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 ${\rm HW}~05$

Lets say the length of the rod is L. The rod is laterally insulated so the PDE takes the form of $u_t = \alpha^2 u_{xx}$. Initial temperature of 20° gives us u(x,0) = 20. One end fixed at 50° gives us u(0,t) = 50. The other end being immersed in 30° liquid gives us $u_x(L,t) = -\lambda[u(L,t) - 30]$ Putting it all together we have:

PDE
$$u_t = \alpha^2 u_{xx} \qquad 0 < x < L \qquad 0 < t < \infty$$
BCs
$$\begin{cases} u(0,t) = 50 \\ u_x(L,t) = -\lambda[u(L,t) - 30] \end{cases} \qquad 0 < t < \infty$$
IC
$$u(x,0) = 20 \qquad 0 \le x \le L$$