Homework 5

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October 2, 2013

Section 3.1 Problems 19,20

19

Solve the logistic equation $\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha y\left(1 - \frac{1}{K}y\right)$ by viewing it as a Bernoulli equation.

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}t} &= \alpha y - \frac{\alpha}{K}y^2 \\
\frac{\mathrm{d}y}{\mathrm{d}t} - \alpha y &= -\frac{\alpha}{K}y^2 \qquad & w &= \frac{1}{y} \\
-\frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}t} + \alpha\frac{1}{y} &= \frac{\alpha}{K} & \frac{\mathrm{d}w}{\mathrm{d}t} &= -\frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}t} \\
\frac{\mathrm{d}w}{\mathrm{d}t} + \alpha w &= \frac{}{K} & \mu(t) &= e^{\int \alpha\,\mathrm{d}t} \\
\mu(t)w &= \int \mu(t)\frac{\alpha}{K}\,\mathrm{d}t & \mu(t) &= e^{\alpha t} \\
e^{\alpha t}w &= \frac{1}{K}\int \alpha e^{\alpha t}\,\mathrm{d}t = \frac{e^{\alpha t} + C}{K} \\
\frac{1}{y} &= \frac{1 + Ce^{-\alpha t}}{K} & y &= \frac{K}{1 + Ce^{-\alpha t}}
\end{aligned}$$

20

What is the limmiting population, $\lim_{t\to\infty} y(t)$, of the United states population using the result obtained in Example 3.1.5?

$$\begin{aligned}
y(t) &= \frac{0.159}{0.00053 + 0.02947e^{-0.03t}} \\
\lim_{t\to\infty} y(t) &= \lim_{t\to\infty} \frac{0.159}{0.00053 + 0.02947e^{-0.03t}} \\
\lim_{t\to\infty} y(t) &= \frac{0.159}{0.00053 + 0.02947e^{-0.03\infty}} \\
\lim_{t\to\infty} y(t) &= \frac{0.159}{0.00053 + 0.02947e^{-\infty}} \\
\lim_{t\to\infty} y(t) &= \frac{0.159}{0.00053 + 0.02947\frac{1}{e^{\infty}}} \\
\lim_{t\to\infty} y(t) &= \frac{0.159}{0.00053 + 0.02947 \cdot 0} \\
\lim_{t\to\infty} y(t) &= 300
\end{aligned}$$

Section 3.2 Problems 7,8

Use equation $T = (T_0 - T_s)e^{kt} + T_s$

7

A thermometer that reads 90°F is placed in a room with temperature 70°F. After 3 min, the thermometer reads 80°F. What does the thermometer read after 5 min?

$$\begin{aligned} T_s &= 70 \\ T_0 &= 90 \\ T(3) &= 80 = (90-70)e^{3k} + 70 \\ \frac{10}{20} &= e^{3k} \\ \ln\frac{1}{2} &= 3k \\ k &= \frac{1}{3}\ln\frac{1}{2} \\ T(5) &= 20e^{\frac{5}{3}\ln\frac{1}{2}} + 70 \approx 76.3°\text{F} \end{aligned}$$

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A thermometer is placed outdoors with temperature 80°F. After 2 min, the thermometer reads 68°F, and after 5 min, it reads 72°F. What was the initial temperature reading of the thermometer?

$$\begin{aligned} T_s &= 80 \\ T(2) &= 68 \\ T(5) &= 72 \\ T(t) - T_s &= (T_0 - T_s)e^{kt} \\ \frac{T(t) - T_s}{e^{kt}} &= T_0 - T_s \\ \frac{T(t) - T_s}{e^{kt}} + T_s &= T_0 \\ \frac{T(2) - 80}{e^{2k}} &= \frac{T(5) - 80}{e^{5k}} = \frac{T(5) - 80}{e^{2k}e^{3k}} \\ 68 - 80 &= \frac{72 - 80}{e^{3k}} \\ e^{3k} &= \frac{-8}{-12} \\ k &= \frac{1}{3}\ln\frac{2}{3} \\ T_0 &= \frac{T(2) - 80}{e^{\frac{2}{3}\ln\frac{2}{3}}} + 80 = \frac{68 - 80}{(2/3)^{2/3}} + 80 = -\frac{12}{(2/3)^{2/3}} + 80 \\ T_0 &\approx 64.3°\text{F} \end{aligned}$$