Part 2 (3 points): Due in class Friday, May 2.

Chapter 8: #33 (just the first part where you calculate R7 and R8), and the problems below.

Schroder exercise 1: Draw the dissection of the 11-gon that corresponds to the bracketing (a1 ((a2 a3 a4 a5) a6) (a7 (a8 a9) a10)).

Schroder exercise 2: Draw the Schroder paths from (0,0) to (6,0). Verify that exactly half have no horizontal steps on the x-axis.

Chapter 8

32. Use the recurrence relation (8.31) to compute the small Schröder numbers s_8 and s_9.

    We are given the s_1 through s_7 on page 310. They are $1, 1, 3, 11, 45, 197, 903, \ldots$

$$
\begin{aligned}
0 &= (n+2)s_{n+2} - 3(2n+1)s_{n+1} + (n-1)s_n \\
s_{n+2} &= \frac{3(2n+1)}{n+2}s_{n+1} + \frac{1-n}{n+2}s_n \\
s_n &= \frac{3(2(n-2)+1)}{(n-2)+2}s_{n-1} + \frac{1-(n-2)}{(n-2)+2}s_{n-2} \\
s_n &= \frac{3(2n-3)}{n}s_{n-1} + \frac{3-n}{n}s_{n-2} \\
s_8 &= \frac{3\cdot 13}{8}\cdot 903 - \frac{5}{8}\cdot 197 \\
&= 4279 \\
s_9 &= \frac{3\cdot 15}{9}\cdot 4279 - \frac{6}{9}\cdot 903 \\
&= 20793
\end{aligned}
$$

33. (just the first part where you calculate R7 and R8)

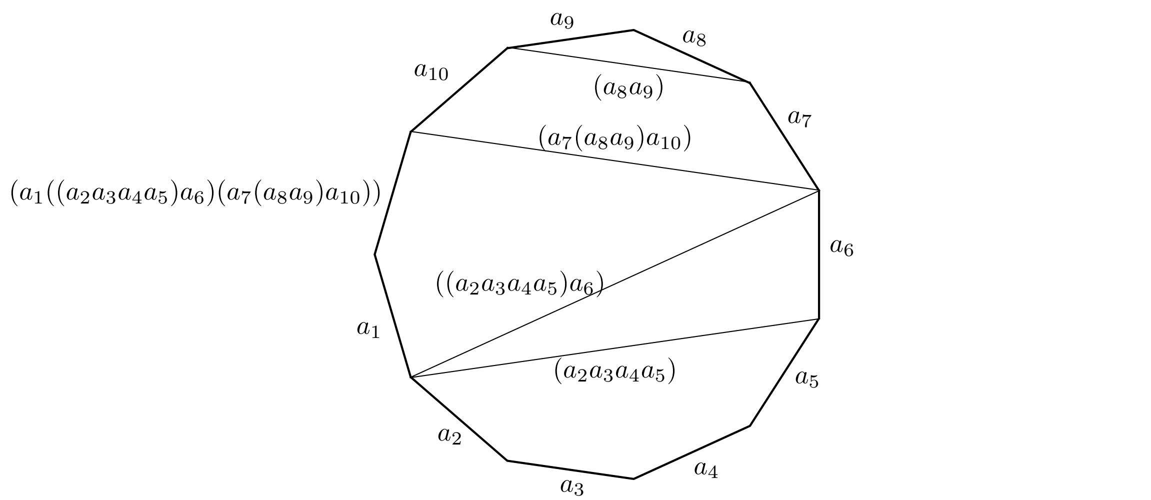
    Use the recurrence relation (8.32) to compute the large Schröder numbers R_7 and R_8. Verify that $R_7 = 2s_8$ and $R_8 = 2s_9$, as stated in Corollary 8.5.8.

