Notes

April 9, 2014

lesson 21

PDE	$u_{tt} = -u_{xxxx}$	$0 < x < 1$	$0 < t < \infty$
BC	$u(0,t) = 0 = u(1,t)$		$0 < t < \infty$
	$u_{xx}(0,t) = 0 = u_{xx}(1,t)$		
IC	$u(x,0) = f(x)$	$0 < x < 1$	
	$u_t(x,0) = g(x)$		

$u = T(t)X(x)$ give $\frac{T''}{-T} = \frac{X''''}{X} = \lambda$
assume $\lambda > 0$ ($\lambda \leq 0$ can be eliminated by using bc)
Have $X'''' - \omega^2 X = 0$ with characteristic roots $\pm\sqrt{\omega}, \pm i\sqrt{\omega}$ and solutions:

$$= C\cos(\sqrt{\omega}x) + D\sin(\sqrt{\omega}x) + E\cosh(\sqrt{\omega}x + F\sinh(\sqrt{\omega}x)$$
$$X'' = \omega(-C\cos(\sqrt{\omega}x) - D\sin(\sqrt{\omega}x) + E\cosh(\sqrt{\omega}x + F\sinh(\sqrt{\omega}x))$$
$$x = 0 \to C = E = 0$$

$$x = 1 \to \quad \left.\begin{aligned} D\sin(\sqrt{\omega}) + F\sinh(\sqrt{\omega}) &= 0 \\ \omega(-D\sin(\sqrt{\omega}) + F\sinh(\sqrt{\omega})) &= 0 \end{aligned}\right\} \begin{aligned} 2F\sinh(\sqrt{\omega}) = 0 &\implies F = 0 \\ 2D\sin(\sqrt{\omega}) = 0 &\implies \begin{array}{c}\text{nontrivial when } \sin(\sqrt{\omega}) = 0 \\ \text{and } \sqrt{\omega} = n\pi \text{ for } n = 1,2,3,\ldots\end{array} \end{aligned}$$

Have $X_n(x) = \sin(n\pi x)$ for $n = 1, 2, 3, \ldots$

$$\frac{T_n{}''}{-T_n} = \omega_n^2 = (n\pi)^4$$
$$T_n'' + (n\pi)^4 T_n = 0$$
$$cos((n\pi)^2 t), \sin((n\pi)^2 t)$$

Have $T_n X_n = a_n \sin((n\pi)^2 t) + b_n \cos((n\pi)^2 t)$ frequencies $\frac{n^2}{2}\pi, n = 1, 2, 3, \ldots$
for pde and bc at $t = 0$

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi x) \to \text{use orthogonality}$$
$$u_t(x,0) = g(x) = \sum_{n=1}^{\infty} a_n (n\pi)^2 \sin(n\pi x)$$

he will give us a homework problem that will look like this

PDE

BC

IC

hopefully the previous problem will help us work out the homework problem

lesson 22

dimensional analysis

object moving through a fluid (air). question: frictional force actin on the object. expect the force to be related to velocity V. want drag force F_D in terms of V velocity of object. A is "characteristic" area associated with object (analysis should hold for similar objects). ρ is fluid density

$$\begin{aligned} F_D \text{ units } & \left[\frac{\text{mass} \cdot \text{length}}{\text{time}^2}\right] \\ V \text{ units } & \left[\frac{\text{length}}{\text{time}}\right] \\ A \text{ units } & \left[\text{length}^2\right] \\ \rho \text{ units } & \left[\frac{\text{mass}}{\text{length}^3}\right] \end{aligned}$$

we are looking for a dimensionless combination

$$\begin{aligned} \frac{F_D}{\rho} & \text{ units } \left[\frac{\text{length}^4}{\text{time}^2}\right] \\ \frac{F_D}{\rho V^2} & \text{ units } \left[\text{length}^2\right] \\ \frac{F_D}{\rho A V^2} & = \text{dimensionless} \\ F_D & = C_D \cdot \rho A V^2 \text{ where } C_D \text{ is dimensionless constant to be measured by experiment} \end{aligned}$$

V^2-law for drag