Notes

February 19, 2014

leftover

sine integral transform (evaluating integrals explicitly)

find:

$$I = \int_0^\infty \frac{\sin(x\omega}{\omega} e^{-\alpha^2 t\omega^2}\,\mathrm{d}\omega, \qquad x > 0$$

convert to

$$I = I(\beta) = \int_0^\infty \frac{\sin(\beta s}{s} e^{-s^2}\,\mathrm{d}s \qquad \text{write } \beta = \frac{x}{\alpha\sqrt{t}} > 0$$

note

$$I(\beta) = \int_0^\infty \frac{\sin(s)}{s} e^{-s^2/\beta^2}\,\mathrm{d}s \qquad \rightarrow \qquad 0 \text{ as } \beta \to 0$$

$$\rightarrow \qquad \frac{\pi}{2} \text{ as } \beta \to +\infty$$

end note

$$I'(\beta) = \frac{\mathrm{d}}{\mathrm{d}\beta}\int_0^\infty \cos(\beta s) e^{-s^2}\,\mathrm{d}s \qquad u = e^{-s^2} \qquad \mathrm{d}v = \cos(\beta s)\mathrm{d}s$$

$$\mathrm{d}u = -2se^{-s^2}\mathrm{d}s \qquad v = \frac{1}{\beta}\sin(\beta s)$$

$$\begin{aligned}
I'(\beta) &= e^{-s^2}\frac{1}{\beta}\sin(\beta s)\Big|_0^\infty - \int_0^\infty \frac{\sin(\beta s)}{\beta}(-2se^{-s^2})\,\mathrm{d}s \\
&= +\frac{2}{\beta}\int_0^\infty \sin(\beta s) s e^{-s^2}\,\mathrm{d}s \\
&= \frac{2}{\beta}\left(-I''(\beta)\right) \\
I''(\beta) &= \int_0^\infty \sin(\beta s) s e^{-s^2}\,\mathrm{d}s = -\frac{\beta}{2} I'(\beta) \\
I'(\beta) &= c_1 e^{-\beta^2/4} \\
I(\beta) &= c_2 - c_1\int_\beta^\infty e^{t^2/4}\,\mathrm{d}t \text{ note the integration starting at } \beta \\
&= c_2 - 0 \\
&= \frac{\pi}{2} - c_1\int_\beta^\infty e^{-t^2/4}\,\mathrm{d}t
\end{aligned}$$

$$I(0) = \frac{\pi}{2} - c_1 \int_0^\infty e^{-t^2/4}\,\mathrm{d}t$$

fact $\int_0^\infty e^{-x^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2}$

$$\frac{1}{2}\int_0^\infty e^{-t^4}\,\mathrm{d}t = \frac{\sqrt{\pi}}{2}$$

$$I(\beta) = \frac{\pi}{2} - \frac{\sqrt{\pi}}{2}\int_x^\infty e^{-x^2/4}\,\mathrm{d}x$$

note error function (erf)

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}\,\mathrm{d}t \to 1 \text{ as } x \to \infty$$
$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,\mathrm{d}t$$

graph on page 79

end note

$$x = 2t \qquad \mathrm{d}x = 2\mathrm{d}t$$
$$0 = \frac{\pi}{2} - c_2\sqrt{\pi} \qquad \frac{\sqrt{\pi}}{2} = c_1$$
$$\begin{aligned}
I(\beta) &= \frac{\pi}{2} - \frac{\sqrt{\pi}}{2}\int_{t/2\cdot 2t?}^\infty 2e^{-t^2}\,\mathrm{d}t \\
&= \frac{\pi}{2} - \frac{\sqrt{\pi}}{2}\cdot 2\cdot\frac{\sqrt{\pi}}{2}\left(\frac{2}{\sqrt{\pi}}\int_{2t}^\infty e^{-u^2}\,\mathrm{d}u\right) \\
&= \frac{\pi}{2} - \frac{\pi}{2}\mathrm{erfc}\left(\frac{\beta}{2}\right)
\end{aligned}$$

solution on p79

$$u(x,t) = A\mathrm{erfc}\left(\frac{x}{2\alpha\sqrt{t}}\right)$$

last homework problem (hw09)

we can do this without paying attention to formula's at all because the idea is so simple.

$$u_x(0,t) = 0 = f(t)$$
$$u_x(1,t) + hu(1,t) = 1 = g(t)$$

introduce $u(x,t) = \omega(x,t) +$ adjustment. This adjustment is chosen to obtain hetorgeneous boundary conditions ($f(t) = g(t) = 0$). Take adjustment to be $+a(t) + bt)x$ because original boundary values (0 and 1) lie on a line.

$$u = \omega + a(t) + b(t)x$$
$$\omega_x(0,t) + b(t) + 0 = f(t)$$
$$(\omega_x(1,t) + b(t)) + h(\omega(1,t) + a(t) + b(t)\cdot 1) = g(t)$$