TNB Wars

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1 Introduction

As you at High Command are aware, we are conducting preliminary research on operation TNB (condoned by General [redacted]). In the interest of security, any sensitive information has been redacted. Our agent, [redacted], has informed us that the [redacted] for the [redacted] are located on Xanadar IV. We will be using an enemy shuttle, provided by our friends at [redacted], to enter the atmosphere. Upon stealing the [redacted] we shall escape the planet back to HQ. At an altitude of 10 km, we will be fired upon by ion cannons.
All presented values have been simplified to 3 significant figures.

Explanation of concepts

Unit Tangent Vector
The unit tangent vector shows the instantaneous direction of travel of a particle at any time. The vector, being a unit vector, disregards the magnitude of the velocity. This report found the unit tangent vector using the equation below:

$$\vec{T}(t) = \frac{\vec{r'}(t)}{|r'\vec{(t)}|}$$

Unit Normal Vector
The unit normal vector is orthogonal to the unit tangent vector, and always points to the the curve that is concave up. To find the unit normal vector, use the following formula:

$$\vec{N}(t) = \frac{\vec{T'}(t)}{|\vec{T'}(t)|}$$

Binormal Vector
The binormal vector is the cross product of the unit tangent and normal vectors. The binormal vector lies in the normal plane. It is orthogonal to the plane defined by these vectors. To find the binormal vector, this report used the definition of the binormal vector:

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

Curvature
The curvature of a function is a measure of how quickly the curve changes direction at that point. It is defined as the rate of change of the tangent vector with respect to the distance traveled along the curve.

$$\kappa(t) = \frac{dT}{ds} = \frac{|\vec{r'}(t) \times \vec{r''}(t)|}{|\vec{r}(t)|^3}$$

Tangential Component of Acceleration
The Tangential component of acceleration is a scalar value, and is defined as the acceleration component tangent to the curve. This report computed the tangential component using the given equation.

$$a_T = \frac{\vec{a}(t) \cdot \vec{v}(t)}{|\vec{v}(t)|}$$

Normal component of Acceleration
The Normal Component of acceleration is a scalar value, and is defined as the acceleration that is orthogonal to the curve. This report computed the tangential component using the given equation:

$$a_N = \frac{|\vec{a}(t) \times \vec{v}(t)|}{|\vec{v}(t)|}$$

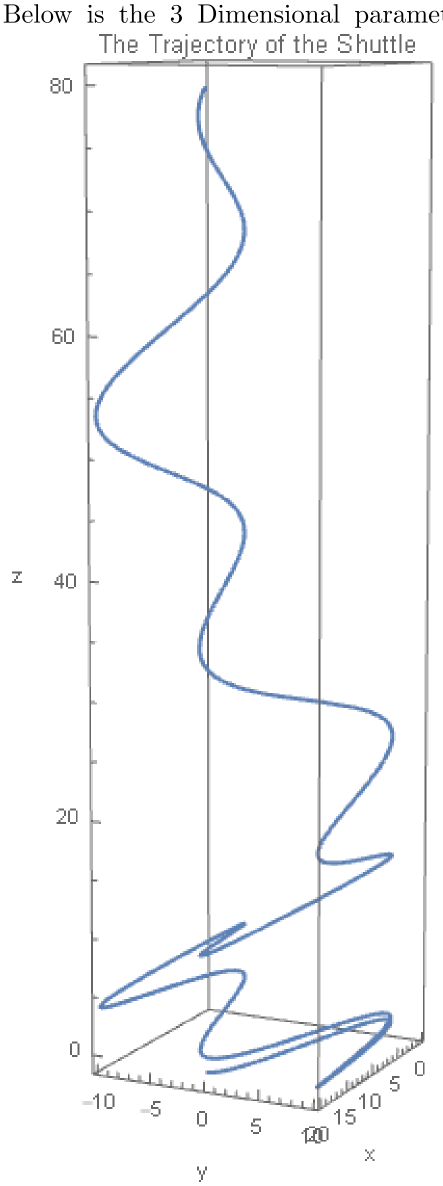
2 The Escape Trajectory

Our skilled pilot has recommended that we our path be defined by the vector:

$$\vec{r}(\mathrm{t})=\langle\ 20[\cos(\tfrac{8\pi}{15})]^2\ ,10\sin(\tfrac{4\pi}{15}\mathrm{t}),\ \tfrac{16}{375}\mathrm{t}^3\ \rangle$$

