Optimization of Solar Energy Production

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This report aims to determine the most efficient layout of *Solar Power Inc*'s solar panels on the equatorial island of Suluclac. The energy produced by each panel is dependent on daily and seasonal weather conditions, and panel angle relative to incoming light. This value was determined using the energy formula provided by the client. This report will show the means by which the maximum energy can be obtained from each panel, and the optimum time to replace the panels to minimize collection losses.

I. Nomenclature

s	=	angle between incoming sun ray and the plane of the equator at noon
u	=	angle between solar panel plane and the ground
t	=	angle proportional to time of day
$W(s,u)$	=	Energy output by solar panels
$C(s,t)$	=	cloudiness function
$I_p(s,t,u)$	=	intensity of radiation contacting the solar panel
$A(s,t)$	=	absorption function
kWh	=	Kilowatt hours
m^2	=	meters squared
D	=	domain of function

II. Introduction

The pacific island of Suluclac, located on the equator, is having solar panels installed by Boulder's *Solar Power Inc*. This report will analyze the panel efficiency in relation to time of year, time of day, and solar panel orientation.
The following information was provided to this report by the client. The Earth has a relatively circular orbit around the Sun; this report assumed the orbit was perfectly circular. The Earth also tilts about its own axis, causing seasonal changes on the planet. This tilt has a maximum of 23° from the normal axis. This tilt has been designated as s. Table 1 relates the values of s to the dates on the Gregorian Calendar.

Table 1 Angle s for Times of Year

Date	$s(°)$	**Increasing/Decreasing**
Dec. 21 2017	$s = -23$	
Dec. 22, 2017 - March 19, 2018	$-23 < s \leq 0$	Increasing with time
March 20, 2018 - June 20, 2018	$0 < s \leq 23$	Increasing with time
June 21, 2018 - Sept. 21, 2018	$23 > s \geq 0$	Decreasing with time
Sept. 22, 2018 - Dec. 20, 2018	$0 > s \geq -23$	Decreasing with time

The solar panels are fixed at an angle u for a 24 hour period, after which they can be readjusted. The variable u ranges from $-90° \leq u \leq 90°$. When $u = 0°$, the solar panel lies flat on the ground. When $u = -90°$, the panel faces due South; when $u = 90°$, it faces due North.
The variable t defines the angle between the sun and ground with respect to the time of day. $t = -90°$ corresponds to dawn, $t = 0°$ corresponds to noon, and $t = 90°$ corresponds to dusk.

To neatly relate s, u, and t, *Solar Power Inc* has provided this report with the energy output function W. The development of W is shown in full in the *Equation Development* section. This analysis will explore this function and determine the optimal configuration for maximum energy generation over the course of a calendar year.

III. Equation Development

The energy function, which is used heavily throughout this report, relates the amount of solar energy that penetrates the atmosphere to the time of day and year, angle of the solar panel relative to the ground, and atmospheric conditions. This function can be described as follows:

$$W(s,u) = \int_{t_{min}}^{t_{max}} A(s,t)C(s,t)I_p(s,u,t)dt$$

The range of values for variables s and u was given by:

$$D = \left\{-0.4 \leq s \leq 0.4, \frac{-\pi}{2} \leq u \leq \frac{\pi}{2}\right\}$$

A. Absorption

Some of this radiation is absorbed by the atmosphere; it is strongly dependent on the distance the sun's rays have traveled through the atmosphere before reaching the ground. The absorption, A, is bounded such that $0 < A(s,t) \leq 1$. Our client has not provided an explicit function for the absorption of energy, but it is accounted for in the final energy equation provided.

B. Cloudiness

The energy produced by the solar panels is dependent on the amount of light that can filter through the atmosphere. Energy generation is dependent on the local cloudiness on Suluclac. A meteorologist has described this cloudiness as:

$$C(s,t) = \frac{3 - (1 + (s - 0.2)^2)\cos^2(t)}{3}$$

Figure 1, seen below, is a 3D representation of the function above. This plot was created using ***Mathematica***.

