Jon Allen
HW 03

$$
\begin{aligned}
&u_t = \alpha^2 u_{xx} && 0 < x < 1 && 0 < t < \infty \\
&u(0,t) = 0 && u_x(1,t) = 1 && u(x,0) = \sin(\pi x) \\
&u_t = 0 = \alpha^2 u_{xx} && \int 0 \,\mathrm{d}x = \int \alpha^2 u_{xx} \,\mathrm{d}x && c_1 = \alpha^2 u_x \\
&\int c_1 \,\mathrm{d}x = \int \alpha^2 u_x \,\mathrm{d}x && c_1 x + c_2 = \alpha^2 U(x) && U(x) = \frac{c_1}{\alpha^2} x + \frac{c_2}{\alpha^2}
\end{aligned}
$$

And simplifying the constants

$$
\begin{aligned}
&U(x) = c_1 x + c_2 && U(0) = 0 = c_2 \\
&U(x) = c_1 x && U'(x) = c_1 \\
&U'(1) = 1 = c_1 && U(x) = x
\end{aligned}
$$

A steady state seems plausible to me. We could interpret this math as a laterally insulated rod with the temperature of one end held at 0°and the other receiving a constant flow of heat. At some point it will stabilize to where the end being held at 0°will be cooling at the same rate that the other end is being heated. And in fact that's what we find with the math, because $U(x) = x$ leads to $U'(x) = 1$.