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Chapter 3

4.

Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, 2n\}$, then there are always two which differ by 1.

proof

Since we want all the integers to differ by more than one, we can only pickevery other integer from $\{1, 2, \ldots, 2n\}$. This gives us a maximum of n integers. Since we are choosing $n+1$ integers, we know that at least two of them must differ by only one.$\square$

5.

Show that if $n+1$ distinct integers are chosen from the set $\{1, 2, \ldots, 3n\}$, then there are always two which differ by at most 2.

proof

Since we want all the integers to differ by more than two, we can only pickevery third integer from $\{1, 2, \ldots, 3n\}$. This gives us a maximum of n integers. Since we are choosing $n+1$ integers, we know that at least two of them must differ by two or less.$\square$

6.

Generalize Exercises 4 and 5.

hypothesis

If $n+1$ distinct integers are chosen from the set $\{1, 2, \ldots, mn\}$ where m is a positive integer then there are always two which differ by at most $m-1$.

proof

We can select at most n integers which have a difference of m or more. Since we are selecting $n+1$ integers then we must have at least two which differ by $m-1$ or less.$\square$

8.

Use the pigeonhole principle to prove that the decimal expansion of a rational number m/n eventually is repeating. For example,

$$\frac{34,478}{99,900} = 0.345125125125\cdots.$$

12.

Show by example that the conclusion of the Chinese rmainder theorem (Application 6) need not hold when m and n are not relatively prime.

Take 3 and 9 for m and n. Take 2 and 4 for a and b. Then we should be able to find an x such that:

$$\begin{aligned} x &= 3p + 2 \\ x &= 9q + 4 \\ 3p + 2 &= 9q + 4 \\ 3p &= 9q + 2 \\ 3 &\mid 3p \\ 3 &\nmid 9q + 2 \\ 3p &\neq 9q + 2 \end{aligned}$$

So we see that x does not exist