Informe 12 Mayo

Delayed CBM

YGK

Undelayed CBM

Useful references followed for this section are Refs. [6, 1].

The objective is clear. What is the relation between SFH and N/O-O/H under the same mass? The simplest model for chemical enrichement has the following assumptions:

i) There is a homogeneous ISM, i.e, the chemical enrichement is produced instantaneously over all the medium.

ii) Closed boxed model. No inflow or outflox of gas.

iii) Stars yield their chemical components instantaneously only on their death.

iv) There is no delay between when a star that yields the element is born and it dies (*This assumption will be the one relaxed in the following section*)

Under these assumptions, the abundance evolution for any **primary production** with mass M_Z, whether it is oxygen, nitrogen, etc, is,

$$\dot{M}_Z(t) = p_Z \dot{M}_*(t) - Z(t)\dot{M}_*(t), \tag{1}$$

where the stellar mass, M_*, and the metal-to-hydrogen fraction $Z \equiv M_Z/M_g$ have been introduced. The former equation conveys both the star-produced yields, p_Z, and the amount of metal trapped in (specially) the newly formed low-mass stars, $Z\dot{M}_*(t)$. For our intent and purposes, p_Z is defined ad hoc, but it is just the integral of yielded metals integrated over the IMF.

In the previous equation, assumption i) has been used to encompass the whole ISM as a unique region and iii) to integrate p_Z to a single time-independent parameter. Now we make us of assumption ii) to relate the gas mass with the stellar mass $M_g(t) = M_g(0) - M_*(t)$, equivalently, $\dot{M}_g(t) = -\Psi(t)$, where $\Psi(t) = \dot{M}_*$ is the SFR in units of $M_\odot \cdot \mathrm{yr}^{-1}$.

Finally, the chemical evolution equation reads:

$$\dot{M}_Z(t) = -(p_Z - Z(t))\dot{M}_g(t), \tag{2}$$

Using the identity $\dot{Z}(t) = \frac{\dot{M}_Z(t)}{M_g} - Z\frac{\dot{M}_g}{M_g}$, it can be written as,

$$\dot{Z}(t) = -p_Z \frac{\dot{M}_g}{M_g} \tag{3}$$

with the usual CBM solution, $Z(t) = -p_Z \ln(M_g(t)/M_g(0))$ with $Z(0) = 0$. It is clear then that Z depends exclusively of M_g, which is directly related to M_* in the CBM. What about secondary production? For the **secondary**

production we only need to substitue $p_Z \to p_{Z'} Z'$, in Eq. 3 where Z' describes another primary chemical element with $Z' = -p_{Z'} \ln(M_g(t)/M_g(0))$.

$$\dot{Z}(t) = p_Z p_{Z'} \ln\left(\frac{M_g(t)}{M_g(0)}\right) \frac{\dot{M}_g(t)}{M_g(t)} \tag{4}$$

Integrating over both sides we get,

$$Z(t) = -p_Z p_{Z'} \int_0^t \ln\left(\frac{M_g(t')}{M_g(0)}\right) \frac{\dot{M}_g(t')}{M_g(t')} dt'. \tag{5}$$

Now this looks like it should depend on the SFR, right? Not quite, if we apply the change of variables $M_g(t) = M$, with $dM = \dot{M}(t)dt$, shows that there is no effect of the SFH,

$$Z(t) = -p_Z p_{Z'} \int_{M_0}^{M_g(t)} \ln\left(\frac{M}{M_0}\right) \frac{1}{M} dM. \tag{6}$$

This means that, for any fixed mass, the secondary production is completely determined by the primary one with the standard result $Z \sim (Z')^2$. Overall, there is only mass dependance.

Delayed CBM

We relax assumption iv) to a bit more general statement: iv)' All stars formed at the same time will die and enrich the ISM after a certain time, τ_Z, has passed. This means that, after analogous calculations, Eq. 3 now reads for **primary production**,

$$\dot{Z}(t) = -p_Z \frac{\dot{M}_g(t - \tau_Z)}{M_g(t)} \tag{7}$$

This delayed differential equation is a much tougher problem. To examine its behavour, we propose a perturbative expansion on τ motivated by the fact that for LIM stars $\tau \leq 0.1$ Gyr. Then, $\dot{M}_g(t-\tau) \approx \dot{M}_g(t) - \tau \ddot{M}_g(t)$ and, for $t > \tau$:

$$Z(t) \approx Z_0(t) - Z_0(\tau) + \underbrace{\tau p_Z \int_\tau^t \frac{\ddot{M}_g(t')}{M_g(t')} dt'}_{\text{SFH contribution} \equiv \xi(t)}. \tag{8}$$

where $Z_0(t)$ indicates that the limits of the integral are 0 and t, instead of τ to t (as if there were no delay). The SFH contribution comes from the second derivate of the gas mass function or, equivalently, from the derivative of the SFR. This expresion can be a little bit obscure as well, so we can put everything together in terms of $\Psi(t)/M_g(t)$, the "gas sSFR". We exploit the identity $d(\Psi/M_g)/dt = \dot{\Psi}/M_g - (\Psi/M_g)^2$.

