# Notes

4 mars, 2015

$$\log x$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-1)^k}{k}$$

radius of convergence is 1 and interval of convergence is $(0,2]$ from last time

in general the $n$th derivative looks like $f^{(n)} = \frac{(-1)^{n+1}(n-1)!}{x^n}$.

we restrict our $[a,b]$ such that $0 < a \leq b$.

now $|f^{(n_1)}(x)| = |f^{(n+1)}(a)| = \frac{(n)!}{a^n}$

now with taylors thm $|R_n(x)| \leq \frac{n!}{a^n}(\frac{|x-1|^n}{(n+1)!}$

take the limit of n to inf and

$$\lim |R_n| = \lim \frac{|x-1|^n}{a^n n} \to 0$$

and so $\log x = \sum\limits_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}(x-1)^k$

### example

$$f(x) = \begin{cases} e^{1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f^{(n)}(x) = \frac{q_n(x)}{x^{3n}} e^{-1/x^2} \qquad x \neq 0$$

with $q_n(x)$ a polynomial of degree less than or equal to 2n first derivative at zero is zero because $\lim_{x\to 0} \frac{f(x)-0}{x-0} = \frac{e^{-1/x^2}}{x}$ which is zero with l'hopitals rule. same with inductive step fr $n$th degree. but then the power series at 0 converges to $P_n(x) = 0$!

## can we approximate a continuous function by a polynomial?

yes, but....

not usually with power series.

## thm

if $f$ is continuous on $[a,b]$ then for any $\epsilon > 0$ there is a polynomial $p_\epsilon$ such that $||f - p_\epsilon||_\infty < \epsilon$

the book does three proofs of this. bernstein, chebychev, and a general proof. stone-weierstrass thm.

## corollary

if $f$ is continuous on $[0,1]$ and $\int_0^1 f(x)x^n \, \mathrm{d}x$ for all $n$ theen $f(x) = 0$.

### proof

$\int_0^1 |f(x)|^2 \, \mathrm{d}x = \lim \int_0^1 f(x)p_n(x) \, \mathrm{d}x$ with $p_n(x)$ being the polynomial approximation from weierstrass approximation theorem.

$$\lim a_n \int f(x)x^n + a_{n-1} \int \cdots + a_0 \int f(x)x^0 \, \mathrm{d}x = 0$$