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HW 22
Given $F(s) = \frac{1}{1+\sqrt{s}}$

(a) Find $f(t)$ by expanding $F(s)$ in reciprocal powers and inverting termwise.

$$\begin{aligned}
\mathcal{L}^{-}1\{F(s)\} &= \mathcal{L}^{-}1\left\{\frac{1}{1+\sqrt{s}}\right\} \\
&= \mathcal{L}^{-}1\left\{\sum_{n=0}^{\infty}(-1)^n s^{n/2}\right\} \\
&= \sum_{n=0}^{\infty}(-1)^n \mathcal{L}^{-}1\left\{s^{n/2}\right\} \\
\mathcal{L}^{-1}\{s^n\} &= \frac{t^{-n-1}}{\Gamma(-n)} \\
&= \sum_{n=0}^{\infty}(-1)^n \frac{t^{-n/2-1}}{\Gamma(-n/2)}
\end{aligned}$$

(b) Reduce all occurrences of the gamma function to ordinary factorials

$$= \sum_{n=0}^{\infty}(-1)^n \frac{t^{-n/2-1}}{(-1-n/2)!} \quad \text{used computer to get this}$$

This result isn't really sane, but it seems to be closest to what you are looking for. I also have this, but it's not right either I think.

$$\begin{aligned}
&= \mathcal{L}^{-}1\left\{\frac{1-\sqrt{s}}{1-s}\right\} \\
&= e^t - \mathcal{L}^{-}1\left\{\frac{\sqrt{s}}{1-s}\right\} \\
&= e^t + \frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{\sqrt{s}+1}\right\} + \frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{\sqrt{s}-1}\right\} \\
\frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{1+\sqrt{s}}\right\} &= e^t + \frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{\sqrt{s}-1}\right\} \\
\mathcal{L}^{-}1\left\{\frac{1}{1+\sqrt{s}}\right\} &= 2e^t + \mathcal{L}^{-}1\left\{\frac{\sqrt{s}+1}{s-1}\right\} \\
&= 2e^t + e^t + \mathcal{L}^{-}1\left\{\frac{\sqrt{s}}{s-1}\right\} \\
&= 2e^t + e^t + \frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{\sqrt{s}+1}\right\} + \frac{1}{2}\mathcal{L}^{-}1\left\{\frac{1}{\sqrt{s}-1}\right\} \\
&= 4e^t + 2e^t + \mathcal{L}^{-}1\left\{\frac{\sqrt{s}+1}{s-1}\right\} \\
f(t) &= \sum_{n=1}^{\infty} 2^n e^t
\end{aligned}$$