PDE C.

PDE.	$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$	for	$0 < x < \infty,$	$0 < t < \infty$
BC.	$\dfrac{\partial u}{\partial x}(0,t) = u(0,t) - \dfrac{1}{\sqrt{\pi t}}$	for		$0 < t < \infty$
IC.	$u(x,0) = 0$	for	$0 < x < \infty$	

Solve PDE C completely by a Laplace transform with respect to t. Use the BC as stated – do not transform to homogeneous BC. (The necessary inverse Laplace transform is not in the textbook table but is on the handout list of transforms.)

$$\begin{aligned}
sU(x) - 0 &= \frac{\mathrm{d}U}{\mathrm{d}x^2}(x) \\
\frac{\mathrm{d}U}{\mathrm{d}x}(0) &= U(0) - \mathcal{L}\left\{\frac{1}{\sqrt{\pi t}}\right\} \\
&= U(0) - \frac{1}{\sqrt{s}} \qquad \text{used computer} \\
0 &= \frac{\mathrm{d}U}{\mathrm{d}x^2}(x) - sU(x) \\
e^{-sx}U(x) &= c_1 \\
U(x) &= c_1 e^{sx} \\
\frac{\mathrm{d}U}{\mathrm{d}x}(x) &= c_1 x e^{sx} \\
\frac{\mathrm{d}U}{\mathrm{d}x}(0) &= c_1 0 e^{s0} = 0 \\
U(0) - \frac{1}{\sqrt{s}} &= 0 = c_1 - \frac{1}{\sqrt{s}} \\
U(x) &= \frac{1}{\sqrt{s}} e^{sx}
\end{aligned}$$

with computer, θ is heavyside step funtion

$$u(x,t) = \frac{\theta(t+x)}{\sqrt{\pi}\sqrt{t+x}}$$