PDE C.

PDE.	$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$	for	$0 < x < \infty,$	$0 < t < \infty$
BC.	$\dfrac{\partial u}{\partial x}(0,t) = u(0,t) - \dfrac{1}{\sqrt{\pi t}}$	for		$0 < t < \infty$
IC.	$u(x,0) = 0$	for	$0 < x < \infty$	

Solve PDE C completely by a Laplace transform with respect to t. Use the BC as stated – do not transform to homogeneous BC. (The necessary inverse Laplace transform is not in the textbook table but is on the handout list of transforms.)

$$\begin{aligned}
sU(x) - 0 &= \frac{\mathrm{d}^2U}{\mathrm{d}x^2}(x) \\
\frac{\mathrm{d}U}{\mathrm{d}x}(0) &= U(0) - \mathcal{L}\left\{\frac{1}{\sqrt{\pi t}}\right\} \\
&= U(0) - \frac{1}{\sqrt{s}} \qquad \text{used computer} \\
0 &= \frac{\mathrm{d}^2U}{\mathrm{d}x^2}(x) - sU(x) \\
0 &= r^2 + 0r - s \\
r &= \frac{\pm\sqrt{4s}}{2} = \pm\sqrt{s} \\
U(x) &= c_1e^{x\sqrt{s}} + c_2e^{-x\sqrt{s}} \\
U'(x) &= c_1\sqrt{s}e^{x\sqrt{s}} - c_2\sqrt{s}e^{-x\sqrt{s}} \\
U'(0) &= c_1\sqrt{s} - c_2\sqrt{s} = c_1 + c_2 - \frac{1}{\sqrt{s}} \\
c_1\sqrt{s} - c_1 &= c_2 + c_2\sqrt{s} - \frac{1}{\sqrt{s}}
\end{aligned}$$

used computer to help find convenient values

$$\begin{aligned}
c_1(\sqrt{s} - 1) &= \frac{1}{s + \sqrt{s}} + \frac{\sqrt{s}}{s + \sqrt{s}} - \frac{1}{\sqrt{s}} \\
c_1(\sqrt{s} - 1) &= \frac{\sqrt{s} + s}{\sqrt{s}(s + \sqrt{s})} - \frac{s + \sqrt{s}}{\sqrt{s}(s + \sqrt{s})} \\
c_1 &= 0 \\
c_2 &= \frac{1}{s + \sqrt{s}} \\
U(x) &= \frac{1}{s + \sqrt{s}}e^{-x\sqrt{s}}
\end{aligned}$$

from handout

$$u(x,t) = e^{x+t}\text{erfc}\left(\sqrt{t} + \frac{x}{2\sqrt{t}}\right)$$