$$\boxed{\xi(t) = \tau p_z \left[\frac{\Psi}{M_g}(t)\Big|_\tau^t + \int_\tau^t \left(\frac{\Psi}{M_g}\right)^2 dt' \right]} \tag{9}$$

Now this looks better, the overall contribution of the SFH comes from the sSFR at each point in time! Not exactly, since the SFR is divided by the gas mass, but with a fixed model the relation between the two is somewhat straightforward. Interestingly, the second term is maximized whenever the sSFR (not the SFR) is constant in time.

For the **secondary production**, we have to modify Eq. 5 and follow the same procedure,

$$\dot{Z}(t) = p_Z p_{Z'} \ln\left(\frac{M_g(t-\tau)}{M_g(0)}\right) \frac{\dot{M}_g(t)}{M_g(t-\tau)} \tag{10}$$

The first order expansion reads,

$$\dot{Z}(t) = p_Z p_{Z'} \left[\underbrace{\ln\left(\frac{M_g(t)}{M_g(0)}\right)\frac{\dot{M}_g(t)}{M_g(t)}}_{\dot{Z}_0} - \left(\frac{\dot{M}_g(t)}{M_g(t)}\right)^2 \tau - \ln\left(\frac{M_g(t)}{M_g(0)}\right)\frac{\ddot{M}_g(t)}{M_g(t)}\tau \right] + O(\tau^2). \tag{11}$$

As in the primary production case, we can avoid the second derivative altogether by applying the integration by parts identity to $d(\dot{M}_g/M_g \ln(M_g/M_0))/dt$. Overall, we have

$$Z(t) = Z_0(t) - Z_0(\tau) - \chi(t)\tau p_Z p_{Z'} \tag{12}$$

where $\chi(t)$ is the SFH integrated contribution,

$$\boxed{\chi(t) \equiv \frac{\dot{M}_g(t')}{M_g(t)} \ln\left(\frac{M_g(t')}{M_g(0)}\right)\Bigg|_{\tau}^{t} + \int_{\tau}^{t} \left(\frac{\dot{M}_g(t')}{M_g(t)}\right)^2 \ln\left(\frac{M_g(t')}{M_g(0)}\right) dt'} \tag{13}$$

What do we take from this? The amount of delayed chemical elements, as it is the case for nitrogen, depends on the “gas sSFR”, either directly, as is in the primary chemical evolution, or weighted by the logarithm of the gas mass fraction, as in the secondary production. One question still remains: how large can this contribution be? Some estimates can be made, but a visual representation is always more comepelling.

Numerical implementation

The exact delayed CBM differential equations can be directly implemented taking into account the boundary condition $\Psi = 0, t < 0$. For the yields, we will fix the values to something akin (not exactly the same since the yield used in that work does not separate between massive and LIM secondary contributions) to [4] with $p_{NP} = 1.6 \cdot 10^{-4}$, $p_O = 3.3 \cdot 10^{-3}$ and the nitrogen secondary production $p_{NS,M} = 0.13 \cdot 0.5 \cdot 10^{-3} = p_{NS,LIM}$ from masssive and low and intermediate mass (LIM) stars. The oxygen yield is extracted directly from their work, the primary nitrogen production has been selected to replicate their N/O plateau and the nitrogen secondary yield was only given in terms of total contribution, here we assume that massive and LIM stars contribute approximately the same.[1]

If we assume a gas evolution $M_g(t) = M_g(0)e^{-Ht}$ we get the results in Fig. 1. As it was expected, the models with larger "gas sSFR" at present time have sligthly lower N/O because they larger amounts of oxygen than a non-starforming galaxy, since the latter will always be closer to the no-delay evolution track. Let's now make use

