Notes

2 fevrier, 2015

from the grader

1. no frillies
2. no paperclips
3. no folding corners
4. staple

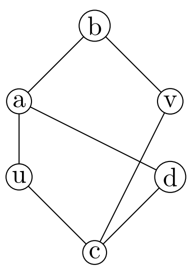
grading scheme

10 available points, 5 for completeness, 5 for 1 or 2 graded problems. 80% is an A

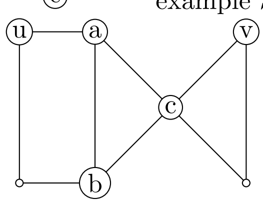
2.5 Menger's Theorem

a $u - v$ **separating set** is a set $S \subseteq V(G)$ such that S separates G with u and v in different components.

exmple



example S are $\{a, c\}, \{b, c\}, \{a, b, c\}$.



$\{a, b\}$ minimally separates u and v in the sense that no subset of $\{a, b\}$ separates u and v but $\{c\}$ is a minimum $u - v$ separating set

theorem

let u and v be non-adjacent vertices in G. the size of a minimum $u - v$ separating set is equal to the number of internally disjoint $u - v$ paths.

thm/corollary (whitney)

a non-trivial graph is k-connected $k \geq 2$ if and only if every pair of vertices has at least k internally disjoint paths between them.

NOTE: adjacency isn't in the second theorem.

PROOF: forward direction:

let $k \geq 2$ and let S be a minimal vertex cut. Take any two different points u, v. any $u - v$ separating has at least size k (it is S). By menger, there are at least k internally disjoint paths from u to v. but menger's theorem doesn't apply if u, v are adjacent. if u and v are adjacent, remove uv (the edge). this reduces connectivity by up to 1. **(check this for homework)**. now repeat the argument on $G - \{uv\}$

this results in at least $k - 1$ internally disjoint paths by menger. of course add in the edge we removed and we have k internally disjoint paths.

reverse direction: any two vertices u, v have k internally disjoint paths between them. let S be a minimal vertex cut. them $G - S = G_1 \cup G_2$ where $G_1 \cap G_2 = \emptyset$. union with dot is disjoint union.

pick $u \in G_1$ and $v \in G_2$. There are at least k internally disjoint $u - v$ paths. so S must contain at least one element of eachpath,hence G is at least $|S|$-connected. S is minimal, so $|S| = k$

issue: check complete graph. if G were complete then our proof fails. $|V(G)| \geq k + 1$ **why? second part of homework**. so $\kappa(G) \geq k + 1 - 1 = k$ because $k(k_n) = n - 1$

□

aside: mengers theorem is often referred to as the max-flow min-cut theorem outside of graph theory.

homework

prove check this and why, also 2.5 #1-5