2.1 Visual Representation of Trajectory

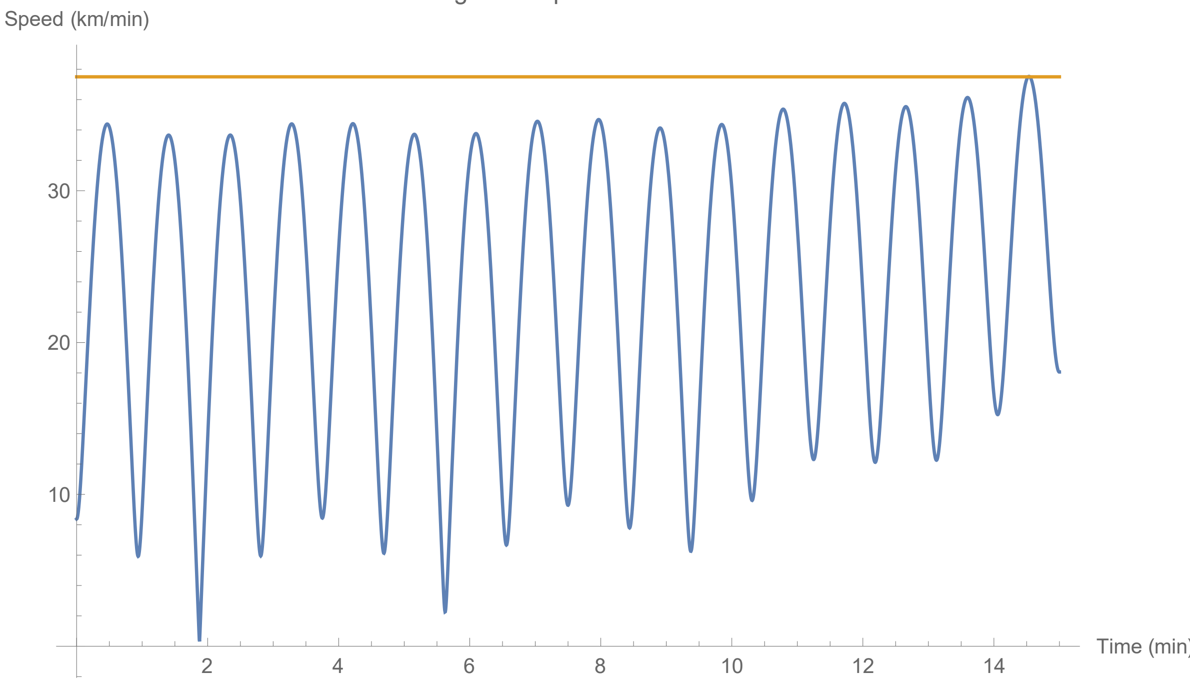
Below is the 3 Dimensional parametric plot of the trajectory given above.



