Notes

January 21, 2015

homework from previous week collected on friday, given to grader on monday. late homework isn't a big deal. get back next monday.

1.2 no 15

converse direction is easy (add in a vertex adjacent to appropriate edges or appropriate vertices)

forward direction not like thm 1.10. hint in book is find contradiction (too many edges, not enough edges)

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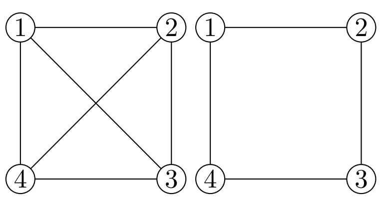
1.3 connected graphs and distance

walk a sequence of vertices $v_1, \ldots, v_r$ such that $v_i v_{i+1}$ is and edge item[length of walk] the number of edges encountered (including repetitions) along the walk

$v_1 v_2 v_3 v_4 v_1 v_4 v_3$ is a walk of length 6



$v_1 v_2 v_3 v_1$ is not a walk

open walk $v_1 \neq v_r$

closed walk $v_1 = v_r$

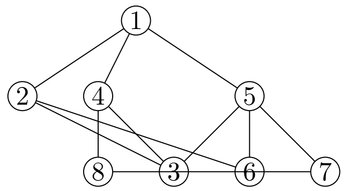
trail no edges are repeated

path no vertices are repeated

distance length of shortest path between two vertices denoted $d(v, u)$

what if there is no path? they are not connected? assign ∞ because distance from vertex to itself is 0. this covers loop case

cycle a closed trail



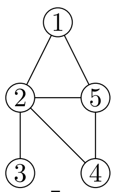
girth length of the smallest cycle denoted $g(G)$

connected if $\forall u, v \in V(G), \exists$ a path from u to v (written $u - v$ path)

adjacency matrix of G

$A = A(G) = [a_{ij}]$ such that $a_{ij} = \begin{cases} 0 & (v_i, v_j) \in E(G) \\ 1 & \text{else} \end{cases}$

example

$G =$ 

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

repeated edges break adjacency matrix. can just change the 1 to the number of edges

questions

1. find $d(v_i, v_j)$

   minimum k such that $a_{ij}^{(k)} \neq 0$

2. $g(G)$

   minimum $\{2k | a_{ii}^{(2k)} \neq 0\} \leq g(G) \leq \min\{2k+1 | a_{ii}^{(2k+1)} \neq 0\}$

3. $deg(v_i)$

   the sum of the row(or column)

4. if G connected

also, what can $A(G)$ tell us about paths in G?
hint: raise A to powers for 1 and 4

homework

1.3 1,2,3,9,13

notation

the a_{ij} entry of A^k is $a_{ij}^{(k)}$

thm

for a finite simple graph G and an integer $k \in \mathbb{N}$ $a_{ij}^{(}k)$ is the number of paths from v_i to v_j of length k.