Notes

April 4, 2014

lesson 20 (continuing from last time

PDE	$u_{tt} = c^2 u_{xx}$	$0 < x < L$	$0 < t < \infty$
BC	$\left.\begin{array}{r} u(0,t) = 0 \\ u(L,t) = 0 \end{array}\right\}$		$0 < t < \infty$
IC	$\left.\begin{array}{r} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{array}\right\}$	$0 < x < L$	

note $c^2 = T/\rho$ tension over mass density.

Separated solutions $u(x,t) = T(t)X(x)$

$$\text{PDE} \qquad \frac{T''}{c^2 T} = \frac{X''}{X} = \lambda$$

$$\text{cases} \begin{cases} \lambda = \mu^2 > 0 & \text{only trivial solutions for } X(x) \text{ from BC} \\ \lambda = 0 & X'' = 0 \; X = c_1 + c_2 x \text{ BC } X(0) = 0 = c_1 \; X(L) = 0 = 0 + c_2 L \to c_2 = 0 \\ \lambda = \mu^2 < 0 & X'' + \mu^2 X = 0 \; X = c_1 \cos(\mu x) + c_2 \sin(\mu x) \text{ BC } X(0) = 0 = c_1 \cdot 1 + c_2 \cdot 0 \to c_1 = 0 \\ & X(L) = 0 = c_2 \sin(\mu L) \to \sin(\mu L) = 0 \end{cases}$$

So $\mu L = n\pi$ give $\mu_n = n\pi/L$ for $n = 1, 2, 3, \ldots$

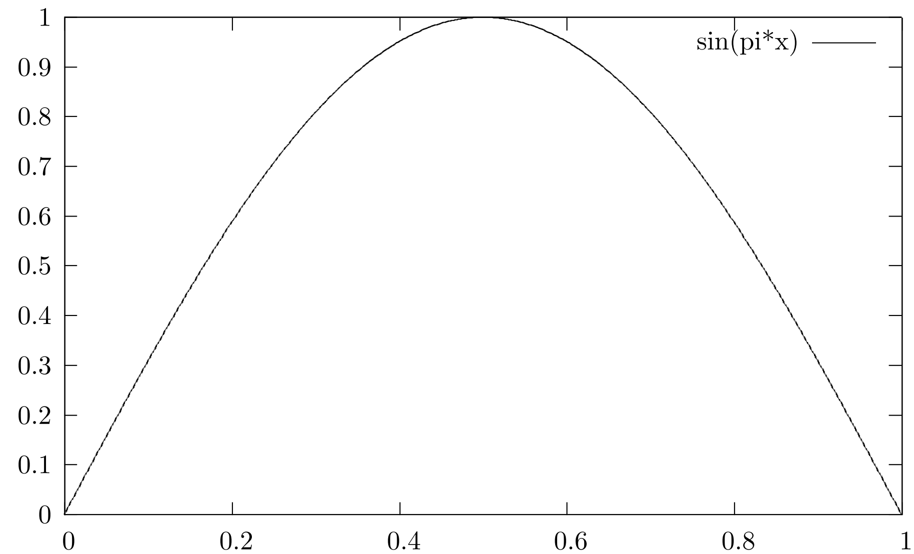
Have $X_n(x) = \sin(\mu_n x) = \sin(n\pi \frac{x}{L})$ nontrivial solutions (eigenfunctions)

