Jon Allen
October 23, 2013

4.3

only need the "form" of the particular solution for 14,20,24. Need the particular solution for 30. Don't forget we need the homogeneous solution too for form of solution.

#14

$$y'' - 4y' + 3y = -e^{-9t}$$

solution

$$\begin{array}{ccc} r^2 - 4r + 3 = 0 = (r-3)(r-1) & r = 1, 3 & y_h = c_1 e^t + c_2 e^{3t} \\ f(t) = -e^{-9t} & S_p = \{e^-9t\} & y_p = Ae^{-9t} \end{array}$$

#20

$$y'' + 4y = 4\cos t - \sin t$$

solution

$$\begin{array}{ccc} r^2 + 4 = 0 & r = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{\pm 4i}{2} = \pm 2i & y_h = c_1 \cos 2t + c_2 \sin 2t \\ f(t) = 4\cos t - \sin t & S = \{\cos t, \sin t\} \cup \{\cos t, \sin t\} = \{\cos t, \sin t\} & y_p = A\cos t + B\sin t \end{array}$$

#24

$$y'' - 4y' + 13y = 2te^{-2t}\sin 3t$$

solution

$$\begin{array}{ccc} r^2 - 4r + 13 = 0 & r = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 13}}{2} = 2 \pm 3i & y_h = e^{2t}\left(c_1 \cos 3t + c_2 \sin 3t\right) \\ f(t) = 2te^{-2t}\sin 3t & S = \left\{e^{-2t}\sin 3t, e^{-2t}\cos 3t, te^{-2t}\sin 3t, te^{-2t}\cos 3t\right\} & \end{array}$$

$$y_p = Ae^{-2t}\sin 3t + Be^{-2t}\cos 3t + Cte^{-2t}\sin 3t + Dte^{-2t}\cos 3t$$

#30

find a general solution to $y'' + 3y' - 4y = -32t^2$

solution

$$r^2+3r-4=0=(r-1)(r+4) \qquad r=1,-4 \qquad y_h=c_1e^t+c_2e^{-4t}$$
$$f(t)=-32t^2 \qquad y_p=A+Bt+Ct^2 \qquad {y_p}'=B+2Ct \quad {y_p}''=2C$$
$$2C+3B+6Ct-4A-4Bt-4Ct^2=(2C+3B-4A)+t(6C-4B)-4Ct^2=-32t^2$$
$$-4C=-32 \qquad C=8 \qquad 6C-4B=0 \qquad B=12 \qquad 2C+3B-4A=0 \qquad \frac{52}{4}=A=13$$
$$y=c_1e^t+c_2e^{-4t}+8t^2+12t+13$$

4.4

#23

find a general solution to $y''+6y'+9y=t^{-1}e^{-3t}, t>0$

solution

$$r^2+6+9=0=(r+3)^2 \qquad r=-3 \qquad y_h=c_1e^{-3t}+c_2te^{-3t}$$
$$W=e^{-3t}(e^{-3t}-3te^{-3t})+te^{-3t}3e^{-3t} \qquad W=e^{-6t} \qquad {u_1}'=-\frac{te^{-3t}}{te^{3t}e^{-6t}}=-1$$
$${u_2}'=\frac{e^{-3t}}{te^{3t}e^{-6t}}=t^{-1} \qquad \int {u_1}'\,\mathrm{d}t=-t \qquad \int\frac{1}{t}\,\mathrm{d}t=\ln|t|, t>0\to\ln t$$
$$y=e^{-3t}\left(c_1+c_2t-t+t\ln t\right)=e^{-3t}\left(c_1+c_3t+t\ln t\right)$$

#28

find a general solution to $y''-10y'+25y=e^{5t}\ln 2t, t>0$

solution

$$r^2-10r+25=0=(r-5)^2 \qquad r=5 \qquad y_h=c_1e^{5t}+c_2te^{5t}$$
$$W=e^{5t}\left(5te^{5t}+e^{5t}\right)-te^{5t}5e^{5t} \qquad W=e^{10t} \qquad {u_1}'=-\frac{te^{5t}e^{5t}\ln 2t}{e^{10t}}=-t\ln 2t$$
$${u_2}'=\frac{e^{5t}e^{5t}\ln 2t}{e^{10t}}=\ln 2t \qquad -\int t\ln 2t\,\mathrm{d}t=-z_1w_1+\int w_1\,\mathrm{d}z_1 \qquad \begin{matrix} z_1=\ln 2t & \mathrm{d}w_1=t\,\mathrm{d}t \\ \mathrm{d}z_1=\frac{1}{t}\mathrm{d}t & w_1=\frac{1}{2}t^2\end{matrix}$$
$$u_1=-\frac{1}{2}t^2\ln 2t+\frac{1}{2}\int t\,\mathrm{d}t \qquad u_1=\frac{1}{2}t^2\left(\frac{1}{2}-\ln 2t\right) \qquad \int\ln 2t\,\mathrm{d}t=z_2w_2-\int w_2\mathrm{d}z_2$$
$$\begin{matrix} z_2=\ln 2t & \mathrm{d}w_2=\mathrm{d}t \\ \mathrm{d}z_2=\frac{1}{t}\mathrm{d}t & w_2=t\end{matrix} \qquad u_2=t\ln t2t-\int\mathrm{d}t \qquad u_2=t\ln 2t-t$$
$$y=e^{5t}\left(c_1+c_2t+\frac{1}{2}t^2\left(\frac{1}{2}-\ln 2t\right)+t^2\ln 2t-t^2\right)$$
$$y=e^{5t}\left(c_1+c_2t+\frac{1}{2}t^2\ln 2t-\frac{3}{4}t^2\right)$$