Notes

25 février, 2015

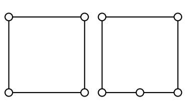
6.2 planarity vs nonplanarity

how to tell if a graph is planar without drawing it in a plane?
on monday we saw some options for finding non-planarity.

1. $e > 3v - 6$
2. can't have $K_{3,3}$ or K_5 as a subgraph
3. didn't get to this last time, but all vertices have degree 6 or more (this follows from one of the above)

if G is a graph then H is a **subdivision** of G if H is obtained from G by adding a vertex on any edge of G.
The process can be iterated.

example



1. H is a subdivision of G implies that H and G have the same number of cycles (faces if planar)
2. H is a subdivision of G implies that $|G| = m$ and $|E(G)| = m$ implies that $|H| = n + c$ and $|E(H)| = m + c$.

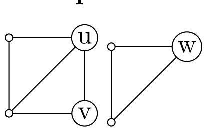
kuratowski's theorem

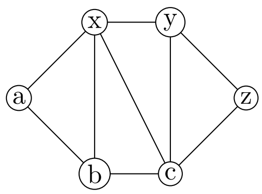
a graph G is planar if and only if G contains no subdivisions of $K_{3,3}$ or K_5 as subgraphs.

remark

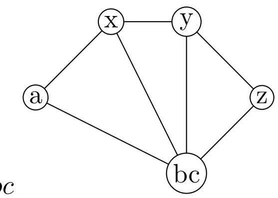
there is an inverse operation to subdivision called **edge contraction**
an **edge contraction** of $uv \in E(G)$ is when uv is replaced by vertex w. The neighbors of w are the neighbors of u and v with duplicates deleted. Then u and v are removed.

example





contract bc



a **minor** of a graph is any graph obtained from edge contraction, edge deletion or vertex deletion

all subgraphs are minors. add in edge contraction to subgraphs and you get a minor.

no connection to minor of a matrix

wagner/kuratowski thrm

a graph is planar if and only if it contains no minors isomorphic to $K_{3,3}$ or K_5.

overarching thrm

if property P is maintained under taking minors then there exists a finite list of excluded/impossible minors.

ie for planarity, the list is $K_{3,3}, K_5$.

what are some minor closed properties? planarity, forest (excluded minor is K_3)

homework

2,3,7,10