Notes

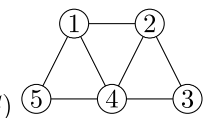
11 février, 2015

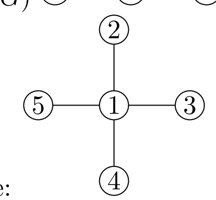
3.3 line graphs

a **line graph** is the result of an operation on another graph

if G is a simple grpah, the line graph $L(G)$ is such that $V(L(G)) = E(G)$. Edges in $L(G)$ occur between vertices that represent edges in G

edges that share a vertice in G are vertices that share an edge in $L(G)$





if S_n is the star graph, what is the line graph of S_n? K_n. small case:

if $|G| = n$ and $|E(G)| = m$ then $|L(G)| = m$ and $|E(L(G))| = ?$

each edge in $L(G)$ corresponds to a choice of two edges from G, in particular, these two edges must be adjacent (share a vertex). let that vertex be v_i and have degree d_i. then we have $\binom{d_i}{2}$ edges corresponding to v_i. $|E(L(G))| = \sum_{i=1}^{n} \binom{d_i}{2}$

every two edges coming of the original vertice make an edge in the new vertice. so we can choose two edges $\binom{d_i}{2}$ different ways

converting line graphs

can we obtain G from $L(G)$? given a graph L can we say $L = L(G)$ for some G?

assume L is connected and non-trivial

theorem

if G_1 and G_2 are connected simple graphs with $L(G_1) \cong L(G_2)$ then $G_1 \cong G_2$ so long as $G_1 \ncong K_3$ or $G_1 \ncong K_1, 3 = S_3$

there is a huge theorem called **kuratowskis** theorem that we will cover later. it's so popular that it has spawned a whole class of theorems called kuratowski type theorems. this one is one of them

theorem

a graph is a line graph of some other graph iff it is not an induced subgraph of 9 specific graphs. see page 142

a kuratowski theorem is a generalized structure theorem. something is true as long as it's not true about a finite number of graphs.

theory of excluded minors (boring).

a generalization of hamiltonicity is a watchmans tour. don't have to visit every "hallway", you just have to be able to see down every hallway. a watchman's tour is a tree so that every edge in G is in the tree or adjacent to the tree.

definition in book: **dominating circuit** is a circuit C of G such that every edge of G is in C or adjacent to C.

example

peterson graph. not hamiltonian. does have dominating circuit, not hamiltonian

theorem

let G be a connected graph, then $L(G)$ is hamiltonian iff G has a diminating circuit.

homework

10,11,12