Notes

January 28, 2015

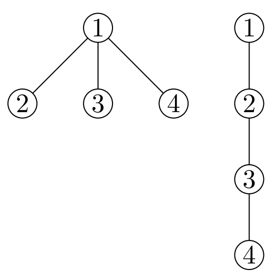
continuing with trees, 2.3

recall that a tree is a graph such that

1. every edge is a bridge
2. there are no cycles
3. order n and size $n-1$

a natural question is to say "how many non isomorphic trees of over n are there?"
early application of graph theory is chemistry. molecules are trees
first few

1. $n = 1$ then we have 1 tree
2. $n = 2$ then we have 1 tree
3. $n = 3$ then we have 1 non-isomorphic tree
4. $n = 4$ then we have

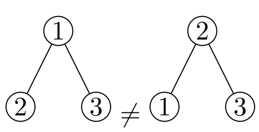


for any n the number of non-isomorphic trees on n-vertices is *a complete mystery*

simpler question

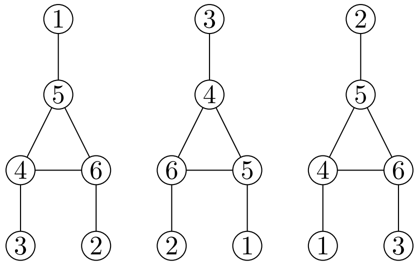
how many non-isomorphic **labelled** trees exists on n-vertices?
a **labelled graph** is a graph where each vertex is distinguished by a label.

example



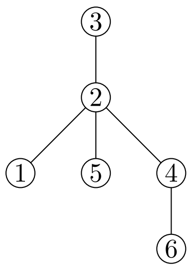
two labeled graphs are different if their edge sets are different

example

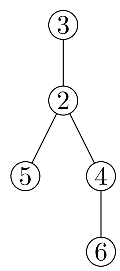


1 and 2 are the same, 3 is different
the solution is given neatly by Prüfer codes

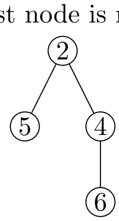
example



1. record the neighbor of the leaf of least valued label
2. erase leaf and repeat



3. last node is not in the code 2



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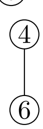

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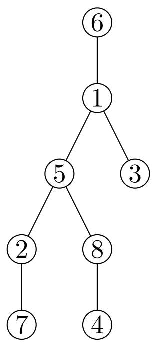
this process is reversible, but it's harder



prüfer code is $(1, 8, 1, 5, 2, 5)$

spanning tree

properties of prüfer codes

1. length is $n-2$
2. i appears at most $\deg(v_i)-1$ times
3. at most $n-2$ *different* entries

Prüfer code is a sequence of $n-2$ integers from $[n]$ where $[n] = \{1, \ldots, n\}$
and so we have $n-2$ spaces with n choices and so n^{n-2} codes

Cayley's Tree theorem

Labelled trees are in bijection with Prüfer codes. ie there are n^{n-2} labeled trees on n vertices (non-isomorphic)

now finally spanning trees

spanning graph of G is a subgraph H such that $V(G) = V(H)$.
weighted graph a graph with numerical labels on the edges
question? how can we find a spanning tree of a weighted graph with least total weight?

greedy algorithm

called Kruskal's algorithm

1. choose any edge of least weight
2. choose any remaining edge of next least weight that doesn't make a cycle
3. repeat