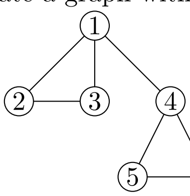
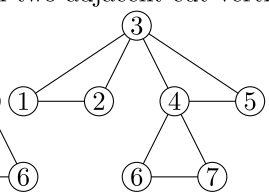
Notes

January 26, 2015

2.2 trees

task:

create a graph with two adjacent cut vertices (connected simple)

notice that removing a cut vertex on the second graph leaves a cut vertex, but not on the first graph.

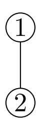
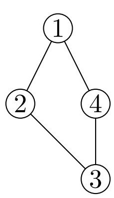
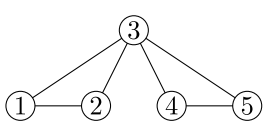
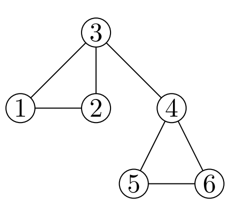
if two cut vertices in a graph G are adjacent and the edge between them is the only trail between them, then that edge is a **cut edge**

that is to say, every path from cut vertex one to cut vertex 2 contains the edge between them.

a **bridge** is a cut edge.

an edge e in G is a bridge if $k(G) < k(G - e)$.

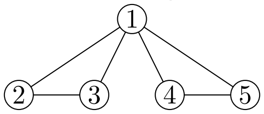
example

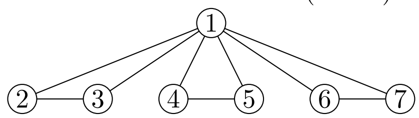
1. 
2. 
3. 
4. 

1 and 4 contain cut edges.

task

in row i create a graph with a cut vertex v such that $k(G-v)=i+1$





notice radial symmetry

question

if e is a bridgeof G what values of n can satisfy $k(G-e)=n+1$

1. even n
2. odd n
3. any n
4. 1

last option is correct (G is connected)

obviously this is different from removing a vertex.

tree is a connected graph, all of whose edges are bridges.

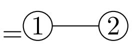
Note: if "connected" is removed, but every edge is still a bridge, then you have a forest.

Note: if a graph is not simple, then you necessarily have an edge that isn't a bridge, and so trees are simple graphs

theorem

if T is a tree and the order of T is n then the size of T is $n-1$

proof

case 1: $T=$ 

$|T|=2, |E(T)|=1=2-1$

now assume it is true for $|T|=k$

Let T be a tree of order $k+1$. since every edge is a bridge, remove any edge e to disconnect the graph

Now we have trees T_1, T_2 such that $T_1 \cup T_2 \cup \{e\} = T$. if $|T_1|=a$ then $|T_2|=k+1-a$

by the inductive hypothesis $|E(T_1)|=a-1$ and $|E(T_2)|=k+1-a-1=k-a$ therefore $|E(T)|=|E(T_1)|+|E(T_2)|+1=a-1+k-a+1=k$

theorem

the converse is also true.

a graph G of order n is a tree iff the size is $n-1$

a graph G is a tree iff G has no cycles

homework

2.2 numbers 1,2,3,10,14,17