Notes

April 7, 2014

lesson 20

vibrating strings on the finite interval.

PDE	$u_{tt} = c^2 u_{xx}$	$0 < x < L$	$0 < t < \infty$
BC	$u(0,t) = 0 = u(L,t)$		$0 < t < \infty$
IC	$\begin{cases} u(x,0) & = f(x) \\ u_t(x,0) & = g(x) \end{cases}$	$0 < x < L$	

looked for separated solutions $u = T(t)X(x)$

found $u_n(x,t) = [a_n \sin(n\pi ct/L) + b_n \cos(n\pi ct/L)] \sin(n\pi x/L)$

text p 157

General solution $u(x,t) = \sum_{n=1}^{\infty} [a)n \sin(n\pi ct/L) + b_n \cos(n\pi ct/L)] \sin(n\pi x/L)$ satisfies PDE (linear/hom) and BC(linear hom)

set $t = 0$ so $u(x,t) = \sum_{n=1}^{\infty} b_n \sin(n\pi x/L) = f(x)$.

Recal $\int_0^L \sin(n\pi x/L)\sin(m\pi x/L)\mathrm{d}x = 0, m \neq n$ and $\int_0^L \sin(n\pi x/L) = L/2$.

$$\int_0^L \sum_{n=1}^{\infty} b_n \sin(n\pi x/L)\mathrm{d}x = \int_0^L f(x)\mathrm{d}x$$
$$\int_0^L \sin(m\pi x/L) \sum_{n=1}^{\infty} b_n \sin(n\pi x/L)\mathrm{d}x = \int_0^L \sin(m\pi x/L) f(x)\mathrm{d}x$$

blah blah, page 157

$$u(x,t) = \sum_{n=1}^{\infty} (a_n \sin(n\pi ct/L) + b_n \cos(n\pi ct/L)) \sin(n\pi x/L$$
$$u_t(x,t) = \sum_{n=1}^{\infty} (n\pi \frac{c}{L} a_n \cos(n\pi ct/L) - n\pi \frac{c}{L} b_n \sin(n\pi ct/L)) \sin(n\pi x/L$$
$$u_t(x,t) = \sum_{n=1}^{\infty} n\pi \frac{c}{L} a_n \sin(n\pi x/L) = g(x)$$

even in something like $\sin(\mu_n \frac{x}{L}$ where $\mu + tan(\mu) = 0$ orthoganality will still be present. Sturm-Lionville theory gives this.

lesson 21

the vibrating beam (4th order PDE)

$$
\begin{array}{llll}
\text{PDE} & u_{tt} = \alpha^2 u_{xxxx} & 0 < x < L & 0 < t < \infty \\
\text{BC} & u(0,t) = 0 = u(L,t) & & 0 < t < \infty \\
 & u_{xx}(0,t) = 0 = u_{xx}(L,t) & & \\
\text{IC} & \begin{cases} u(x,0) & = f(x) \\ u_t(x,0) & = g(x) \end{cases} & 0 < x < L &
\end{array}
$$

HW will involve exer 1 on p 166-167 $u(0,t) = 0, u_{xx}(1,t) = 0, u_x(1,t) = 0, u_{xxx}(1,t) = 0$ free end p 166.
set $u = T(t)X(x)$

$$\frac{T''}{-\alpha^2 T} = \frac{X''''}{X} = \lambda \qquad \text{separation constant}$$

$$X'''' - \lambda X = 0 \qquad \text{consider bc}$$

assume $\lambda > 0$ since $\lambda \leq 0$ cannot satisfy bc

$$
\begin{aligned}
\lambda &= \omega^2 > 0 \\
X^{(4)} - \omega^2 X &= 0 \\
X &= e^{rx} \\
X^{(4)} &= r^4 e^{rx} - \omega^2 e^{rx} \\
r^4 - \omega^2 &= 0 \\
r^2 &= \pm\omega \\
r &= \pm\sqrt{\omega}, \pm i\sqrt{\omega} \\
e^{\pm\sqrt{\omega}x}&, e^{\pm i\sqrt{\omega}x} \\
X(x) &= C\cos(\sqrt{\omega}x) + D\sin(\sqrt{\omega}x) + E\cosh(\sqrt{\omega}x) + F\sinh(\sqrt{\omega}x
\end{aligned}
$$

apply bc

$$
\begin{aligned}
X(0) &= 0 = C + E \\
X'' &= -C\omega\cos(\sqrt{\omega}x) - D\omega\sin(\sqrt{\omega}x) + E\omega\cosh(\sqrt{\omega}x) + F\omega\sinh(\sqrt{\omega}x) \\
X''(0) = 0 &= -C\omega + E\omega \\
C &= E = 0
\end{aligned}
$$

now at $x = L$