Notes

23 février, 2015

Theorem: if $f(x) = \sum a_n x^n$ has radius of convergence R then so does $\sum n a_n x^{n-1}$ and further $f'(x) = \sum n a_n x^{n-1}$ and $\int_0^x f(t)\ \mathrm{d}t = \sum \frac{a_n}{n+1} x^{n+1}$

examples

$\sum\limits_{n=1}^{\infty} \frac{2^n}{n5^n} = \sum\limits_{n=1}^{u} \left(\frac{2}{5}\right)^n \frac{1}{n}$

$\sum\limits_{n=1}^{\infty} \frac{x^n}{n} = \int_0^x \sum\limits_{n=0} t^n = \int \frac{x^{n+1}}{n+1} = \int_0^x \frac{1}{1-t} = -\ln(1-t)|_0^x = -\ln(1-x) = -\ln(1-\frac{2}{5}) = \ln 5 - \ln 3$

$\sum\limits_{n=0} (n+1)x^n = \sum\limits_{n=1} nx^n$ antiderivative is $\sum\limits_{n=0} x^n = \frac{1}{1-x}$ and our answer is $\frac{1}{(1-x)^2}$

$\sum \frac{n+1}{2^n} = \frac{1}{1-\frac{1}{2}}^2 = \frac{1}{\frac{1}{2^2}} = 4$

approximate pi

$\tan^{-1} x = \int_0^x \frac{1}{1+t^2}\ \mathrm{d}t$

$\sum (-t^2)^n = \frac{1}{1+t^2} = \frac{1}{1-(-t^2)}$ as long as $-1 < -t^2 < 1 \Leftrightarrow -1 < t < 1$

and so $\tan^{-1} x = \int_0^x \sum\limits_{n=0}^{\infty} (-1)^n t^2 n\ \mathrm{d}t$