\bar{n}} = \frac{\Lambda P}{2\bar{n}^2}\left(\bar{n}^2 + \bar{x}_A^2 - 2\bar{x}_A\bar{n}\cdot\frac{\partial\bar{x}_A}{\partial\bar{n}}\right) = \frac{\Lambda P}{2\bar{n}^2}\left((\bar{n}-\bar{x}_A)^2 + 2\left(1-\frac{\partial\bar{x}_A}{\partial\bar{n}}\right)\bar{n}\bar{x}_A\right) > 0,$$

and this indicates that the profit function increases with $\bar{n}$ when the price is fixed. The increasing property of the profit function with $\bar{n}$ implies that with the increase of $\bar{n}$, the profit under optimal price also increases.

Next, to derive the monotonicity of the optimal price with respect to $\bar{n}$, recall that the optimal price P_A^* is uniquely solved by

$$F(P) \equiv \bar{n}^2 - \bar{x}_A(P)^2 - 2P\bar{x}_A(P)\cdot\bar{x}_A' = 0, \tag{EC.14}$$

where we denote $\bar{x}_A' = \partial\bar{x}_A(P)/\partial P$ in the following results. By taking the first-order derivative of both sides of Equation (EC.14) with respect to $\bar{n}$, we have

$$\bar{n} - \frac{\partial\bar{x}_A}{\partial\bar{n}}\cdot(\bar{x}_A + \bar{x}_A'P) - \frac{\partial\bar{x}_A'}{\partial\bar{n}}\cdot\bar{x}_A P = \frac{\partial P}{\partial\bar{n}}\cdot\bar{x}\bar{x}_A', \tag{EC.15}$$

where

$$\frac{\partial\bar{x}_A'}{\partial\bar{n}} = \frac{\mu}{h\Lambda}\cdot\frac{(\bar{n}^2+\bar{x}_A^2)\bar{x}_A f\frac{\partial f}{\partial\bar{n}} + 4\Lambda\bar{x}_A^3(\bar{n}^2-\bar{x}_A^2)\frac{\partial f}{\partial\bar{n}} - 2\bar{x}_A f\bar{n}(f+2\Lambda\bar{x}_A^2) + 2f\frac{\partial\bar{x}_A}{\partial\bar{n}}(f\bar{n}^2+2\Lambda\bar{x}_A^2)}{[(\bar{n}^2+\bar{x}_A^2)f + 2(\bar{n}^2-\bar{x}_A^2)\Lambda\bar{x}_A^2]^2}$$

and $f \equiv 2\mu\bar{n} - \Lambda(\bar{n}^2-\bar{x}_A^2)$. When $\bar{n}$ goes to zero, $\bar{x}_A(\le\bar{n})$ also approaches zero, and this induces that $(\partial\bar{x}_A'/\partial\bar{n})|_{\bar{n}\to 0} = 0$. Combined with the results $\partial\bar{x}_A/\partial\bar{n} \in (0,1)$ and $\partial\bar{x}_A(P)/\partial P > 0$ ($\Leftrightarrow \bar{x}_A' > 0$) that we have proved previously, we find that the left-hand of Equation (EC.15) is negative when $\bar{n}$ approaches to zero, and then $\partial P/\partial\bar{n} < 0$ since we have that $\partial\bar{x}_A(P)/\partial P > 0$ and $\bar{x}_A > 0$. While if $\bar{n}$ goes to infinity, the left hand of Equation (EC.15) is positive due to the finiteness of the second and third terms, and then $\partial P/\partial\bar{n} > 0$. This concludes that the optimal price decreases in service demand when $\bar{n}$ is sufficiently small and then increases when $\bar{n}$ is sufficiently large. □

EC.2.3. Proof of Lemma 2

We define $\underline{x}(P) = 2(V-P)/k$ and denote $\bar{x}_H(P)$ as the solution of $U_A(\bar{x}_H(P)) = 0$, which is equivalent to

$$(V-P)\bar{x}_H(P) = \frac{h\Lambda(\bar{n}^2 - \bar{x}_H^2(P))}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_H^2(P))]}. \tag{EC.16}$$

Similar with the analysis of agent-service case, there exists a unique solution $\bar{x}_H(P)$ of (EC.16).

Combined with the previous analysis, we then discuss the following two cases:

Case (1) When $\bar{x}_H(P) \le \underline{x}(P) \Leftrightarrow k \le 2(V-P)/\bar{x}_H(P)$, there exists a unique solution of $U_T(\hat{x}) = U_A(\hat{x})$ and we have that $U_T(x) \ge U_A(x)$ if and only if $x \le \hat{x}$. Therefore, consumers with service demand $x \in [0, \hat{x}]$ prefer to use SST and prefer to choose sales agents service otherwise. Note that $\hat{x}$ is uniquely determined by

$$\frac{h\Lambda(\bar{n}^2 - \hat{x}^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \hat{x}^2)]} = \frac{1}{2}k\hat{x}^2.$$

