Jon Allen
HW 06

$$\phi(x) = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \qquad A_n = 2\int_0^1 \phi(x)\sin(n\pi x)\,\mathrm{d}x$$

$$1 = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \qquad A_n = 2\int_0^1 \sin(n\pi x)\,\mathrm{d}x$$

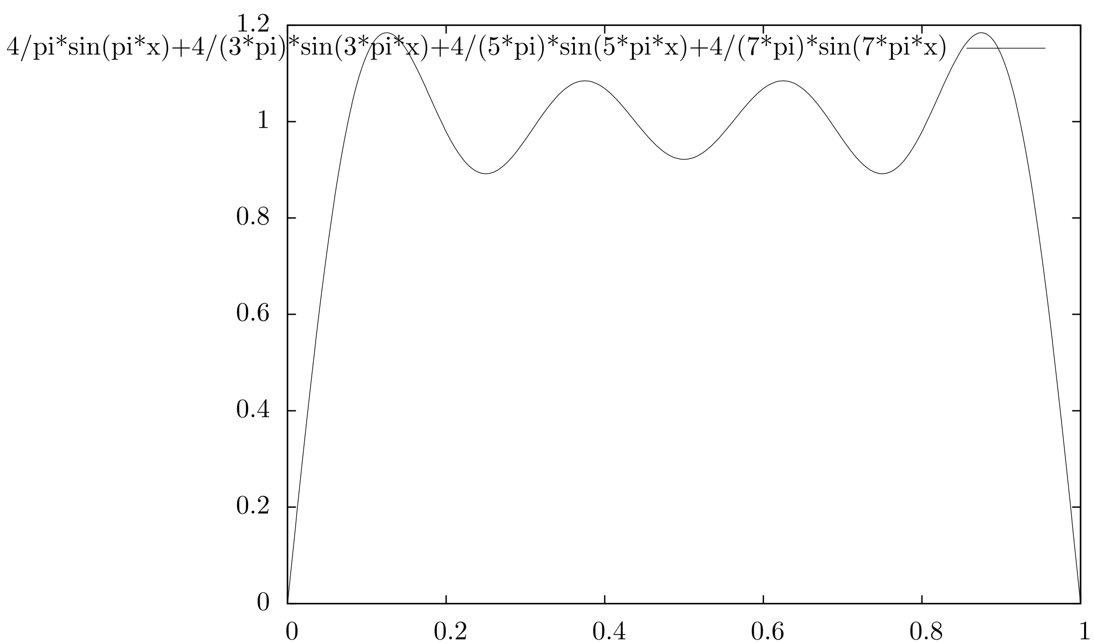
$$u = n\pi x, \quad \mathrm{d}u = n\pi \qquad A_n = \frac{2}{n\pi}\int_0^1 \sin(u)\,\mathrm{d}u$$

$$A_n = \frac{2}{n\pi}\left[-\cos u\right]_0^1 = \frac{2}{n\pi}\left[-\cos(n\pi x)\right]_0^1 \qquad A_n = \frac{2}{n\pi}\left[-\cos(n\pi)+\cos(0)\right] = \frac{2}{n\pi}\left(1-\cos(n\pi)\right)$$

$$A_n = \frac{2}{n\pi}\left(1-(-1)^n\right)$$

So A_n is zero for all even ns and $\frac{4}{n\pi}$ for odd.

$$\begin{aligned}\phi(x) &= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi}\sin((2n-1)\pi x)\\ &= \frac{4}{\pi}\left(\sin(\pi x) + \frac{1}{3}\sin(3\pi x) + \frac{1}{5}\sin(5\pi x) + \frac{1}{7}\sin(7\pi x) + \cdots\right)\end{aligned}$$



Jon Allen
HW 07

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} A_n e^{-(n\pi\alpha)^2 t} \sin(n\pi x) \\
A_n &= \frac{2}{n\pi}(1-(-1)^n) \\
u(x,t) &= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} e^{-((2n-1)\pi\alpha)^2 t} \sin((2n-1)\pi x)
\end{aligned}
$$

This seems a little simple, but all the work was really already done in HW 06.

Jon Allen
HW 08

$$
\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} A_n e^{-(n\pi\alpha)^2 t} \sin(n\pi x) \\
u(x,0) &= \sum_{n=1}^{\infty} A_n \sin(n\pi x) \\
&= \sin(2\pi x) + \frac{1}{3}\sin(4\pi x) + \frac{1}{5}\sin(6\pi x) \\
A_2 &= 1 \\
A_4 &= \frac{1}{3} \\
A_6 &= \frac{1}{5} \\
\text{all other } A_n &= 0 \\
u(x,t) &= e^{-4(\pi\alpha)^2 t}\sin(2\pi x) + \frac{1}{3}e^{-16(\pi\alpha)^2 t}\sin(4\pi x) + \frac{1}{5}e^{-36(\pi\alpha)^2 t}\sin(6\pi x)
\end{aligned}
$$

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HW 09
Transform

$$\begin{array}{lll} \text{PDE} & u_t = u_{xx} & 0 < x < 1 \\ \text{BCs} & \begin{cases} u_x(0,t) = 0 \\ u_x(1,t) + hu(1,t) = 1 \end{cases} & 0 < t < \infty \\ \text{IC} & u(x,0) = \sin(\pi x) & 0 \le x \le 1 \end{array}$$

into a new problem with zero BCs; Is the new PDE homogeneous?

Cribbing from the text we "seek a solution of the form":

$$\begin{aligned} u(x,t) &= A(t)[1-x] + B(t)x + U(x,t) \\ &= S(x,t) + U(x,t) \\ S(x,t) &= A(t)[1-x] + B(t)x \\ S_x &= B(t) - A(t) \end{aligned}$$

New BCs become

$$\begin{aligned} S_x(0,t) = 0 &= B(t) - A(t) \\ B(t) &= A(t) \\ S_x(1,t) + hS(1,t) = 1 &= B(t) - A(t) + hB(t) \\ &= 1 = hB(t) \\ \frac{1}{h} &= B(t) = A(t) \end{aligned}$$

Now we have

$$\begin{aligned} u(x,t) &= \frac{1}{h}[1-x] + \frac{x}{h} + U(x,t) = \frac{1}{h} + U(x,t) \\ u_t &= U_t \\ u_x &= U_x \\ u_{xx} &= U_{xx} \\ U(x,0) &= u(x,0) - \frac{1}{h} = \sin(\pi x) - \frac{1}{h} \\ 1 &= u_x(1,t) + hu(1,t) = U_x(1,t) + h(\frac{1}{h} + U(1,t)) \\ 0 &= U_x(1,t) + hU(1,t) \end{aligned}$$

And putting it all together we have:

$$\begin{array}{lll} \text{PDE} & U_t = U_{xx} & 0 < x < 1 \\ \text{BCs} & \begin{cases} U_x(0,t) = 0 \\ U_x(1,t) + hU(1,t) = 0 \end{cases} & 0 < t < \infty \\ \text{IC} & U(x,0) = \sin(\pi x) - \frac{1}{h} & 0 \le x \le 1 \end{array}$$

This new PDE is homogeneous.