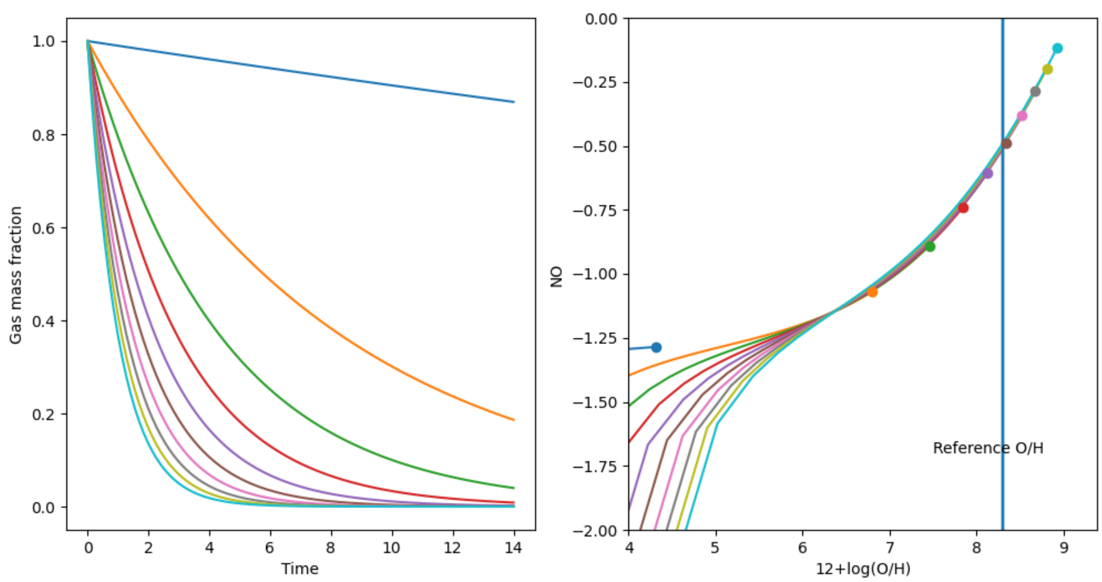

Figure 1: Numerical results for the delayed CBM model. In the left panel, the gas mass fraction (of $M_g(0)$) as a function of time. In the right panel, the O/H-N/O plot.

of the previous section developments. Since in the observed galaxies we would not have SFH information at such small timescales the best approximation for the parameters χ and ξ comes from considering only the information that we have available (present time values and integrated over the whole time interval). Nonetheless, in this example, every parameter is correlated (the functions involved, the gas sSFR and the logarithm of gas mass, are completely determined by a single point in the gas fraction-time diagram), so the gas sSFR at a given time is a good enough proxy to totally determine the final NO[2]. Since we want to control for mass, we plot the gas sSFR versus NO for $12 + O/H = 8.3$ (indicated as a vertical blue line in Fig. 1, right panel.) The results are displayed in Fig. 2.

[1]Esta últma suposición no es cierta, pero los valores concretos no tienen mucha relevancia, están para que se obtengan aproximadamente lo observado. De hecho, es muy probable que las estrellas LIM tengan una contribución mucho mayor a las masivas, pero eso solo apoyaría incluso más los resultados que se van a presentar en esta sección.

[2]This will always be the case for models with monotonically increasing or decreasing gas sSFRs, since there is no "degeneration".

The values are only representative and should only be taken qualitatively. As we can see, the correlation is perfect but the variations are small and we can conclude that, if real galaxies follow an evolution similar to this model, all relevant information can be extracted form the lattest (gas) sSFR, something that it has been assumed repeatedly ([3, 2, 5, 7]). In general it is assumed that the SFR evolution proceeds in a burst-like manner in which succesive bursts cause the dispersion observed here [2, 5, 7]. If that is the case, the different bursts' intensities could lead to different scatters in the O/H-N/O plane (are bursts different depending on the large scale structure? Could be a way to challenge this hypothesis.)

There is one last interesting thing to consider, whether the SFH of a galaxy is highly correlated will determine, as in our gas model, if we can retrieve any additional information from the integrated SFH in χ, ξ or if the only relevant parameter is (gas) sSFR. In any case, the effects can be considered second or even third order corrections, so disentangling them could be tricky or unfeasible.

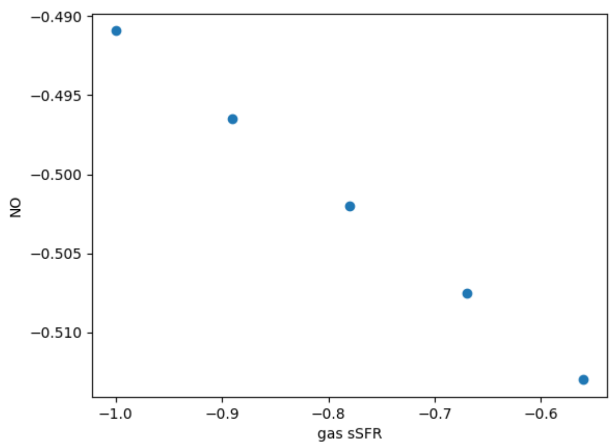

Figure 2: Numerical results for the delayed CBM model. Gas sSFR versus NO for fixed $12 + log(O/H) = 8.3$ and the gas model e^{-Ht}.

References

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