Notes

November 19, 2014

6.1.4

6.1.6 the chain rule

assume that $f : [a,b] \to [c.d]$ is differentiable at $x_0 \in [a,b]$ and $g : [c,d] \to \mathbb{R}$ is differentiable at the point $f(x_0)$, then $g \circ f : [a,b] \to \mathbb{R}$ is differentiable at x_0 and $(g \circ f)$

proof

from 6.1.4(3) because f is differentiable at x_0, we know the there exists a function φ such that φ is continuous at $x_0, \varphi'(x_0) = f'(x_0)$, and $f(x) = f(x_0) + \varphi(x)(x - x_0)$ similarly $g(y) = g(f(x_0) + \psi(y)(y - f(x_0))$ because g is differentiable at $f(x_0)$ ψ is continuous at $f(x_0)$ and $\psi(f(x_0) = g'(f(x_0)$

we want the same condition for $g \circ f$. $g \circ f(x) = g \circ f(x_0) + \eta(x)(x - x_0)$ for some $\eta(x)$ continous at x_0 and $\eta(x_0) = (g \circ f)'(x_0)$. tat $y = f(x)$. $g(f(x)) = g(f(x_0)) + \psi(f(x))(f(x) - f(x_0))$ and $g(f(x)) = g(f(x_0)) + \psi(f(x))\varphi(x)(x - x_0)$

6.1.7

example

let $f(x) = \sin x$ and $f^{-1}(x) = \arcsin x$. on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ it is injective. on $[\frac{\pi}{2}, \frac{3\pi}{2}]$ $f(x)$ is injective. the inverses are not the same functions. the derivitives though will have the same values.

proof

by corollary 6.1.4 we know that $f(x) = f(x_0) + \varphi(x)(x - x_0)$ when φ is continuous at x_0. we want to prove that $f^{-1}(y) = f^{-1}(f(x_0)) + \psi(y)(y - f(x_0))$ with ψ continuous at $f(x_0)$ an $\psi(f(x_0)) = \frac{1}{f'(x_0)}$. since f, f^{-1} are inverses if $y = f(x)$ then $x = f^{-1}(y)$ and let $y_0 = f(x_0) \Leftrightarrow x_0 = f^{-1}(y_0)$ and $y - y_0 = \varphi(f^{-1}(y)(f^{-1}(y) - f^{-1}(y_0) \to f^{-1}(y) - f^{-1}(y_0)$

is $\frac{1}{\varphi(f^{-1}(y)}$ continous at $f(x_0)$? $\varphi(f^{-1}(f(x_0))) = f'(x_0) \neq 0$.

from monday

where is $\sqrt{1 - \sin x}$ differentiable?

$$\begin{aligned}\lim_{h \to 0} \frac{\sqrt{1 - \sin(x_0 + h)} - \sqrt{1 - \sin x_0}}{h} &= \lim \frac{1 - \sin(x_0 + h) - (1 - \sin x_0)}{h(\sqrt{1 - \sin(x_0 + h)}\sqrt{1 - \sin x_0})} \\ &= -\cos x_0 \frac{1}{2\sqrt{1 - \sin x_0}}\end{aligned}$$

if $\sin x_0 = 1$ then $\cos x_0 = 0$ and $\lim \frac{\sqrt{1-\sin(x_0+h)}}{h} = \lim \frac{\sqrt{1-\sin^2(x_0+h)}}{h\sqrt{1+\sin(x_0+h)}} = \lim \frac{\sqrt{\cos^2(x_0+h)}}{h\sqrt{1+\sin(x_0+h)}} = \lim \frac{|\cos(x_0+h)-\cos x_0|}{h\sqrt{1+\sin(x_0+h)}}$