Notes

23 février, 2015

no class 3/4, 3/6

6.1 planarity

if a graph can be drawn on a plane without any crossings, it is called **planar**

note that if it is planar, you can draw it without crossings with all straight lines.

examples

K_3, K_4

drawing:

geogebra.org

more

we often think of planar graphs as collections of polygons (remember we can draw all planar graphs in straight lines)

picture on board is pentagon glued to one side of a square, and a triangle glued to another when we have polygons glued together, then we can count things. things like

1. faces, eg the polygons

   for faces we have several bounded, and one unbounded

   4

2. edges

   10

3. vertices

   8

note that $V - E + F = 2$

this generalizes off the plane, but the idea of "faces" kind of breaks down and we have to looks at cycles and such.

theorem

if a graph is planar then $v - e + f = 2$

proof

by induction on edges. if you add an edge, then you are adding a vertex or a face

if $E = 0$ and G is connected then $G \cong K_1$. $1 - 0 + 1 = 2$ and so check. assume this is true for $e = k$. suppose G is a tree with $e = k + 1$. Now remove an edge that is part of a cycle and we have $e = k$ and number of faces is reduced by 1. Now $v - (e - 1) + (f - 1) = 2$ by inductive hypothesis. adding the edge back in and we have $v - e + f = 2$. Removing an edge not in a cycle reduced the number of vertices and $v - e + f = 2$ similar to above.

theorem

if G is planar and $|G| \geq 4$ then $E \leq 3V - 6$. Proof crux: every face has at least 3 edges on it's boundary

contrapositive: if $E > 3V - 6$ and $|G| \geq 4$ then G is not planar.

homework

6.1 numbers 1,2,5