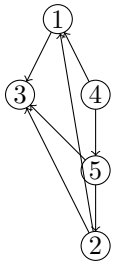
Notes

13 février, 2015

1.4 directed graphs

digraphs are graphs with orientations (arrows) on the edges
every term from graphs has a directed version for digraphs.
one notable lexicographical difference: edges are called **arcs**.

examples



a digraph is **weakly connected** if the underlying graph is connected.

a digraph is **strongly connected** if for every $u, v \in V(G)$, there exists a directed $u - v$ path and a directed $v - u$ path.

thrm

a digraph is strongly connected iff it contains a closed spanning directed walk.

notes

in proving things about digraphs, the degree of a vertex is nuanced. eg arcs in and arcs out might be different numbers. so we say: let D be a digraph and $v \in V(D)$ and we say $\mathbf{id}(v)$ is the number of incoming arcs to v and $\mathbf{od}(v)$ is the number of outgoing arcs

4.1 directed graphs

if D is a simple digraph of size m then $\sum \mathrm{od}(v) = \sum \mathrm{id}(v) = m$

eulerian digraphs

eulerian circuit that is directed

thm

a digraph D has an eulerian circuit iff $\text{od}(v) = \text{id}(v)$ for all $v \in V(D)$

proof

basically identical to undirected case

need everything to be even so we can go in and out, now we need to be able to go in and out similarly

thm

a digraph D has an eulerian path iff $\text{id}(v) = \text{od}(v)$ for all vertices but two called u, v and $\text{od}(u) = \text{id}(u) + 1$ and $\text{id}(v) = \text{od}(v) + 1$

Homework

2,7