It follows that

$$\hat{x} = \sqrt{\frac{\sqrt{8hk\Lambda^2\mu\bar{n}^2 + (2h\Lambda + k\mu\bar{n}(2\mu - \Lambda\bar{n}))^2} - 2h\Lambda + k\mu\bar{n}(\Lambda\bar{n} - 2\mu)}{2k\Lambda\mu}}.$$

Case (2) When $\bar{x}_H(P) > \underline{x}(P) \Leftrightarrow k > 2(V-P)/\bar{x}_H(P)$, we have that $U_T(x) \ge 0 \ge U_A(x)$ if $x \in [0, \underline{x}(P)]$, thus consumers prefer to use SST service when $x \in [0, \underline{x}(P)]$; when $x \in [\bar{x}_H(P), \bar{n}]$, we find that $U_A(x) \ge 0 \ge U_T(x)$, thus consumers prefer to choose sales-agent service; while when their service demand $x \in [\underline{x}(P), \bar{x}_H(P)]$, consumers' utility under both service channels are negative thus they balk. □

EC.2.4. Proof of Lemma 3

Combined with the equilibrium results of Lemma 2, when $k \le \bar{k}$, all consumers place orders, either by using SST or sales-agent service. Thus the system throughput is

$$TH_H = \Lambda \int_0^{\bar{n}} t\frac{1}{\bar{n}}dt = \frac{\Lambda}{2}\bar{n}.$$

When $k > \bar{k}$, consumers join with demand $x \in [0, \underline{x}(P)] \cup [\bar{x}_H(P), \bar{n}]$, then the system throughput is

$$TH_H = \Lambda\left(\frac{\underline{x}(P)}{\bar{n}}\int_0^{\underline{x}(P)} \frac{t}{\underline{x}(P)}dt + \frac{\bar{n} - \bar{x}_H(P)}{\bar{n}}\int_{\bar{x}_H(P)}^{\bar{n}} \frac{t}{\bar{n} - \bar{x}_H(P)}dt\right) = \frac{\Lambda}{2}\cdot\frac{\bar{n}^2 - \bar{x}_H^2(P) + \underline{x}^2(P)}{\bar{n}}.$$ □

EC.2.5. Proof of Proposition 2

From the results of Proposition 1, the profit of the system is

$$\Pi_H(P) = \begin{cases} \frac{\Lambda}{2}\bar{n}P - \mathcal{C}, & \text{if} \quad k \le \bar{k} \\ \frac{\Lambda}{2}\frac{\bar{n}^2 - \bar{x}_H^2(P) + \underline{x}^2(P)}{\bar{n}}P - \mathcal{C}, & \text{if} \quad k > \bar{k}. \end{cases}$$

The condition $k \le \bar{k} = 2(V-P)/\bar{x}_H(P)$ is equivalent to

$$\frac{k}{2} \le \frac{V-P}{\bar{x}_H(P)}, \tag{EC.17}$$

where $\bar{x}_H(P)$ uniquely solves (2), and combining with the increasing property of $\bar{x}_H(P)$ with respect to P, the right-hand of (EC.17) is decreasing in P. Therefore, there exists a unique price threshold $\bar{P}$, when $P \le \bar{P}$, the inequality (EC.17) is established, and $\bar{P}$ uniquely solves $k/2 = (V-P)/\bar{x}_H(P)$. Then the profit function turns to

$$\Pi_H(P) = \begin{cases} \frac{\Lambda}{2}\bar{n}P - \mathcal{C}, & \text{if} \quad P \le \bar{P}, \quad \text{case (1)}, \\ \frac{\Lambda}{2}\frac{\bar{n}^2 - \bar{x}_H^2(P) + \underline{x}^2(P)}{\bar{n}}P - \mathcal{C}, & \text{if} \quad P > \bar{P}, \quad \text{case (2)}. \end{cases}$$

For case (1), the profit function is increasing in P, thus the optimal price is $P_H^* = \bar{P}$. While in case (2), given k, the first-order derivative of $\Pi_H(P)$ with respect to P is

$$G(P) \equiv \frac{\partial \Pi_H(P)}{\partial P} = \frac{\Lambda}{2\bar{n}}\left(\bar{n}^2 + \frac{4(V-P)(V-3P)}{k^2} - \bar{x}_H^2(P) - 2P\bar{x}_H(P)\cdot\frac{\partial \bar{x}_H(P)}{\partial P}\right). \tag{EC.18}$$

Recall that the inconvenience parameter $k = 2(V-P)/\bar{x}_H(P)$, which induces that

$$G(P) = \frac{\Lambda}{2\bar{n}}\left(\bar{n}^2 - \frac{2P}{V-P}\bar{x}_H^2(P) - 2P\bar{x}_H(P)\cdot\frac{\partial \bar{x}_H(P)}{\partial P}\right). \tag{EC.19}$$

