Show that $\int_0^1 X_m(x)X_n(x)\,\mathrm{d}x = 0$ for ALL $m \neq n$ by integrating by parts. This calculation should make no explicit reference to trigonometric or hyperbolic functions.

$$\begin{aligned}
\frac{X''(x)}{X(x)} &= \lambda \\
X'(0) &= 0 \\
X'(1) - X(1) &= 0 \\
\int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= \int_0^1 X_m(x)\frac{1}{\lambda_n}{X_n}''(x)\,\mathrm{d}x \\
u = X_m(x) &\quad \mathrm{d}v = {X_n}''(x)\mathrm{d}x \\
\mathrm{d}u = {X_m}'(x)\mathrm{d}x &\quad v = {X_n}'(x) \\
\int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= \frac{1}{\lambda_n}\left[X_m(x){X_n}'(x) - \int {X_m}'(x){X_n}'(x)\,\mathrm{d}x\right]_0^1 \\
u = {X_m}'(x) &\quad \mathrm{d}v = {X_n}'(x)\mathrm{d}x \\
\mathrm{d}u = {X_m}''(x)\mathrm{d}x &\quad v = X_n(x) \\
\lambda_n \cdot \int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= \left[X_m(x){X_n}'(x) - {X_m}'(x)X_n(x) + \int {X_m}''(x)X_n(x)\,\mathrm{d}x\right]_0^1 \\
&= \left[X_m(x){X_n}'(x) - {X_m}'(x)X_n(x)\right]_0^1 + \int_0^1 \lambda_m X_m(x)X_n(x)\,\mathrm{d}x \\
(\lambda_n - \lambda_m) \cdot \int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= \left[X_m(1){X_n}'(1) - {X_m}'(1)X_n(1)\right] - \left[X_m(0){X_n}'(0) - {X_m}'(0)X_n(0)\right] \\
X'(1) - X(1) = 0 &\rightarrow X'(1) = X(1) \\
\int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= \frac{[X_m(1)X_n(1) - X_m(1)X_n(1)] - [X_m(0)\cdot 0 - 0 \cdot X_n(0)]}{\lambda_n - \lambda_m} \\
\int_0^1 X_m(x)X_n(x)\,\mathrm{d}x &= 0
\end{aligned}$$