# Traffic Signal Scheduling Problem Formulation

## Indices and Sets

| | |
|---|---|
| $i \in \{0, 1, \ldots, I-1\}$ | Intersections |
| $s \in \{0, 1, \ldots, S-1\}$ | Streets |
| $k \in \{0, 1, \ldots, V-1\}$ | Cars |
| $l \in \{0, 1, \ldots, |P_k|-2\}$ | Position in car path $P_k$ |
| $p \in \{0, 1, \ldots, |in\_streets_i|-1\}$ | Slot index at intersection $i$ |
| $r \in \{0, 1, \ldots, |street\_events_s|-1\}$ | Service slot index on street $s$ |

## Parameters

$$B_s, E_s \text{ start and end intersections of street } s$$
$$L_s \text{ length (travel time) of street } s$$
$$D \text{ duration of simulation}$$
$$F \text{ fixed bonus for reaching destination}$$
$$P_k \text{ path of car } k \text{ as list of streets}$$
$$in\_streets_i \text{ streets incoming to intersection } i$$
$$street\_events_s \text{ events for street } s \text{ (cars crossing)}$$

## Decision Variables

**Intersection Scheduling Variables:**

| | |
|---|---|
| $y_{i,j} \in \{0,1\}$ | whether street $j$ at intersection $i$ has green light |
| $T_{i,j} \in [0, D]$ | duration of green light |
| $x_{i,j,p} \in \{0,1\}$ | whether street $j$ is assigned to slot $p$ at $i$ |
| $u_{i,p} \in \{0,1\}$ | whether slot $p$ at intersection $i$ is used |
| $b_{i,p} \in [0, D \cdot |in\_streets_i|]$ | start time of slot $p$ |
| $d_{i,p} \in [0, D]$ | duration of slot $p$ |
| $C_i \in [0, D \cdot |in\_streets_i|]$ | total cycle time at intersection $i$ |

**Car Variables:**

$$arr_{k,l} \in [0, D \cdot V] \qquad \text{arrival time of car } k \text{ at street } P_k[l]$$
$$\tau_{k,l} \in [0, D \cdot V] \qquad \text{crossing time of car } k \text{ at street } P_k[l]$$
$$z_{k,l} \in \{0,1\} \qquad \text{whether car } k \text{ crosses street } P_k[l]$$
$$fin_k \in [0, D \cdot V] \qquad \text{finish time of car } k$$
$$f_k \in \{0,1\} \qquad \text{whether car } k \text{ finishes within } D$$
$$rscore_k \in [0, D] \qquad \text{residual score for car } k$$
$$sk_k \in [0, F + D] \qquad \text{total score for car } k$$

**Service Slot Variables:**

$$\tau_{s,r} \in [0, D \cdot V] \qquad \text{crossing time at service slot } r \text{ of street } s$$
$$R_{s,r} \in [0, D] \qquad \text{residual time in slot } r$$
$$Q_{s,r} \in [0, V] \qquad \text{number of cycles before slot } r$$
$$A_{s,local,r} \in \{0,1\} \qquad \text{assignment of event to slot } r$$

## Constraints

### Intersection Slot Assignment:

$$T_{i,j} \geq 1 \cdot y_{i,j}, \qquad\qquad T_{i,j} = 0 \cdot (1 - y_{i,j}) \qquad (1)$$

$$\sum_p x_{i,j,p} = y_{i,j}, \qquad\qquad \sum_j x_{i,j,p} = u_{i,p} \qquad (2)$$

$$d_{i,p} = \sum_j x_{i,j,p} T_{i,j}, \qquad b_{i,0} = 0, \quad b_{i,p+1} = b_{i,p} + d_{i,p} \qquad (3)$$

$$C_i = b_{i,|in\_streets_i|} \qquad (4)$$

### Car Arrival and Crossing Times:

$$arr_{k,0} = 0 \qquad\qquad (5)$$
$$arr_{k,l} = \tau_{k,l-1} + L_{P_k[l]} \qquad\qquad \forall l \geq 1 \qquad (6)$$
$$\tau_{k,l} \geq arr_{k,l} \qquad\qquad (7)$$
$$\tau_{k,l} \leq arr_{k,l} + M(1 - z_{k,l}) \qquad \text{(big-M enforcement)} \qquad (8)$$
$$z_{k,l} \leq y_{E_{P_k[l]},j} \qquad \text{link to green light} \qquad (9)$$

**Finish Times and Scores:**

$$fin_k = \tau_{k,|P_k|-2} + L_{P_k[|P_k|-1]} \qquad\qquad\qquad\qquad \text{if } f_k = 1$$

$$\tag{10}$$

$$f_k \leq z_{k,l} \qquad\qquad\qquad\qquad \forall l \qquad (11)$$

$$f_k = 1 \Rightarrow fin_k \leq D, \quad f_k = 0 \Rightarrow fin_k \geq D+1 \qquad (12)$$

$$rscore_k = \begin{cases} D - fin_k, & \text{if } f_k = 1 \\ 0, & \text{if } f_k = 0 \end{cases} \qquad (13)$$

$$sk_k = F \cdot f_k + rscore_k = \begin{cases} F + (D - fin_k), & \text{if } fin_k \leq D \\ 0, & \text{if } fin_k > D \end{cases} \qquad (14)$$

**Service Slot and FIFO Ordering:**

$$\tau_{s,r} = a_{i,j} + R_{s,r} + Q_{s,r}C_i \qquad\qquad\qquad (15)$$

$$\tau_{s,r} \geq \tau_{s,r-1} + 1 \qquad\qquad \forall r \geq 1 \qquad (16)$$

$$\tau_{k,l} = \sum_r A_{s,local,r}\tau_{s,r} \qquad\qquad \forall \text{events on street } s \qquad (17)$$

## Objective

$$\max \sum_{k=0}^{V-1} sk_k$$