Notes

September 10, 2014

2.6.3 the nested intervals lemma

I_n's need to be closed:

$$I_n = (0, \frac{1}{n})$$

this is not closed

2.6.I

Let $\{a_n\}_{n=1}^{\infty}$ be a bounded sequence. we define $b_n = \sup_{k\geq n}\{a_k\}$. $b_1 = \sup_{i\geq 1}\{a_i\}, b_2 = \sup_{i\geq 2}\{a_i\}, \ldots$. Let X, Y be sets, $X \subseteq Y$. $\sup X \leq \sup Y$, $\inf X \geq \inf Y$. Because $\{a_n\}_{n=1}^{\infty}$ is bounded, $\{b_n\}_{n=1}^{\infty}$ is bounded. b_n is a decreasing sequence. $\lim_{n\to\infty} b_n = \inf b_n$.

definition: the limsup of a bounded sequence $\{a_n\}$ is $\inf_{n\in\mathbb{N}} \left(\sup_{k\geq n}\{a_k\}\right) = \lim_{n\to\infty\left(\sup_{k\geq n}\{a_k\}\right)}$ and it always exists.

example

let $a_n = (-1)^n$. $b_n = 1 \forall n$ and $\lim_{n\to\infty} b_n = 1$. $\limsup_{n\to\infty}(-1)^n = 1$

definition: the $\liminf_{n\to\infty} a_n = \sup_{n\in\mathbb{N}} (\inf_{k\geq n} a_n)$ always exists.

we can extend this concept to unbounded sequences and allow the values $\pm\infty$ for $\limsup$ and $\liminf$

increasing sequence of supremum negative infinity: all negative infinity

proposition

let $\{a_n\}_{n=1}^{\infty}$ be a sequence. then $\lim_{n\to\infty} a_n$ exists $\Leftrightarrow \liminf_{n\to\infty} a_n$

we are assuming that $\lim_{n\to\infty} a_n = L \in \mathbb{R}, |a_n - L| < \epsilon \forall \epsilon > 0$. for $\epsilon > 0 \exists N, a_n \in (L-\epsilon, L+\epsilon)$ if $n \geq N$