

Notes

February 10, 2014

homework

for number 35 write down subsets in lexicographic order, and complements (this will be reverse lexicographic order)

number 33 has a theorem in the book

pigeonhole principle

this probably won't be on the test, but "it will blow my mind"

so if we have 11 cups and 12 markers than distributing the markers we have at least one cup with 2 markers. or if you have n holes and $n + 1$ pigeons then you will have at least one hole with more than one pigeon.

aside

we have 24 people in the class. what are the chances that anyone has the same birthday?

in math mathyness

abstract pigeonhole principle

let x and y be finite sets and $f : x \Rightarrow y$. If $|x| > |y|$ then f is not injective. If $|x| = |y|$ then f is surjective if and only if f is injective (that is bijective).

example

given a sequence $a_1, \dots, a_m, a_i \in \mathbb{N} \quad \forall i$. \exists consecutive a_i 's whose sum is divisible by m .

proof

consider $a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots, a_1 + a_2 + \dots + a_m$. Suppose none of these are divisible by m . So the remainders of each sum when divided by m are in $\{1, \dots, m - 1\}$. So $\exists k < l$ such that $a_1 + \dots + a_k$ and $a_1 + \dots + a_l$ have the same remainder r . So $\exists b, c \in \mathbb{N}$ such that

$$\begin{aligned} a_1 + \dots + a_l &= cm + r \\ -(a_1 + \dots + a_k &= bm + r) \\ a_{k+1} + \dots + a_l &= (c - b)m \end{aligned}$$

So $a_{k+1} + \dots + a_l$ is divisible by m .

note

what if a_i 's $\in \mathbb{Z}$?

back to the birthday problem

use the subtraction principle. The number of ways for 24 people to have a pair with the same birthday is the total number of birthday distributions minus the number of ways to have distinct birthdays. So 365^{24} is the total number of ways for people to have birthdays. And $\frac{365!}{(365-24)!}$ is the number of ways to have distinct birthdays. And the number of ways for 24 people to have a pair of the same birthday is $365^{24} - \frac{365!}{(365-24)!}$.