Notes

September 17, 2014

2.7a

show that $(a_n) = \left(\frac{n\cos^n(n)}{\sqrt{n^2+2n}}\right)_{n=1}^{\infty}$ has a convergent sequence
$n/\sqrt{n^2+2n} < 1$ and $|\cos^n n| \leq 1$

2.7b

$$n + \cos(n\pi)\sqrt{n^+1}$$

bounded below because even terms are increasing, odd terms bounded by 0 and $1-\sqrt{2}$. odd subsequence is bounded, so there is a convergent subsequence

2.8

if a sequence is convergent to L then $\forall\varepsilon > 0 \exists N \in \mathbb{N}$ such that $|a_n - L| < \varepsilon$ if $n \geq N$
where cauchy sequence is $\forall\varepsilon > 0 \exists N \in \mathbb{N}$ such that $|a_n - a_m| < \varepsilon \forall n, m \geq N$

theorem

every convergent sequence is cauchy
every cauchy sequence of reals converges to a real
not every cauchy sequence of rationals converges to a rational, although it will converge to a real.
definition:
a subset S is said to be complete if every Cauchy sequence (a_n) in S (that is $a_n \in S$) converges to a point in S
reals are complete, rationals are not

completeness theorem

every cauchy sequence of real numbers converges. so $\mathbb{R}$ is complete

proof

let (a_n) be cauchy. then $\forall\varepsilon > 0 \exists N$ such that if $n, m \geq N$ then $|a_n - a_m| < \varepsilon$.
step 1
(a_n) is bounded.
given $\varepsilon = 1, \exists N_1$ succh that $|a_n - a_m| < 1 \forall n, m \geq N_1$. in particulare $|a_n - a_{N_1}| < \forall n \geq N_1$ so $-1 + a_{N_1} < a_n < 1 + a_{N_1}$ and $a_1, \ldots a_{N_1}$ is finite so it is bounded.

step 2
BW saays there is a subsequence (a_{n_k}) converging to $L \in \mathbb{R}$.
step 3
the whole sequence onverges to some L
$|a_n - L| = |a_n - a_{n_k} + a_{n_k} - L| \leq |a_n - a_{n_k}| + |a_{n_k} - L| < 2\varepsilon$

example

contractive sequences $|a_{n+1} - a_n| \leq \lambda \left| a_n - a_{n-1} \right|, \lambda \in (0, 1)$
limit of λ^n as n aproaches infinity is 0.
limit of $|a_{n+1} - a_n| = 0$, telescoping $|a_n - a_n| = |a_n - a_{m-1} + a_{m-1-\cdots-a_{n+1}a_{n+1}-a_n} \leq$