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HW 17
Find the inverse transform u(x,t). Hint use mathematica.

$$
\begin{aligned}
U(t) &= \frac{2}{\pi}\frac{\sin(\omega)}{\omega}e^{-(\omega\alpha)^2 t} \\
\mathcal{F}_c{}^{-1}[U] &= u(x,t) \\
&= \int_0^\infty U(t)\cos(\omega x)\,\mathrm{d}\omega \\
&= \frac{2}{\pi}\int_0^\infty \frac{\sin(\omega)\cos(\omega x)}{\omega e^{(\omega\alpha)^2 t}}\,\mathrm{d}\omega \\
&= \frac{2}{\pi}\int_0^\infty \frac{1}{\omega}e^{-\omega^2\alpha^2 t}\sin(\omega)\cos(\omega x)\mathrm{d}\omega
\end{aligned}
$$

Punching the above integral into Mathematica we get the following:

$$
u(x,t) = \frac{1}{2}\left[\operatorname{erf}\left(\frac{x+1}{2\sqrt{\alpha^2 t}}\right) - \operatorname{erf}\left(\frac{x-1}{2\sqrt{\alpha^2 t}}\right)\right]
$$