What is the solution to the inital-value problem

$$\text{PDE} \quad u_{tt} = u_{xx} \quad -\infty < x < \infty \quad 0 < t < \infty$$

$$\text{ICs} \quad \begin{cases} u(x,0) &= 0 \\ u_t(x,0) &= xe^{-x^2} \end{cases} \quad -\infty < x < \infty$$

Graph the solution $u(x,t)$ for vairous values of time.

$$\begin{aligned}
u(x,t) &= \frac{1}{2}\left[f(x-ct)+f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\xi)\,\mathrm{d}\xi \\
f(x) &= 0 \\
g(x) &= xe^{-x^2} \\
c &= \pm 1 \\
u(x,t) &= \pm\frac{1}{2}\int_{x\mp t}^{x\pm t} \xi e^{-\xi^2}\mathrm{d}\xi \\
-\frac{1}{2}\int_{x+t}^{x-t} \xi e^{-\xi^2}\mathrm{d}\xi &= \frac{1}{2}\int_{x-t}^{x+t} \xi e^{-\xi^2}\mathrm{d}\xi \\
\mu = -\xi^2 &\quad \mathrm{d}\mu = -2\xi\mathrm{d}\xi \\
u(x,t) &= -\frac{1}{2\cdot 2}\int_{x-t}^{x+t} -2\xi e^{-\xi^2}\mathrm{d}\xi \\
&= \frac{1}{4}\int_{-(x+t)^2}^{-(x-t)^2} e^{\mu}\mathrm{d}\mu \\
&= \frac{1}{4}\left[e^{-(x-t)^2} - e^{-(x+t)^2}\right]
\end{aligned}$$