Notes

April 11, 2014

lesson 21

vibrating beam - done (hw in prep-later)

lesson 22

dimensionless form - practice in past hw - discussion of dimensional analysis.

lesson 23

classification of pde's

hw 33 lesson 23 exercise 3. hw 34 lesson 23 exercise 4. hw 35 lesson 23 exercise 5. due fri april 18.

general second order linear PDE (in x,y).

$$Au_{xx} + Bu_{xy} + C_{yy} + Du_x + Eu_y + F_u = G$$

$A, \dots, G$ are functions of x, y. Basic question:can we simplify by changing variable:$\xi = \xi(x, y)$ and $\eta = \eta(x, y)$?

new equations in new variables look like:

$$\hat{A}u_{\xi\xi} + \hat{B}u_{\xi\eta} + \hat{C}u_{\eta\eta} + \hat{D}u_{\xi} + \hat{E}u_{\eta} + \hat{F}u = \hat{G}$$

$$\begin{aligned}
u_x &= u_\xi \xi_x + u_\eta \eta_x \\
u_y &= u_\xi \xi_y + u_\eta \eta_y
\end{aligned}$$

the following are mentioned on page 177 exercise 2

$$\begin{aligned}
u_{xx} &= u_{\xi\xi}{\xi_x}^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}{\eta_x}^2 + u_\xi \xi_{xx} + u_\eta \eta_{xx} \\
u_{xy} &= \frac{\partial}{\partial y}(u_x) \\
&= \frac{\partial}{\partial y}(u_\xi \xi_x + u_\eta \eta_x) \\
&= \frac{\partial}{\partial y}(u_\xi)\xi_x + u_\xi \xi_{xy} + \frac{\partial}{\partial y}(u_\eta)\eta_x + u_\eta \cdot \eta_{xy} \\
&= (u_{\xi\xi}\xi_y + u_{\xi\eta\eta_y})\xi_x + u_\xi \xi_{xy} + (u_{\eta\xi}\xi_y + u_{\eta\eta}\eta_y)\eta_x + u_\eta ? \\
&= u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \eta_x\xi_y) + u_{\eta\eta} \cdot \eta_x\eta_y + u_\xi \xi_{xy} + u_\eta \eta_{xy} \\
u_{yy} &= u_{\xi\xi}{\xi_y}^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}{\eta_y}^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}
\end{aligned}$$

the hat equations is obtained by expanding out the non-hat equations with the above. So $\hat{A} = u_{\xi\xi}(A{\xi_x}^2 + B\xi_x\xi_y + C{\xi_y}^2)$ and so on.

try setting $\hat{A} = 0 = \hat{C}$ 2 conditions to determine $\xi(,y), \eta(x,y)$

$$\hat{A} = 0 = A\xi_x^2 + B\xi_x\xi_y + C\xi_y^2$$
$$\hat{C} = 0 = A\eta_x^2 + B\eta_x\eta_y + C\eta_y^2$$

notice that a curve $\xi(x,y)$ =constant.

$$\frac{\mathrm{d}}{\mathrm{d}x}(\xi(x,y)) = \xi_x + \xi_y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\xi_x}{\xi_y}$$

these classifications come from the general quadratic in two variables:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

which gives you ellipse($B^2 - 4AC < 0$), hyperboles($B^2 - 4AC > 0$), parabolas($B^2 - 4AC = 0$)