The first-order derivative of $G(P)$ with respective to P is

$$\frac{\partial G(P)}{\partial P} = -\left[\frac{2V\bar{x}_H^2(P)}{(V-P)^2} + \frac{2(V+P)\bar{x}_H(P)}{V-P}\cdot\frac{\partial \bar{x}_H(P)}{\partial P} + 2P\left(\left(\frac{\partial \bar{x}_H(P)}{\partial P}\right)^2 + \bar{x}_H(P)\cdot\frac{\partial^2 \bar{x}_H(P)}{\partial^2 P}\right)\right] < 0. \tag{EC.20}$$

The above inequality holds as we have shown that $\bar{x}(P)$ convexly increases in P. This indicates that $G(P)$ decreases in P. We then consider two cases:

(1) If $G(\bar{P}) \le 0$, then $G(P) < 0$ for all $P \in [\bar{P}, V]$, which means that $\Pi_H(P)$ is decreasing in $[\bar{P}, V]$. Therefore, the optimal price is $P_H^* = \bar{P}$.

(2) If $G(\bar{P}) > 0$, since $G(V) \to -\infty$, then there exists a threshold of price $\hat{P} > \bar{P}$, when $P \in [\bar{P}, \hat{P}]$, $G(\bar{P}) > 0$; otherwise, $G(\bar{P}) \le 0$, which means that $\Pi_H(P)$ is concave in $[\bar{P}, V]$. Therefore, the optimal price is $P_H^* = \hat{P}$.

Recall that $\bar{P}$ uniquely solves $k/2 = (V-P)/\bar{x}_H(P)$, the solution $\bar{P}$ decreases in k since $(V - \bar{P})/\bar{x}_H(\bar{P})$ decreases in P and $k/2$ increases in k. Combined with the decreasing property of $G(P)$

in P, we have $G(\bar{P})$ decreases in $\bar{P}$, it indicates that $G(\bar{P})$ is monotonically increasing in k. In addition, we have $G(0) > 0$. Therefore, there exists a unique threshold of consumers inconvenience parameter of SST $\hat{k}$ which satisfies $G(\bar{P}(\hat{k})) = 0$, and when $k \leq \hat{k}$, $G(\bar{P}(\hat{k})) \leq 0$; otherwise, $G(\bar{P}(\hat{k})) > 0$.

Therefore, we conclude that if $k \leq \hat{k}$, $\Pi_H(P)$ increases in $[0, \bar{P}(k)]$ and decreases in $[\bar{P}(k), V]$, the optimal price $P_H^* = \bar{P}(k)$. If $k > \hat{k}$, $\Pi_H(P)$ is increasing in $[0, \bar{P}(\hat{k})]$ and concave in $P \in [\bar{P}(k), V]$, and the maximum value point is $\hat{P}$, thus the optimal price is $P_H^* = \hat{P} > \bar{P}(\hat{k})$.

From the above results, if $k \leq \hat{k}$, the optimal price $P_H^* = \bar{P}(k)$, and we have previously proved that $\bar{P}(k)$ is decreasing in k. If $k > \hat{k}$, the optimal price is solved by $G(P) \equiv 0$. As k increases, the first-order derivative of $G(P)$ with respect to k is

$$\frac{\partial G(P)}{\partial k} = -\frac{8(V-P)(V-3P)}{k^3}.$$

If the optimal price satisfies $\hat{P} < V/3$, then $G(P)$ decreases in k. Combined with the decreasing property of $G(P)$ with P, the optimal price $\hat{P}$ is decreasing in k. Recall that $P_H^* = \bar{P}(\hat{k})$ when $k = \hat{k}$, which implies that $\hat{P} < \bar{P}(\hat{k})$ when $k > \hat{k}$, and this is in contradiction with the results in Proposition 2: the optimal price $\hat{P} > \bar{P}(\hat{k})$. Therefore, the optimal price satisfies $\hat{P} > V/3$ for any $k > \hat{k}$, and with k increases, $G(P)$ increases, thus $\hat{P}$ is increasing in k. We also prove that the optimal price exceeds $V/3$ since the minimal optimal price satisfies $P_H^* > V/3$.

We next focus on the impact of k on the optimal profit. When $k \leq \hat{k}$, the optimal profit is $\Pi_H^* = \Lambda \bar{n} \bar{P}(k)/2 - \mathcal{C}$, where $\bar{P}(k)$ decreases in k, thus the optimal profit also decreases in k. When $k > \hat{k}$, the optimal profit is $\Pi_H^* = \frac{\Lambda}{2\bar{n}}[\bar{n}^2 - \bar{x}_A(\hat{P})^2 + \underline{x}(\hat{P})^2]\hat{P} - \mathcal{C}$. Since we have proved that when $k > \hat{k}$, the profit function $\Pi_H(P)$ is concave in P and the optimal price $\hat{P}$ uniquely solves $\partial \Pi_H / \partial P = 0$, and $\partial \Pi_H / \partial P$ is decreasing in P. In addition, we have also proved that the optimal price $\hat{P}$ increases in $k \in [\hat{k}, \infty)$, this implies that Π_H^* decreases with k. □

