Notes

March 3, 2014

$$L = \text{length}$$
$$\alpha^2 = \text{diffusion}$$
$$= \left[\frac{\text{cm}^2}{\text{sec}}\right]$$

homework, in #20 we are deriving when change of variables will and won't work. #21 is deriving a little more complicated version for the remaining cases.

oh snap the equation i couldn't read last time was the gamma function. $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}$. We understan $n!$ what is $x!$

$$n! = \int_0^\infty e^{-t}t^{x-1}\,\mathrm{d}t$$
$$x! = \int_0^\infty e^{-t}t^{x}\,\mathrm{d}t,\ x > -1$$
$$\Gamma(n+1) = n!$$
$$\Gamma\left(\frac{1}{2}\right) = \pi^{1/2}$$
$$\Gamma(x+1) = x\Gamma(x)$$
$$t^p \to \frac{\Gamma(p+1)}{s^{p+1}}$$

okay on to dlmf.nist.gov handbook of mathematical functions? written/editted by by abramowitz and stegan. part of a government project to get a standardized reference of mathematical functions. update to it is NIST Handbook of Mathematical Functions. Which is precisely this web site. *THE* reference for special functions. reference we want is 5.2 and 5.3, famma function. 5.12 beta function is relevant to homework exercises.

lesson 13

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obtained $U(x,s) = u_0\left(\frac{1}{s} - \frac{\sqrt{s}}{s(\sqrt{s}+1)}e^{-x\sqrt{s}}\right)$. Text claims that $u(x,t) = u_0(1 - (\text{erfc}(\frac{x}{2\sqrt{2}}) - \text{erfc}(\sqrt{t} + \frac{x}{2\sqrt{t}})e^{x+t})$. The text tables are not adequate. Mathematica does not handle it.

note:

$$\frac{\sqrt{s}}{s(\sqrt{s}+1)}e^{-a\sqrt{s}} \qquad (a > 0)$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{s}(\sqrt{s}+1)} e^{-a\sqrt{s}} \\
&= (\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{s}+1}) e^{-a\sqrt{s}}
\end{aligned}
$$

case 1

$$F(s) = \frac{1}{\sqrt{s}} e^{-\sqrt{s}} \to f(t)? \text{ in table}$$

approach

$$G(s) = e^{-\sqrt{s}} \to g(t)$$

note

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}s}(e^{-\sqrt{s}}) &= e^{-\sqrt{s}} \cdot -\frac{1}{2}\frac{1}{\sqrt{s}} = -\frac{1}{2}F(s) \\
\frac{\mathrm{d}}{\mathrm{d}s}(e^{-\sqrt{s}}) &= \frac{\mathrm{d}}{\mathrm{d}s}(G(s)) \to -tg(t) = -\frac{1}{2}f(t) \\
G'(s) &= -\frac{1}{2}s^{-1/2}e^{-\sqrt{s}} \to -tg(t)
\end{aligned}
$$

$$\vdots$$

$$
\begin{aligned}
4sG''(s) + 2G'(s) - G(s) &= 0 \\
4sG''(s) - t^2 g(t) \mid_{t=0} + 2G'(s) - G(s) &= 0 \\
4\frac{\mathrm{d}}{\mathrm{d}t}\left[t^2 g(t)\right] + 2(-tg(t) - g(t) &= 0 \\
4t^2\frac{\mathrm{d}g}{\mathrm{d}t} + 6tg(t) - g(t) &= 0 \\
\frac{\mathrm{d}g}{\mathrm{d}t} + \left(\frac{3}{2t} - \frac{1}{4t^2}\right) g &= 0
\end{aligned}
$$

integrating factor

$$\mu = t^{3/2} \cdot e^{1/4t}$$