Notes

November 7, 2014

5.3.J

Let $m(x, y) = \max\{x, y\}$. prove that m is continuous and then show that $h(x) = \max\{f(x), g(x)\}$ is continuous if $f(x)$ and $g(x)$ are continuous.

case 1, $m(x, y) = x, m(x_1, y_1) = x_1$ then $|m(x, y) - m(x_1, y_1)| = |x - x_1|$. take $r = \varepsilon$ and then $||(x, y) - (x_1, y_1)|| < r$ then $|m(x, y) - m(x_1, y_1)| < \varepsilon$. similar with $m(x, y) = x$ and $m(x_1, y_1) = y_1$ is similar

now if $m(x, y) = x$ and $m(x_1, y_1) = y_1$ then $-\varepsilon < x - y_1 < \varepsilon$.

$$
\begin{aligned}
-r < x - x_1 < r \\
-r < y - y_1 < r \\
x > y \\
y_1 > x_1 \\
-y_1 < -x_1 \\
(x - y_1) < (x - x_1)
\end{aligned}
$$

So if $x - x_1 < \varepsilon$ then $(x - y_1) < \varepsilon$.

$$
\begin{aligned}
y - y1 < x - y1 \\
\text{if } y - y1 > -\varepsilon \\
\text{then } x - y1 > -\varepsilon
\end{aligned}
$$

so $r = \varepsilon$ will give us $|x - y1| < \varepsilon$

now let $\varepsilon \circ f(x) = \varepsilon_1(f(x) = (f(x), 0)$ and $\varepsilon_2 \circ g(x) = \varepsilon_2(g(x) = (0, g(x))$. and $h(x) = m \circ (\varepsilon \circ f(x) + \varepsilon \circ g(x))$

because sum and composition of continuous functions are continuous, then h is continuous. induction gives us continuity for $\max\{f_1(x), \ldots, f_m(x)\}$

5.4H,I& 5.5F

let $f : \mathbb{R} \to \mathbb{R}$ be periodic and continuous, ie $\exists d > 0$ such that $\forall x \in \mathbb{R}, f(x + d) = f(x)$.

show it attains max min

look at $y \in [x, x + d]$ then $\exists a, b \in [x, x + d]$ such that $f(a) \leq f(y) \leq f(b)$. periodicity gives us same properties on all of $\mathbb{R}$

show that it is bounded and uniformally continuous

find a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x+1, y) = f(x, y) \forall (x, y)$ but f is not bounded or does not attain it's max