Notes

January 16, 2015

error fixup

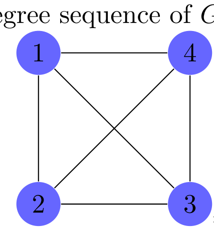
neighborhood is $N_G(V_i) = \{V_j | (v_i, v_j) \in E(G)\}$

degree sequences

these will be ascending, book is descending

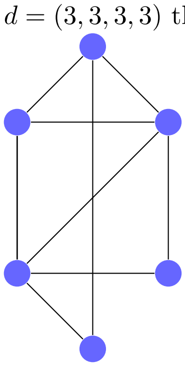
definitionn

if G is finite with $V(G) = \{v_1, \ldots, v_n\}$ such that $d_i = \deg(v_i) \leq \deg(v_j)$ for $i \leq j$ then $(d_1, \ldots, d_n)$ is the degree sequence of G.



$= k_4$

$d = (3, 3, 3, 3)$ three regular graph



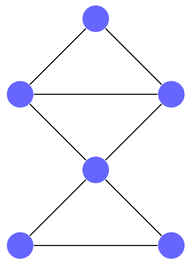
$d = (2, 3, 3, 3, 3, 4)$

Havel, Hakimi thm

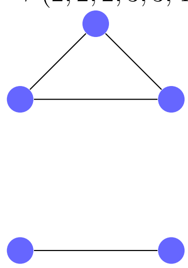
if $(d_1, \ldots, d_n)$ is a non decreasing sequence with $d_n \geq 1$ (avoid the empty graph) it is a degree sequence iff $(d_1, \ldots, d_{n-d_n-1}, d_{n-d_n} - 1, \ldots, d_{n-1} - 1)$ is a degree sequence

example

given



$\rightarrow (2,2,2,3,3,4) \rightarrow (2,1,1,2,2)$



$(1,1,2,2,2)$

proof

$\Rightarrow$ careful vertex deletion

$\Leftarrow$ let G have a degree sequeence $*$ then $\deg(v_i) = \begin{cases} d_i & i = 1, \ldots, n - d_n - 1 \\ d_i - 1 & i = n - d_n, \ldots, n - 1 \end{cases}$

add a vertex to G and add edges between the new vertex and all vertices of degree $d_i - 1$

the degree of the new vertex is $n - 1 - (n - d_n) + 1 = d_n$

the new graph has degree sequence $(d_1, \ldots, d_n)\square$

claim

havel-hakimi can be used to verify, refute the degree sequenceness of any nondecreasing sequence of integers

i.e. we can say rather quickly that (polynomial time) if $(2,3,3,5,5,5,5,5,5,6)$ is graphical

$n = 10, 10-6-1 = 3, d_3 = 3, n-d_n = 4, d_4-1 = 4, d_9-1 = 5$ $(2,3,3,4,4,4,4,4,4) \cdots \rightarrow \ldots (1,1,2,2,2,2)$

question, if two degree sequences are the same, are the graphs isomorphic? no!

homework 1.2

6a,b,7,10,15