Notes

September 26, 2014

assignment

Read Section 3.2 (up to Theorem 3.2.10) Section 3.2: # 14, 16, 18, 19, 21.

last time

group G and subgroup H where $x \sim y \Leftrightarrow xy^{-1} \in H$, for $a \in G$ the

$$\begin{aligned}[a] &= \{x \in G | x \sim a\} \\ &= \{x \in G | xa^{-1} \in H\} \\ &= \{x \in G | x \in Ha\} \\ &= Ha\end{aligned}$$

Note that $[e] = H$.

claim

there is a one to one correspondence $H \xrightarrow{\varphi} Ha$ and $h \to ha$.

lagrange's theorem

let G be a finite group and H a subgroup of G. Then $|H|$ is a divisor of $|G|$.

proof

consider on G the previous equivalence relation. recall that equivalence classes partition G.

let t be the number of distinct equivalence classes. note that we also proved that any two equivalence classes have the same cardinality. then $t \cdot |H| = |G|$. so $|H|$ divides $|G|$

corollary

let G be finite and $a \in G$ then the order of a divides $|G|$. Take the smallest subgroup that contains a: $< a >= \{e, a, a^2, \ldots, a^{ord(a)-1}\}$. Then $|H| = ord(a)$ and divides $|G|$.

corollary

say that G is finite, and the number of elements in G is prime, then G is cyclic.

choose $a \in G, a \neq e$, then $| < a > |p$ and so $| < a > | = p$. Because $< a > \subseteq G$ and so $G =< a >$.

example

$\mathbb{Z}_n^* = \{[a] | (a,n) = 1\}$ and $|\mathbb{Z}_n^*| = \varphi(n)$

take $\mathbb{Z}_8^* = \{[1],[3],[5],[7]\} = \{e,a,b,ab\} = \{a^i b^j | a^2 = b^2 = e, ab = ba\}$. Note that $x^2 = [1]$ for all $x \in \mathbb{Z}_8^*$. so then every element has order 2 except identity which always has order 1.

also note that this is not cyclic as there is no element of order 4.

example

$S_3 = \{(1),(12),(23),(31),(123),(132)\}$

let $(12) = a, (123) = b$ then $(123)^2 = (132), ab = (123)(12) = (13), a^2b = (23)$ and $S_3 = \{e,a,a^2,ab,a^2b,b\}$

$S_3 = \{a^i b^j | a^3 = e, b^2 = e, ba = a^2 b\}$

proposition

let G be a group and $\{H_i\}_{i\in I}$ be a family of subgroups of G then $\cap_{i\in I} H_i$ is a subgroup

proof

exercise, write it down.

unions are different, take $S_3, H = \{(1),(12)\}, K = \{(1),(123),(132)\}$ then the union is not a subgroup. Note that $|H \cup G| = 4$ and $6 = |G|$ but $4|6$ is not true