Notes

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edge ideals

consider $\mathbb{R}$ and polynomials in X with coefficients in $\mathbb{R}$.

example

$$x^2 + 1$$
$$x - 2$$
$$\pi$$
$$3x^3 - 5x + 7$$

we denote the set of all of these as $\mathbb{R}[x]$.
some polynomials "do fun things"

1. factor

$$x^2 - 1 = (x + 1)(x - 1)$$
$$x^2 - 2x + 1 = (x - 1)^2$$
$$x^3 - x^2 + 4x - 4$$
$$etc$$

we say all plynomials in $\mathbb{R}[x]$ with the property that $x - 1$ divides it, this entire set is an ideal.

this is denoted $\langle x - 1\rangle \subseteq \mathbb{R}[x]$

if we want all polynomials divisable by several things, then $\langle x-1, x^2+1, x^4-2\rangle$. This is "or" or union. the "and" or intersection would be formed by just multiplying the divisors.

in general $\langle f_1, \ldots, f_r\rangle = \{\sum_{i=1}^{r} p_i f_i | p_i \in \mathbb{R}[x]\}$

we can do this in many variables like $\mathbb{R}[x, y, z]$ or $\mathbb{R}[x_1, \ldots, x_n]$ or even $\mathbb{R}[x_1, \ldots]$

all ideals have many invariants, eg, dimesnion, projective dimension, injective dimension, height, resolutions, betti numbers, etc.

let G be a graph and label the vertices $x_1, \ldots, x_n$. then create an ideal in $\mathbb{R}[x_1, \ldots, x_n]$. $I = \langle x_i x_j | x_j$

example