Notes

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f_n converges pointwise to f if $\lim f_n(x) = f(x)$ for all x in the domain of f_n
the L^∞-norm is $||f||_\infty = \sup_{x \in \mathrm{dom} f} |f(x)|$

lebesque

f_n converges to f uniformally if $\lim ||f_n - f||$

maybe in exam

if $f_n : [a, b] \to \mathbb{R}$ is continuous and $f_n \to f$ uniformally then f is continuous. counterexample is $f_n(x) = x^n$ if $x \in [0, 1]$ does not converge uniformally

Lebesque

uniform convergence and integrals

thrm 8.3.1

if$f_n \to f$ is uniform and f_n are Riemann integrablethen f is Riemann and $\lim \int_a^b f_n = \int_a^b f kk$

corollary 8.2.2

$C(K) = \{f : K \to \mathbb{R}$ that are continuous$\}$. $C(K)$ is a complete space with the uniform norm,ie every cauchy sequence f_n of functions in $C(K)$ converges to a function in $C(K)$.

example

let $f_n(x) = f(x) + \frac{1}{n}\sin(\frac{x}{n} \in [a, b]$ where $a \neq 0$ error gets smaller and smaller but sin function wildly varies. so $f_n \to f$ is uniform all are continuous. but thef_n' goes nuts. if we want $f_n \to f$ to converge and to preserve derivitives then we need to explicitly require $f_n' \to f'$ be uniform

8.4 series

we say a series of functions converges pointwise (or uniformally) ifthe partial sums converge pointwise (or uniformly)

power series

a special type of series is the power series. $\sum\limits_{n=0}^{\infty} a_n x^n$. these are easy to study because the ratio test works for them.

$\lim_{n\to\infty} \frac{|a_{n+1}x^{n+1}|}{|a_n x^n|} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| |x|$ and so it converges if $|x| < \frac{1}{\lim_{n\to\infty} \frac{a_{n+1}}{a_n}}$. check $\frac{x^n}{n!}$

weierstrass-m

power series

hadamard's thrm

given a power series $\sum a_n x^n$ there is R in $[, +\infty) \cup +\infty$ so that the series converges for all x with $|x| < R$ and diverges for all x with $|x| > R$. moreover the series converges uniformally oneach closedinterval $[a, b] \subset (-R, R)$

$R = \frac{1}{\lim_{n\to\infty} \sqrt[n]{|a_n|}}$ root test?