Notes

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weierstrass m-test

if $f_k : S \to \mathbb{R}^n$ and for all k there is M_k with $||f_k||_\infty \leq M_k$ then if $\sum M_k$ converges then the series $\sum f_n$ converges uniformly.

that is to say, if we can bound f_n by M_n and the series of M_n converges, then f_n converges.

example

if $\sum_{n=1}^{\infty} |a_n| < \infty$ then $\sum_{n=1}^{\infty} a_n \cos(nx)$ converges uniformly on $\mathbb{R}$.

$|a_n \cos(nx)| \leq |a_n|$ for any x. $a_n = M_n \ldots \square$

note that $\sum a_n \cos(nx)$ is continuous because $a_n \cos(nx)$ is continuous and $\sum a_n \cos(nx)$ converges uniformly.

theorem

if $f_n(x)$ is continuous and $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly to $F(x)$ then $F(x)$ is continuous (the sums are continuous if f_n is continuous)

example

$\sum_{n=1}^{\infty} x^n e^{-nx}$ converges uniformly on $[0, A]$ for any $A > 0$.

$|x^n e^{-nx}| \leq |x^n| \leq A^n$. $f'(x) = (nx^{n-1}e^{-nx}) - nx^n e^{-nx} = ne^{-nx}x^{n-1}(1-x) = 0$ so critical points are $0, 1, A$.

f'' shows us that 1 is concave down so it is a max. and so $x^n e^{-nx} \leq e^{-n}$ on $[0, \infty)$.

so $\sum e^{-n} = \sum \left(\frac{1}{e}\right)^n$ which is geometric and so it converges to $\left(\frac{1}{1-\frac{1}{e}}\right) - 1$ by m-test we converge uniformly on $[0, \infty)$