Jon Allen
HW 04

$$
\begin{array}{ccc}
u_t = \alpha^2 u_{xx} & 0 < x < 1 & 0 < t < \infty \\
u_x(0,t) = 0 & u_x(1,t) = 0 & u(x,0) = \sin(\pi x) \\
u_t = 0 = \alpha^2 u_{xx} & \displaystyle\int 0 \,\mathrm{d}x = \int \alpha^2 u_{xx} \,\mathrm{d}x & c_1 = \alpha^2 u_x \\
\displaystyle\int c_1 \,\mathrm{d}x = \int \alpha^2 u_x \,\mathrm{d}x & c_1 x + c_2 = \alpha^2 U(x) & U(x) = \dfrac{c_1}{\alpha^2} x + \dfrac{c_2}{\alpha^2}
\end{array}
$$

Simplify constants

$$
\begin{array}{ccc}
U(x) = c_1 x + c_2 & U'(x) = c_1 & U'(0) = U'(1) = 0 \\
c_1 = 0 & U(x) = c_2 &
\end{array}
$$

If the problem is interpreted as the temperature of a rod, then the rod is completely insulated. The amount of heat in the system never changes. We know that the amount of heat initially is $\int_0^1 \sin(\pi x) \,\mathrm{d}x$. Because $U(x) = c_2$ we know that when the system reaches a steady state, the amount of heat is $\int_0^1 c_2 \,\mathrm{d}x$.

$$
\begin{aligned}
\int_0^1 c_2 \,\mathrm{d}x &= \int_0^1 \sin(\pi x) \,\mathrm{d}x \\
[c_2 x]_0^1 &= \left[-\frac{1}{\pi} \cos(\pi x) \right]_0^1 \\
c_2 &= -\frac{1}{\pi}(-1) - -\frac{1}{\pi}(1) = \frac{2}{\pi}
\end{aligned}
$$

So our steady state is $U(x) = \frac{2}{\pi}$