hyperplanes

distance from origin to $H(\mathbf{x}_0, \mathbf{a})$. we are looking for a vector in the same direction as $\mathbf{a}$. Find $L(\mathbf{0}, \mathbf{a}) \cap H(\mathbf{x}_0, \mathbf{a})$. Now $\mathbf{x} \in L \cap H \Rightarrow x = t\mathbf{a}$ and $\mathbf{a} \cdot \mathbf{x} = \mathbf{a} \cdot \mathbf{x}_0 \Rightarrow \mathbf{a} \cdot (t\mathbf{a}) = \mathbf{a}\mathbf{x}_0 \to t(\mathbf{a} \cdot \mathbf{a}) = \mathbf{a}\mathbf{x}_0 \to t = \frac{\mathbf{a} \cdot \mathbf{x}_0}{||a||^2}$

and now take $t\mathbf{a} = \text{proj}_a(\mathbf{x}_0)$ and $||t\mathbf{a}|| = \frac{|a \cdot \mathbf{x}_0|}{||a||^2}||a| = \frac{|a \cdot \mathbf{x}_0|}{||a||}$

if we have another point $\mathbf{y}_0$ not on the hyperplane and we take $\text{proj}_a(\mathbf{x}_0 - \mathbf{y}_0)$ then we have the distance from the hyperplane to $\mathbf{y}_0$