Notes

September 5, 2014

proposition 2.5.1

last time we got to $|a_nb_n - AB| \leq ... \leq |a_n||b_n - B| + |a_n - A||B|$, where $|b_n - B| = |a_n - A| = \epsilon$. $L - 1 \leq a_n \leq L + 1$.

let $\{a_n\}_{n=1}^{\infty}$ be a convergent sequence of real numbers, them $\{a_n : n \in \mathbb{N}\}$ is bounded above and below.

proof

let $L = \lim_{n\to\infty} a_n$. set $\epsilon = 1$ then there is a $N_1 \in \mathbb{N}$ such that $|a_n - L| < 1$ if $n \geq N_1$. Hence for $n = N_1, N_1 + 1, N_1 + 2, ...$ etc $L - 1 \leq a_n \leq L + 1$. N_1 is a fixed natural set. $B = \{a_z, a_2, ..., a_{N_1-1}\}$. let $M = \max B, m = \min B$. $\forall n \geq 1, \min\{L - 1, m\} \leq a_n \leq \max\{L + 1, M\}$.

example 2.4Ac

$$\begin{aligned}
0 \leq \lim_{n\to\infty} \frac{3^n}{n!} &= \lim_{n\to\infty} \frac{3 \cdot 3 \cdot 3 \cdot ... \cdot 3 \cdot 3 \cdot 3}{n(n-1)(n-2)...4 \cdot 3 \cdot 2 \cdot 1} \\
&= \lim_{n\to\infty} \frac{3 \cdot 3 \cdot 3 \cdot ... \cdot 3 \cdot 3 \cdot 3 \cdot 27}{n(n-1)(n-2)...4 \cdot (3 \cdot 2 \cdot 1)} \\
&\leq \lim_{n\to\infty} \frac{3}{n} \cdot 1 \cdot ... \cdot 1 \cdot \frac{9}{2}
\end{aligned}$$

so by squeeze it's zero

excercise

if $a_n \leq b_n \forall n$ then $\lim_{n\to\infty} a_n \leq \lim_{n\to\infty} b_n$