Notes

February 26, 2014

lesson 13

laplace transform for $ft)$ on $0 \leq t < \infty$

$$\mathcal{L}\{f\} = F(x) = \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$$

inverse transform

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) e^{st}\,\mathrm{d}s$$

note $F(s)$ will typically be defined on $s \geq s_0$ (as described in introductory courses). $F(x)$ is analytic on $\mathrm{Re}(s_s) \geq s_0$ on half-planes in $\mathbb{C}$.

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$$\begin{array}{rccc} PDE & u_t = u_{xx} & 0 \leq x < \infty, & 0 < t < \infty \\ BC & u_x(0,t) - u(0,t) = 0 & & 0 < t < \infty \\ IC & u(x,0) = u_0 & & \end{array}$$

u_x is temperature gradient. $u_x = u$. $-cu_x$ is heat flow (in positive direction). when $u > 0$ heat flows out of the rod and if $u < 0$ then heat is flowing into rod. if the BC had a $+$ instead of a $-$ we would have an unstable condition where more heat means the heat increases at a greater rate and boom. extra credit for this neh?

$$U(x,s) = \mathcal{L}\{u(x,t)\}$$
$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\begin{array}{rc} PDE & sU(x,s) - u(x,0) = U_{xx}(x,s) \\ & \dfrac{\mathrm{d}^2U}{\mathrm{d}x^2} - sU = -u_0 \\ BC & U_x(0,s) - U(0,s) = 0 \end{array}$$

solution of DE. Can assume s is positive (and large)

$$U(x,s) = c_1 \text{ etc from pg 102}$$