What is the solution to the inital-value problem

$$\begin{array}{llll} \text{PDE} & u_{tt} = u_{xx} & -\infty < x < \infty & 0 < t < \infty \\ \text{ICs} & \begin{cases} u(x,0) &= e^{-x^2} \\ u_t(x,0) &= 0 \end{cases} & -\infty < x < \infty & \end{array}$$

What does the solution look like for various values of time?

$$\begin{aligned} u(x,t) &= \frac{1}{2}\left[f(x-ct)+f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\xi)\,\mathrm{d}\xi \\ f(x) &= e^{-x^2} \\ g(x) &= 0 \\ c &= \pm 1 \\ u(x,t) &= \frac{1}{2}\left[e^{-(x\mp t)^2} + e^{-(x\pm t)^2}\right] \\ &= \frac{1}{2}\left[e^{-(x-t)^2} + e^{-(x+t)^2}\right] = \frac{1}{2}\left[e^{-(x+t)^2} + e^{-(x-t)^2}\right] \end{aligned}$$