Jon Allen
HW 21
CASE 2: $\alpha_2\beta_1 - \alpha_1\beta_2 + \beta_1\beta_2 = 0$
Given the problem:

PDE	$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + f(x,t)$	$0 < x < 1,$	$0 < t < \infty$
BC	$g_1(t) = \alpha_1 \dfrac{\partial u}{\partial x}(0,t) + \beta_1 u(0,t)$		$0 < t < \infty$
	$g_2(t) = \alpha_2 \dfrac{\partial u}{\partial x}(1,t) + \beta_2 u(1,t)$	${\alpha_1}^2 + {\beta_1}^2 \neq 0$	${\alpha_2}^2 + {\beta_2}^2 \neq 0$
IC	$u(x,0) = \phi(x)$	$0 < x < 1$	

Assume $\alpha_2\beta_1 - \alpha_1\beta_2 + \beta_1\beta_2 = 0$. A change of vriables of the form $u = w + a(t)x + b(t)(1-x)$ cannot convert the problem to homogeneous BC for w for arbitrary $g_1(t), g_2(t)$. Consider the change of variables

- $u(x,t) = w(x,t) + a(t)x^p + b(t)(1-x)^p$ with $p > 1$

Here $a(t), b(t)$ and p are to be determined so that $w(x,t)$ satisfies the homogeneous BC:

BC	$\alpha_1 \dfrac{\partial w}{\partial x}(0,t) + \beta_1 w(0,t) = 0$	$0 < t < \infty$
	$\alpha_2 \dfrac{\partial w}{\partial x}(1,t) + \beta_2 w(1,t) = 0$	

(a) Show that there always exist values $p > 1$ such that the homogeneous BC for $w(x,t)$ can be achieved (that is, a solution for $a(t), b(t)$ can be found) for *arbitrary* functions $g_1(t), g_2(t)$ in the original problem. This will involve conditions on p in terms of $\alpha_1, \alpha_2, \beta_1, \beta_2$.

(b) :