Notes

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3.8 #10

$N \leq G$ and $m = [G : N] = |G/N|$

if $x \in G/N$ then order$(x)|m$. then $\forall a \in G, aN \in G/N$. let $x \in aN$ and then order$(aN)|m$ and $(aN)^m = N$.

$a^m N = N \leftrightarrow a^m \in N$

simple group

G is simple iff 'normal' subgroups.

3.8 #6

two subgroups intersecting is a subgroup. $x \in H \cap N$ means $x \in H \cap N$ so for all $a \in H$we have $axa^{-1} \in N$ and $axa^{-1} \in H$ (from closure of H).

chapter 4

polynomials with coefficients in fields

field definition

a set with two operations on it. $(K, +, \cdot)$ is a field if $+, \cdot$ are binary operations on K such that $(K, +)$ is an abelian group and $K^* = K \setminus \{0\}$ and $(K*, \cdot)$ is an abelian group and $(a+b) \cdot c = ac+bc$ and $a(b+c) = ab+ac$.

examples

$(\mathbb{R}, +, \cdot)$ is a field. $(\mathbb{Q}, +, \cdot)$ is a field, and so is $\mathbb{C}$. $(\mathbb{Z}, +, \cdot)$ is not a field because $(\mathbb{Z}^*, \cdot)$ is not a group. $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a field with respect to usual mult and addition.

polynomial definition

let K be a field, $f(x) = a_m x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0$ where x is indeterminate and $a_i \in K$. we say that $f(x)$ is a polynomial with coefficients in K. if $a_m \neq 0$ then we define deg $f = m$. conventional problem with degree of zero. defined to be $-\infty$.

$K[x] =$ the set of all polynomials with coefficients in K. on $K[x]$ we define two operations. if $f(x)$ is deg m and $g(x)$ is degree n then $f(x) + g(x) =$ as usual and $f(x)g(x) =$ as usual.

obeservation

given a polynomial $f(x) \in K[x]$, the polynomial function associated with $f(x)$ is the function defined from $K \to K$ that takes $c \to f(c)$.

there is a difference between a polynomial and a polynomial function. they are different objects. lets take $K = \mathbb{Z}_p$. where p is prime. This is a field.

let $f(x) = x^p - x \in K[x]$. $\text{def} f = p$. polynomial function assosiated with $f(x)$ is $[a] \to [a]^p - [a] = [a^p - a]$ but for $a \in \mathbb{Z}$ we have $a^p \equiv a \mod p$ and so the function is zero.

finite fields make confusing the polynomial and the function dangerous.

observation

$f(x), g(x) \in K[x]$, if $f(x) \neq 0$ and $g(x) \neq 0$ then the product is not zero and the degree of the product is the sum of the degrees of $f(x)$ and $g(x)$

claim $a, b \neq 0$ then $ab \neq 0$. assume $ab = 0$ then $a^{-1}ab = a^{-1}0$ because $a \neq 0$ and so $b = 0$ because $a^{-1}0 = a^{-1}(0 + 0)$ minus $a^{-1}0$ from both sides.