EC.2.6. Proof of Proposition 3

Consider the case when $k > \bar{k} = \frac{2(V-P)}{\bar{x}_H(P)} \Leftrightarrow \bar{x}_H(P) > \frac{2(V-P)}{k}$. Recall the consumers' equilibrium strategy described in Lemma 2. For any fixed price P, the system's profit is

$$\Pi_H(P) = \frac{\Lambda\left(\bar{n}^2 - \bar{x}_H(P)^2 + \underline{x}(P)^2\right)}{2\bar{n}} P - \mathcal{C}.$$

The first-order derivative of the profit function with respect to $\bar{n}$ is

$$\frac{\partial \Pi_H(P)}{\partial \bar{n}} = \frac{\Lambda P}{2\bar{n}^2}\left(\bar{n}^2 + \bar{x}_H^2(P) - \underline{x}^2(P) - 2\bar{x}_H(P)\bar{n} \cdot \frac{\partial \bar{x}_H(P)}{\partial \bar{n}}\right).$$

We would like to find an interval of $\bar{n}$ that satisfies $\partial\Pi_H/\partial\bar{n} < 0$, and this is equivalent to

$$\bar{n}^2 + \bar{x}_H^2(P) - 2\bar{x}_H(P)\bar{n}\cdot\frac{\partial\bar{x}_H(P)}{\partial\bar{n}} < \underline{x}^2(P). \tag{EC.21}$$

Since we have proved that $\bar{n}^2 + \bar{x}_H^2(P) - 2\bar{x}_H(P)\bar{n}\cdot\partial\bar{x}_H(P)/\partial\bar{n} > 0$ in Proposition 1, and $\underline{x}(P) = 2(V-P)/k$ decreases in k, while the left-hand side of inequality (EC.21) is independent of k.

When k approaches zero, $\underline{x}(P) \to \infty$, and inequality (EC.21) is satisfied. Conversely, as k approaches infinity, $\underline{x}(P) \to 0$, and inequality (EC.21) is no longer valid. Since $\underline{x}(P)$ is continuous and decreasing in k, we conclude that for any fixed $\bar{n}$, there exists a unique threshold k_1. Inequality (EC.21) is satisfied if and only if $k \le k_1$, where k_1 uniquely solves $\bar{n}^2 + \bar{x}_H^2(P) - 2\bar{x}_H(P)\bar{n}\cdot\frac{\partial\bar{x}_H(P)}{\partial\bar{n}} = \underline{x}^2(P)$. This implies that, for large k, inequality (EC.21) is not satisfied, and the optimal profit increases with $\bar{n}$. Next, we explore the conditions that $\bar{n}$ must satisfy for inequality (EC.21) to be valid when k is small.

Recall that the initial condition of the above analysis is $k > \bar{k} = 2(V-P)/\bar{x}_H(P) \Leftrightarrow \bar{x}_H(P) > 2(V-P)/k$, which implies that $\bar{x}_H(P) > \underline{x}(P) \Rightarrow \bar{x}_H^2(P) > \underline{x}^2(P)$. Furthermore, we have proved in Proposition 1 that $\bar{x}_H(P)$ increases with $\bar{n}$, which requires a lower bound for $\bar{n}$ to ensure that inequality (EC.21) is satisfied combining the initial condition. In addition, as $\bar{n}$ approaches infinity, the left-hand side also goes to infinity, and thus inequality (EC.21) does not hold. Since the left-hand side of inequality (EC.21) is continuous in $\bar{n}$, we conclude that when the consumers' inconvenience parameter of SST is sufficiently low ($k \le k_1$), there exists an interval $[\bar{n}_a, \bar{n}_b]$ for $\bar{n}$ within which the system's profit decreases with $\bar{n}$ for any fixed price P. Consequently, the profit under the optimal price also decreases with $\bar{n}$ for an intermediate range of $\bar{n}$. □

EC.2.7. Proof of Theorem 1

Note that the optimal price P_A^* under the agent-service system is independent of k and solved by $F(P) \equiv 0$. While $P_H^* = \bar{P}(k)$ if $k \le \hat{k}$; otherwise, $P_H^* = \hat{P}$ which is solved by $H(P) \equiv 0$. When $k = 0$, the optimal price of the hybrid-service system is $P_H^* = \bar{P}(0)$ solved by $(V - \bar{P}(0))/\bar{x}(\bar{P}(0)) = 0$, which gives the optimal price $P_H^* = \bar{P}(0) = V > P_A^*$. If $k = \hat{k}$, the optimal price $P_H^* = \bar{P}(\hat{k})$ is solved by $H(\bar{P}(\hat{k})) = 0$, while the optimal price of the agent-service system P_A^* solved by $F(P) = 0$ and $F(P) > H(P)$ since we have proved that the optimal price P_H^* always exceeds $V/3$. Therefore, the two roots of these functions satisfies $\bar{P}(\hat{k}) < P_A^*$ since both $F(P)$ and $H(P)$ decrease in P. In addition, we have also proved that $\hat{P}$ increases in k and is less than P_A^* due to $F(P) > H(P)$. Combined with the decreasing property of P_H^* in $k \in [0, \hat{k}]$, there exists a unique threshold of consumers' inconvenience parameter $\underline{k}$ and if $k \le \underline{k}$, $P_H^* \ge P_A^*$; otherwise, $P_H^* < P_A^*$. Moreover, $\underline{k}$ uniquely solves by $P_A^* = \bar{P}(k)$.

