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HW 09
Transform

$$\begin{array}{lll} \text{PDE} & u_t = u_{xx} & 0 < x < 1 \\ \text{BCs} & \begin{cases} u_x(0,t) = 0 \\ u_x(1,t) + hu(1,t) = 1 \end{cases} & 0 < t < \infty \\ \text{IC} & u(x,0) = \sin(\pi x) & 0 \le x \le 1 \end{array}$$

into a new problem with zero BCs; Is the new PDE homogeneous?

Cribbing from the text we "seek a solution of the form":

$$\begin{aligned} u(x,t) &= A(t)[1-x] + B(t)x + U(x,t) \\ &= S(x,t) + U(x,t) \\ S(x,t) &= A(t)[1-x] + B(t)x \\ S_x &= B(t) - A(t) \end{aligned}$$

New BCs become

$$\begin{aligned} S_x(0,t) &= 0 = B(t) - A(t) \\ B(t) &= A(t) \\ S_x(1,t) + hS(1,t) &= 1 = B(t) - A(t) + hB(t) \\ &= 1 = hB(t) \\ \frac{1}{h} &= B(t) = A(t) \end{aligned}$$

Now we have

$$\begin{aligned} u(x,t) &= \frac{1}{h}[1-x] + \frac{x}{h} + U(x,t) = \frac{1}{h} + U(x,t) \\ u_t &= U_t \\ u_x &= U_x \\ u_{xx} &= U_{xx} \\ U(x,0) &= u(x,0) - \frac{1}{h} = \sin(\pi x) - \frac{1}{h} \\ 1 &= u_x(1,t) + hu(1,t) = U_x(1,t) + h(\frac{1}{h} + U(1,t)) \\ 0 &= U_x(1,t) + hU(1,t) \end{aligned}$$

And putting it all together we have:

$$\begin{array}{lll} \text{PDE} & U_t = U_{xx} & 0 < x < 1 \\ \text{BCs} & \begin{cases} U_x(0,t) = 0 \\ U_x(1,t) + hU(1,t) = 0 \end{cases} & 0 < t < \infty \\ \text{IC} & U(x,0) = \sin(\pi x) - \frac{1}{h} & 0 \le x \le 1 \end{array}$$

This new PDE is homogeneous.