1. Prove that if $A \subseteq \mathbb{R}$ and for $A \in \mathbb{R}$ we have $A + \lambda = \{a + \lambda : a \in A\}$ then $m * (A) = m * (A + \lambda)$

   proof

   First we notice that because $A \subseteq \mathbb{R}$ and $\lambda \in \mathbb{R}$ then $A + \lambda \in \mathbb{R}$. Now using the definition of the outer measure we have

   $$m * (A + \lambda) = \inf \left\{ \sum_{i=1}^{\infty} b_i - a_i : A + \lambda \subseteq \bigcap_{i=1}^{\infty} (a_i, b_i) \right\}$$

   Now we can rewrite this a little bit based on the definition of $A + \lambda$ to get

   $$m * (A + \lambda) = \inf \left\{ \sum_{i=1}^{\infty} (b_i + \lambda) - (a_i + \lambda) : A \subseteq \bigcap_{i=1}^{\infty} (a_i, b_i) \right\}$$

   Our lambdas cancel, so we are just left with

   $$m * (A + \lambda) = \inf \left\{ \sum_{i=1}^{\infty} b_i - a_i : A \subseteq \bigcap_{i=1}^{\infty} (a_i, b_i) \right\}$$

   But $m * (A)$ is defined to be

   $$m * (A) = \inf \left\{ \sum_{i=1}^{\infty} b_i - a_i : A \subseteq \bigcap_{i=1}^{\infty} (a_i, b_i) \right\}$$

   And so we have $m * (A) = m * (A + \lambda)$ as desired. $\square$

2. Prove that if $m * (A) = 0$ then $m * (A \cup B) = m * (B)$ for any set $B \subseteq \mathbb{R}$

   proof

   In class we showed that $m * \left(\bigcup_{i=1}^{\infty} A_i \right) \leq \sum_{i=1}^{\infty} m * (A_i)$ which means that $m * (A \cup B) \leq m * (A) + m * (B) = 0 + m * (B) = m * (B)$. Obviously $B \subseteq A \cup B$ and so we know from lecture that $m * (B) \leq m * (A \cup B)$. Put them together and we have $m * (B) \leq m * (A \cup B) \leq m * (B)$ which means $m * (B) = m * (A \cup B)\square$