Recall from Proposition 2 that the optimal price decreases with k if $k \leq \hat{k}$ and increases otherwise. Combining this with the above analysis: there exists only one intersection $\underline{k}$ between the optimal prices P_A^* and P_H^*, and it is straightforward to see that $\underline{k} < \hat{k}$. $\square$

EC.2.8. Proof of Proposition 4

If $k \leq \hat{k}$, the hybrid-service system achieves the full market coverage, and it is obvious that $TH_H^*(P_H^*) = \Lambda\bar{n}/2 > TH_A^*(P_A^*)$ and $TH_H^*(P_H^*)$ is independent of k. While if $k > \hat{k}$, $P_H^* = \hat{P}$, we have that

$$TH_H^*(P_H^*) - TH_A^*(P_A^*) = \frac{\Lambda(\bar{x}_A^2(P_A^*) - \bar{x}_H^2(\hat{P}) + \underline{x}^2(\hat{P}))}{2\bar{n}} > 0,$$

the above inequality holds since it has been proven that $P_A^* > \hat{P}$ when $k > \hat{k}$ (see Theorem 1) and $\bar{x}_H(P)$ increases in P. In addition, $\bar{x}_H(\hat{P})$ increases in k since we have proved that $\hat{P}$ increases with k, while $\underline{x}(\hat{P}) = 2(V - \hat{P})/k$ decreases in k, and this also indicates that the system throughput decreases in k when $k > \hat{k}$. $\square$

EC.2.9. Proof of Proposition 5

It is obvious that $\bar{N}_H^* = \bar{n}/2 < \bar{N}_A^* = [\bar{n} + \bar{x}_A(P_A^*)]/2$, when $k \leq \hat{k}$, while if $k > \hat{k}$, it follows that

$$\bar{N}_H^* - \bar{N}_A^* = \frac{(\bar{x}_H(\hat{P}) - \bar{n})(\bar{x}_A(P_A^*) - \bar{x}_H(\hat{P})) + \underline{x}(\hat{P})(\underline{x}(\hat{P}) - \bar{n}) - \bar{x}_H(\hat{P})\bar{x}_A(P_A^*)}{2(\bar{n} - \bar{x}_H(\hat{P}) + \underline{x}(\hat{P}))} < 0.$$

The above inequality holds as it has been proven that $P_A^* > \hat{P}$ and thus $\bar{x}_A(P_A^*) > \bar{x}_H(\hat{P})$. $\square$

EC.2.10. Proof of Theorem 2

Recall the profit functions under two systems with a fixed price P. First, we suppose that $\mathcal{C} = 0$. If $k \leq \bar{k}$, $\Pi_H(P) - \Pi_A(P) = \frac{\Lambda\bar{x}_A^2}{2\bar{n}}P = \frac{\Lambda\bar{x}_H^2}{2\bar{n}}P \geq 0$; otherwise, $\Pi_H(P) - \Pi_A(P) = \frac{\Lambda\underline{x}^2}{2\bar{n}}P \geq 0$. This indicates that given any fixed price P and zero SST cost, the profit of hybrid-service system is always larger than that of the agent-service system, thus the system's profit of hybrid-service system under profit-maximization problem is also larger than that of agent-service system when $\mathcal{C} = 0$.

In addition, we have proved that the optimal profit under hybrid-service system decreases in k in Proposition 1. When k goes to infinity, the hybrid-service system degenerates into the agent-service system since there is no consumer who selects the SST service. Consequently, the supermarket achieves the same profit in the two systems. However, with a non-zero fixed SST cost, the optimal profit under hybrid-service system is less than it under the agent-service system when k goes to infinity. When k goes to zero, there is no effort cost when selecting the SST service, thus all consumers in hybrid-service system use the SST service (where a full market coverage is reached) and the supermarket charges the price equal to consumers surplus, i.e., $P_H^* = V \geq P_A^*$. Also, the system throughput also exhibits $TH_H^* \geq TH_A^*$ regarding the full market coverage compared to the partial market coverage. Hence, we conclude that the hybrid-service system achieves a higher profit than it of the agent-service system with a relatively small SST cost $\mathcal{C}$ when k goes to zero, i.e., $\Pi_H^* \geq \Pi_A^*$. Combined with the decreasing property of Π_H^* in k, there exists a unique threshold k_p such that $\Pi_H^* \geq \Pi_A^*$ when $k \leq k_p$ and $\Pi_H^* < \Pi_A^*$ when $k > k_p$. □

EC.2.11. Proof of Theorem 3

When $k > \hat{k}$, we have proved that $P_H^* = \hat{P} < P_A^*$ and thus $\bar{x}_H(\hat{P}) < \bar{x}_H(P_A^*)$. Therefore, the second term of the bracket in Equation (15) is larger than CS_A^* and then the hybrid-service system achieves a higher consumer surplus. In addition, the consumer surplus of the hybrid-service system decreases in k due to the decreasing property of consumers' utility using SST while consumers' utility of agent-service system is independent in k. As k tends to infinity, no consumers choose the SST, resulting the hybrid-service system to degenerate into the agent-service system. Consequently, both systems exhibit the same behavior and result in equal consumer surplus.

