Notes

March 7, 2014

worksheet

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multinomial theorem

$$(x_1 + x_2 + \cdots + x_t)^n = \sum \binom{n}{r_1}\binom{n-r_1}{r_z}\cdots\binom{n-r_1-\cdots-r_{t-1}}{r_t} {x_1}^{r_1}{x_2}^{r_2}\ldots{x_t}^{r_t}$$

combinatorial proof sketch

"would be sufficient for an exam"

$$(x_1 + \cdots + x_t)(x_1 + \cdots + x_t)\ldots(x_1 + \cdots + x_t)$$

when you multiply out, each term is a word in the letters $x_1, \ldots, x_t$

$$\underbrace{x_2x_5x_2x_1x_3x_1}_{n}$$

so the coefficient of ${x_1}^{r_1}{x_2}^{r_2}\ldots{x_t}^{r_t}$ counts # words of length n with $\underbrace{r_ix_i}_{\text{letter distribution}}$'s

example 39

find the coefficient of ${x_1}^3x_2{x_3}^4{x_5}^2$ in $(x_1 + \cdots + x_5)^{10}$. $\frac{10!}{3!1!4!2!}$

other example

what is $\sum_{n_1+n_2+n_3=n} \binom{n}{n_1,n_2,n_3}(-1)^{n_2}$. use multinomial thm, plug in 1,-1,1 for x's

other example

what is $\sum_{n_1+n_2+n_3=n} \binom{n}{n_1,n_2,n_3,n_4}(-1)^{n_2+n_4} = (1-1+1-1)^n = 0$

chapter 6

principle of inclusion-exclusion

$$|A_1 \cup A_2 \cup A_3 \cdots \cup A_n| = |\cup_{i=1}^n A_i| = \sum_{i=1}^n |A_1| - \sum_{i\neq j} |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap \cdots \cap A_n|$$

application 1

find # integers between 1 and 1000. that are not divisible by 5,6, or 8. A_i =set of integers between 1-1000 divisible by i. consider A_5, A_6, A_8

we are looking for $A_5 \cup A_6 \cup A_8$

$|A_5| + |A_6| + |A_8| - |A_5 \cap A_6| - |A_5 \cap A_8| - |A_6 \cap A_8| + |A_5 \cap A_6 \cap A_8|$