

Jon Allen  
HW 02

$$\begin{aligned}u_t &= \alpha^2 u_{xx} - \beta u \\ u(0, t) &= 1\end{aligned}$$

$$\begin{aligned}0 &< x < 1 \\ u(1, t) &= 1\end{aligned}$$

Setting the rate of change over time to zero

$$\begin{aligned}u_t &= 0 \\ 0 &= \alpha^2 r^2 + 0r + -\beta\end{aligned}$$

$$\begin{aligned}0 &= \alpha^2 u_{xx} - \beta u \\ r &= \frac{\pm \sqrt{-4\alpha^2(-\beta)}}{2\alpha^2}\end{aligned}$$

Using the condition from the text that  $\beta > 0$

$$\begin{aligned}r &= \pm \sqrt{\frac{4\alpha^2\beta}{4\alpha^4}} = \pm \sqrt{\frac{\beta}{\alpha^2}} = \pm \frac{\sqrt{\beta}}{\alpha} \\ U(x) &= c_1 e^{x\sqrt{\beta}/\alpha} + c_2 e^{-x\sqrt{\beta}/\alpha}\end{aligned}$$

Since it is stated that heat is lost we know that the conservation condition  $U'(0) = U'(L)$  will not hold. This is because the energy in the system is not conserved.