Solving Nonstationary Non-Markovian Queueing Systems

A Transfer Learning Neural Network Approach

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Joint work with Spyros Garyfallos, Amazon Connect, AWS

AMLC 2025

Queueing Theory

Long waiting line at the AMLC registration desk.

- A mathematical study of waiting lines or queues (a branch of operations research).
- Applications: customer call centers, healthcare, manufacturing systems, transportation, cloud computing, food services, etc.
- Entities in a queueing system:
  - Customers: consumers, patients, jobs, data packets, riders, etc.
  - Servers: agents, doctors, nurses, beds, machines, cloud, drivers, etc.

Real-World Service Systems

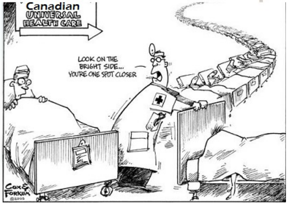
Complex dynamics:

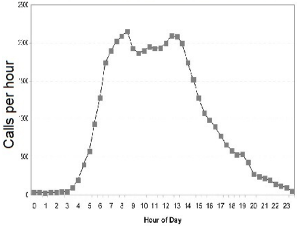
- **Time-varying demand:** Customer arrivals fluctuate throughout the day
- **Time-varying capacity:** Staffing levels adjust to match demand
- **Non-Markovian structure:** Service and abandonment times don't follow exponential distributions
- **Multiple performance objectives:** Need to satisfy concurrent service-level constraints
- **Others:** Complex network topologies, multiple customer classes, customer/server behavior.

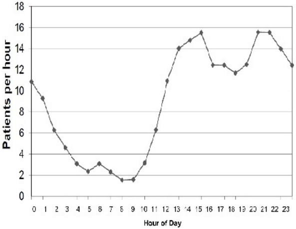
Key Question:

- **Performance prediction:** *How can we accurately predict system performance in such complex systems?*
- **Capacity planning:** *How can we optimally plan service capacity to meet service-level targets under nonstationary non-Markovian dynamics?*
- **Dynamic control:** *What real-time actions should be taken when service-level constraints are at risk of being violated?*

Time-Varying Demand

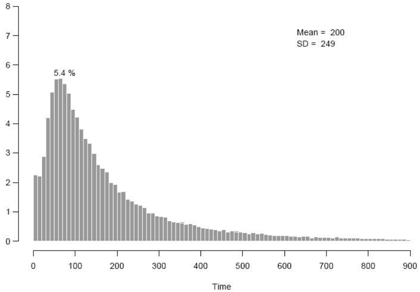


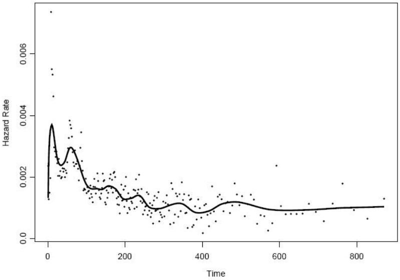




- **Call centers:** Peak hours during business days, lunch breaks
- **Healthcare:** Emergency department arrivals follow daily patterns
- **Cloud services:** Traffic varies by time of day, day of week

