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HW 14
Lesson 9 problem 4. Find general series solution for PDE and BC's.

$$\begin{array}{lll} PDE & u_t = u_{xx} + \sin(\pi x) & 0 < x < 1 \qquad 0 < t < \infty \\ BCs & \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases} & 0 < t < \infty \\ IC & u(x,0) = 0 & 0 \le x \le 1 \end{array}$$

We need to find the coefficients $T_n(t)$ in

$$u(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin(n\pi x)$$

Substituting into the original problem we have

$$\sum_{n=1}^{\infty} T_n(t) \sin(n\pi x) = -\sum_{n=1}^{\infty} (n\pi)^2 T_n(t) \sin(n\pi x) + \sum_{n=1}^{\infty} f_n(t) \sin(n\pi x)$$

$$\sum_{n=1}^{\infty} T_n(t) \sin 0 = 0$$

$$\sum_{n=1}^{\infty} T_n(t) \sin(n\pi) = 0$$

$$\sum_{n=1}^{\infty} T_n(0) \sin(n\pi) = 0$$

We can rewrite the pde to get

$$\sum_{n=1}^{\infty} T_n(t) \sin(n\pi x) + (n\pi)^2 T_n(t) \sin(n\pi x) - f_n(t) \sin(n\pi x) = 0$$