Notes

April 7, 2014

cores maps: to_partition and to_bounded_partition
questions about homework
generating functions for $h_0, h_1, h_2, \ldots$ is $h(x) = h_0 + h_1 x + h_2 x^2 + h_3 x^3 + \ldots$

last time

$h_n =\#$ of combinations of e_1, e_2, e_3, e_4 with infinite repetition. solution $e_1 + e_2 + e_3 + e_4 = n$

$$\underbrace{(1+x+x^2+\ldots)}_{x^k \text{ means } e_1=k}(1+x+x^2+\ldots)(1+x+x^2+\ldots)(1+x+x^2+\ldots) = \tfrac{1}{(1-x)^4} = \sum_{n=0}^{\infty}()$$

example

what is the gf for sol'ns to $e_1 + e_2 + e_3 + e_4 = n$ where $e_1 \geq 2, e_2$ is even, $4 \leq e_3 \leq 7, e_4$ is positive.

$$\begin{aligned}
\text{gf} &= (x^2+x^3+\ldots)(1+x^2+x^4+\ldots)(x^4+x^5+x^6+x^7)(x+x^2+\ldots) \\
&= x^2(1+x+x^2+\ldots)(1+(x^2)+(x^2)^2+\ldots)x^4(1+x+x^2+x^3)x(1+x+x^2\ldots) \\
&= x^7\frac{1}{(1-x)^2}\frac{1}{1-x^2}\frac{1-x^4}{1-x} = \frac{x^7(1-x^4)}{(1-x)^3(1-x^2)}
\end{aligned}$$

14. (a)

$$(x+x^3+x^5+\ldots)^4 = x^4(1+x^2+(x^2)^2+\ldots)^4 = \frac{x^4}{(1-x^2}^4$$

(b)

$$(1+x^3+x^6+\ldots)^4 = \frac{1}{(1-x^3)^4}$$

new generating functions from old ones
start with $\frac{1}{1-x} = 1 + x + x^2 + \ldots$. Take derivative= $\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \ldots$. So we have the generating function for $g_n = n + 1$. Now we multiply both sides by x to get $\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \ldots$ which is the generating function for $h_n = n$

generating functions on findstat

look at statistics database and at the number of inversions of inversions. an inversion is when a object in the permutation is ahead of it's natural place. eg 321 has 3 inversions.

object we can write generating function for that we can't count

let P_n be the number of integer partitions of n, that is the number of ways to write n as a sum of decreasing positive integers. eg partitions of 4:

$$\begin{aligned} 4 \\ 3+1 \\ 2+2 \\ 2+1+1 \\ 1+1+1+1 \end{aligned}$$

so $P_4 = 5, P_1 = 1, P_2 = 2, P_3 = 3$

what is a formula for this? don't know (at least not a closed formula)

what is a generating function for P_n

$$\begin{aligned} \text{g.f.} &= \underbrace{(1 + x^1 + x^2 + \dots)}_{\#1\text{'s}} \underbrace{(1 + x^3 + x^6 + x^9 + \dots)}_{\#3\text{'s}} \dots \\ &= \sum_{n=0}^{\infty} P_n x^n = \frac{1}{1-x} \frac{1}{1-x^2} \frac{1}{1-x^3} \cdots \end{aligned}$$

section 7.3

for a sequence $h_0, h_1, h_2, \dots$ the exponential generating function is the infinite series $h_0 + h_1 x + h_2 \frac{x^2}{2!} + h_3 \frac{x^3}{3!} + h_4 \frac{x^4}{4!} + \dots = \sum_{n=0}^{\infty} h_n \frac{x^n}{n!}$

example

the gf for 1,1,1,... $1 + x + x^2/2! + x^3/3! + \dots = e^x$

example

$1, a, a^2, \dots \to e^{ax}$

example

find the exp gf for the k-permutations of [n]: p(n,0),p(n,1),$\ldots = \frac{n!}{(n-k)!}$ it's $(1 + x)^n$