Non-Markovian Distributions

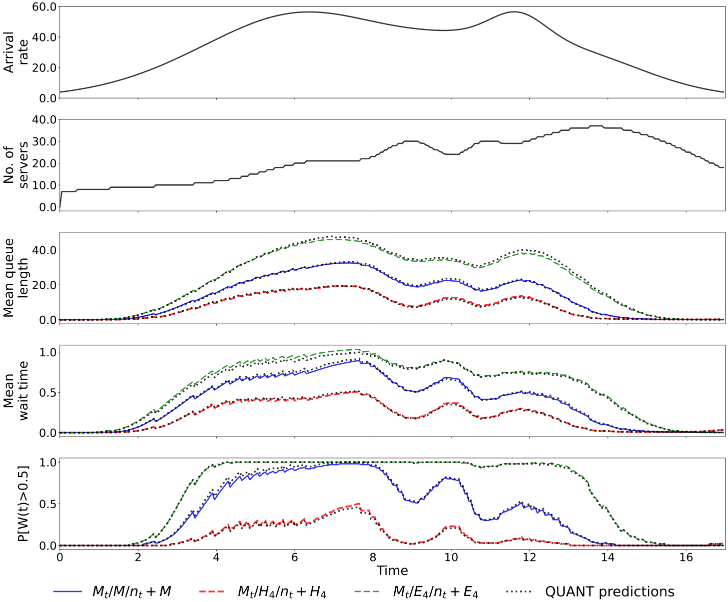




Real service and abandonment times are NOT exponential:

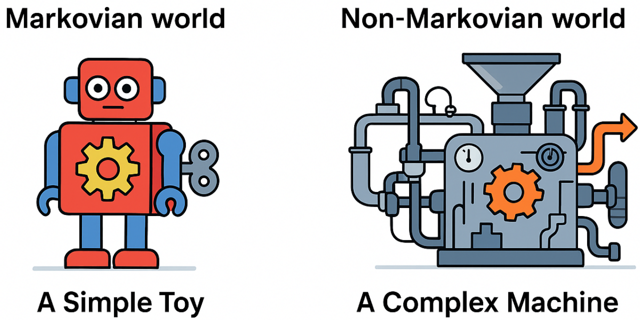
- Call durations often have high variability (lognormal)
- Customer abandonment times show complex patterns
- Distribution shape significantly impacts performance

Markovian vs. Non-Markovian: Performance Divergence



Markovian Model vs. Non-Markovian Model

Methodological complexity:



Markovian Model vs. Non-Markovian Model

Markovian queueing systems:

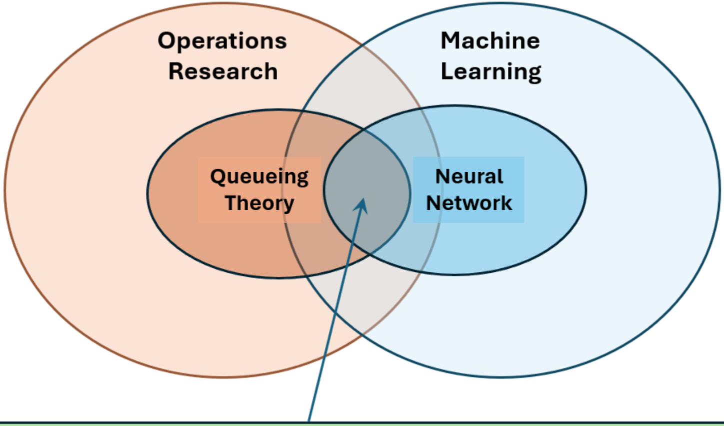
- Memoryless property: Future depends only on current state
- State = simple queue length
- Exact solutions via uniformization and matrix methods
- Computationally tractable

Non-Markovian queueing systems:

- Must track customer ages, residual service times
- State space becomes infinite-dimensional
- No closed-form solutions
- Heavy-traffic approximations are complex and often inaccurate
- Simulation is slow and computationally expensive

QUANT:
QUeueing Analysis via Neural-Network Transfer Learning

Where Are We?



QUANT – **QU**eueing **A**nalysis via **N**eural-network **T**ransfer learning

The $G_t/GI/n_t + GI$ Queueing Model

System Components:

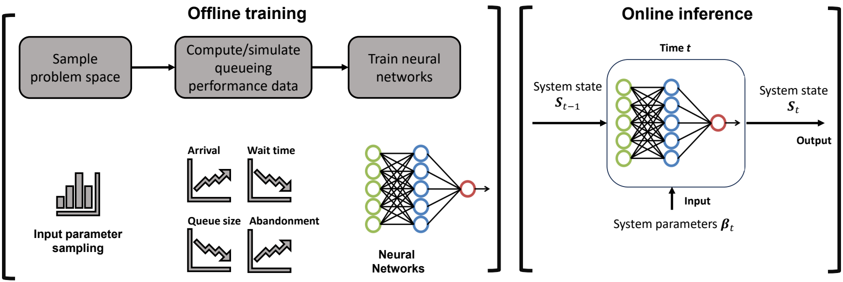
- G_t: General arrival process with nonstationary rate $\lambda(t)$
- GI: General independent service times (distribution G, mean $1/\mu$)
- n_t: Time-varying number of servers $n(t)$
- $+GI$: Customer abandonment (distribution F, mean $1/\theta$)

