Verify that the equation

$$3u_{xx} + 7u_{xy} + 2u_{yy} = 0$$

is hyperbolic for all x and y and find the new *characteristic coordinates*.

$$B^2 - 4AC = 7^2 - 4 \cdot 3 \cdot 2 = 49 - 24 = 25 > 0$$

Since $B^2 - 4AC > 0$ the equation is hyperbolic. Now for the characteristic coordinates.

$$\begin{aligned}
\xi &= \xi(x,y)\\
\eta &= \eta(x,y)\\
u(x,y) \to u(\xi,\eta) &= u(\xi(x,y),\eta(x,y))\\
u_{xx} &= u_{\xi\xi}{\xi_x}^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}{\eta_x}^2 + u_\xi\xi_{xx} + u_\eta\eta_{xx}\\
u_{yy} &= u_{\xi\xi}{\xi_y}^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}{\eta_y}^2 + u_\xi\xi_{yy} + u_\eta\eta_{yy}\\
u_{xy} &= u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_\xi\xi_{xy} + u_\eta\eta_{xy}\\
\overline{A} = 0 &= A{\xi_x}^2 + B\xi_x\xi_y + C{\xi_y}^2\\
\overline{C} = 0 &= C{\eta_x}^2 + B\eta_x\eta_y + C{\eta_y}^2\\
\overline{A} = 0 &= A\left[\frac{\xi_x}{\xi_y}\right]^2 + B\left[\frac{\xi_x}{\xi_y}\right] + C\\
\overline{C} = 0 &= A\left[\frac{\eta_x}{\eta_y}\right]^2 + B\left[\frac{\eta_x}{\eta_y}\right] + C\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\xi_x}{\xi_y} = \frac{-B - \sqrt{B^2 - 4AC}}{2A} = \frac{-7 - \sqrt{25}}{6} = -2\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\eta_x}{\eta_y} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{-7 + \sqrt{25}}{6} = -\frac{1}{3}\\
y &= -2x + c_1 \quad \xi = y + 2x = c_1\\
y &= -\frac{1}{3}x + c_2 \quad \eta = y + \frac{1}{3}x = c_2
\end{aligned}$$

$$\begin{aligned}
\xi &= \text{constant} = c = f(x,y)\\
\frac{\partial}{\partial x}c &= \frac{\partial}{\partial x}f(x,y)
\end{aligned}$$