Notes

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section 6.2.4

let $f : [a, b] \to \mathbb{R}$ be differentiable, then

1. if f' is (strictly) positive, f is (strictly) increasing
2. same for negative
3. if $f'(x) = 0$ then $\forall x \in [a, b]$, f is constant

proof

suppose f' is strictly positive, let $x, y \in [a, b]$ such that $x < y$. then by mean value theorem $\exists c \in (x, y)$ such that $f'(c) = \frac{f(y)-f(x)}{y-x}$ and so $(y - x)f'(c) = f(y) - f(x)$ and so $y - x > 0$ and $f'(c) \geq 0 (> 0)$ so $f(y) - f(x) \geq 0$ an $f(y) \geq f(x)$

exercise 6.2.L

a function is convex (lies below that line segment (x,f(x) to (y,f(y))) if $f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y)$ for all x, y in $[a, b]$ and all $t \in [0, 1]$

a)

if f is differentiable and f' is increasing then f is convex.

define $z = tx + (1 - t)y$

note that $x \leq z \leq y$. and there exists $c_1 \in (x, z), c_2 \in (z, y)$ and $f'(c_1) = \frac{f(z)-f(x)}{z-x}$ and $f'(c_2) = \frac{f(y)-f(z)}{y-z}$ and $c_1 < c_2$ and so $f'(c_1) \leq f'(c_2)$ and $\frac{f(z)-f(x)}{z-x} \leq \frac{f(y)-f(z)}{y-z}$ and on through until $f(z)(y-z)+f(z)(z-x) \leq f(y)(z - x) + f(x)(y - z)$ and sub t back in for

$$f(z)(y - x) \leq f(y)[tx + (1 - t)y - x] + f(x)[y - tx - (1 - t)y]$$

and algebra to get definition