Performance Metrics to Predict:

- Mean waiting time: $\mathbb{E}[W(t)]$
- Mean queue length: $\mathbb{E}[Q(t)]$
- Mean busy servers: $\mathbb{E}[B(t)]$
- Tail probability of delay: $\mathbb{P}(W(t) \leq \tau)$ (e.g., $\tau = 60$ sec)

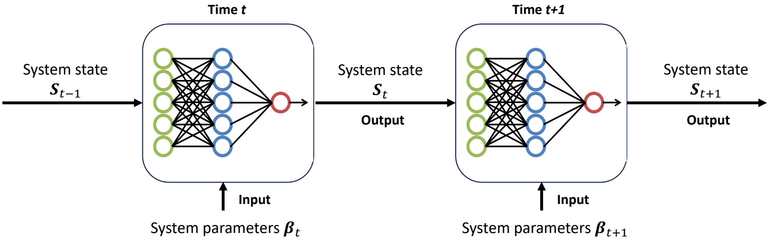
Goal: Predict these metrics over entire horizon $t \in [0, T]$ for any $\lambda(t)$ and $n(t)$

The QUANT Framework



- Goal: Predicts queue performance under time-varying demand and capacity
- Method: Neural network models trained on exact and simulated data
  - Offline phase: generate data and train networks
  - Online phase: fast prediction in real time
  - Inputs: demand rate, staffing level, current system state
  - Outputs: waiting time, queue length, busy servers, service level.

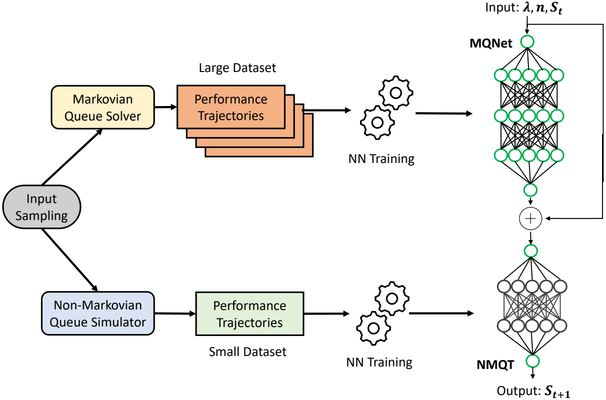
QUANT Architecture: Macroscopic Level



Recurrent Structure:

- One-step prediction: $\mathbf{S}_{t+1} = \mathcal{F}(\mathbf{S}_t, \boldsymbol{\beta}_{t+1})$
- State: $\mathbf{S}_t = (\mathbb{E}[W(t)], \mathbb{E}[Q(t)], \mathbb{E}[B(t)])$
- Input: $\boldsymbol{\beta}_{t+1} = (\lambda(t+1), n(t+1))$
- Roll out predictions recursively over time horizon
- Each step takes $\sim$1 millisecond on GPU

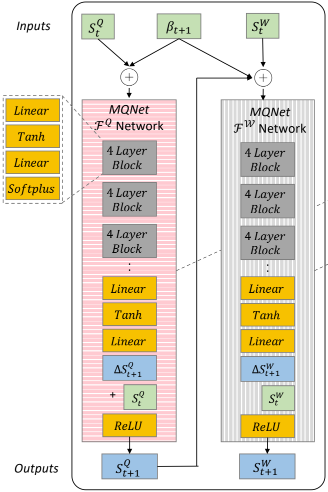
QUANT Architecture: Mesoscopic Level

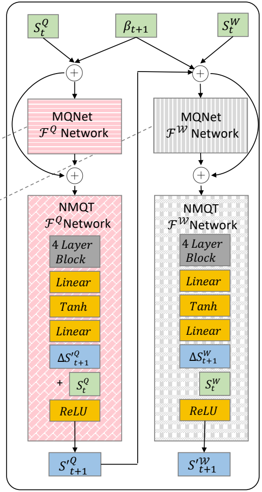


Two-Module Design:

