Optimization and Operations Research in Mitigation of a Pandemic

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1 Introduction

Nine applications in Operations Research during the pandemic:
•Reopen Economy - Algorithm-based production planning/Dispatching in a pandemic (Decision problem)
•Inventory and Risk Pooling of Medical Equipment and Supply-Chain-Network Management
•Monte Carlo/Computer Simulation Methods for Drastic and Rare Scenario Analyses
•New Norm - Operation/Optimization under Social Distance Constraints
•Indoor GPS and Tracking by Sensor Network Localization for Contact-Tracing (LoT)
•Dynamic and Equitable Region Partitioning for Hospital or Health care Services (Computational geometry problem)
•Efficient Public Goods Allocation under Tight Capacity Restriction via Market Equilibrium Mechanisms/Platforms
•Identifications and Protective Measures for High-risk Groups in a Pandemic (Data Science problem)
•Machine Learning, e.g. Logistic Regression, of Multiple Social Features to Reduce Pandemic Fatality

2 Reopen Economy-Production Planning and Operation: Machine Decisioning Solution

2.1 Regular Production Plan: Integer Linear Programming Solver

The objective setting can be production cost, holding cost, delay penalty, etc.; The decision variables can be production quantity (when and where), materials inventory level, delivery quantity, etc.; The constraints are order satisfaction constraints, material constraints, capacity constraints, policy rules, etc.;
During a pandemic, production challenges appears, such as reduced demand, uncertainty in supply chain, longer production leadtime, facility relocation, shortage in critical materials, and longer transfer cycle time.
Possible solution scenario: Alternative BOM (Bill of materials), Alternative delivery or substitution of product, priority resettings...

2.2 Multi-stage algorithm for production planning: agile, flexible and robust

A case from cardinal operations: Background: multiple factories, complicated operational rules, time-consuming computation.
Summary:
Basic constraints: capacity, material, plant attributes;
Priority requirements: raw material assignment priority, order priority;
Auxiliary analysis: delay warning and analysis.
Statistics:
Number of products $\geq 10^4$
Number of plants $\approx 10^2$
Number of production lines $\geq 10^2$. In this case, traditional integer optimization solver may be no longer tractable, so new algorithm:
Stage 1: Relaxed LP Solve a smaller LP problem as the initial solution.
Stage 2: Full Scale Relaxed LP Fix the solution from stage 1 as parameters, solve a full-scale LP problem which satisfied all the priority requirements.
Stage 3: LP Rounding Round up or round down the solution to ensure

feasibility in real-life.
Advantages: Flexible (rolling on daily basis), emergency response, root cause analysis.
Performance: modeling and solving time $\leq$ 3 hours. Number of decision variables $\geq 10^7$. Number of constraints: $\geq 10^7$.

3 Inventory and Risk Pooling of Ventilators and Supply Chain Network Management

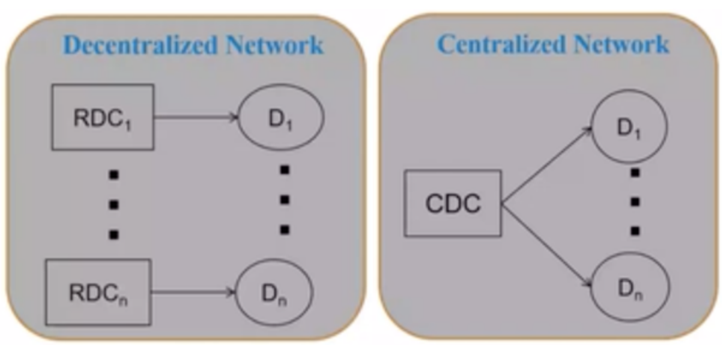
The War of Ventilators: Decentralization vs Centralization, Why centralization is better from the perspective of operations research?
Beat uncertainty through safety stock: How many to order?
Consider Gauss distribution, the order quantity q^* satisfies:

$$Prob(D \leq q^*) = 0.99 \implies F(q^*) = 0.99 \tag{1}$$

Consider $D \sim N(10000, 9000^2)$, The needed quantity of ventilators is

$$q^* = \mu + F_{norm}^{-1}(0.99) * \sigma = 10000 + 2.326 * 9000 = 30937 \tag{2}$$

where $F_{norm}^{-1}(0.99) * \sigma$ is the safety stock.



Inventory Network Management: Risk Pooling via Centralization, how many ventilators to order for COVID-19? Suppose the number of patients is random and follows a certain distribution $F(\cdot)$, and the desired guarantee level is 99%.

Complex Supply-Chain Network Management, forecasting, inventory replenishment, demand fulfillment, vehicle routing. The challenges of supply-chain include factory shutdown, limited and unstable transportation resources, volatile demand, interruption of warehouse capacity and tolerance level...To make centralized inventory replenishment and efficiently allocate inventory to RDCs facing the challenge of coronavirus, and due to such challenge, the existing violent change of supply-chain network and resources, the solution should take in emergent changes in business and data, and make quick, agile, and robust responses. Therefore, machine decision is needed to see the trends in data.

How to evaluate a drastic but rare scenario outcome?

→ Sample testing: random sampling, focused or importance sampling

→ Robust decisioning and optimization

→ Computer-simulation-based stress tests:

Simulation allows us to quickly and inexpensively acquire knowledge concerning a problem that is usually gained through experience. Monte Carlo simulation is an important and flexible tool for modeling situations in which uncertainty is a key factor. Analyze how a health-care system fare in drastic and rare scenarios based on rare event simulation.