Use $\frac{T''(t)}{c^2 T(t)} = -\mu_n{}^2 \; T''(t) + c^2 \mu_n{}^2 T(t) = 0$

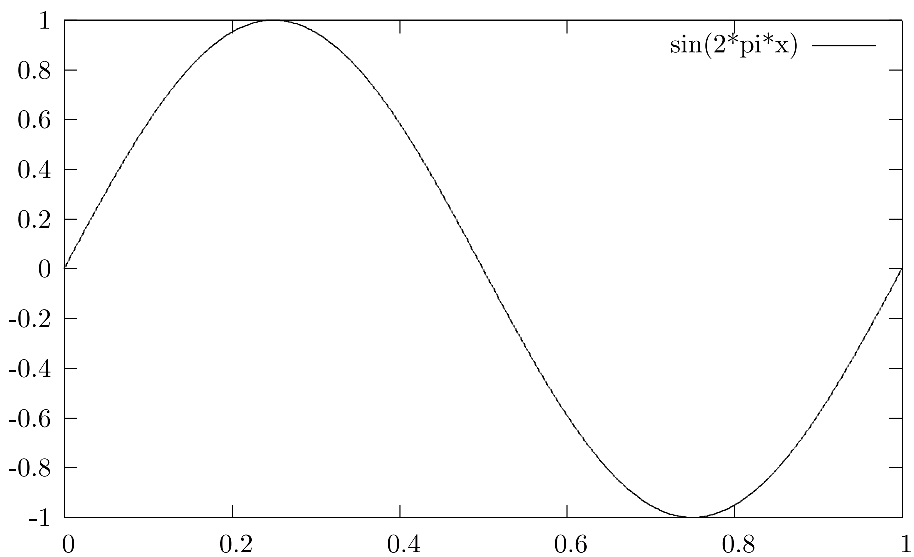
$T(t) = a_n \cos(c\mu_n t) + b_n \sin(c\mu_n t)$

Separated solutions $u_n(x,t) = \left[a_n \cos(n\pi \frac{ct}{L}) + b_n \sin(n\pi \frac{x}{L})\right] \sin(n\pi \frac{x}{L}$.

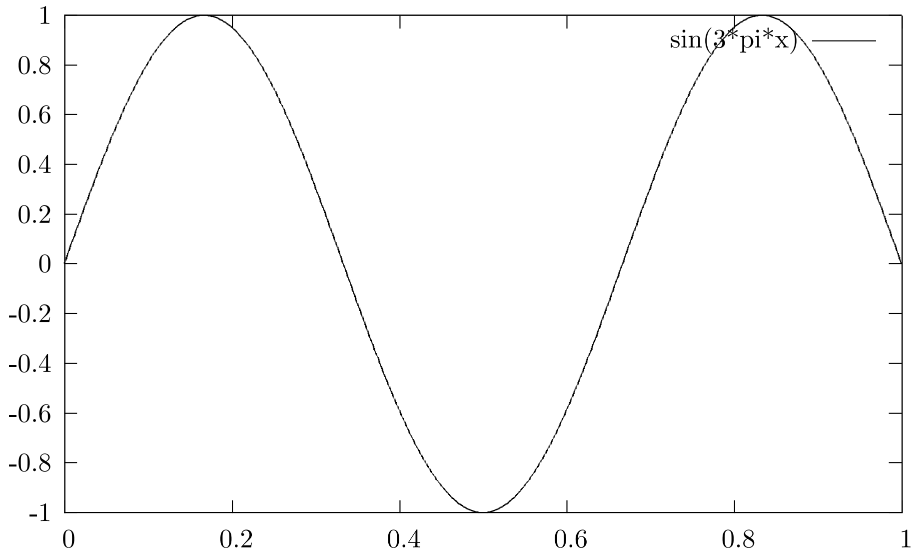
$\sin(2\pi\omega t)$ where ω is frequency (oscillations/second) Hz. Frequency $\omega_n = n\frac{c}{2L} = n\sqrt{\frac{T}{\rho}} \cdot \frac{1}{2L}$ for $n = 1, 2, 3, \ldots$


sin(pi*x)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

n=1


sin(2*pi*x)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1

n=2



n=3

All solutions have periods ω_1, this is why same pitch happens when plucked in different places.

page 154

Mersenne Laws for Strings

mid-1600s, First person to determine frequency of a pitch. middle c is $256/2^8$ Hz.

1. Frequency is proportional to root of tension $\alpha\ \sqrt{T}$
2. Frequency is inversely proportional to length. $\alpha\ 1/L$
3. Freqency is inversely proportion to root of density. $\alpha\ \frac{1}{\sqrt{\rho}}$

Fix L, ρ, and for low T freq= $k_0\sqrt{\frac{T}{\rho}}\frac{1}{L}$

$\sin(\omega t)$

don't forget to add -shell-escape to the plugin