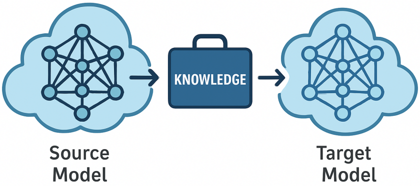
- Markovian Queue Network (MQNet): Pre-trained on $M_t/M/n_t + M$ systems (exact results)
- Non-Markovian Queue Transformer (NMQT): Learns corrections for non-Markovian $G_t/G/n_t + GI$ (simulated results)

QUANT Architecture: Microscopic Level





Why Transfer Learning?



Features Characterizing A Queueing System

1. **System Structure**
   - Topology: single, pooled, parallel, tandem, or networked queues
   - Capacity: number of servers; time-varying or fixed
2. **Operational Rules**
   - Service discipline: FCFS, LCFS, Priority, Processor Sharing
   - Routing: fastest-server-first, join-shortest-queue
   - Customer behavior: balking, abandonment, retrial
3. **Random Elements: Averages**
   - Arrival rate: constant or time-varying
   - Average service time, average abandonment time
4. **Random Elements: Distribution beyond Means**
   - Arrival process: Poisson vs. non-Poisson
   - Service- and abandonment-time distribution: exponential vs. general

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Markovian Model vs. Non-Markovian Model

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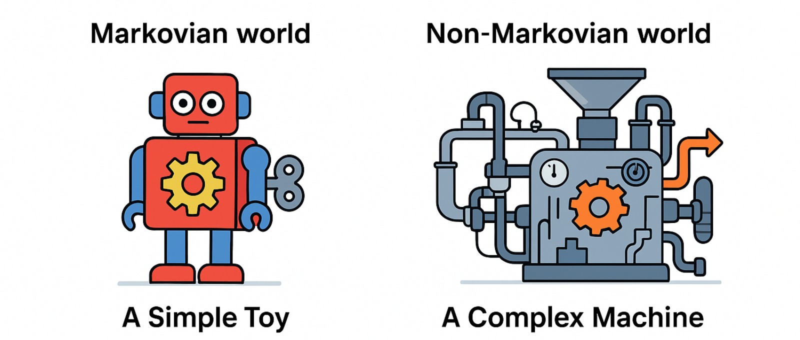


Transfer Learning: The Key Innovation

Why Transfer Learning?

- Markovian systems: Exact solutions available via MQSolver (cheap!)
- Non-Markovian systems: Only simulation available (expensive!)
- Generate 10,000 trajectories for Markovian training: $\sim$7 CPU-days
- Generate 10,000 trajectories for non-Markovian: $\sim$100 CPU-days

Our Solution:

1. Pre-train MQNet on 10,000 Markovian trajectories
2. Fine-tune with only 100 non-Markovian trajectories (1% of data!)
3. Achieves near-optimal accuracy with 99% less simulation
4. Total training: $\sim$8 CPU-days vs. $\sim$100 CPU-days

$\Rightarrow$ **Orders of magnitude reduction in training cost**

Data Generation Strategy

Synthetic Trajectory Generation:

- Time-series composition of Gaussian processes
- Creates realistic nonstationary patterns: single-peak, multi-peak, high-volatility
- Covers operational bounds: $\lambda \in [0, 100]$, $n \in [1, 100]$