When $k \leq \hat{k}$, the consumer surplus of the hybrid-service system is given by

$$\begin{aligned} CS_H^* &= \frac{\Lambda}{\bar{n}}\left(\frac{1}{2}(V - \bar{P}(k))\hat{x}^2 - \frac{1}{6}k\hat{x}^3 + \int_{\hat{x}}^{\bar{n}}\left((V - \bar{P}(k))t - \frac{h\Lambda(\bar{n}^2 - t^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - t^2)]}\right)dt\right) \\ &= \frac{\Lambda}{\bar{n}}\left(\frac{1}{2}k\hat{x}^2\left(\frac{h\Lambda(\bar{n}^2 - \bar{x}_H(\bar{P}(k))^2)}{2\mu(V - P)[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_H(\bar{P}(k))^2)]} - \frac{1}{3}\hat{x}\right) + \int_{\hat{x}}^{\bar{n}}\left((V - \bar{P}(k))t - \frac{h\Lambda(\bar{n}^2 - t^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - t^2)]}\right)dt\right), \end{aligned} \tag{EC.22}$$

and the second equation of (EC.22) is valid since we have $k/2 = (V - \bar{P})/\bar{x}_H(\bar{P})$ and $(V - P)\bar{x}_H(P) = (h\Lambda(\bar{n}^2 - \bar{x}_H^2(P)))/(\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_H^2(P))])$. Recall that $\hat{x}$ uniquely solves

$$\frac{h\Lambda(\bar{n}^2 - \hat{x}^2)}{\mu[2\mu\bar{n} - \Lambda(\bar{n}^2 - \hat{x}^2)]\hat{x}^2} = \frac{1}{2}k, \tag{EC.23}$$

where the left hand of Equation (EC.23) decreases in $\hat{x}$ and the right hand of it increases in k, and this indicates that $\hat{x}$ decreases in k.

In addition, the first term in the bracket of (EC.22) increases with k, thus the consumer surplus for those who choose the SST service also increases in k. Also, it is obvious that the second term of (EC.22) increases with k due to the decreasing property of both $\bar{P}(k)$ and $\hat{x}$ with respect to k. Therefore, the consumer surplus in the hybrid-service system increases with k when $k \leq \hat{k}$.

When k approaches to zero, there is no effort cost for using SST, thus all consumers choose SST, and the utility function is $U_T(x) = (V - P)x$. The supermarket extracts all consumers' surplus and charges the highest price $P_H^* = V$. Hence, the consumer surplus in the hybrid-service system goes to zero. Recall that as k goes to infinity, both systems achieve equal consumer surplus. Combined with the fact that the consumer surplus increases when $k \leq \hat{k}$ and decreases otherwise, and the consumer surplus is independent of k in the agent-service system, we conclude that there exists a threshold of inconvenience parameter $k_c(< \hat{k})$, which satisfies $CS_H^*(k_c) = CS_A^*(k_c)$. □

EC.2.12. Proof of Lemma EC.1

Given the products' arrival rate λ, consumers' utility increases in demand x. Note also that $U_A^{(2)}(0) < 0$ and $U_A^{(2)}(\bar{n}_1) = (V_1 + \alpha V_2 - \widetilde{P})\bar{n}_1 > 0$, there exists a unique solution $\hat{x}$ of

$$U_A^{(2)}(\bar{x}_A^{(2)}) = (V_1 + \alpha V_2 - \widetilde{P})\bar{x}_A^{(2)} - h\frac{\lambda}{\mu(\mu - \lambda)} = 0 \quad \text{and} \quad \lambda = (1+\alpha)\Lambda \cdot \frac{\bar{n}_1 + \bar{x}_A^{(2)}}{2} \cdot \frac{\bar{n}_1 - \bar{x}_A^{(2)}}{\bar{n}_1},$$

and $\bar{x}_A^{(2)}$ is a function of full price $\widetilde{P}$. When consumers' demand excesses it, they are more likely to place orders, which also means that consumers with demand $x \in [\bar{x}_A^{(2)}, \bar{n}_1]$ of product 1 will place orders and balk if $x \in [0, \bar{x}_A^{(2)})$. Therefore, the effective arrival rate of all products under equilibrium is $\lambda = (1+\alpha)\Lambda \cdot (\bar{n}_1 + \bar{x}_A^{(2)}) \cdot (\bar{n}_1 - \bar{x}_A^{(2)})/(2\bar{n}_1)$ since consumers demand follows uniform distribution. Note that under equilibrium, we have $\hat{U}_A(\bar{x}_A^{(2)}) = 0$, which is equivalent to

$$V_1 + \alpha V_2 - \widetilde{P} = \frac{(1+\alpha)h\Lambda(\bar{n}_1^2 - (\bar{x}_A^{(2)})^2)}{\mu\bar{x}_A^{(2)}[2\mu\bar{n}_1 - (1+\alpha)\Lambda(\bar{n}_1^2 - (\bar{x}_A^{(2)})^2)]}. \tag{EC.24}$$