$$
\begin{aligned}
&R_n = R_{n-1} + \sum_{k=0}^{n-1} R_k R_{n-1-k}, \quad (n \geq 1) \\
&R_0 = 1 \\
&R_1 = 1 + 1 = 2 \\
&R_2 = 2 + 2 + 2 = 6 \\
&R_3 = 6 + 6 + 4 + 6 = 22 \\
&R_4 = 22 + 22 + 12 + 12 + 22 = 90 \\
&R_5 = 90 + 90 + 44 + 36 + 44 + 90 = 270 + 88 + 36 = 394 \\
&R_6 = 394 + 394 + 180 + 132 + 132 + 180 + 394 = 1806 \\
&R_7 = 1806 + 1806 + 788 + 540 + 484 + 540 + 788 + 1806 = 8558 \\
&R_8 = 8558 + 8558 + 3612 + 2364 + 1980 + 1980 + 2364 + 3612 + 8558 = 41586
\end{aligned}
$$

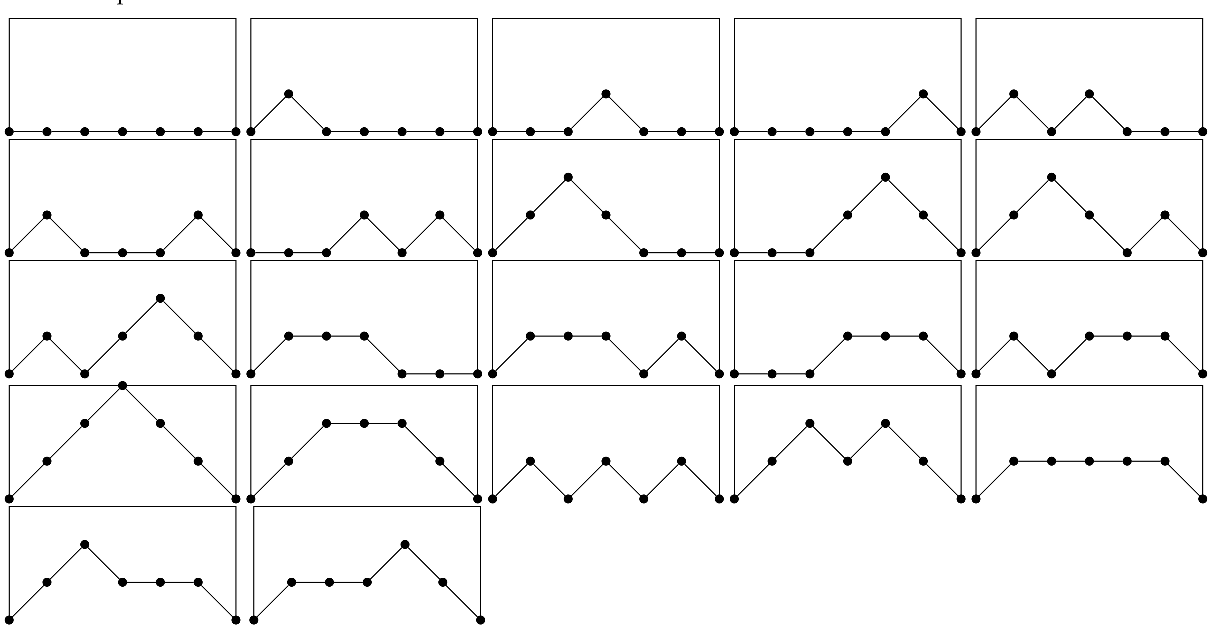
And since I accidentally did #32 anyhow, I could as well do the second part. $41586/2 = 20793$ and $8558/2 = 4279$

Schröder Exercises

1. Draw the dissection of the 11-gon that corresponds to the bracketing $(a_1((a_2a_3a_4a_5)a_6)(a_7(a_8a_9)a_{10}))$.



2. Draw the Schroder paths from $(0,0)$ to $(6,0)$. Verify that exactly half have no horizontal steps on the x-axis.



#1,2,3,4,5,6,6,8,9,12,14→11 out of 22 have horizontal steps on x-axis