Notes

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solution $w(\xi_0, \eta_o; x, y)$ is related to riemann function $v(\xi_0, \eta_0; x, y)$ for original problem by $v = e^{b(x-\xi_0)+a(y-\eta_0)} w(\xi_0, \eta_0; x, y)$.

Introduced $x = (\xi_0 - x)(\eta_0 - y)$

$$w(x, y) = h(z)$$
$$zh''(z) + h'(z) + (c + ab)h(z) = 0$$

with $h(0) = 1$ want $h(z)$ on $z \geq 0$ try $h(z) = \sum_{n=0}^{\infty} h_n z^n$

$$\begin{aligned} 0 &= z \sum_{n=0}^{\infty} h_n n(n-1) z^{n-2} + \sum_{n=0}^{\infty} h_n n z^{n-1} + (c+ab) \sum_{n=0}^{\infty} h_n z^n \\ &= \sum_{n=0}^{\infty} h_n n^2 z^{n-1} + \sum_{n=0}^{\infty} (c+ab) h_n z^n \\ &= \sum_{n+1=1}^{\infty} h_{n+1}(n+1)^2 z^n + \sum_{n=0}^{\infty} (c+ab) h_n z^n \end{aligned}$$

for $n \geq 0$ $(n+1)^2 h_{n+1} = (ab - c)h_n, h_0 = 1$

$n \geq 1$

$$h_n = \frac{ab-c}{n^2} h_{n-1} = \frac{ab-c}{n^2} \cdot \frac{ab-c}{(n-1)^2} \dots$$

so $h(z) = \sum_{n=0}^{\infty} \frac{(ab-c)^n}{n!n!} z^n$ that is $w(x, y) = \sum_{n=0}^{\infty} \frac{(ab-c)^n}{n!n!} (\xi_0 - x)^n (\eta_0 - y)^n$

note series converges for $|z| < \infty$

note $J_0(x) = \sum_{n=0}^{\infty} \frac{(-x^2/4)^n}{n!n!}$ and $I_0(x) = \sum_{n=0}^{\infty} \frac{(x^2/4)^n}{n!n!}$ so $h(z)$ can be written as J_0 or I_0 depending on sign of $ab - c$

lesson 30

vibrating drumhead

PDE	$u_{tt} = c^2(u_{xx} + u_{yy})$	$0 \leq r \leq 1$	$0 < \theta < 2\pi$	
BC	$u(1, \theta, t) = 0$		$0 < \theta < 2\pi$	$t > 0$
IC	$u(r, \theta, 0) = f(r, \theta)$			
	$u_t(r, \theta, 0) = g(r, \theta)$			

$\nabla^2 u = u_{xx} + u_{yy}$ (cartesian) is laplacian operator $= u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}$ (polar) remember $x = r\cos(\theta)$ and y is multiple of r also.

we will use separation of variables

$$u = U(r,\theta)T(t)$$
$$U = R(r)\Theta(\theta)$$

eigenfunction $U = R(r)$. note that this is a circle. nodel line. add in Θ and get radial nodel lines ($U = R(r)\Theta(\theta)$)

chladni came up with sprinkling sand on surface of these things.

$$u_r = u_x \cos(\theta)$$

$$\text{PDE} \qquad \frac{T''(t)}{T(t)} = \left[U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta}\right] = \text{separation constant} = -\lambda^2 \text{ will assume less than 0}$$

$$T'' + c^2\lambda^2 T = 0 \leftarrow \text{trig solution}$$
$$U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} + \lambda^2 U = 0$$
$$U = R(r)\Theta(\theta)$$
$$R''(r) + \frac{1}{r}R'(r) + \frac{1}{r^2}R(r)\frac{\Theta''}{\Theta} + \lambda^2 R = 0$$

notice

$$\frac{\Theta''(\theta)}{Theta(\theta)} = \text{function of } r$$
$$= \text{function of } \theta$$
$$= \text{constant}$$
$$\Theta'' + \mu^2\Theta = 0$$

Θ must be 2π periodic

$$\cos(\mu\theta), \sin(\mu\theta)$$

by periocity

$$\mu = 1, 2,$$
$$\Theta(\theta) = a_n\cos(n\theta) + b_n\sin(n\theta)$$