Jon Allen
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5.1

#13

A 6-lb object stretches a spring 6 in. If the object is lifted 3 in. above the equilibrium position and released, determine the time required for the mass to return to its equilibrium position. What is the displacement of the object at t = 5 s? If the object is released from its equilibrium position with a downward initial velocity of 1 ft/s, determine the time required for the object to return to its equilibrium position.

solution

$$s = \frac{1}{2} \qquad F = 6 \qquad a = g = 32$$
$$\alpha = -\frac{1}{4} \qquad \beta = 0 \qquad F = ks = ma$$
$$k = 12 \qquad m = \frac{3}{16} \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12 \cdot 16}{3}} = 8$$
$$x(t) = -\frac{1}{4}\cos 8t \qquad 0 = -\frac{1}{4}\cos 8t \qquad t = \frac{1}{8}\arccos 0 = \frac{\pi}{16} \approx 0.196\text{s}$$
$$x(5) = -\frac{1}{4}\cos 40 \approx 0.167\text{ft} \qquad \alpha = 0 \qquad \beta = 1$$
$$x(t) = \frac{1}{8}\sin 8t \qquad 0 = \frac{1}{8}\sin 8t \qquad t = \frac{1}{8}\arcsin 0 = \frac{\pi}{8} \approx 0.393\text{s}$$

#14

A 16-lb weight stretches a spring 8 in. If the weight is lowered 4 in. below the equilibrium position and released, find the time required for the weight to return to the equilibrium position. What is the displacement of the weight at t = 4 s? If the weight is released from its equilibrium position with an upward initial velocity of 2 ft/s, determine the time required for the weight to return to the equilibrium position.

solution

$$s = \frac{2}{3} \qquad F = 16 \qquad a = g = 32$$
$$\alpha = \frac{1}{3} \qquad \beta = 0 \qquad F = ks = ma$$
$$k = 24 \qquad m = \frac{1}{2} \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{48} = 4\sqrt{3}$$
$$x(t) = \frac{1}{3}\cos(4\sqrt{3} \cdot t) \qquad 0 = \frac{1}{3}\cos(4\sqrt{3} \cdot t) \qquad t = \frac{1}{4\sqrt{3}}\arccos 0 = \frac{\pi}{8\sqrt{3}} \approx 0.227\text{s}$$
$$x(4) = \frac{1}{3}\cos(16\sqrt{3}) \approx -0.282\text{ft} \qquad \alpha = 0 \qquad \beta = -2$$
$$x(t) = -\frac{1}{2\sqrt{3}}\sin(4\sqrt{3} \cdot t) \qquad 0 = -\frac{1}{2\sqrt{3}}\sin(4\sqrt{3} \cdot t) \qquad t = \frac{1}{4\sqrt{3}}\arcsin 0 = \frac{\pi}{4\sqrt{3}} \approx 0.453\text{s}$$

5.2

#3

$$\frac{1}{4}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0, \quad x(0) = -\frac{1}{2}, \quad x'(0) = 1$$

solution

The mass $m = \frac{1}{4}$slugs, the spring constat $k = 1$lb/ft, and the damping coefficient $c = 2$. The object starts 6 inches above equilibrium and is thrown down with an initial velocity of 1ft/s.