Notes

March 10, 2014

lesson 13

$U(x,s) = \frac{u_0}{s} - \frac{u_0}{s(\sqrt{s}+1}e^{-x\sqrt{s}} \to u(x,t) = u_0\left[1 - (\text{erfc}(\frac{x}{2\sqrt{t}}) - \text{erfc}(\sqrt{t} + \frac{x}{2\sqrt{t}})e^{x+t}\right]$

mixup, he wrote $\frac{u_0}{s} - \frac{u_0\sqrt{s}}{s(\sqrt{s}+1)}e^{-\sqrt{s}x}$ but want $F(s) = \frac{1}{s(\sqrt{s}+1)}e^{-a\sqrt{s}} \to f(t) =?$ and then $F(\frac{s}{a^2}) = \frac{a^3}{s(\sqrt{s}+a)}e^{-\sqrt{s}} \to a^2 f(a^2 t)$ concentrate on $G(s) = \frac{e^{-s}}{s(\sqrt{s}+a)} = \frac{(\sqrt{s}-a)e^{-\sqrt{s}}}{s(s-a^2)}$ and then $(s-a^2)G(s) = \frac{1}{\sqrt{s}}e^{-\sqrt{s}} - \frac{a}{s}e^{-\sqrt{s}}$ step 3 solve $a'(t) - a^2 g(t) = \dots$

number 19 from handout

$$f(t) = \text{erfc}\left(\frac{a}{2\sqrt{t}}\right)$$

in process of inverting (notes from 2 lectures back) $F(s) = e^{-\sqrt{s}}$ we obtained a DE for $f(t)$.

$$\begin{aligned}
f(t) &= c_1 t^{-3/2} e^{-\frac{1}{4t}} \\
\lim_{s\to\infty} s e^{-\sqrt{s}} &= f(0) \\
\lim_{s\to 0^+} e^{-\sqrt{s}} &= 1 = \int_0^\infty f(t)\,\mathrm{d}t \\
\int_0^\infty f(t)\,\mathrm{d}t &= c\int_0^\infty t^{-3/2} e^{-\frac{1}{4t}}\,\mathrm{d}t = 1 \\
\Gamma(x) &= \int_0^\infty e^{-t} t^{x-1}, \mathrm{d}t \\
u &= \frac{1}{4t} \\
t &= \frac{1}{4u} \\
\mathrm{d}t &= -\frac{1}{4u^2}\mathrm{d}u \\
\int_0^\infty f(t)\,\mathrm{d}t &= c\int_0^\infty 4u^{3/2} e^{-u} \frac{-1}{4u^2}\,\mathrm{d}t \\
1 &= c\int_0^\infty e^{-u} u^{-1/2} 4^{1/2}\,\mathrm{d}u \\
&= 2c\int_0^\infty e^{-u} u^{-1/2}\,\mathrm{d}u = 2c\Gamma\left(\frac{1}{2}\right) \\
c &= \frac{1}{2\Gamma(1/2)} = \frac{1}{2\sqrt{\pi}}
\end{aligned}$$

$$
\begin{aligned}
F(s) &= e^{-\sqrt{s}} \to f(t) \\
F'(s) &= -\frac{1}{2\sqrt{s}} e^{-\sqrt{s}} \to -tf(t) \\
\frac{1}{\sqrt{s}} &\to 2tf(t) = 2t\frac{1}{2\sqrt{\pi}} t^{-3/2} e^{-\frac{1}{rt}} \\
e^{-\sqrt{s}} &\to \frac{1}{2\sqrt{\pi}} t^{-3/2} e^{-1/(4t)} \\
\frac{1}{\sqrt{s}} &\to \frac{1}{\sqrt{\pi}} t^{-1/2} e^{-1/(4t)} \\
F(s) &\to f(t) \\
\frac{1}{s} F(s) &\to \int_0^t f(u)\,\mathrm{d}u \\
\frac{1}{s} e^{-\sqrt{s}} &\to \frac{1}{2\sqrt{gp}} \int_{n=0}^{u=t} e^{-1/(4u)} u^{-3/2}\,\mathrm{d}u \\
&\vdots \\
g'(t) - a^2 g(t) &= \frac{1}{\sqrt{\pi t}} e^{1/(4t)} - a \cdot \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right)
\end{aligned}
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multiply by $e^{-a^2 t}$ which is integrating factor

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{-a^t}\right) =
$$