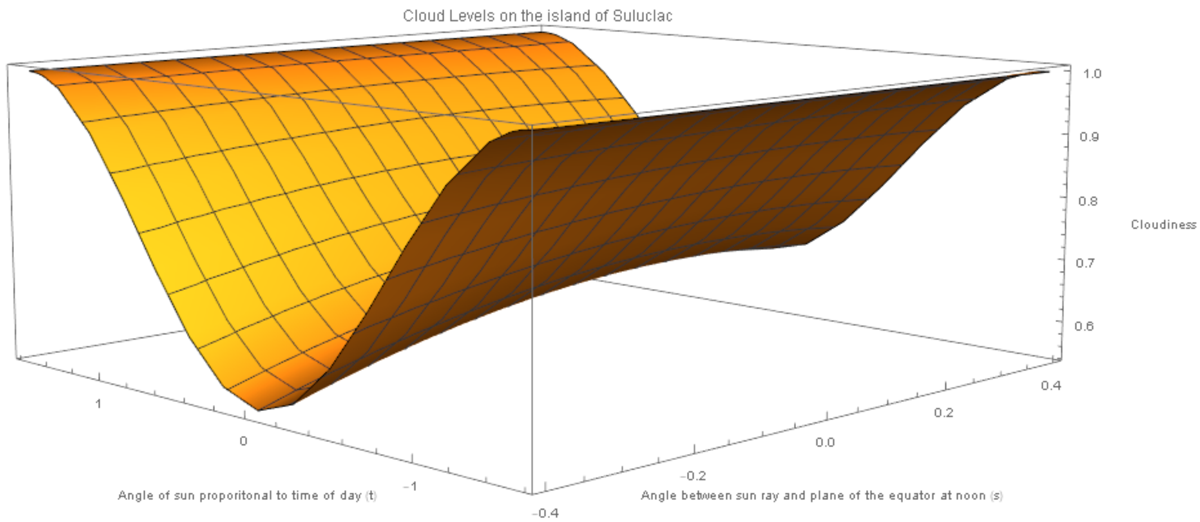

Fig. 1 Cloud Levels on Suluclac

A cloudiness value of 1 corresponds to no cloud cover, while 0 corresponds to complete cloud cover. As such, it is clear that the cloud cover on any given day is at a maximum at noon and a minimum at sunrise and sunset. Over the course of the year, the overall cloud cover per day will be highest during the winter months. As the year progresses into spring and summer, the cloudiness will gradually decrease until mid-spring, when it slightly increases until the summer solstice. Entering into fall and winter, the cloudiness level will change in the opposite manner to the first 6 months of the year. Average cloud cover will decrease slightly until mid-fall, and resume increasing until reaching its maximum on the winter solstice.

C. Intensity

The intensity of radiation, I, is needed to find how much energy is present at a point under ideal conditions. It is given by:

$$I_p(s, t, u) = I_0(\cos(s)\cos(u)\cos(t) - \sin(s)\sin(u))$$

This is assuming no interference from outside sources, which is far from the reality of the situation. The intensity of the radiation reaching the panel will be determined by the values of A and C, found above.

D. Energy Function

Using the individual functions above, all necessary information to determine energy generation will be known. The functions can then be expressed by the energy equation provided by *Solar Power Inc*, shown below.

$$W(s, u) = 1 + (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)\cos(u) + (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)\sin(u)$$

This is the function that will determine the amount of energy collected by a solar panel on any given day of the year. It is necessary to determine the value of this function for every day of the year, which will be accomplished by plotting the function across all possible values of s and u.

E. Converting Angles

This report has expressed final values in terms of radians. As such, the conversion process is included for reader awareness.

$$x \text{ radians} = x \text{ degrees} \cdot \frac{\pi}{180}$$

IV. Maximizing the Energy Function

In order to produce the most energy over the course of a year, the maximum points on the energy function are required.

A. Critical Points of the Energy Function

To visualize the relationship between W, s, and u, a 3D plot of the energy function was created. To further aid in visualization, a contour plot of the function was generated.