2.2 Instantaneous Speed

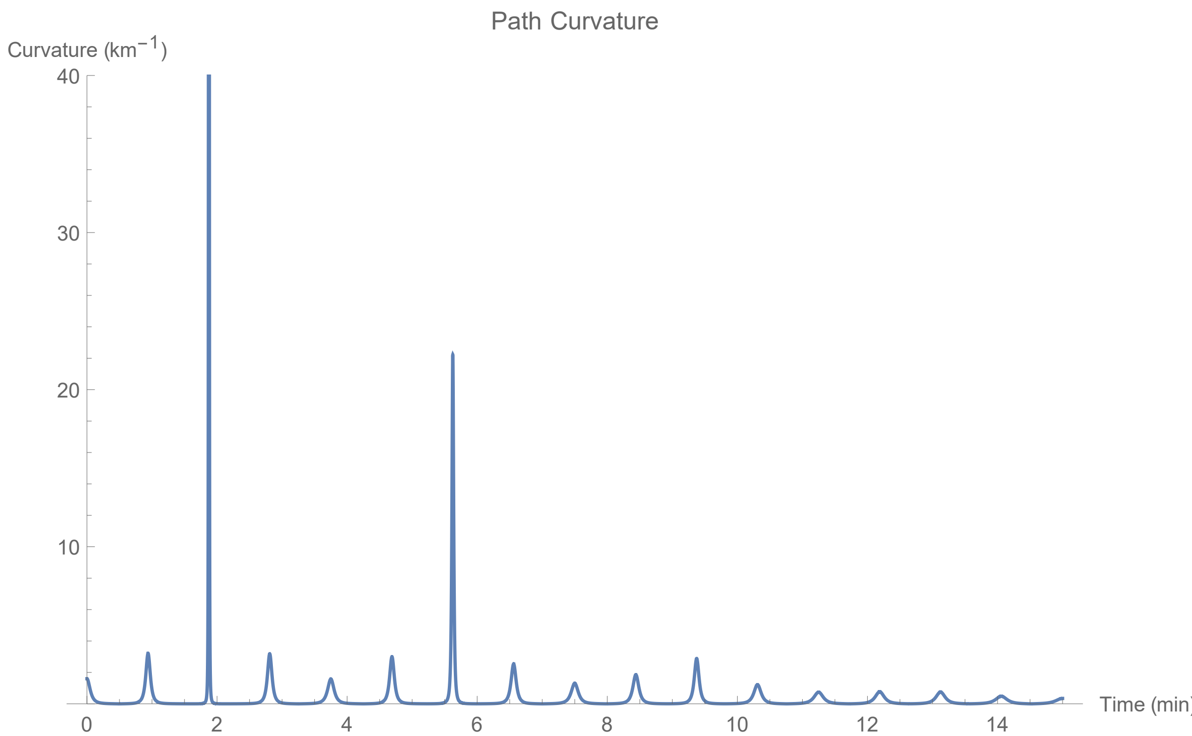
The instantaneous speed of the shuttle during the first 15 minute of flight is plotted below. This graph was found by taking the absolute value of the velocity vector. The velocity vector was found by taking the derivative of the position vector. The orange horizontal line on the plot represents a speed of 37.5 $\frac{km}{hr}$

Figure 1: Speed vs Time



2.3 Curvature

This report has produced a graph of the curvature of the suggested path (below). The graph was derived using the curvature equation provided in the introduction. The scale on this graph has been limited to show the most information possible. In reality, the maximum curvature is roughly 1800 km^{-1}.
It is evident that this path includes some points of extreme curvature, particularly at shuttle launch. For further analysis of curvature on shuttle integrity, see section 3.2.



2.4 Distances and Speeds Related to the Trajectory

The total distance along the shuttle route was found by integrating the speed function (equivalent to the absolute value of the velocity vector) from t=0 to t=15 minutes.

Total distance traveled = 357 km

The direct distance was computed using the initial point (defined by $\vec{r}(0)$) and the final point (defined by $\vec{r}(15)$). These two points were substituted into the Distance Formula.

Direct distance between start and end points = 80 km

The average shuttle speed was calculated by dividing the total distance traveled by the total time.

Average Speed = 23.8 $\frac{km}{min}$

Hence, the average speed of the shuttle does exceed 20 $\frac{km}{min}$ during the first 15 minutes.

3 Shuttle Integrity

This section evaluates whether the shuttle can survive traveling on the provided trajectory.
The shuttle that the Alliance has commandeered is much more delicate than the military spacecraft that are commonly used in this ongoing Rebellion. Excessive speed, curvature, or acceleration could tear the shuttle apart and, as a result, lose the war.