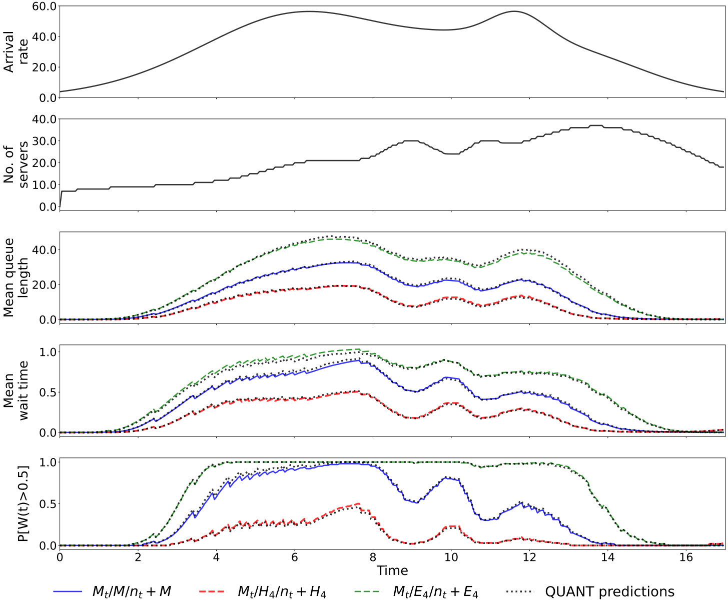
MQSolver: Exact Markovian Solution

- Exploits birth-death process structure
- Uniformization for time-inhomogeneous systems
- Computes exact $\mathbb{P}(W(t) \leq \tau)$ for all t, τ
- Runtime: 60 seconds per trajectory (vs. 900s for simulation)

Non-Markovian Simulation:

- 10,000 replications for smooth estimates
- Only 100 trajectories needed for transfer learning

QUANT Prediction: An Example



Runtime Performance

Method	Runtime (seconds)
QUANT (GPU)	**0.200**
QUANT (CPU)	0.350
MQSolver	60
Simulation	900

Speedup Analysis:

- **300× faster** than exact Markovian solver
- **4,500× faster** than simulation
- Enables real-time optimization with thousands of evaluations
- Critical for production capacity planning systems

Prediction Accuracy and Robustness

Configuration	TAMSE[1]
Base Markovian System	
Queue/Busy metrics	0.09999
Waiting time	0.00131
End-to-end	**0.07032**
By System Scale	
Small scale (0-41 servers)	0.08621
Medium scale (41-72 servers)	0.07559
Large scale (72-100 servers)	0.06377
By Distribution (Non-Markovian)	
$M_t/H_2/n_t + H_2$ (SCV=4)	0.07144
$M_t/E_4/n_t + E_4$ (SCV=1/4)	0.07092

Key Findings:

- Consistent accuracy (error < 0.1) across all configurations
- Minimal degradation from Markovian to non-Markovian
- Scale-invariant performance

[1] TAMSE: time-averaged mean square error

Transfer Learning Efficiency

How much non-Markovian data is actually needed?

Fine-tuning Data	TAMSE[2]	Relative Error
Baseline (10,000 trajectories, no transfer)	0.0703	–
50% (5,000)	0.0709	0.8%
10% (1,000)	0.0714	1.6%
1% (100)	**0.0734**	**4.4%**
0.5% (50)	0.0872	24.0%

Key Insight:

- Robust performance down to **1% of data (100 trajectories)**
- Accuracy within 5% of full dataset
- Sharp degradation below this threshold
- **Practical guideline: Use 100-500 trajectories for new systems**

[2] TAMSE: time-averaged mean square error

QUANT-based Optimal Staffing

Optimal Staffing under Service-Level Constraints

Problem: Given $\lambda(t)$ and constraints, find optimal $n^*(t)$

Example Constraints:

