Notes

6 février, 2015

wednesday was eulerian graphs (bridges of königsberg).
cycles are not circuits and trails are not paths

3.2 hamiltonian paths

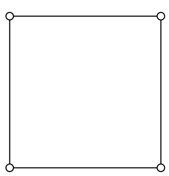
Euler is E, edges is E. Hamiltonian graphs is H, vertices is...not

a **Hamiltonian path** or **cycle** is a path or cycle that meets every vertex of G exactly once.

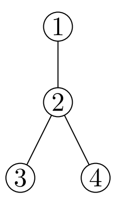
a graph with hamiltonian cycle is called **hamiltonian**

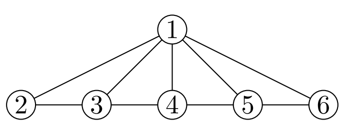
example

1. Hamiltonian:



2. Not hamiltonian



3. is hamiltonian 

theorem (Ore)

if G is a graph of order $n \geq 3$ and $\forall u, v$ vertices $\deg(u) + \deg(v) \geq n$ then G is hamiltonian

NOTE: completely nonconstructive proof

proof

assume for a contradiction that for all $u, v \in V(G)$, $\deg(u) + \deg(v) \geq n$ but G is not hamiltonian.

without loss of generality, we can assume that G is "maximal" with this property. why?

G is finite therefore G is a subgraph of some complete graph $G \leq K_n$. But K_n is hamiltonian. Since G is not hamiltonian and K_n is then. somewhere added enough edges to G to make it hamiltonian.

Add edge xy to G. The $G \cup \{xy\}$ is hamiltonian, so there is an $x - y$ path in G. We have to use the xy edge in the hamiltonian cycle, else the graph would already be hamiltonian. In fact the $x - y$ path is hamiltonian.

let the $x - y$ path be $x = v_1, \ldots, v_n = y$. If x is adjacent to v_i then y cannot be adjacent to v_{i-1}. why? because then you would have a hamiltonian cycle. $v_1 v_i \ldots v_n v_{i-1} v_1$

Therefore, for every neighbor of x we can eliminate a possible neighbor of y. This means that $\deg(y) \leq (n-1) - \deg(x)$. because $n - 1$ is max possible deg and $\deg(x)$ are the things that can't be adjacent to y. Now $\deg(y) + \deg(x) \leq n - 1$ which is a contradiction because $\deg(y) + \deg(x) \geq n$

corollary

if G of order $n \geq 3$ has the property that for all $v \in V(G)$ then $\deg(v) \geq \frac{n}{2}$, then G is hamiltonian.

proof

$\forall u, v \in V(G)$ then $\deg(u) + \deg(v) \geq n$

independent sets

a subset of $V(G)$ is called **independent** if no vertices are adjacent to one another.

the maximal cardinality of independent sets is called the **independent number**. it is denoted $\alpha(G)$

what is $\alpha(K_n)$? 1

theorem chvátal-erdös

let G be a graph of order $n \geq 3$. if connectedness$\kappa(G) \geq \alpha(G)$, then G is Hamiltonian.

homework

1,7,14