

# Notes

April 26, 2014

homework questions catalan problems (1,2,36), could prove formula directly but that can be hard, easier is write bijection from objects to catalan numbers. 1)b to d, 2)a to d, 36)c to d

## worksheet

prove # length  $2n$  ballot seq =  $\frac{1}{n+1} \binom{2n}{n}$

a)  $\binom{2n}{n}$

b)  $A_n + U_n = \binom{2n}{n}$

c)  $a_k = -1$ ,  $a_1 + a_2 + \dots + a_{k-1} = 0$

d) is  $k$  odd or even? odd

e) one more one, one less negative one. this process gives all possible sequences of  $n+1$  1's and  $n-1$  -1's. it is really a bijection from  $A_n + U_n$  to  $n+1$  1's and  $n-1$  -1's. so  $|U_n| = \binom{2n}{n+1}$ ,  $A_n = \binom{2n}{n} - \binom{2n}{n+1}$   
this leads to a catalan number.

f)

## sterling numbers

### warm up

how many ways are there to partition  $\{1, 2, \dots, p\}$  into  $k$  indistinguishable boxes?

$k$  choices for each object in set.  $k^p$ . but the boxes here are distinguishable. divide by the number of permutations to eliminate distinguishability.  $\frac{k^p}{k!}$ .

### question

what if no box may be left empty? (choose  $k$  elements, then distribute)

### answer

use inclusion exclusion. let  $A_i$  = number of distributions where box  $i$  is empty.