separable

$$g(y)\,\mathrm{d}y = f(t)\,\mathrm{d}t \qquad g(y)y' = f(t) \qquad \int g(y)\,\mathrm{d}y = \int f(t)\,\mathrm{d}t$$

first order linear

$$\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y = q(t) \qquad \mu(t) = e^{\int p(t)\,\mathrm{d}t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mu(t)y\right) = \mu(t)\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)\mu(t)y \qquad \mu(t)y = \int \mu(t)q(t)\,\mathrm{d}t$$

exact

$$M(t,y)\,\mathrm{d}t + N(t,y)\,\mathrm{d}y = 0 \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\int M(t,y)\,\mathrm{d}t + \phi(y) = f(t,y) \qquad \phi'(y) = N(x,y) - \frac{\mathrm{d}}{\mathrm{d}y}\left(\int M(t,y)\,\mathrm{d}t\right)$$

$$\int M(t,y)\,\mathrm{d}t + \int \phi'(y)\,\mathrm{d}y = f(t,y)$$

Solution is $f(t,y) = C$

bernoulli

$$\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y = q(t)y^n \qquad \frac{1}{y^n}\frac{\mathrm{d}y}{\mathrm{d}t} + p(t)y^{1-n} = q(t)$$

$$w = y^{1-n} \qquad \frac{\mathrm{d}w}{\mathrm{d}t} = (1-n)\frac{1}{y^n}\frac{\mathrm{d}y}{\mathrm{d}t}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} + (1-n)p(t)w = (1-n)q(t)$$

Solve as first order linear, then back substitute

homogeneous

$$M(t,y)\,\mathrm{d}t + N(t,y)\,\mathrm{d}y = 0 \qquad M(xt,xy) + N(xt,xy) = x^n\left(M(t,y) + N(t,y)\right)$$

$$\mathrm{d}y = w\,\mathrm{d}t + t\,\mathrm{d}w \qquad \mathrm{d}t = w\,\mathrm{d}y + y\,\mathrm{d}w$$

Substitute with $y = wt$ if $N(t,y)$ is simpler and $t = wy$ if $M(t,y)$ is simpler. Solve as a separable equation

population

growth and decay

$$y(t) = y_0 e^{kt}$$

Logistical equation

$$y = \frac{ry_0}{ay_0 + (r - ay_0)e^{-rt}}$$

Newton's law of cooling

$$T(t) = (T_0 - T_s)\, e^{kt} + T_s$$

Newton's laws of motion

Acceleration is $a = g = 9.8\text{m/sec}^2 = 32\text{ft/sec}^2$ and position is s and velocity is v.

$$\begin{aligned} v &= v_0 + at \\ s &= v_0 t + \frac{1}{2} a t^2 \\ v^2 &= {v_0}^2 + 2as \end{aligned}$$

trigonometric identities

$$\begin{array}{ccc} \sin x = \dfrac{1}{\csc x} & \sin(-x) = -\sin x & \cos(-x) = \cos x \\ \sin(u \pm v) = \sin u \cos v \pm \cos u \sin v & \cos(u \pm v) = \cos u \cos v \mp \sin u \sin v & \sin 2u = 2 \sin u \cos u \\ \cos 2u = \cos^2 u - \sin^2 u & \cos 2u = 2\cos^2 u - 1 & \cos 2u = 1 - 2\sin^2 u \\ \sin^2 u = \dfrac{1 - \cos 2u}{2} & \cos^2 u = \dfrac{1 + \cos 2u}{2} & \end{array}$$

$$\begin{array}{cc} \sin u \pm \sin(v) = 2 \sin\left(\dfrac{u \pm v}{2}\right) \cos\left(\dfrac{u \mp v}{2}\right) & \cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right) \\ \cos u - \cos v = -2 \sin\left(\dfrac{u+v}{2}\right) \sin\left(\dfrac{u-v}{2}\right) & \sin u \sin v = \dfrac{1}{2}[\cos(u-v) - \cos(u+v)] \\ \cos u \cos v = \dfrac{1}{2}[\cos(u-v) + \cos(u+v)] & \sin u \cos v = \dfrac{1}{2}[\sin(u+v) + \sin(u-v)] \end{array}$$

integration rules

$$\int e^{au} \sin(bu)\, \mathrm{d}u = e^{au} \frac{a \sin(bu) - b\cos(bu)}{b^2 + a^2} \qquad \int e^{au} \cos(bu)\, \mathrm{d}u = e^{au} \frac{b \sin(bu) + a\cos(bu)}{b^2 + a^2}$$