$$\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta$$

let $n \neq q$

$$\begin{aligned}
\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta &= \int_0^\pi \sin(n\theta)\sin(q\theta)\left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\mathrm{d}\theta \\
&= \left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\int_0^\pi \sin(n\theta)\sin(q\theta)\,\mathrm{d}\theta \\
&= \left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\left(-\frac{(q-n)\sin(q\pi+n\pi)+(-q-n)\sin(q\pi-n\pi)}{2q^2-2n^2}\right) \\
&= \left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\cdot 0 \\
&= 0
\end{aligned}$$

now let $(n,m)=(q,p)$

$$\begin{aligned}
\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta &= \left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\int_0^\pi \sin(n\theta)\sin(q\theta)\,\mathrm{d}\theta \\
&= \left[\int_0^1 J_n(k_{n,m}r)^2 r\,\mathrm{d}r\right]\int_0^\pi \sin(n\theta)^2\,\mathrm{d}\theta \\
&= -\frac{\sin(2\pi n)-2\pi n}{4n}\left[\int_0^1 J_n(k_{n,m}r)^2 r\,\mathrm{d}r\right] \\
&= \frac{\pi}{2}\left[\int_0^1 J_n(k_{n,m}r)^2 r\,\mathrm{d}r\right] \\
&= \frac{\pi}{2}\left[\frac{1}{2}(J_n'(k_{n,m}))^2\right] = \frac{\pi}{4}J_n'(k_{n,m})^2 \quad \text{formula from class}
\end{aligned}$$

let $n=q$ and $m \neq p$

$$\begin{aligned}
\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta &= \left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]\int_0^\pi \sin(n\theta)^2\,\mathrm{d}\theta \\
&= \frac{\pi}{2}\left[\int_0^1 J_n(k_{n,m}r)J_q(k_{q,pr)r\,\mathrm{d}r}\right]
\end{aligned}$$

we take as given that $k_{n,m} \neq k_{q,p}$ and use the formula from my notes

$$\begin{aligned}
a &= k_{n,m} \\
b &= k_{q,p} \\
\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta &= \frac{\pi}{2}\left[\int_0^1 J_n(ar)J_n(br)r\,\mathrm{d}r\right] \\
&= \frac{\pi}{2}\left[\frac{r}{b+a}\frac{1}{b-a}(aJ_n'(ar)J_n(br)-bJ_n(ar)J_n'(br))\right]_0^1 \\
&= \frac{\pi}{2}\left[\frac{1}{b+a}\frac{1}{b-a}(aJ_n'(a)J_n(b)-bJ_n(a)J_n'(b))\right. \\
&\qquad \left.-\frac{0}{b+a}\frac{1}{b-a}(aJ_n'(ar)J_n(br)-bJ_n(ar)J_n'(br))\right] \\
J_n(a) &= J_n(b) = 0
\end{aligned}$$

$$\int_0^\pi \int_0^1 E_{n,m}(r,\theta)E_{q,p}(r,\theta) r \mathrm{d}r \mathrm{d}\theta = 0$$