Notes

March 14, 2014

very interesting use of laplace transforms. straight from the text

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duhamel's principle.

easy problem

PDE	$w_t = w_{xx}$
BC	$w(0,t) = 0$
	$w(1,t) = 1$
IC	$w(x,0) = 0$

subsubsection*hard problem

			$z = u + v$
PDE	$u_t = u_{xx}$	$v_t = v_{xx}$	$z_t = z_{xx}$
BC	$u(0,t) = 0$	$v(0,t) = 0$	$z(0,t) = 0$
	$u(1,t) = g(t)$	$v(1,t) = 0$	$z(1,t) = g(t)$
IC	$u(x,0) = 0$	$v(x,0) = \phi(x)$	$z(x,0) = \phi(x)$

$\mathcal{L}$ with respect to time

$$sW(x,s) - \underbrace{w(x,0)}_{\to 0} = W_{xx}(x,s)$$

$$W_{xx} - SW = 0 \text{ on } 0 < x < 1 \qquad U_{xx} - sU = 0$$

$$W(0,s) = 0 \text{ and } W(1,s) = \frac{1}{s} \qquad U(0,s) = 0 \text{ and } U(1,s) =$$

$$W = c_1 \sinh(\sqrt{s}x) + c_2 \cosh(\sqrt{s}x)$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - sy = 0 \qquad y = e^{rx}$$

$$r^2 - s = 0$$

$$y = a_1 e^{-\sqrt{s}x} + a_2 e^{\sqrt{s}x}$$

$$\sinh(z) = \frac{1}{2}(e^z - e^{-z})$$

$$
\begin{aligned}
\cosh(z) &= \frac{1}{2}(e^z + e^{-z}) \\
W &= c_1 \sinh(\sqrt{s}x) \\
\frac{1}{s} &= c_1 \sinh(\sqrt{s}) \\
W(x,s) &= \frac{1}{s}\frac{\sinh(\sqrt{s}x)}{\cosh(\sqrt{s})} \\
U &= c_1 \sinh(\sqrt{s}x) \\
G(s) &= c_1 \sinh(\sqrt{s}) \\
U(x,s) &= G(s)\frac{\sinh(\sqrt{s}x}{\sinh(\sqrt{s})} = G(s)sW(x,s)
\end{aligned}
$$

note: $sW(x,s) - \underbrace{w(x,0)}_{\to 0} = \mathcal{L}\{w_t\}$

$$
\begin{aligned}
u(x,t) &= \int_0^t g(t-u)w_t(x,u)\,\mathrm{d}u \\
&= g(t-u)w(x,u)\mid_0^t - \int_0^t (-g'(t-u)w(x,u)\,\mathrm{d}u
\end{aligned}
$$

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$$
w(x,t) = x + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{-(n\pi)^2 t}\sin(n\pi x) \text{ from eigenfunction expansion}
$$

homework #27 lesson 14, exercise 4 $g(t) = \sin(t)$. take $\alpha^2 = 1$. due friday, 28 march.