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8.1

compute the Laplace transforms

#5

$$f(t) = 2\sin 2t$$

solution

$$\begin{aligned}
\mathcal{L}\{2\sin 2t\} &= 2\int_0^\infty e^{-st}\sin 2t\,\mathrm{d}t \qquad \text{use maxima to do indefinite integral}\\
&= 2\cdot\lim_{M\to\infty}\left[\frac{1}{s^2+4}e^{-st}(-s\cdot\sin 2t - 2\cos 2t)\right]_{t=0}^{M}\\
&= \frac{2}{s^2+4}\left[0-(-s\cdot\sin 0 - 2\cos 0)\right] = \frac{4}{s^2+4}
\end{aligned}$$

#6

$$f(t) = \begin{cases} 1, & \text{if} \quad 0 \le t \le 2\\ 0, & \text{if} \quad t > 2\end{cases}$$

solution

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)\,\mathrm{d}t = \int_0^2 e^{-st}\,\mathrm{d}t + \int_2^\infty 0\,\mathrm{d}t = \left[-\frac{1}{s}e^{-st}\right]_{t=0}^{2} + 0 = -\frac{1}{s}(e^{-2s}-1) = \frac{1}{s} - \frac{1}{se^{2s}}$$

#7

$$f(t) = \begin{cases} 0, & \text{if} \quad 0 \le t \le 1\\ 1, & \text{if} \quad t > 1\end{cases}$$

solution

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)\,\mathrm{d}t = \int_0^1 0\,\mathrm{d}t + \int_1^\infty e^{-st}\,\mathrm{d}t = 0 + \lim_{M\to\infty}\left[-\frac{1}{s}e^{-st}\right]_{t=1}^{M} = -\frac{1}{s}(0-e^{-s}) = \frac{1}{se^s}$$

#16

$$f(t) = \cos kt$$

solution

$$\begin{aligned}
\mathcal{L}\{\cos kt\} &= \int_0^\infty e^{-st}\cos kt\,\mathrm{d}t \qquad \text{use maxima to calculate indefinite integral}\\
&= \lim_{M\to\infty}\left[\frac{1}{s^2+k^2}e^{-st}(k\cdot\sin kt - s\cdot\cos kt)\right]_{t=0}^{M} = \frac{1}{s^2+k^2}\left[0-(k\cdot\sin 0 - s\cdot\cos 0)\right] = \frac{s}{s^2+k^2}
\end{aligned}$$

#23

$$\mathcal{L}\{e^{-t}\sin 5t\}$$

solution

$$\mathcal{L}\{\sin 5t\} = \frac{5}{s^2+25}$$
$$\mathcal{L}\{e^{-t}f(t)\} = F(s+1) = \frac{5}{(s+1)^2+25} = \frac{5}{s^2+2s+26}$$

8.2

#10

$$F(s) = \frac{1}{s^2+12s+61}$$

solution

$$\frac{1}{s^2+12s+61} = \frac{1}{(s+6)^2+25}$$
$$\mathcal{L}\{e^{-6t}f(t)\} = F(s+6) \qquad \mathcal{L}\{\sin 5t\} = \frac{5}{s^2+25}$$
$$\mathcal{L}\{\frac{1}{5}e^{-6t}\sin 5t\} = \frac{1}{(s+6)^2+25} \qquad f(t) = \frac{1}{5}e^{-6t}\sin 5t$$

#11

$$F(s) = \frac{s}{s^2-5s-14}$$

solution

$$\frac{s}{s^2-5s-14} = \frac{s}{(s-7)(s+2)} \qquad \frac{1}{(s-7)(s+2)} = \frac{A}{s-7} + \frac{B}{s+2}$$
$$A(s+2)+B(s-7) = 1 = s(A+B)+2A-7B \qquad A = -B \quad 2A+7A = 1$$
$$F(s) = \frac{1}{9}\left(\frac{s}{s-7} - \frac{s}{s+2}\right) = \frac{1}{9}\left(\frac{s-7+7}{s-7} - \frac{s+2-2}{s+2}\right)$$
$$= \frac{1}{9}\left(1+7\frac{1}{s-7} - 1 + 2\frac{1}{s+2}\right) = \frac{7}{9}\left(\frac{1}{s-7}\right) + \frac{2}{9}\left(\frac{1}{s+2}\right) \qquad f(t) = \frac{7}{9}e^{7t} + \frac{2}{9}e^{-2t}$$