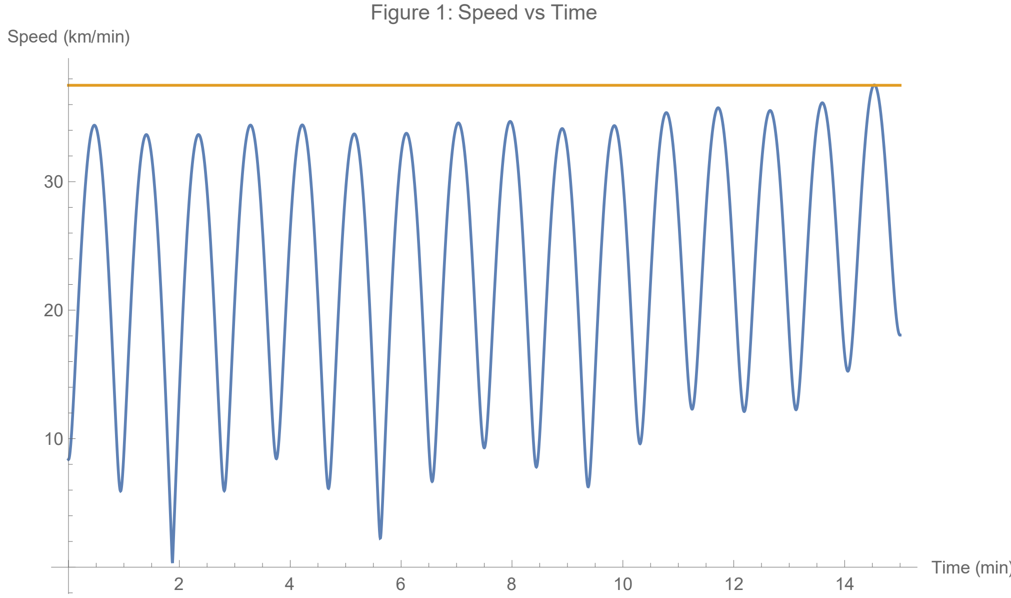
3.1 Excessive Speeds

The shuttle will be destroyed if its speed exceeds 37.5 $\frac{km}{min}$ for more than 2 seconds.
To find if the speed is in excess, this report found the values of t for which:

$$|\mathbf{v}(t)| = 37.5\ \frac{km}{min}$$

This was accomplished using the **FindRoot** function in Mathematica. From the graph below, the speed is equal to 37.5 $\frac{km}{min}$ when t $\approx$ 14.5 minutes.

Figure 1: Speed vs Time



Using the method described above, this report found the speed was in excess of safety standards at:

$$\text{t} = 14.5219 \text{ minutes t} = 14.5488 \text{ minutes t} = 15.397 \text{ minutes}$$

However, this report is only concerned with the first 15 minutes of travel. Hence, the third time point is irrelevant.

To calculate the total time the shuttle travels in excess of 37.5 $\frac{km}{min}$, the difference between the two relevant time points is required. This result is then converted from minutes into seconds.

