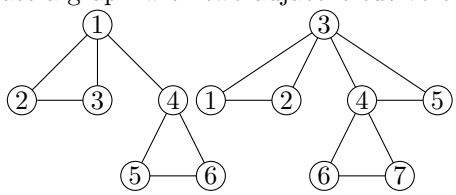
Notes

January 26, 2015

2.2 trees

task:

create a graph with two adjacent cut vertices (connected simple)



notice that removing a cut vertex on the second graph leaves a cut vertex, but not on the first graph.

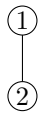
if two cut vertices in a graph G are adjacent and the edge between them is the only trail between them, then that edge is a **cut edge**

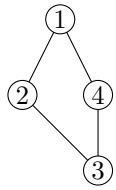
that is to say, every path from cut vertex one to cut vertex 2 contains the edge between them.

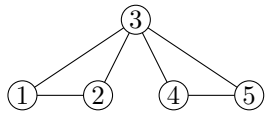
a **bridge** is a cut edge.

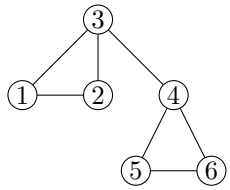
an edge e in G is a bridge if $k(G) < k(G - e)$.

example

1. 

2. 

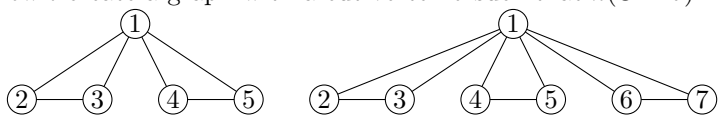
3. 

4. 

1 and 4 contain cut edges.

task

in row i create a graph with a cut vertex v such that $k(G - v) = i + 1$



notice radial symmetry

question

if e is a bridgeof G what values of n can satisfy $k(G - e) = n + 1$

1. even n
2. odd n
3. any n
4. 1

last option is correct (G is connected)

obviously this is different from removing a vertex.

tree is a connected graph, all of whose edges are bridges.

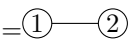
NOTE: if "connected" is removed, but every edge is still a bridge, then you have a forest.

NOTE: if a graph is not simple, then you necessarily have an edge that isn't a bridge, and so trees are simple graphs

theorem

if T is a tree and the order of T is n then the size of T is $n - 1$

proof

CASE 1: $T =$ 

$|T| = 2, |E(T)| = 1 = 2 - 1$

now assume it is true for $|T| = k$

Let T be a tree of order $k + 1$. since every edge is a bridge, remove any edge e to disconnect the graph

Now we have trees T_1, T_2 such that $T_1 \cup T_2 \cup \{e\} = T$. if $|T_1| = a$ then $|T_2| = k + 1 - a$

by the inductive hypothesis $|E(T_1)| = a - 1$ and $|E(T_2)| = k + 1 - a - 1 = k - a$ therefore $|E(T)| = |E(T_1)| + |E(T_2)| + 1 = a - 1 + k - a + 1 = k$

theorem

the converse is also true.

a graph G of order n is a tree iff the size is $n - 1$

a graph G is a tree iff G has no cycles

homework

2.2 numbers 1,2,3,10,14,17