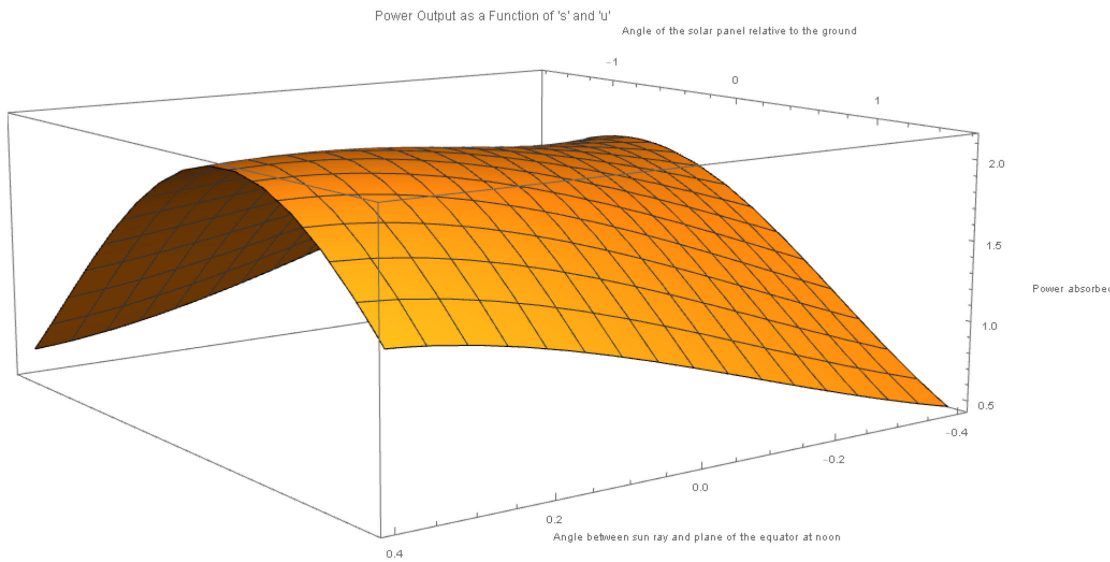

Fig. 2 Energy Generated

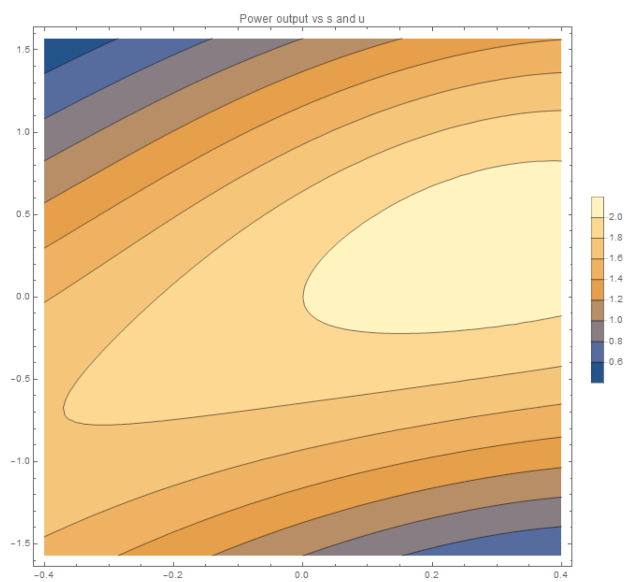

Fig. 3 Energy Function Contour

Using these plots, and the ***Mathematica*** *Find Root* function, the points at which the gradient vector was the zero vector were found. The contour plot was used to give an initial estimate the locations of possible critical points. Looking at the contour plot it was determined that a possible critical point in the domain occurs at $(0.4, 0.5)$. Using *Find Root*, it was determined that the actual location of the critical point is at $(0.32, 0.32)$. To classify this point as a critical point it must pass the second derivative test.

$$D = W_{ss}W_{uu} - [W_{su}]^2$$

Where $W_{ss/ss/uu}$ are the partial derivatives of the energy function with respect to the subscripts.
When $D > 0$ it classifies the point as a critical point; If $D < 0$ it classifies the point as a saddle point. If $D = 0$, the test is indeterminate. If $D > 0$ and $W_{ss} > 0$ the point is a minimum, if $W_{ss} < 0$ the point is a maximum. When evaluated, $(0.32, 0.32)$ returned a value of 3.99694, which is greater than zero declaring $(0.32, 0.32)$ as a critical point. Evaluating $W_{ss}[0.32, 0.32]$ returned a value of -1.132253 therefore (0.32, 0.32) is a maximum.

B. Extreme Points on the Energy Function Domain Edges

In addition to the 4 corners and maximum value of the energy function, critical points could lie directly along the boundary of the function in the given domain. Our Energy function is bounded by a rectangle, whose length is the domain of s, and width is the domain of u. It is impossible to evaluate all critical points by approximating near a given point, due to the infinite amount of points that exist on the boundary. Instead, s or u can be set to a fixed value while letting the other vary, and taking the partial derivative of the resultant single-variable function, and setting it equal to zero. Using the ***Mathematica*** *Find Root* function, critical points were readily evaluated. The four critical points found along the boundary are (-0.4, -0.74), (0.4, 0.36),(-0.77, -π/2) and (0.45, π/2). Using the 3D plot of the energy function and the second derivative test, the points were classified as maxima, minima or saddle points. Just by looking at the plot it is evident that (-0.4, -0.74) is a saddle point; along the boundary line, slope decreases from that point, but increases while moving towards the absolute maximum. (0.4, 0.36) is a maximum because the slope decreases in all directions from that point. Points (-0.77, -π/2) and (0.45, π/2) lie outside of the boundary so they can be disregarded. Double checking our observations using the second derivative test, it was determined that (-0.77, -π/2) is a saddle point since the determinant is less than zero. (0.4, 0.36) has a determinant greater than zero and using the second derivative test W_{ss} evaluated to a negative value, indicating that (0.4, 0.36) is a maximum. Looking again at the 3d plot, the s boundary did not need to be evaluated; along this boundary, the slope does not change signs.

