8.4

D. Does $\sum\limits_{n=1}^{\infty} \frac{1}{x^2+n^2}$ converge uniformly on the whole real line?

E. Show that if $\sum\limits_{n=1}^{\infty} |a_n| < \infty$, then $\sum\limits_{n=1}^{\infty} a_n \cos nx$ converges uniformly on $\mathbb{R}$.

F. (a) Let $f_n(x) = \frac{x^2}{(1+x^2)^n}$ for $x \in \mathbb{R}$. Evaluate the sum $S(x) = \sum\limits_{n=0}^{\infty} f_n(x)$.

(b) Is this convergence uniform? For which values $a < b$ does this series converge uniformly on $[a, b]$?

H. Suppose that $a_k(x)$ are continuous functions on $[0, 1]$, and define $s_n(x) = \sum\limits_{k=1}^{n} a_k(x)$. Show that if (s_n) converges uniformly on $[0, 1]$, then (a_n) converges uniformly to 0.

J. Let (f_n) be a sequence of functions defined on $\mathbb{N}$ such that $\lim\limits_{k\to\infty} f_n(k) = L_n$ exists for each $n \geq 0$. Suppose that $||f_n||_\infty \leq M_n$, where $\sum\limits_{n=0}^{\infty} M_n < \infty$. Define a function $F(k) = \sum\limits_{n=0}^{\infty} f_n(k)$. Prove that $\lim\limits_{k\to\infty} F(k) = \sum\limits_{n=0}^{\infty} L_n$. HINT: Think of f_n as a function g_n on $\{\frac{1}{k} : k \geq 1\} \cup 0$. How will you define $g_n(0)$?

8.5

A.

B.