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October 30, 2013

4.7

#8

$$x^2y'' - 5xy' + 10y = 0$$

solution

$$\begin{aligned}
0 &= y(t)'' - y(t)' - 5y(t)' + 10y(t) \\
0 &= r^2 - 6r + 10 \\
r &= \frac{6 \pm \sqrt{36 - 4 \cdot 10}}{2} = 3 \pm i \\
y(t) &= e^{3t}\left(c_1 \cos t + c_2 \sin t\right) \\
y(x) &= x^3\left(c_1 \cos(\ln x) + c_2 \sin(\ln x)\right)
\end{aligned}$$

#12

$$x^2y'' + 7xy' + 9y = 0$$

solution

$$\begin{aligned}
0 &= y(t)'' - y(t)' + 7y(t)' + 9y(t) \\
r &= -3 \\
y(t) &= c_1e^{-3t} + c_2te^{-3t} \\
y(x) &= c_1x^{-3} + c_2x^{-3}\ln x = \frac{c_1 + c_2\ln x}{x^3}
\end{aligned}$$

#22

$$x^2y'' - 5xy' + 9y = x^3, \quad x > 0$$

solution

$$\begin{aligned}
e^{3t} &= y(t)'' - y(t)' - 5y(t)' + 9y(t) \\
0 &= r^2 - 6r + 9 \\
r &= 3 \\
y_h(t) &= c_1e^{3t} + c_2te^{3t} \\
y_p(t) &= At^2e^{3t} \\
y_p(t)' &= 3At^2e^{3t} + 2Ate^{3t} \\
y_p(t)'' &= 9At^2e^{3t} + 6Ate^{3t} + 6Ate^{3t} + 2Ae^{3t} \\
&= 9At^2e^{3t} + 12Ate^{3t} + 2Ae^{3t} \\
e^{3t} &= 9At^2e^{3t} + 12Ate^{3t} + 2Ae^{3t} - 6\left(3At^2e^{3t} + 2Ate^{3t}\right) + 9\left(At^2e^{3t}\right)
\end{aligned}$$

$$1 = A\left(9t^2 - 18t^2 + 9t^2 + 12t - 12t + 2\right), \quad A = \frac{1}{2}$$
$$y(t) = c_1 e^{3t} + c_2 t e^{3t} + \frac{1}{2} t^2 e^{3t}$$
$$y(x) = x^3 \left(c_1 + c_2 \ln x + \frac{1}{2}(\ln x)^2\right)$$

#27

$$x^2 y'' + xy' + 4y = 8, \quad x > 0$$

solution

$$8 = y(t)'' - y(t)' + y(t)' + 4y(t)$$
$$0 = r^2 + 4$$
$$r = \frac{0 \pm \sqrt{0 - 4 \cdot 4}}{2} = \pm 2i$$
$$y_h(t) = c_1 \cos 2t + c_2 \sin 2t$$
$$y_p(t) = A, \quad y_p(t)' = y_p(t)'' = 0$$
$$8 = 0 + 4A, \quad A = 2$$
$$y(t) = c_1 \cos 2t + c_2 \sin 2t + 2$$
$$y(x) = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x) + 2$$