Notes

17 avril, 2015

9.4 chromatic plynomials

the chromatic poly counts the number of λ-colorings of a graph G

notation $P(G, \lambda)$

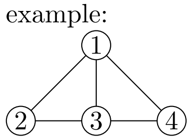
polynomial starts at zero and then becomes positive. once positive it's positive forever. discrete poly (integer), not continuous.

leading coefficient is positive from end ending positive

zeros are factors

two colorings C, C' are distinct if $\exists v \in V(G)$ such that $C(v) \neq C'(v)$

example:



let $\lambda = 6$. how many ways can we color G with 6 colors? 1 has 6, 3 has 5, 4 choices for 2 and 4. so 480.

we have shown:

$P(G, 6) = 480$

the enumeration of choices above, we can get $P(G, \lambda) = \lambda(\lambda - 1)(\lambda - 2)^2$

properties

that should make sense

1. chromatic number: $\chi(G) = 3$
2. what is the smallest λ such that $P(G, \lambda) > 0$? 3o
3. convention: $P(G, 0) = 0 \forall G$

formally, if $P(G, \lambda)$ is the chrompoly of G then $\chi(G) = \min_{\lambda}(P(G, \lambda) > 0)$

exercise

find the $P(K_n, \lambda)$

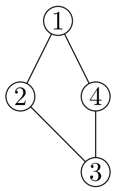
$P(K_3, \lambda) = \lambda(\lambda - 1)(\lambda - 2)$

$P(K_4, \lambda) = \lambda(\lambda - 1)(\lambda - 2)(\lambda - 3)$

$P(K_n, \lambda) = \prod_{i=0}^{n-1} (\lambda - i) = \frac{\lambda!}{(\lambda - n)!}$

if E_n is the empty graph (no edges) on n vertices $P(E_n, \lambda) = \lambda^n$

exercise



$P(C_4, \lambda)$

$P(C_4, \lambda) = \lambda(\lambda - 1)^2(\lambda - 2)$ unless the two adjacent to the starting count are the same then $\lambda(\lambda - 1)^3$
this leads to sums in $P(G, \lambda)$ and so we have graph theoretically two options

1. if they are the same edge contraction

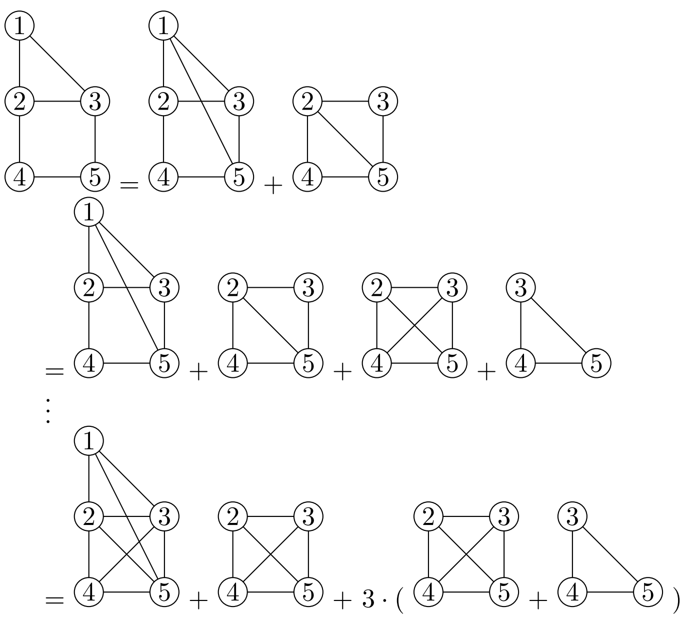
2. if they are different then we can insert an edge with no change

either choice leads to a complete graph

theorem

if $uv \notin E(G)$ and H is the graph $G + uv$ with uv contracted, then $P(G, \lambda) = P(G + uv, \lambda) + P(H, \lambda)$
we are going to draw graphs instead of using this notation

example



homework is $1, 6, 7$