Moreover, the right hand of Equation (EC.24) decreases in $\bar{x}_A^{(2)}$, which ensures the uniqueness of solution $\bar{x}_A^{(2)}$. □

EC.2.13. Proof of Lemma EC.2

We define $\underline{x}^{(2)} = 2(V_1 + \alpha V_2 - \widetilde{P})/(k(1+\alpha)^2)$ and denote $\bar{x}_H^{(2)}$ as the solution of $U_A^{(2)}(\bar{x}_H^{(2)}) = 0$, which is equivalent to

$$V_1 + \alpha V_2 - \widetilde{P} = \frac{(1+\alpha)h\Lambda(\bar{n}_1^2 - (\bar{x}_H^{(2)})^2)}{\mu\bar{x}_H^{(2)}[2\mu\bar{n}_1 - (1+\alpha)\Lambda(\bar{n}_1^2 - (\bar{x}_H^{(2)})^2)]}. \tag{EC.25}$$

Similar to the analysis of the agent-service case, there exists a unique solution $\bar{x}_H^{(2)}(P)$ of (EC.25). Combined with the previous analysis, we then discuss the following two cases:

Case (1) When $\bar{x}_H^{(2)} \leq \underline{x}^{(2)} \Leftrightarrow k \leq 2(V_1 + \alpha V_2 - \widetilde{P})/((1+\alpha)^2\hat{x})$, there exists a unique solution of $\hat{U}_T(\hat{x}^{(2)}) = \hat{U}_A(\hat{x}^{(2)})$ and we have that $\hat{U}_T(x) \geq \hat{U}_A(x)$ when $x \leq \hat{x}^{(2)}$ and $\hat{U}_T(x) < \hat{U}_A(x)$ when $x > \hat{x}^{(2)}$. Therefore, consumers with the demand of product 1 $x \in [0, \hat{x}^{(2)}]$ prefer self-service and with service demand $x > \hat{x}^{(2)}$ prefer to choose sales agents service. Note that $\hat{x}^{(2)}$ is uniquely determined by

$$\frac{(1+\alpha)h\Lambda(\bar{n}_1^2 - (\hat{x}^{(2)})^2)}{\mu[2\mu\bar{n}_1 - (1+\alpha)\Lambda(\bar{n}_1^2 - (\hat{x}^{(2)})^2)]} = \frac{1}{2}k(1+\alpha)^2(\hat{x}^{(2)})^2.$$

It follows that

$$\hat{x}^{(2)}=\sqrt{\frac{\sqrt{[2h\Lambda+k(1+\alpha)\mu\bar{n}_1(2\mu-(1+\alpha)\Lambda\bar{n}_1)]^2-8h\Lambda^2\bar{n}_1^2k(1+\alpha)^2\mu}-[2h\Lambda+k(1+\alpha)\mu\bar{n}_1(2\mu-(1+\alpha)\Lambda\bar{n}_1)]}{2k(1+\alpha)^2\mu\Lambda}}.$$

Case (2) When $\bar{x}_H^{(2)}>\underline{x}^{(2)}\Leftrightarrow k>2(V_1+\alpha V_2-\widetilde{P})/((1+\alpha)^2\hat{x})$, we have that $\hat{U}_T(x)\geq 0\geq \hat{U}_A(x)$ if $x\in[0,\underline{x}^{(2)}]$, thus consumers prefer self-service when $x\in[0,\underline{x}^{(2)}]$; when $x\in[\hat{x}^{(2)},\bar{n}_1]$, we find that $\hat{U}_A(x)\geq 0\geq \hat{U}_T(x)$, resulting in consumers prefer the sales-agent service. However, when demand $x\in[\underline{x}^{(2)},\hat{x}^{(2)}]$, consumer utility in both service channels is negative, leading them to balk.

□

EC.2.14. Proof of Proposition EC.1

We begin by analyzing the profit-maximization problem under the Hybrid-service system.

$$\max_{P,\mu}\quad \Pi_H(P,\mu)=P\cdot TH_H(P,\mu)-\mathcal{C}-c\mu^2, \tag{EC.26}$$
$$s.t.\ \ \bar{x}_H(P,\mu)\ \text{ uniquely solves (2)},$$

where the throughput $TH_H(P,\mu)$ is given by Lemma 3.

Case (1), when $k\leq\bar{k}$, $TH_H(P,\mu)=\Lambda\bar{n}/2$, the profit function is $\Pi_H(P,\mu)=P\cdot\Lambda\bar{n}/2-\mathcal{C}-c\mu^2$, which is concave in μ, the optimal service capacity $\mu^H=0$, which is independent in k.