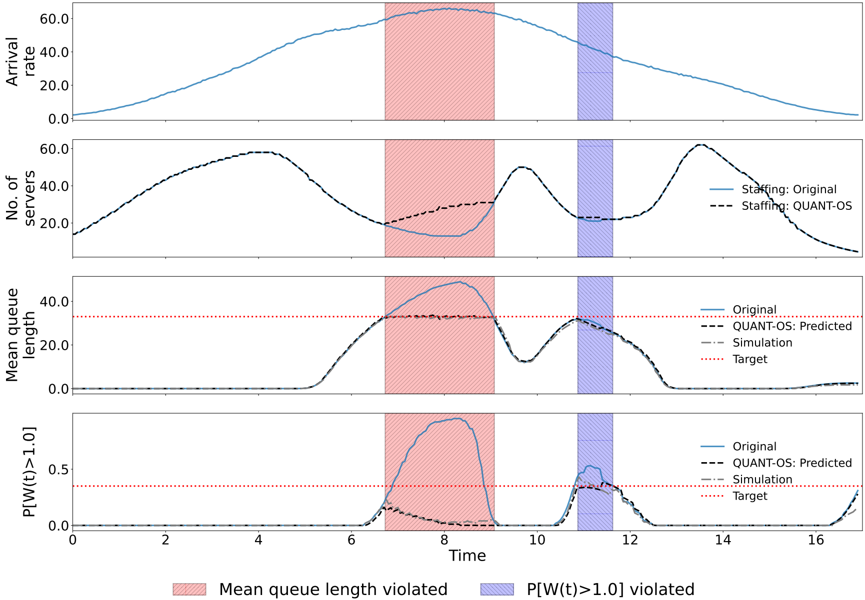
- Mean waiting time: $\mathbb{E}[W(t)] \leq w$ (e.g., $w = 30$ seconds)
- Mean queue length: $\mathbb{E}[Q(t)] \leq q$ (e.g., $q = 10$)
- Tail probability: $\mathbb{P}(W(t) > \tau) \leq \alpha$ (e.g., $\tau = 60$ seconds, $\alpha = 20\%$)
- Meeting multiple constraints simultaneously!

Three Optimization Methods:

1. **Binary Search:** Exploits monotonicity, ∼8 iterations
2. **Batch Evaluation:** Parallel GPU evaluation, 1 pass
3. **Gradient-Based:** Auto-differentiation through QUANT, ∼35 iterations

All methods complete in seconds (vs. hours for simulation-based optimization)

QUANT-OS: A Multi-Constraint Example



Optimization Performance

Method	Iterations	Time (s)	Success
Binary Search	8.2	1.2	100%
Batch Evaluation	1.0	0.8	100%
Gradient-Based	34.5	10.1	97.2%

Validation Against Simulation:

Constraint Type	Pred. Error	Violation Rate
$\mathbb{E}[W] \leq w$	0.082-0.107	2.5-3.5%
$\mathbb{E}[Q] \leq q$	0.080-0.096	2.0-3.0%
$\mathbb{P}(W > \tau) \leq \alpha$	0.131	5.0%
Multi-constraint	0.140	6.0%

$\Rightarrow$ **Over 94% constraint satisfaction in all cases**

Robustness Across Configurations

Configuration	Convergence	Satisfaction	Pred. Error
$M_t/M/n_t + M$	99.2%	98.5%	0.121
$M_t/E_4/n_t + E_4$	98.5%	97.0%	0.224
$M_t/H_2/n_t + H_2$	97.8%	95.5%	0.360
Small scale	96.7%	96.5%	0.156
Medium scale	97.9%	97.0%	0.140
Large scale	98.4%	98.5%	0.132

Key Findings:

- Consistent performance across all distributions tested
- Works for both high and low variability systems
- Scale-independent (works for small and large systems)
- Maintains $> 95\%$ satisfaction even for high-variability systems

Conclusions

Summary:

- Recurrent neural network for nonstationary non-Markovian queues with time-varying capacity
- Transfer learning achieves 99% data reduction
- Real-time multi-constraint optimization in seconds

Future directions:

- Multiple customer classes with priority scheduling
- Network topologies using Graph Neural Networks
- Reinforcement learning with QUANT as environment model
- Multi-objective optimization beyond constraint satisfaction

Thank You!

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Paper: Transfer Learning Neural Networks for Nonstationary Non-Markovian Queueing Systems. *Submitted to INFORMS Journal on Computing.*

Website: `yliu48.github.io`