

Verify that the equation

$$3u_{xx} + 7u_{xy} + 2u_{yy} = 0$$

is hyperbolic for all x and y and find the new *characteristic coordinates*.

$$B^2 - 4AC = 7^2 - 4 \cdot 3 \cdot 2 = 49 - 24 = 25 > 0$$

Since $B^2 - 4AC > 0$ the equation is hyperbolic. Now for the characteristic coordinates.

$$\begin{aligned}\xi &= \xi(x, y) \\ \eta &= \eta(x, y) \\ u(x, y) &\rightarrow u(\xi, \eta) = u(\xi(x, y), \eta(x, y)) \\ u_{xx} &= u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_{\xi}\xi_{xx} + u_{\eta}\eta_{xx} \\ u_{yy} &= u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_{\xi}\xi_{yy} + u_{\eta}\eta_{yy} \\ u_{xy} &= u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_{\xi}\xi_{xy} + u_{\eta}\eta_{xy} \\ \bar{A} = 0 &= A\xi_x^2 + B\xi_x\xi_y + C\xi_y^2 \\ \bar{C} = 0 &= C\eta_x^2 + B\eta_x\eta_y + C\eta_y^2 \\ \bar{A} = 0 &= A\left[\frac{\xi_x}{\xi_y}\right]^2 + B\left[\frac{\xi_x}{\xi_y}\right] + C \\ \bar{C} = 0 &= A\left[\frac{\eta_x}{\eta_y}\right]^2 + B\left[\frac{\eta_x}{\eta_y}\right] + C \\ \frac{dy}{dx} = \frac{\xi_x}{\xi_y} &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} = \frac{-7 - \sqrt{25}}{6} = -2 \\ \frac{dy}{dx} = \frac{\eta_x}{\eta_y} &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{-7 + \sqrt{25}}{6} = -\frac{1}{3} \\ y &= -2x + c_1 \quad \xi = y + 2x = c_1 \\ y &= -\frac{1}{3}x + c_2 \quad \eta = y + \frac{1}{3}x = c_2\end{aligned}$$

$$\xi = \text{constant} = c = f(x, y)$$

$$\frac{\partial}{\partial x}c = \frac{\partial}{\partial x}f(x, y)$$