Case (2), when $k>\bar{k}$, $TH_H(P,\mu)=\Lambda(\bar{n}^2-\bar{x}_H^2(P)+\underline{x}^2(P))/(2\bar{n})$, the profit function is $\Pi_H(P,\mu)=P\Lambda/(2\bar{n})[(\bar{n}^2-\bar{x}_H^2(P)+\underline{x}^2(P))]-\mathcal{C}-c\mu^2$.

We now focus on Case (2). The optimal service capacity satisfied the condition $\frac{\partial\Pi_H}{\partial\mu^H}=0$, which gives

$$\frac{\partial\Pi_H}{\partial\mu^H}=-\frac{P\Lambda}{\bar{n}}\bar{x}_H\frac{\partial\bar{x}_H}{\partial\mu^H}-2c\mu^H=0, \tag{EC.27}$$

to derive $\frac{\partial\mu^H}{\partial k}$, we take the partial derivative of (EC.27) with k,

$$-\frac{\Lambda}{\bar{n}}\left[\frac{dP}{dk}\cdot\bar{x}_H\cdot\frac{\partial\bar{x}_H}{\partial\mu^H}+P\cdot\left(\left(\frac{\partial\bar{x}_H}{\partial\mu^H}\cdot\frac{d\mu^H}{dk}+\frac{\partial\bar{x}_H}{\partial P}\cdot\frac{dP}{dk}\right)\cdot\frac{\partial\bar{x}_H}{\partial\mu^H}+\bar{x}_H\cdot\left(\frac{\partial^2\bar{x}_H}{\partial(\mu^H)^2}\cdot\frac{d\mu}{dk}+\frac{\partial^2\bar{x}_H}{\partial\mu^H\partial P}\cdot\frac{dP}{dk}\right)\right)\right]-2c\cdot\frac{d\mu^H}{dk}=0,$$

we then derive

$$\frac{d\mu^H}{dk}=-\frac{\frac{\Lambda}{\bar{n}}\cdot\frac{dP}{dk}\cdot\left(\bar{x}_H\cdot\frac{\partial\bar{x}_H}{\partial\mu^H}+P\cdot\left(\frac{\partial\bar{x}_H}{\partial P}\cdot\frac{\partial\bar{x}_H}{\partial\mu^H}+\bar{x}_H\cdot\frac{\partial^2\bar{x}_H}{\partial\mu^H\partial P}\right)\right)}{\frac{\Lambda}{\bar{n}}\cdot P\cdot\left(\left(\frac{\partial\bar{x}_H}{\partial\mu^H}\right)^2+\bar{x}_H\cdot\frac{\partial^2\bar{x}_H}{\partial(\mu^H)^2}\right)+2c}. \tag{EC.28}$$

From Equation (2), taking the first-order derivative of μ, we have

$$\frac{\partial\bar{x}_H}{\partial\mu}=-\frac{(V-P)(\bar{x}_Hg+2\bar{n}\bar{x}_H\mu)}{(V-P)(\mu g+2\Lambda\bar{x}_H^2\mu)+2h\Lambda\bar{x}_H}<0,$$

where $g = 2\mu\bar{n} - \Lambda(\bar{n}^2 - \bar{x}_H^2)$, and

$$\frac{\partial^2 \bar{x}_H}{\partial \mu^2} = -\frac{(V-P)^2 y + 4h\Lambda\bar{x}_H^2(V-P)(2\bar{n} + \Lambda\bar{x}_H\frac{\partial \bar{x}_H}{\partial \mu})}{\left[(V-P)(\mu g + 2\Lambda\bar{x}_H^2\mu) + 2h\Lambda\bar{x}_H\right]^2} > 0,$$

where $g' = 2\bar{n} + 2\Lambda\bar{x}_H\frac{\partial \bar{x}_H}{\partial \mu}$ and $y = \frac{\partial \bar{x}_H}{\partial \mu}(g^2\mu + 2\bar{n}\mu^2 g - 2\bar{x}_H^2\Lambda\mu g - 4\bar{n}\bar{x}_H^2\mu^2\Lambda) + 2\bar{x}_H^3 g'\Lambda\mu - \bar{x}_H g^2 - 2\bar{n}\bar{x}_H\mu^2 g' - 2g\Lambda\bar{x}_H^3 < 0$ and $2\bar{n} + \Lambda\bar{x}_H\frac{\partial \bar{x}_H}{\partial \mu}) < 0$.

Given the above results, in equation (EC.28), the denominator is positive, and we have shown that $\bar{x}_H$ increases with P and decreases with μ. Moreover, in Case (2), the optimal price is increasing in k, this implies that the denominator is negative. Therefore, based on Equation (EC.28), it follows that $\frac{d\mu^H}{dk} > 0$.

Recall that as the inconvenience parameter k approaches infinity, the hybrid service degenerates into the agent-only service, implying that $\mu^H|_{k\to\infty} = \mu^A$. Combined with the fact that $\mu^H = 0$ when k is small, and that μ^H is increasing in k, we conclude that $\mu^H \leq \mu^A$ for all k. $\square$