Notes

March 28, 2014

result from last time (D'Alemberts' solution)

$$\begin{array}{rlr} \text{PDE} & u_{tt} = c^2 u_{xx} & -\infty < x < +\infty \qquad 0 < t < \infty \\ \text{IC} & u(x,0) = f(x) & \\ & u_t(x,0) = g(x) & \\ \hline & u(x,t) = \frac{1}{2}(f(x-ct) + f(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\mathrm{d}s & \end{array}$$

18 properties of this solution

Case 1.

$$\text{IC} \qquad \left.\begin{array}{r} u(x,0) = f(x) \\ u_t(x,0) = 0 \end{array}\right\} \text{ Solution } u(x,t) = \frac{1}{2}\left[\underbrace{f(x-ct)}_{\text{wave moving right}} + \underbrace{f(x+ct)}_{\text{wave moving left}}\right]$$

Case 2.

$$\text{IC} \qquad \left.\begin{array}{r} u(x,0) = 0 \\ u_t(x,0) = g(x) \end{array}\right\} \text{ Solution } u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,\mathrm{d}s$$

value of g over a widening interval

see graphs on pages 139-141 (155-157)

$$\begin{array}{rrll} \text{PDE} & u_{tt} = c^2 u_{xx} & 0 < x < \infty & 0 < t < \infty \\ \text{BC} & u(0,t) = 0 & & 0 < t < \infty \\ \text{IC} & u(x,0) = f(x) & 0 < x < \infty & \\ & u_t(x,0) = g(x) & & \end{array}$$

as last time $u(x,t) = \phi(x-ct) + \psi(x+ct)$ general solution – IC's and BC are not used.

Match IC:

$$\begin{array}{rl} \phi(x) + \psi(x) = f(x) & 0 < x < \infty \\ -c\phi'(x) + c\psi'(x) = g(x) & \to -\phi(x) + +\psi(x) \\ & = \frac{1}{c}\int_0^x g(s)\,\mathrm{d}s + K \end{array}$$

$$\phi(x) = \frac{1}{2}f(x) - \frac{1}{2c}\int_0^x g(s)\,\mathrm{d}s + \frac{k}{2}$$
$$\psi(x) = \frac{1}{2}f(x) + \frac{1}{2c}\int_0^x g(s)\,\mathrm{d}s + \frac{k}{2}$$
$$u(x,t) = \phi(x-ct) + \psi(x+ct)$$

k cancel out. $x+ct > 0$ so $\psi(x+ct)$ is no problem. $x-ct$ changes sign.

What is $\phi(x-ct)$ when $x-ct<0$?

now lets look at the boundary condition

BC $\quad u(0,t) = 0 = \phi(-ct) + \psi(ct)$ for $\quad 0 < t < \infty$

for $\quad -\infty < x < 0, \quad \phi(x) = -\psi(-x)$

for $\quad x - ct > 0,$
$$u(x,t) = \phi(x-ct) + \psi(x+ct) = \frac{1}{2}\left[f(x-ct) + f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,\mathrm{d}s$$

for $\quad x - ct < 0,$
$$u(x,t) = -\psi(x-ct) + \psi(x+ct) = \frac{1}{2}\left[f(x-ct) + f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,\mathrm{d}s$$

more on page 143(159)

homework #28 & #29 due next friday (first friday of april) lession 17 exercise 3 and exercise 4