separable

$$g(y)\,\mathrm{d}y = f(t)\,\mathrm{d}t \qquad\qquad g(y)y' = f(t)$$

$$\int g(y)\,\mathrm{d}y = \int f(t)\,\mathrm{d}t$$

first order linear

$$\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y = q(t) \qquad\qquad \mu(t) = e^{\int p(t)\,\mathrm{d}t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mu(t)y\right) = \mu(t)\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)\mu(t)y \qquad\qquad \mu(t)y = \int \mu(t)q(t)\,\mathrm{d}t$$

exact

$$M(t,y)\,\mathrm{d}t + N(t,y)\,\mathrm{d}y = 0 \qquad\qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

$$\int M(t,y)\,\mathrm{d}t + \phi(y) = f(t,y) \qquad\qquad \phi'(y) = N(x,y) - \frac{\mathrm{d}}{\mathrm{d}y}\left(\int M(t,y)\,\mathrm{d}t\right)$$

$$\int M(t,y)\,\mathrm{d}t + \int \phi'(y)\,\mathrm{d}y = f(t,y)$$

Solution is $f(t,y) = C$

bernoulli

$$\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y = q(t)y^n \qquad\qquad w = y^{1-n)}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} + (1-n)p(t)w = (1-n)q(t)$$

Solve as first order linear, then back substitute

homogeneous

$$M(t,y)\,\mathrm{d}t + N(t,y)\,\mathrm{d}y = 0 \qquad\qquad M(xt,xy) + N(xt,xy) = x^n\left(M(t,y) + N(t,y)\right)$$

substitute with $y = wt$ if $N(t,y)$ is simpler and $t = wy$ if $M(t,y)$ is simpler

$$\mathrm{d}y = w\,\mathrm{d}t + t\,\mathrm{d}w \qquad\qquad \mathrm{d}t = w\,\mathrm{d}y + y\,\mathrm{d}w$$

Solve as a separable equation