Notes

December 1, 2014

note on homework

differentiable with bounded f' means lipschitz and differentiable

lipschitz mean continuous and differentiable means continuous

continuous means continuous at x_0 and differentiable at x_0 means continous at x_0

analytic(c^ω)$\to c^\infty \to \cdots \to c^{m+1} \to c^m \to \cdots \to c^2 \to c^1 \to$ differentiable

riemann integrable

$f : [a, b] \to \mathbb{R}$ bounded. then f is Riemann integrable if $\sup L(f, p) = \inf U(f, p) = \int_a^b f(x)\, \mathrm{d}x$

reimann condition

if f is riemann integrable $\Leftrightarrow \forall \varepsilon > 0 \exists P_\varepsilon$ such that $U(f, P_\varepsilon - L(f, P_\varepsilon) < \varepsilon$.

6.3.6 theorem

not covered, just variations on the riemann condition

examples

which are integrable, which are not?

$$f(x) = \chi_{\mathbb{Q}_n[0,1]} = \begin{cases} 1 & x \in \mathbb{Q}_{n[0,1]} \\ 0 & \text{otherwise} \end{cases}$$

because any interval contains a rational then inf is 1 and since any interval has an irrational then sup is 0.

there are other ways to do integrals such at riemann-stieltjes, lebesque, denjoy,feynmann

thm 6.3.7

if f is monotone on $[a, b]$ then f is integrable on $[a, b]$

thm 6.3.8

any continous function on $[a, b]$ is riemann integrable on $[a, b]$

proof

function is compact, and so uniformally continuous. definition of unif cont.

choose a partition p_ε such that $\delta_i < r$ (r =radius of continuity ball) $\forall i = 0, \ldots, n-1$ then on each $[x_i, x_{i+1}]$ we have $\max f(x) - \min f(x) < \varepsilon$ and so back to book proof

fundamental thm of calculus

if f is riemann integrable on$[a, b]$ then $F(x) = \int_a^x f(x)\,\mathrm{d}x$ is continuous. when f is continuous at x_0 then F is differentiable at x_0 and $F'(x_0) = f(x_0)$