

Continue with problem 4 by finding the *alternative* canonical form

$$u_{\alpha\alpha} - u_{\beta\beta} = \Psi(\alpha, \beta, u, u_\alpha, u_\beta)$$

$$\alpha(\xi, \eta) = \xi + \eta$$

$$\beta(\xi, \eta) = \xi - \eta$$

$$\alpha_\xi = 1 \quad \alpha_\eta = 1$$

$$\beta_\xi = 1 \quad \beta_\eta = -1$$

$$u_\xi = u_\alpha \alpha_\xi + u_\beta \beta_\xi = u_\alpha + u_\beta$$

$$u_{\xi\eta} = u_{\alpha\alpha} \alpha_\eta + u_{\alpha\beta} \beta_\eta + u_{\beta\alpha} \alpha_\eta + u_{\beta\beta} \beta_\eta = u_{\alpha\alpha} - u_{\beta\beta}$$

Since $u_{\xi\eta} = 0$ we know $u_{\alpha\alpha} - u_{\beta\beta} = 0$