Notes

April 14, 2014

homework due on 25 april now

lesson 23

classification of pde's

PDE	$Au_{xx}+Bu_{xy}+Cu_{yy}+Du_x+Eu_y+Fu=G$	$A=A(x,y),\dots,G=G(x,y)$
Idea:	New variables $\xi=\xi(x,y),\eta=\eta(x,y)$	
PDE	$\hat{A}u_{\xi\xi}+\hat{B}u_{\xi\eta}+\hat{C}u_{\eta\eta}+\hat{D}u_\xi+\hat{E}u_\eta+\hat{F}u=\hat{G}$	$\hat{A},\dots,\hat{G}$ on page 177
Idea:	$\hat{A}=0=A\xi_x^2+B\xi_x\xi_y+C\xi_y^2$	
	$\hat{C}=0=A\eta_x^2+B\eta_x\eta_y+C\eta_y^2$	
curves:	$\xi=$ constant have $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{\xi_x}{\xi_y}\to A(\dfrac{\mathrm{d}y}{\mathrm{d}x})^2-B\dfrac{\mathrm{d}y}{\mathrm{d}x}+C=0$	
curves:	$\eta=$ constant have $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{\eta_x}{\eta_y}\to A(\dfrac{\mathrm{d}y}{\mathrm{d}x})^2-B\dfrac{\mathrm{d}y}{\mathrm{d}x}+C=0$	

in agreement with the text we take

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2A}\left(B-\sqrt{B^2-4AC}\right)\qquad \xi\equiv\text{const}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2A}\left(B+\sqrt{B^2-4AC}\right)\qquad \eta\equiv\text{const}$$

define hyperbolic pde as $B^2-4AC>0$ and parabolis is $B^2-4AC=0$ and elliptic is $B^2-4AC>0$

example

$$\begin{aligned}
u_{tt}&=c^2u_{xx} && t=y\\
c^2u_{xx}-u_{yy}&=0\\
A&=c^2\quad b=0\quad c=-1\quad D=E=F=G=0\\
\xi\equiv\text{const}&\quad\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2C^2}\left[0-\sqrt{0+4C^2}\right]\\
&=-\frac{1}{c}\\
y&=-\frac{x}{c}+\text{const determines }\xi\equiv\\
\eta\equiv\text{const}&\quad\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2C^2}\left[0+\sqrt{0+4C^2}\right]
\end{aligned}$$

$$
\begin{aligned}
y &= \frac{x}{c} + \text{const determines } \eta \equiv \\
&= \frac{1}{c} \\
\xi &= x + cy \\
\eta &= x - cy
\end{aligned}
$$

what is new pde?

$$
\begin{aligned}
\hat{A}u_{\xi\xi} + \hat{B}u_{\xi\eta} + \hat{C}u_{\eta\eta} + \ldots &= \hat{G} \\
0 + \hat{B} + 0 + 0 &= 0 \quad \text{from 23.6 on page 177}
\end{aligned}
$$

new pde

$$
\begin{aligned}
4c^2 u_{\xi\eta} &= 0 \quad \text{or} \\
u_{\xi\eta} &= 0 \text{ this has general solutioon}
\end{aligned}
$$

example

$$
\begin{aligned}
y^2 u_{xx} - x^2 u_{yy} &= 0 \\
A = y^2 \quad b &= 0 \quad c = -x^2 \quad D = E = F = G = 0 \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2y^2}\left[0 - \sqrt{0 + 4y^2x^2}\right] = -\frac{x}{y} \\
x^2 + y^2 &= a_1 \\
\xi &= x^2 + y^2 \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2y^2}\left[0 + \sqrt{0 + 4y^2x^2}\right] = \frac{x}{y} \\
x^2 - y^2 &= a_2 \\
\eta &= x^2 - y^2
\end{aligned}
$$

note typo in text around page 179, formulas on bottom of page not possible, sign errors

$$
\begin{aligned}
\hat{A} &= \hat{C} = 0 \\
\hat{B} &= 2y^2 2x2y + 0 + (-2x^2)(2y)(-2y) \\
&\vdots \\
16x^2y^2 u_{\xi\eta} + \hat{D}u_\xi + \hat{E}u_\eta &= 0
\end{aligned}
$$

change coefficients from xy to $\xi\eta$