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HW 06

$$\phi(x) = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \qquad A_n = 2\int_0^1 \phi(x)\sin(n\pi x)\,\mathrm{d}x$$

$$1 = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \qquad A_n = 2\int_0^1 \sin(n\pi x)\,\mathrm{d}x$$

$$u = n\pi x, \quad \mathrm{d}u = n\pi \qquad A_n = \frac{2}{n\pi}\int_0^1 \sin(u)\,\mathrm{d}u$$

$$A_n = \frac{2}{n\pi}\left[-\cos u\right]_0^1 = \frac{2}{n\pi}\left[-\cos(n\pi x)\right]_0^1 \qquad A_n = \frac{2}{n\pi}\left[-\cos(n\pi) + \cos(0)\right] = \frac{2}{n\pi}\left(1 - \cos(n\pi)\right)$$

$$A_n = \frac{2}{n\pi}\left(1 - (-1)^n\right)$$

So A_n is zero for all even ns and $\frac{4}{n\pi}$ for odd.

$$\begin{aligned}\phi(x) &= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi}\sin((2n-1)\pi x)\\ &= \frac{4}{\pi}\left(\sin(\pi x) + \frac{1}{3}\sin(3\pi x) + \frac{1}{5}\sin(5\pi x) + \frac{1}{7}\sin(7\pi x) + \cdots\right)\end{aligned}$$