Notes

January 30, 2015

2.3 homework

1,2,15,17,19

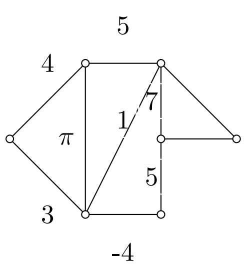
recap/finish

spanning subgraph H of G such that $V(H) = G(H)$

weighted graph one in which the edges are given a value

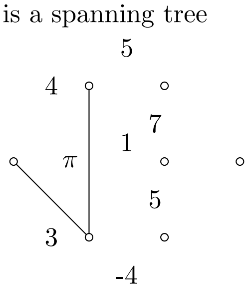
greedy algorithm gives a minimal spanning tree

example



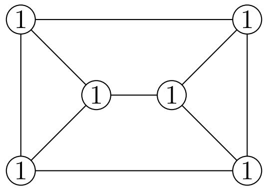
algorithm

choose an edge of least weight, choose another edge of least weight not creating a cycle, repeat, stop when H is a spanning tree



a vertex cut is a subset $S \subseteq V(G)$ such that $k(G - S) > k(G)$

if S is of minimal cadinality, we say G is $|S|$-connected. if we have a cut vertex the graph is 1-connected. if G is disconnected, then it is 0-connected. nonseparable means more than 2-connected

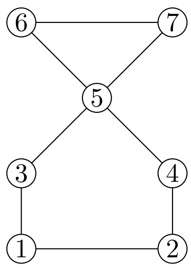


the notation for connectedness is $\kappa(G)$

question: what is $\kappa(k_n)$? (connectedness of a complete graph order n) removing any vertex gives k_{n-1} and so on. so we just define it to be $\kappa(k_n)$ is $n-1$. analog is $0! = 1$.

edge connectedness: an edge cut is a set $S \subseteq E(G)$ such that $k(G-S) > k(G)$.

if S is of minimal cardinality, then we say G is $|S|$-edge-connected, denots $\lambda(G) = |S|$

example  $\lambda(G) = 2$

theorem

for every simple graph $\kappa(G) \leq \lambda(G) \leq \delta(G)$

proof

case 1: G is disconnectd, then $\kappa(G) = \lambda(G) = 0$ and $\delta(G) = 0$

case 2: G is the complete graph. what is the $\lambda(k_n)$? it is $n-1$. we can remove $n-1$ to disconnect a single vertex. removing less leaves something connected because $\delta(G) = n-1$. and so $\kappa(k_n) = n-1 = \lambda(k_n) = \delta(k_n)$

case 3: every thing else. in this case $\delta(G) \leq n-2$ because it's not complete, and so there is a vertex of degree less than $n-1$. pick a vertex v such that $\deg(v) = \delta(G)$. remove all incident edges to disconnect the graph, and so $\lambda(G) \leq \deg(G) = \delta(G) \leq n-2$. showing the last inequality is harder.

corollary

if G is a graph such that the order of G is n and the size is m $m \geq n-1$ then $\kappa(G) \leq \left\lfloor \frac{2m}{n} \right\rfloor$

proof

$\sum d(v) = 2m$ so the average degree is $\frac{2m}{n}$. By theorem $\kappa(G) \leq \delta(G)$ and since $\frac{2m}{n}$ is average $\delta(G) \leq \lfloor 2m/n \rfloor$

2.4 homework

1,2,9,13,14