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HW 01

$$u_t = \alpha^2 u_{xx} + 1 \qquad 0 < x < 1$$
$$u(0,t) = 0 \qquad u(1,t) = 1$$

Temperature is steady when it doesn't change with time

$$u_t = 0 \qquad 0 = \alpha^2 u_{xx} + 1$$
$$\int 0 \, \mathrm{d}x = \int \alpha^2 u_{xx} + 1 \, \mathrm{d}x \qquad c_1 = \alpha^2 u_x + x$$
$$\int c_1 \, \mathrm{d}x = \int \alpha^2 u_x + x \, \mathrm{d}x \qquad c_1 x = \alpha^2 U(x) + \frac{x^2}{2} + c_2$$
$$U(x) = -\frac{x^2}{2\alpha^2} + \frac{c_1}{\alpha^2}x - \frac{c_2}{\alpha^2} \qquad U(0) = 0 = -\frac{c_2}{\alpha^2}$$
$$U(1) = 1 = -\frac{1}{2\alpha^2} + \frac{c_1}{\alpha^2} \qquad \alpha^2 = c_1 - \frac{1}{2}, \quad c_1 > \frac{1}{2}$$
$$U(x) = -\frac{x^2}{2(c_1 - \frac{1}{2})} + \frac{c_1}{c_1 - \frac{1}{2}}x \qquad U(x) = -x^2\frac{1}{2c_1 - 1} + \frac{c_1 - \frac{1}{2} + \frac{1}{2}}{c_1 - \frac{1}{2}}x$$
$$U(x) = -x^2\frac{1}{2c_1 - 1} + x\left(1 + \frac{1}{2c_1 - 1}\right) \qquad U(x) = -c_3x^2 + c_3x + x, \quad c_3 > 0$$

Intuitively we know that the conservation condition won't hold. Because there is a heat source, the total energy in the system isn't constant. But let's verify.

$$U'(x) = -2c_3x + c_3 + 1$$
$$U'(0) = c_3 + 1$$
$$U'(L) = U'(1) = -2c_3 + c_3 + 1 = -c_3 + 1$$
$$U'(0) = U'(L)$$
$$c_3 + 1 = -c_3 + 1$$
$$c_3 \neq -c_3$$

Because $c_3 > 0$ we see that $U'(0)$ is never equal to $U'(L)$