

## Chapter 5

**5.**

Expand  $(2x - y)^7$  using the binomial theorem.

**7.**

Use the binomial theorem to prove that

$$3^n = \sum_{k=0}^n \binom{n}{k} 2^k.$$

Generalize to find the sum

$$\sum_{k=0}^n \binom{n}{k} r^k$$

**8.**

Use the binomial theorem to prove that

$$2^n = \sum_{k=0}^n (-1)^k \binom{n}{k} 3^{n-k}.$$

**10.**

Use *combinatorial* reasoning to prove the identity (5.2).

**11.**

Use *combinatorial* reasoning to prove the identity (in the form given)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

(*Hint:* Let  $S$  be a set with three distinguished elements  $a, b$ , and  $c$  and count certain  $k$ -subsets of  $S$ .)

**14.**

**15.**

**16.**

**17.**

**21.**

**27.**

**28.**