

Continue with problem 3 by finding the new canonical equation.

$$u_{\xi\eta} = \Phi(\xi, \eta, u, u_\xi, u_\eta)$$

$$\begin{aligned}\xi &= y + 2x \\ \eta &= y + \frac{1}{3}x \\ A &= 3, \quad B = 7, \quad C = 2 \\ D &= E = F = G = 0 \\ \xi_x &= 2, \quad \xi_y = 1 \\ \xi_{xx} &= \xi_{xy} = \xi_{yy} = 0 \\ \eta_x &= \frac{1}{3}, \quad \eta_y = 1 \\ \eta_{xx} &= \eta_{xy} = \eta_{yy} = 0 \\ \overline{A} &= \overline{C} = 0\end{aligned}$$

We solved for  $\overline{A} = \overline{C} = 0$  to get  $\xi, \eta$ . Also, because all the second derivatives are zero along with  $D, E, F$  we quickly see that  $\overline{D} = \overline{E} = 0$ . And of course  $\overline{F} = F = \overline{G} = G = 0$ .

$$\begin{aligned}\overline{B} &= 2A\xi_x\eta_x + B(\xi_x\eta_y + \xi_y\eta_x) + 2C\xi_y\eta_y \\ &= 2 \cdot 3 \cdot 2 \cdot \frac{1}{3} + 7 \left( 2 \cdot 1 + 1 \cdot \frac{1}{3} \right) + 4 \\ &= \frac{12}{3} + 14 + \frac{7}{3} + 4 = \frac{19 + 18 \cdot 3}{3} \\ \overline{B}u_{\xi\eta} &= 0 \\ \overline{B} \neq 0 &\rightarrow u_{\xi\eta} = 0\end{aligned}$$