# Notes

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## 4.2 5a,6

### 5a

$\gcd(f(x), f'(x)) = d(x) \neq 1$ assume $f(x)$ does not have repeatable factors

consider $p(x)$ to be an irreduceable factor of $d(x)$.

then we can say $f(x) = a(x)p(x), f'(x) = b(x)p(x)$ where $g(x), b(x) \in F[x]$.

$f'(x) = a'(x)p(x) + a(x)p'(x)$ and the following holds. $a(x)p'(x) = b(x)p(x) - a'(x)p(x)$

$p(x)|a(x)p'(x) \to p(x)|a(x)$ because $p(x)|h_1(x)h_2(x) \to p(x)|h_1(x)$ or $p(x)|h_2(x)$

$\exists cc(x) \in F[x]$ where $a(x) = c(x)p(x)$ and $f(x) = a(x)p(x) = c(x)p(x)p(x) = c(x)p(x)^2$ which is a contradiction

note that $p(x)|p'(x)$ is possible. $p(x) \in \mathbb{Z}_p[x], p(x) = x^{p^2} - x^p - 1$ and $p'(x) = 0$.

## 4.3 existence of roots

$p(x) \in K[x] \setminus \{0\}$.

construction $m$ $K[x]$. for $f(x), g(x) \in K[x]$ $f \equiv g(x) \mod p(x) \Leftrightarrow p(x)|[f(x) - g(x)]$. this is an equivalence relation on $K[x]$.

now we examine the equivalence class of $f(x)$. It consists of all polynomials in $K[x]$ such that $[f(x)] = \{g(x)|g(x) \in K[x], f(x) \equiv g(x) \mod p(x)\}$. we write $f(x) = p(x)q(x)+r(x)$ where $r(x) = 0$ or $\deg r < \deg p$. and then $[f(x)] = [r(x)]$. the set of equivalence classes is denoted $K[x]/\langle p(x)\rangle$

### properties

1. $f(x) \equiv g(x) \mod p(x)$ and $h(x) \equiv l(x) \mod p(x)$ then $f(x) + h(x) \equiv g(x) + l(x) \mod p(x)$ and $f(x)h(x) \equiv g(x)l(x) \mod p(x)$
2. $f(x)h(x) \equiv g(x)h(x) \mod p(x)$ and $\gcd(p(x), h(x)) = 1$ then $f(x) \equiv g(x) \mod p(x)$.

**on** $K[x]/\langle p(x)\rangle$

define

1. $[f(a)] + [g(x)] = [f(a) + g(a)]$
2. $[f(x)] \cdot [g(x)] = [f(x) \cdot g(x)]$

these are well defined operations, check all details

$[f(x) + g(x)] = [f'(x) + g'(x)]$?

**example**

$K = \mathbb{R}, p(x) = x^2 + 1 \in \mathbb{R}[x]$.

$\mathbb{R}[x]/\langle p \rangle = \mathbb{R}[x]/\langle x^2 + 1 \rangle$

$[x]^2 = -[1]$.