Notes

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fundamental theorem of calculus

we step up one degree of regularity with integration. non-continuous-¿continuous, or continuous-¿differentiable

example

$f(x) = \begin{cases} 1 & x \in [0,1) \\ 2 & x \in [1,2] \\ 0 & \text{otherwise} \end{cases}$

$$\begin{aligned} F(x) = \int_0^x f(t)\,\mathrm{d}t &= \begin{cases} x \in [0.1) & \int_0^x 1\,\mathrm{d}t = x \\ x = 1 & \int_0^1 1\,\mathrm{d}t = 1 \\ 1 \leq x \leq 2 & \int_0^1 1\,\mathrm{d}t + \int_1^x 2\,\mathrm{d}t = 1 + (2x) - 2 = 2x - 1 \\ 2 \leq x & \int_0^1 1 + \int_1^2 2 = 3 \end{cases} \\ &= \begin{cases} 0 & x \leq 0 \\ x & x \in [0.1] \\ 2x - 1 & x \in [1,2] \\ 3 & x \geq 2 \end{cases} \end{aligned}$$

and this is not differentiable at 1

FTC 2

integration by parts

reverse of the product rule

$$(fg)' = f'g + fg'$$

by FTC

$$\begin{aligned} F(b)G(b) - F(a)G(a) &= \int_a^b (fg)'(x)\,\mathrm{d}x \\ &= \int_a^b f'g(x)\,\mathrm{d}x + \int_a^b fg'(x)\,\mathrm{d}x \end{aligned}$$

u-substitution, change of variable

reverse of chain rule

8.1 sequences of functions

difference between pointwise and uniform limit is that in pointwise limit you fix $x = x_0$ and in uniform convergence, the entire function converges

if $f_n \to f$ uniformly on $[a, b]$ then $f_n \to f$ pointwise on $[a, b]$

uniform convergence is a stronger condition

8.1.A

if we fix x_0 then pointwise limit is 0 so we assume that $x_0 \neq 0$. exponentials always win so $x_0 ne^{-nx_0} = 0$ and so the pointwise limit is 0.

do all the x get to zero at the same rate?

what is the maximum? $f_n'(x) = ne^{-nx} + xne^{-nx}(-n) = 0$ and $x = \frac{1}{n}$ is critical point.

critical point is a maximum

$f_n(\frac{1}{n}) = e^{-1}$

so max height does not depend on n

if $\varepsilon < \frac{e^-1}{2}$ then $\sup_{x \geq 0} |f_n(x) - f(x)| \not< \varepsilon$

8.2 and 8.3

uniform convergence and continuity

if $f_n(x) \to f(x)$ uniformally and $f_n(x)$ are all continuous then $f(x)$ is continuous

uniform convergence preserves continuity. pointwise convergence does not

inuform convergence and integrals

if $f_n : [a, b] \to \mathbb{R}$ are Riemann integrable and converge uniformally to $f : [a, b] \to \mathbb{R}$ then f is riemann integrable and integral of limit of $f_(x)$ is integral of $f(x)$

uniform continuity preserves integrals

pointwise continuity does not preserve integrals

the L^∞ norm

$f : [a, b] \to \mathbb{R}$ is defined as $\sup_x \in [a, b]|f(x)| = ||f||_\infty$

$f_n \to f$ uniformally $\Leftrightarrow ||f_n - f||_\infty \to_{n\to\infty} 0$