$$t = (14.5488 - 14.5219) \times 60$$

$$t = 1.61s$$

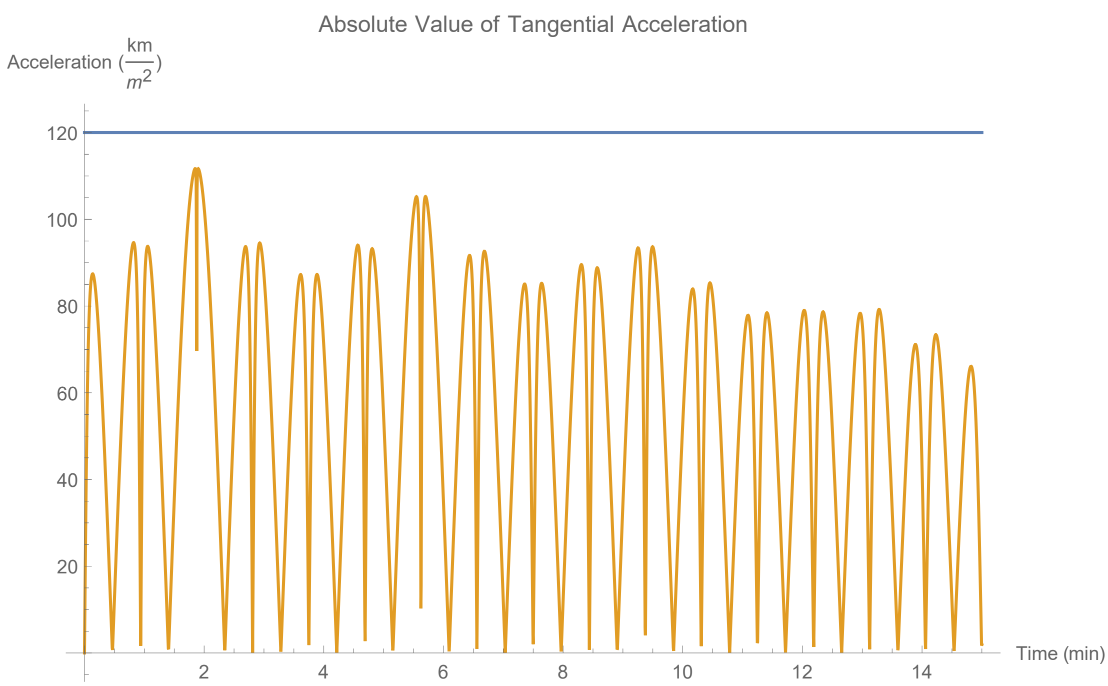
3.2 Maximum Curvature

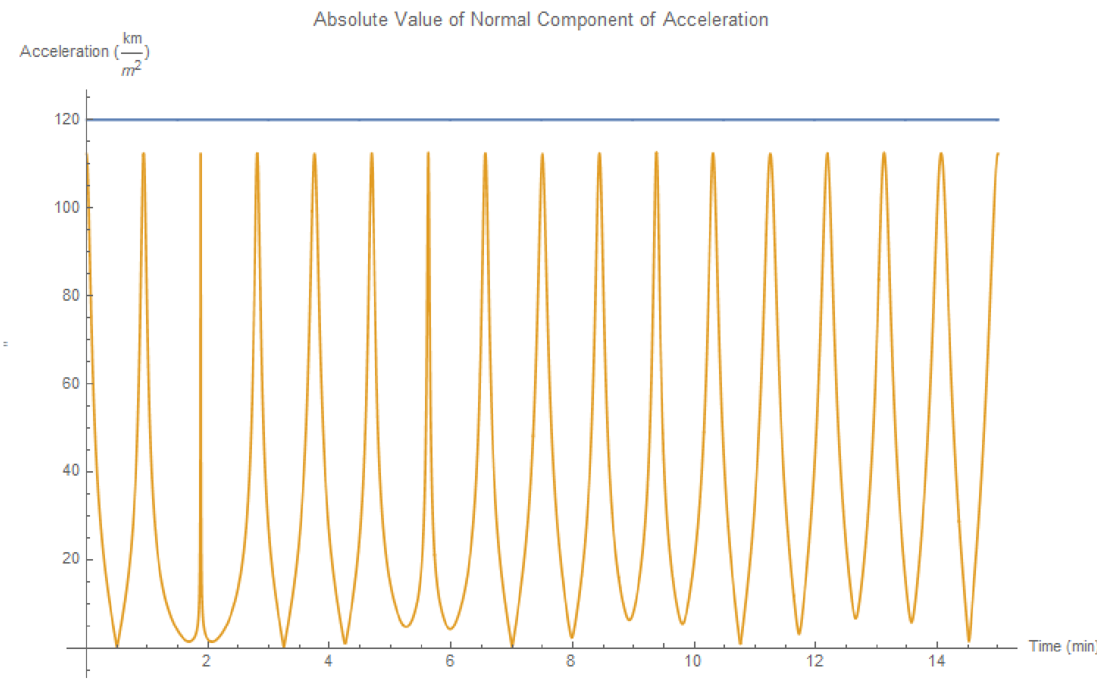
The maximum curvature during the first 15 minutes of flight was found using the **FindMaximum** function in Mathematica. From the graph below, it is clear that the maximum curvature occurs when t $\approx$ 2 minutes. Using this as a reference for the function, the result is:

$$\textbf{Maximum Curvature} = \textbf{1800 km}^{-1}$$

3.3 Components of Acceleration

The tangential and normal components of acceleration were found using the equations given in the **Introduction**. The results have been plotted against the fatal acceleration of 120 $\frac{km}{m2}$. This report encourages High Command to review the attached code to view the entire process used to obtain these plots.





3.4 Safety Observations

The shuttle will be destroyed if: the speed exceeds 37.5 $\frac{km}{min}$ for more than 2 seconds; the curvature of the trajectory cannot exceed 2000 km^{-1} at any time in the first 15 minutes; or the magnitudes of the tangential and normal components of acceleration exceed 120 $\frac{km}{min}$ in the first 15 minutes.
Using the results found in the previous sections, it can be seen that: the shuttle speed only exceeds the maximum for 1.87 seconds (3 significant figures); the maximum curvature experienced is 1800 km^{-1} (3 significant figures); and, the maximum tangential and normal components of acceleration do not exceed the fatal value of 120 $\frac{km}{m2}$. These results all lie within the shuttle safety standards outlined by our ace pilot.
Hence, this report finds that the shuttle's trajectory **will not result in a fiery explosion.** This has greatly pleased those on the proposed mission.

4 Evading the Ion Cannons

Xanadar IV has powerful ion cannons dotted across its surface. Luckily, only three of these cannons are within range of the suggested trajectory. Our spies have acquired crucial information about the location and firing power of these cannons; this information forms the basis for the analysis in this section.
The shuttle can only withstand one collision with an ion beam, as long as the beam is not in the osculating or normal planes of the trajectory at impact.

