Notes

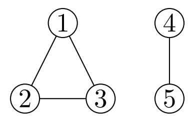
January 23, 2015

skipping 1.4

a graph is connected if for all u, v in $E(G)$ there is a $u - v$ path

how can you tell if a graph is connected with the adjacency matrix? raise it to powers until it hits a nonzero entry. but you have to raise to infinity to prove non connectedness. this is inefficient at best.

there is a very easy way to do this

what happens when graph is disconnected? 

$$A(G) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

notice blocks of zeros on upper right and lower left

this depends on vertex labelling, note homework problem on isomorphic graphs and adjacency matrices

reduced row echelon form? determine negates when you switch columns or rows? same eigenvalues (?unsure). spectrum of graph

definition

$k(G)$ is the number of components of a graph

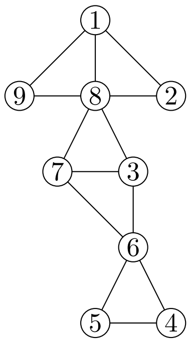
example

from above, $k(G)$ is 2

if G is connected $k(G) = 1$

cut vertex vertex v such that $k(G) < k(G - v)$

example



8 and 6 are cut vertices

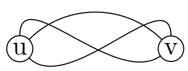
nonseparable a connected graph with no cut vertices

theorem

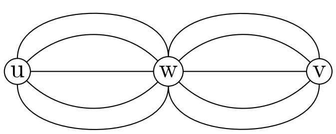
if G is nonseparable and is simple and $|G| \geq 3$ then every pair of vertices lies on a cycle.
another way of thinking of this is that there are two paths between any two vertices.

proof

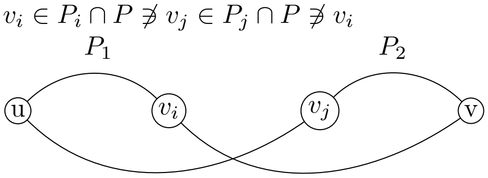
picture:



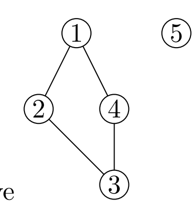
G has no cut vertices and $|G| \geq 3$ since G is connected, there is a $u - v$ path P. Let $\mathcal{P} = \{p_1, \ldots, p_r\}$ be all the $u - v$ paths different from P. why is $\mathcal{P} \neq \emptyset$? nonseperability



if $P_i \cap P = \{u, v\}$ for some i, then we are done. if $P_i \cap P \ni w \forall i$, then w is a cut vertex. so $\nexists w \in P_i \cap P \forall i$. let $v_i \in P_i \cap P \not\ni v_j \in P_j \cap P \not\ni v_i$



claim: we can choose v_i, v_j such that $\{v_i, v_j\} \leq N_G(u)$ **proof:** by nonsep $\deg(u) \geq 2$. if every $u - v$ path used edge uw then w is a cut verte, therefore there are at least two $u - v$ paths starting from u going through distinct neighbors.

fromthe claim, let v_i, v_j be as before. then we have 

$vi = 1, v_j = 3, u = 2, v = 3$
repeat for v_i, v_j to find two disjoint $u - v$paths making a cycle

corollary 1

a connected graph G of order 3 or more is nonsep iff every pair of vertices has two internally disjoint paths between them

corollary 2

let u, vbe two distinct vertices in a nonseparable graph G. if H is obtained by adding a vertex to G and connecting it to u and v G then H is nonsep

corollary 3

if G is nonsep and order 4 or more and $u, v \subseteq V(G)$ with $|U| = |V| = 2$ and $U \cap V = \emptyset$ then G contains two internally disjoint paths from vertices in U to vertices in V.

internally disjoint:

two$u - v$ paths P_1 and P_2 are internally disjoint if $P_1 \cap P_2 = \{u, v\}$