3.1

6. (a) Let U and V

8. Let $\mathbf{v}_1, \ldots, \mathbf{v}_k \in \mathbb{R}^n$ and let $\mathbf{v} \in \mathbb{R}^n$. Prove that $\text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k) = \text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k, \mathbf{v})$ if and only if $\mathbf{v} \in \text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$.

   Define $A = \text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$ and $B = \text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_k, \mathbf{v})$

9. Determine the intersection of the subspaces $\mathcal{P}_1$ and $\mathcal{P}_2$ in each case:

   (b) $\mathcal{P}_1 = \text{Span}\left((1,2,2),(0,1,1)\right), \mathcal{P}_2 = \text{Span}\left((2,1,1),(1,0,0)\right)$

   (c) $\mathcal{P}_1 = \text{Span}\left((1,0,-1),(1,2,3)\right), \mathcal{P}_2 = \{\mathbf{x} : x_1 - x_2 + x_3 = 0\}$

11. Suppose V and W are orthogonal subspaces of $\mathbb{R}^n$, i.e., $\mathbf{v} \cdot \mathbf{w} = 0$ for every $\mathbf{v} \in V$ and every $\mathbf{w} \in W$. Prove that $V \cap V^\perp = \{\mathbf{0}\}$

15. Let A be an $m \times n$ matrix. Let $V \subset \mathbb{R}^n$ and $W \subset \mathbb{R}^m$ be subspaces.

   (a) Show that $\{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} \in W\}$ is a subspace of $\mathbb{R}^n$.

   (b) Show that $\{\mathbf{y} \in \mathbb{R}^m : \mathbf{y} = A\mathbf{x} \text{ for some } \mathbf{x} \in V\}$ is a subspace of $\mathbb{R}^m$

16. Suppose A is a symmetric $n \times n$ matrix. Let $V \subset \mathbb{R}^n$ be a subspace with the property that $A\mathbf{x} \in V$ for every $\mathbf{x} \in V$.