Notes

March 10, 2014

homework

#30: $\{12, 13, 14, 23, 24, 34\}$ is only antichain. Not 1234 or $\emptyset$ since they are comparable to everything.

our counting box from way back

	none	unlimited	restricted supply
ordered	$\frac{n!}{(n-r)!}$	n^r	$\frac{n!}{(n-r)!x_1!x_2!\ldots x_n!}$
unordered	$\frac{n!}{(n-r)!r!} = \binom{n}{r}$	$\binom{r+n-1}{r}$	*use inclusion-exclusion
	$k_1 = k_2 = \cdots = k_n = 1$	$k_1 = k_2 = \cdots = k_n = r$ (or ∞)	$1 \leq k_i$

unify bottom row into the folowing problem:

What is the number of r-combinations taken from the multiset $\{k_1 \cdot a_1, k_2 \cdot a_2, \ldots, k_n \cdot a_n\}$?

$= \{\underbrace{a_1, a_1, \ldots, a_1}_{k_1 \text{ copies}}, \underbrace{a_2, \ldots, a_2}_{k_2}, \ldots, \underbrace{a_n, \ldots, a_n}_{k_n}\}$

pick r elts from above

example

in how many ways could you choose 12 pieces of candy if there are at the store: 13 butterscotch, 4 root beer barrels, 8 lemon heads, 5 cinnamon.

our multiset is $\{13 \cdot b, 4 \cdot r, 8 \cdot l, 5 \cdot c\}$. butterscotch$> 12 = \infty$ so $\{\infty \cdot b, 4 \cdot r, 8 \cdot l, 5 \cdot c\}$ lest $S = \{12 - \text{combinations of } \{\infty \cdot b, \infty \cdot r, \infty \cdot l, \infty \cdot c\}\}$

$$|S| = \binom{12+4-1}{12}$$
$$= \binom{15}{12}$$
$$\text{Let } A_r = \{12 - \text{combinations in } S \text{ with } \geq 5\ r\text{'s}\}$$
$$\text{Let } A_l = \{12 - \text{combinations in } S \text{ with } \geq 9\ l\text{'s}\}$$
$$\text{Let } A_c = \{12 - \text{combinations in } S \text{ with } \geq 6\ c\text{'s}\}$$

$|S| - |A_r \cup A_l \cup A_c|$ do this with inclusion exclusion

$A_r = \{rrrrr \text{ and 7 others}\}$ =7-combinations from 4 types (infinite supply). $|A_r| = \binom{7+4-1}{7}$. $|A_l| = \binom{3+4-1}{3}$, $|A_c| = \binom{6+4-1}{6}$. similarly with intersections.

Rewrite example as follows: what is the # of nonnegative integral solutions to

$$x_1 + x_2 + x_3 + x_4 = 12$$

where $0 \leq x_2 \leq 4, x_3 \leq 8, x_4 \leq 5, x_1 \leq \infty$

exam question? equivalently to $y_1 + y_2 + y_3 + y_4 =$ where $\leq y_1 \leq \quad \leq y_2 \leq \quad \leq y_3 \leq$

equivalently to $y_1 + y_2 + y_3 + y_4 =$ where $4 \leq y_1 \leq \infty \quad 1 \leq y_2 \leq 5 \quad -1 \leq y_3 \leq 7 \quad 2 \leq y_4 \leq 7$

so we've done $y_1 = x_1 + 4, y_2 = x_2 + 1, y_3 = x_3 - 1, y_4 = x_4 + 2$