Homework 1

Jon Allen

September 12, 2014

2.6

F. Let a, b be positive real numbers. Set $x_0 = a$ and $x_{n+1} = ({x_n}^{-1} + b)^{-1}$ for $n \geq 0$.

(a) Prove that x_n is monotone decreasing.

(b) Prove that the limit exists and find it.

G. Let $a_n = (\sum_{k=1}^{n} 1/k) - \log n$ for $n \geq 1$. **Euler's constant** is defined as $\gamma = \lim\limits_{n\to\infty} a_n$. Show that $(a_n)_{n=1}^{\infty}$ is decreasing and bounded below by zero, and so this limit exists. HINT: Prove that $1/(n+1) \leq \log(n+1) - \log n \leq 1/n$

M. Suppose that $(a_n)_{n=1}^{\infty}$ has $a_n > 0$ for all n. Show that $\limsup {a_n}^{-1} = (\liminf a_n)^{-1}$.