Notes

January 14, 2015

graph theory birthday

leibniz and the geometry of postion (maybe called topology now)
200 years before algebraists, at this time it's all analysis
he says maybe you only care about relations, not distance

next

mid 18th century, bridges of knigsberg.
knigsberg has river around it, with bridges throughout. check drawing online
game is to cross every bridge exactly once and return to starting place, enter euler
"very little relationship to mathematics" but changes his mind later. then solves every bridge problem ever. 21 paragraphs.
he also solved the optimal number of sails to have on a sailing vessel.
anachronistic to say that he invented graph theory. however the bridges problem is considered the original graph theory problem.
in modern theory/notation each land bit is a point and each bridge is an edge.

definition

a graph is written $G = G(V, E)$ where V, E are a pair of sets such that E is a subset of $V \times V$ where E can have repetitions.

abstract examples

$V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1, v_2), (v_1, v_2), (v_1, v_1), \ldots\}$

definitions

simple graph

a graph is simple if E has no repetitions and excludes (v_i, v_i)

multigraph

a graph that is not simple is called a multigraph

knigsberg is an example of this

incident/adjacent

two vertices v_i and v_j are called incident or adjacent if (v_i, v_j) is in E

neighborhood of v_i

if $v_i \in V$ then $N_G(v_i) = \{(v_i, v_j) \in E\}$

order

$|V(G)|$ is the order of G

size

$|E(G)|$ is the size of G

path

a path is a graph like ._._._. (a line with discrete points on it. this one is order 4, size 3)

a path on n vertices is denoted P_n but instructor will often call $P_{n-1} = P_n$ because P_n had P_{n-1} edges

trivial graph

if $|V(G)|$ (order) is one and $|E(G)|$ (size) is zero then we have a trivial graph

this is opposed to not having any vertices or having multiple vertices and no edges

empty

if $|E(G)|$ size is zero then we have an empty graph

complete graph

$E(G) = \{V \times V - (v_i, v_i)\}$ then G is a complete graph

example

$V = \{(v_1, v_2, v_3\}$ with a triangle graph.

the trivial graph is a complete graph

notation

K_n is complete graph on n vertices. K_1 is a point, K_2 is a line, K_3 is a triangle, K_4 is a square with an x

order of neighborhood

$|N_G(v_i)|$ is the degree of a vertex (valence)

theorem

if G is finite and simple, then

$$\sum_{v_i \in V} |N_G(v_i)| = 2|E|$$

proof

adding each degree counts one end of each edge
each two vertices to each edge

homework

define: path, cycle, isomorphism, subgraph, regular graph, bipartate graph, complement
numbers: 2,3,4,11,13,18