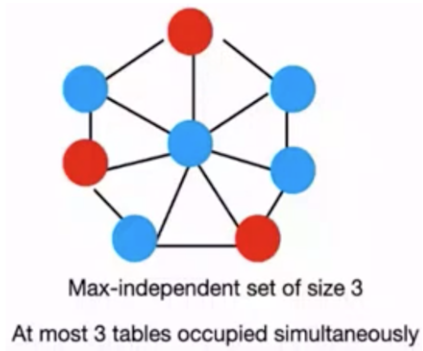
Stress test was first used in banking. The design of a simulation system for financial networks should consider estimating the distribution of losses for a banking network, and then test the losses incurred by the initial shock and the losses resulting from the contagion process.

4 Social Distancing: Mathematical Implication and Solution

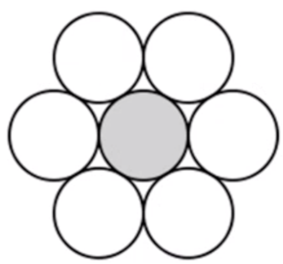
To accommodate people in finite space with sufficient distance from each other, the mathematical representation of social distancing can be:

$$||x_i - x_j||_2 \geq 6 \tag{3}$$

for nonconvex in position variable x. There are two scenarios:
When people accommodated discretely, such as indoor space in theater, restaurant, school, etc. It is a combinatorial/discrete optimization problem. The Max-Independent set model can be shown as:



Another scenario is when people accommodated continuously, such as outdoor space on beach or square. It is non-convex continuous optimization. **The Max-Independent set problem** Given a graph G, find a subset of



vertices of maximum cardinality such that no two vertices in the subset are directly connected. IP formulation of Max-Independent set probrem:

$$max \Sigma_i x_i \tag{4}$$

such that,

$$x_i + x_j \leq 1 (i, j) \in E \tag{5}$$

$$x_i \in 0, 1 \forall i \tag{6}$$

The kissing problem Given a unit sphere, find the maximum number of unit spheres in d dimension that kiss the center sphere. Measure distance by Euclidean Norm:

$$dist(x_i, x_j) = ||x_i - x_j||_2 \tag{7}$$

Need for safe distance δ

$$||x_i - x_j||_2 \geq \delta \quad (8)$$

The quadratic constraint is non-convex and results in hardness. There is no closed form solution for dimension d and the upper bounds can be provided. [2, 4]

Dimension	$K(d)$
1	2
2	6
3	12
4	24
8	240
24	196560

New Extension: Humanized Arrangement? What if people from the same family can be seated together, then they do not need to obey social distancing. Potentially, more people are accommodated. However, independent set like above fails to capture the extension. We adjust the problem to Max-Independent Set Problem with clusters. The solution can be adding another subscript such that one stands for people, the other stands for family. This can be formulated as 0-1 integer programming. **Think about intuitive and heuristic approach?** starting with accommodate large groups to the corner and then individual hospitality...

5 Indoor GPS and Tracking by Sensor Network Localization

Identifying trajectory during the pandemic: outdoors-using GPS, indoors-using indoor signal anchors...

Sensor Network Localization(SNL) Given m anchor points $a_1, ..., a_m \in \mathbb{R}^d$ whose locations are known and n sensors points $x_1, ..., x_n \in \mathbb{R}^d$ whose locations we wish to determine. Furthermore, we are given the Euclidean distance $d_{i,j}$ between a_k and x_j for some k,j and $d_{i,j}$ between x_i and x_j for some i,j. The Sensor Localization Problem is to find a realization of $x_1, ..., x_n$ such that

$$||a_k - x_j||^2 = d_{kj}, d_{kj} \tag{9}$$

where $||a_k - x_j||^2$ is distance between anchor and sensor,

$$||x_i - x_j||^2 = d_{kj}, d_{kj} \tag{10}$$

where $||x_i - x_j||^2$ is distance between sensor and sensor, it is hard to track even for d=1. However, it can be formulated and relaxed as SDP feasibility problem. [1]

Real-time Sensor Localization Problem works under milder conditions. It is a real-time version of sensor localization problem and retrieves moving trajectory and predicts. It is a combination of ESDP for tracking and gradient method for error minimization. [3] The objects move subject to linear differential equation:

$$\frac{dX(t)}{dt} = AX(t) + Zt + C \tag{11}$$

and a least square problem can be

$$min_{\gamma} \Sigma_{i=1}^{n} \eta_i \gamma_i^2 \tag{12}$$

such that

$$\gamma_i \geq \frac{X(t_i) - X(t_i - 1)}{t_i - t_{i-1}} - AX(t_i) - Zt_i - C, \forall i \tag{13}$$

where $A, C, Z \in \Lambda$

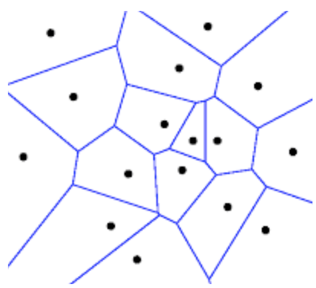
Simple Distance Checking and Enforcing e.g. high-tech solution for distancing alarming, primitive distancing enforcement, card-play table redesign, one-way or two-way for pedestrian environments...

6 Dynamic Hospital Service Region Partition

Computational Geometry Theory and Algorithm: **input data:** hospital location and capacity, pandemic density distribution prediction; **Planning:** partition the city into multiple regions such that,

(1) each region has a hospital nearby
(2) each hospital will not be overrun
(3) can be easily adjusted by input data change

Plane-Geometry Problem Statement and Theorem: suppose n points are scattered inside a convex polygon P in 2-D with m vertices. Does there exist a partition of P into n sub-regions satisfying the following:
(1) Each sub-region is a convex polygon - travel convenience
(2) Each sub-region contains one point - service center for the region
(3) All sub-regions have equal area - load balance.
Not only such an equitable partition always exists but also we can find it exactly in running time O(NnlogN) where $N = m + n$.



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