Notes

May 7, 2014

final

turn in to mel, math dept secretary by noon on thursday.

last time

finding the n^{th} derivative of $J_v(x)$. $J_v(x)$ is bessel function 1st kind, order v. Satisfies $z^2w''(z) + zw'(z) + (z^2 - v^2)w(z) = 0$. showed

1. $(b^2 - a^2)\int J_v(ar)J_v(br)r\,\mathrm{d}r = r\left(aJ_v{}'(ar)J_v(br)b - J_v(ar)J_v{}'(br)\right)$
2. $\int (J_v(ar))^2 r\,\mathrm{d}r$

$$\begin{aligned}
\int J_v(ar)J_v(br)r\,\mathrm{d}r &= \frac{r}{b+a}\frac{1}{b-a}\left(aJ_v{}'(ar)J_v(br) - bJ_v(ar)J_v{}'(br)\right)\\
\int J_v(ar)^2 r\,\mathrm{d}r &= \frac{r}{2a}\frac{\mathrm{d}}{\mathrm{d}b}(\text{ above })\\
&= \frac{r}{2a}\left[aJ_v{}'(ar)rJ_v{}'(ar) - J_v(ar)J_\nu{}'(ar) - aJ_\nu(ar)J_\nu{}''(ar)r\right]\\
&= \frac{r}{2a}\left[ar(J_\nu{}'(ar))^2 - J_\nu(ar)\left(arJ_\nu{}''(ar) + J_\nu{}'(ar)\right)\right]\\
&= \frac{r}{2a}\left[ar(J_\nu{}'(ar))^2 - (J_\nu(ar)^2(\frac{\nu^2-(ar)^2}{ar}\right]
\end{aligned}$$

book page 239

$$\begin{aligned}
\int_0^1 J_0(k_{oi}r)^2 r\,\mathrm{d}r &= \frac{1}{2}J_1{}^2(k_{oi})\\
\int_0^1 J_n(k_{nm}r)r\,\mathrm{d}r &= \frac{1}{2}\left[(J_n{}'(k_{nm})^2 + (J_n(k_{nm}))^2\right] - \frac{n^2}{2k_{nm}{}^2}(J_n(k_{nm}))^2 = \frac{1}{2}(J_n'(k_{nm}))^2\\
\int_0^1 J_0(k_{0m}r)^2 r\,\mathrm{d}r &= \frac{1}{2}J_0'(k_{om})^2
\end{aligned}$$