PDE D.1

PDE.	$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2}$	for	$0 < x < \infty,$	$0 < t < \infty$
BC.	$w(0,t) = 1$	for		$0 < t < \infty$
IC.	$w(x,0) = 0$	for	$0 < x < \infty$	

Solve PDE D.1 by Laplace transforming with respect to t. In particular, show that the Laplace transform of the solution $w(x,t)$ is $W(x,s) = \frac{1}{s}e^{-x\sqrt{s}}$ and then obtain the solution $w(x,t)$ (use tables).

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\begin{aligned}
sW(x) - 0 &= \frac{\mathrm{d}^2 W}{\mathrm{d}x^2} \\
W(0) &= \mathcal{L}\{1\} = \frac{1}{s} \\
0 &= \frac{\mathrm{d}^2 W}{\mathrm{d}x^2} - sW(x) \\
0 &= r^2 + 0r - s \\
r &= \frac{\pm\sqrt{4s}}{2} = \pm\sqrt{s} \\
W(x) &= c_1 e^{x\sqrt{s}} + c_2 e^{-x\sqrt{s}} \\
W(0) &= c_1 + c_2 = \frac{1}{s} \\
c_1 = 0 \quad c_2 &= \frac{1}{s} \\
W(x) &= \frac{1}{s} e^{-x\sqrt{s}}
\end{aligned}
$$

from handout

$$
w(x,t) = \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right)
$$