Notes

March 12, 2014

$$
\begin{aligned}
G(s) &= \frac{1}{s}\frac{1}{\sqrt{s}+a}e^{-\sqrt{s}} \\
(s-a^2)G(s) &= \frac{1}{\sqrt{s}}e^{-\sqrt{s}} - \frac{a}{s}e^{-\sqrt{s}}
\end{aligned}
$$

assuming $g(0) = 0$

$$
\frac{\mathrm{d}g}{\mathrm{d}t} - a^2 g(t) = \frac{1}{\sqrt{\pi t}}e^{-1/4t} - a\mathrm{erfc}(\frac{1}{2\sqrt{t}})
$$

with $g(0) = 0$ integrating factor $\omega = e^{-a^2 t}$

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{-a^2 t}g(t)\right) = \frac{e^{-a^2 t}}{\sqrt{\pi t}e^{-1/4t}} - ae^{-a^2 t}\mathrm{erfc}(\frac{1}{2\sqrt{t}}) \qquad \text{decays exponentially}
$$

instead of integrating from $0 \to t$ rewrite as $t \to \infty$ with arbitrary constant

$$
\begin{aligned}
\int_0^t &= -\int_t^\infty + C \\
\int_0^t \frac{\mathrm{d}}{\mathrm{d}t}\left(e^{-a^2 t}g(t)\right) &= -\int_t^\infty \frac{e^{-a^2 t}}{\sqrt{\pi t}}e^{-1/4t} - ae^{-a^2 t}\mathrm{erfc}(\frac{1}{2\sqrt{t}}) \\
e^{-a^2 t}g(t) - 0 &= -\int_t^\infty \frac{1}{\sqrt{\pi t}}e^{-a^2 u - 1/4u}\,\mathrm{d}u + \int_t^\infty ae^{-a^2 u}\mathrm{erfc}(\frac{1}{2\sqrt{u}})\,\mathrm{d}u + C \\
\int_t^\infty e^{a^2 u}\mathrm{erfc}\left(\frac{1}{2\sqrt{u}}\right)\,\mathrm{d}u &= \int_t^\infty \mathrm{erfc}\left(\frac{1}{2\sqrt{t}}\mathrm{d}(\frac{-1}{a^2}e^{-a^2 u})\right) \\
&= \mathrm{erfc}(\frac{1}{2\sqrt{u}})\frac{-1}{a^2}e^{-a^2 u}\Big|_t^\infty + \frac{1}{a^2}\int_t^\infty e^{-a^2 u}\mathrm{d}(\mathrm{erfc}(\frac{1}{2\sqrt{u}})) \\
&= \frac{1}{a^2}e^{-a^2 t}\mathrm{erfc}(\frac{1}{2\sqrt{t}}) + \frac{1}{a^2}\int_t^\infty e^{-a^2 u}\left(\frac{-2}{\sqrt{\pi}}e^{-1/4u}\underbrace{\frac{1}{2}(-\frac{1}{2})u^{-3/2}\mathrm{d}u}_{\mathrm{d}(1/2\sqrt{u})}\right) \\
e^{-a^2 t}g(t) - 0 &= -\int_t^\infty \frac{1}{\sqrt{\pi u}}e^{-a^2 u - \frac{1}{4u}} + \frac{1}{a}e^{-a^2 t}\mathrm{erfc}(\frac{1}{2\sqrt{t}} + \frac{1\ 2}{4a\sqrt{\pi}}\int_t^\infty e^{-a^2 u - 1/4u}\frac{\mathrm{d}u}{u^{3/2}} \qquad \vdots \\
g(t) &= \frac{1}{a}\mathrm{erfc}\left(\frac{1}{2\sqrt{t}}\right) - \frac{1}{a}e^{a^2 t + a}\cdot\mathrm{erfc}\left(a\sqrt{t} + \frac{1}{2\sqrt{t}}\right) + C
\end{aligned}
$$

$$
\begin{aligned}
g(0) = 0 = 0 - \frac{1}{a} e^{a} \cdot 0 + C \\
C = 0
\end{aligned}
$$

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$$
\begin{aligned}
g(t) &= \int_0^t e^t \cdot h(u) \mathrm{d}u \\
f(t) &= g'(t) \\
g(t) &= e^t \cdot \int_0^t h(u) \mathrm{d}u
\end{aligned}
$$