4.1 Parameterizing the Beams

To perform analysis on beam trajectory, it is essential to find express the beam's position as a function of time. This can be done by parameterizing the beam using the equation below.

$$\vec{r}(t) = P_0 + t \cdot \vec{v}$$

Using the acquired information the parameterizations of the beams are shown below

Parameterization of the first beam:

$\vec{r}_{B1} = \langle$ 40 - 40(-7.5+t), 7(-7.5+t), 15(-7.5+t) $\rangle$

Parameterization of the second beam:

$\vec{r}_{B2} = \langle$ 20(−8+t), −30+20(−8+t), $\frac{135}{32}$(−9 + t) $\rangle$

Parameterization of the third beam:

$\vec{r}_{B3} = \langle$ 25−20(−9 + t), 20+(−20+5 $\sqrt{3}$)(−9+t), $\frac{640}{27}$(− 9+t) $\rangle$

4.2 Beam Collision with the shuttle

By setting the parameterization of the shuttle's path and the parameterization of each beam's path equal to one another, it is possible to determine if any of the three ion beams will collide with the shuttle. Using the Mathematica function **Reduce** it was determined that only the third beam will collide with the shuttle at t = 10 minutes and at the position $(5,5\sqrt{3},\frac{640}{27})$

4.3 Beam collision when shuttle is not present

To determine if a beam will hit the path of the shuttle when the shuttle is not present, the **Reduce** function was used again. However, instead of parameterizing the beam with t, the beam was parametrized with the variable **v** instead. Upon evaluation, it was found that the first beam does not intersect the trajectory at at all. However, the second beam does intersect when t = $\frac{45}{9}$ minutes, and **v** = 9. This indicates that the beam from the second ion cannon will intersect the path of the shuttle at a time of t=5.625 minutes. In addition we already know that the third beam intersects at t = 10 minutes. Taking the time of impact for the second beam, the calculated time value of 5.625 minutes can be placed into the parameterization of the path of the shuttle to determine where the beam intersected the path. The result is (20,-10,4.23).

4.4 Equations of planes

Under the condition that a beam collides with a plane (which it does) both equations of the oscillating and normal planes need to be found. Using the scalar equation of a plane(shown below) the equation of the plane can be found easily.

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Where a, b and c are components of the normal vector, and x_0, y_0, and z_0 are the coordinates of a point in the plane.

Osculating Plane Equation:
22.7758 = 0.0617**x**+0.7293**y**+0.6814**z**

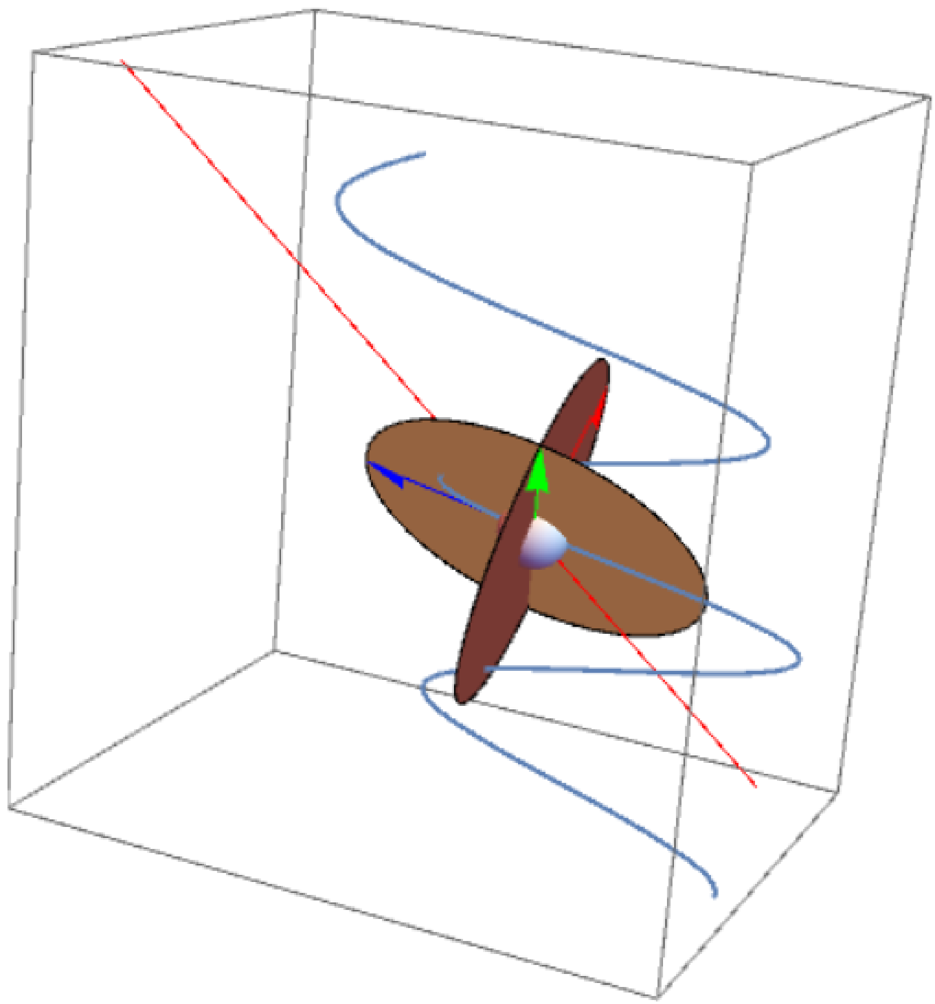
Normal Plane Equation
11.9561=0.2665**x**−0.6699**y**+0.6929**z**

