Notes

September 15, 2014

2.7 subsequences

subsequences have infinite increasing sequence of index n.

deciding convergence

1. squeeze thm.
2. monotone convergence thm: if a sequence is monotone increasing/decreasing and bounded above/below, then it converges.
3. if it is the sum, product, square root, quotient, etc, of convergent sequences, then it is convergent.
4. cesàro sums converge "better" than original sequence

2.7.2 bolzano-weierstrass theorem

every bounded sequence of real numbers has a convergent subsequence.

think of it as a fixup of the monotone convergent sequence, when the sequence isn't monotone

sequence $\{a_n\}$ is bounded by $B \in \mathbb{R}$ so $-B \leq a_n \leq B$.

if $\{a_n\}$ only takes finitely many values, then nescessarily, one of them can be taken innfinitely many times. this is our constant subsequence, which is convergent.

if $\{a_n\}$ takes infinitely many values, then split $[-B, B]$ into halves, $[-B, 0], [0, B]$. One of these halves contains infinitely many values of the sequence. Call this half I_1. Split I_1 into halves, call the one with infinitely many values I_2 and so on. $\{I_n\}$ is a sequence of invervals and each I_n contains infinitely many values of a_n. $I_{n+1} \subseteq I_n$. $|I_n| = \frac{B}{2^{n-1}} \to (n \to \infty) \to 0$

by nestedintevals theorem $\cap_{n=1}^{\infty} I_n \neq \emptyset$, $[a, b] \subseteq \cap_{n=1}^{\infty} I_n$ Because $|I_n| \to 0$ then $a = b$ and it convergences on this point. because each I_n contains some $x_n \in \{a_n\}$ and we can choose them such that