Notes

April 9, 2014

generating function is $\sum_{n=0}^{\infty} h_n x^n$ and exponential generating function is $\sum_{n=0}^{\infty} h_n x^n/n!$

theorem

let S be the multiset $\{n_1 \cdot a_1, n_2 \cdot a_2, n_3 \cdot a_3, dots, n_k \cdot a_k,\}$and h_n be the number of n-permutations of S. The exponential generating function for $h_0, h_1, \ldots$ is $g^{(e)}(x) = f_{n1}(x) f_{n2}(x) f_{n3}(x) \ldots f_{nk}(x)$

proof

see page 224(237)

example

find $g^{(e)}$ for the number of n-digit numbers with digits 3,5,7 where the # of 5's is even the number of 3's is more than 1 and at most 4 sevens.

$$(x^2/2! + x^3/3! + \ldots)(1 + x^2/2! + x^4/4! + \ldots)(1 + x + x^2/2! + x^3/3! + x^4/4!)$$

$$\left(\sum_{n=0}^{\infty} x^{n+2}/(n+2)!\right)\left(\sum_{n=0}^{\infty} x^{2n}/(2n)!\right)\left(\sum_{n=0}^{4} x^n/(n)!\right)$$

$$(e^x - 1 - x)(\frac{e^x + e^{-x}}{2})(...)$$

find the number of ways to color the squares of a 1xn chessboard with red blue and green so that the number of red squares is odd and the number of green squares is positive.

$$\frac{e^x - e^{-x}}{2} e^x \sum_{n=0}^{\infty} x^{n+1}/(n+1)!$$

$$\frac{e^x - e^{-x}}{2} e^x (e^x - 1)$$

$$\frac{e^x - e^{-x}}{2} (e^{2x} - e^x)$$

$$\frac{e^{3x} - e^{2x} - e^x + 1}{2}$$

$$\frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} (3x)^n/n! - (2x)^n/n! - x^n/n!$$