4.5 Collision with beam

Although a collision with an ion beam is not ideal, the shuttle can sustain a direct collision from an ion beam as long as it is not orthogonal to the normal or oscillating plane. To determine whether the third beam collides orthogonally, the tangential, normal, and bi-normal vectors are needed. These vectors were found earlier using the definitions of tangential, normal and bi-normal unit vectors. After finding the vectors, the dot product was used to determine if they were orthogonal to the planes.

$$a.b = |a||b|\cos\theta$$

Knowing that two orthogonal vectors make a 90 degree angle, the dot product of two orthogonal vectors is 0. Applying this to the problem at hand, it can be concluded that if the dot product of the third beam and either the oscillating or normal plane at time t = 10 minutes is 0, then the shuttle will be destroyed. To calculate the dot product the Mathematica function **Dot** was used. Both dot products of the third beam with each plane were not equal to zero, which means that the shuttle is able to withstand the collision with the third beam and continue the mission.



Displayed above is the shuttles escape trajectory at the time of impact with the third beam. The shuttle is represented by the sphere, the normal and oscillating planes are represented by the circular disks, the beam from the ion cannon is the red line, and the tangential,normal and bi normal vectors represented by the arrows. The blue arrow is the tangential vector,the green arrow is the normal vector and the red arrow is the bi normal vector.

5 Conclusion

Greetings High Command. This report details the analysis of the flight plan brought forth by [redacted]. This report has strong reason to believe the mission will be successful, but has some concerns that should be known.

Given the condition of the aging shuttle, it will not withstand velocity in excess of 37.5 km/min. Based on the calculated trajectory, there will be 1.6 seconds of flight during which the speed exceeds the recommended safety cap. This will occur in the last 30 seconds prior to escaping the atmosphere, and should not affect the mission's success.

Another unfortunate side effect of using such an old vessel is that it is only able to withstand a certain amount of acceleration before disintegrating from excessive force. Fortunately for our brave pilot, the curvature of her chosen flight path, and its resultant acceleration vectors, will remain within operational limits. It will by no means be an enjoyable ride, but mission success is more critical than comfort.

Good news never comes without its dark side. Assuming our reconnaissance information is accurate and our pilot follows the exact path specified, the vessel will be struck by an ion cannon at precisely 10 minutes into its flight. However, the impact is non-fatal, as the beam does not lie in neither the osculating nor normal planes of the shuttle.

In an attempt to avoid any beam collisions, this report has recalculated the trajectory vector by altering the shuttle velocity during travel. Given the current shuttle's decrepit state, any increase in velocity would destroy the vessel due to the excessive forces experienced (refer to maintaining shuttle integrity). It would be advantageous to avoid the guaranteed impact and flight stresses by reducing the shuttle velocity throughout the path. However, this would give ample time for enemy fighters to scramble and destroy the shuttle. Hence, as the impact from the third cannon is non fatal, it would be most advantageous to continue with the original plan and absorb the impact from the third cannon.