C. Energy at Critical Points

The time at which maximum energy is collected can be determined by evaluating each maximum critical point. Two maximums were found as stated earlier, these are (0.4, 0.36) and (0.32, 0.32). evaluating these two points gives us a total Energy output of $2.13\frac{kWh}{m^2day}$ and $2.12\ \frac{kWh}{m^2day}$ therefore maximum energy collecting occurs at (0.4, 0.36). To determine when the least amount of energy collection occurs, one can evaluate the minimum points. (-0.4, π/2) is a global minimum in the domain with a value of $0.46304\ \frac{kWh}{m^2day}$. The rest of the critical points lie in between the global maximum and minimum values, but are shown below.

Coordinate of critical point on contour plot	Energy collected ($\frac{kWh}{m^2day}$)
(0.32, 0.32)	2.13
(-0.4, -π/2)	1.54
(-0.4, π/2)	0.46
(0.4, -π/2)	0.61
(0.4, π/2)	1.39
(-0.4, 0.74)	1.06
(0.4, 0.36)	2.12
(-0.4,-0.74)	1.79

V. Optimizing Solar Panel Angle

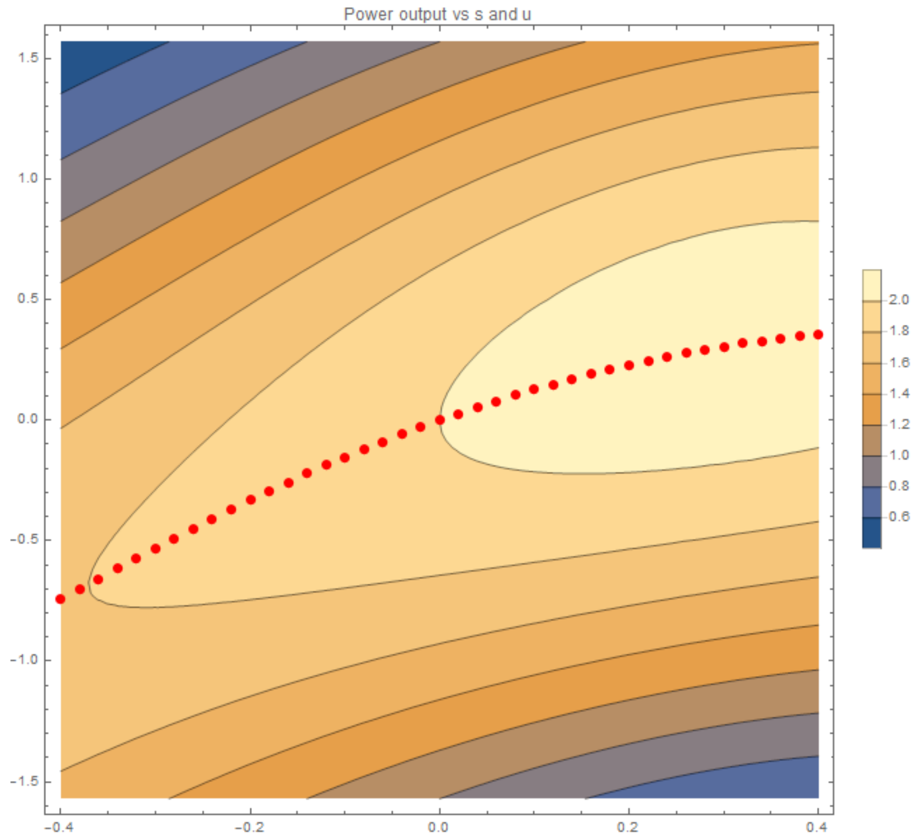

Fig. 4 Optimized path of energy production with respect to u and contour graph of W

The figure above shows the ideal angle at which energy should be collected during the year, tracing out the maximum values of the energy function. *Solar Power Inc* will decide the appropriate frequency at which the panels should be adjusted, according to the cost of labor against the amount of energy gained by adjusting the panels.
This plot makes sense as the optimized values of u follow the highest values at any given point on the contour plot. As the goal of this report is to maximize the energy collected over the course of a year, the maximum energy collection per day is required. This can be achieved by fixing s, and finding the u that corresponds to a maximum. The plot above clearly shows the ideal values of u follow the maximum possible energy outputs for each value of s.

VI. Obtaining Maximum Solar Energy

From the analysis above, it can be seen that the most energy will be obtained during the months leading up to and immediately following the summer solstice. The more direct angle of sunlight reaching the solar panels, in combination with the decreased amount of cloudiness, leads to a noticeable increase in energy collected per day; this is assuming that the panels are regularly adjusted to obtain the maximum amount of energy each day.

VII. Replacing Solar Panels

Unfortunately nothing lasts forever, including the company's cutting edge solar panels. The performance of the panels will deteriorate and will need to be replaced once per year. This is a lengthy process during which the panels will be off-line, and no energy will be collected. Intuitively, this process should be done during the time of year in which the least amount of energy can be collected. The increased cloud cover and smaller angle of sunlight striking the panels during the winter months will cause this to be the time of year in which energy collection rates are minimized. As such, the replacement process should be timed so that it is halfway through completion on the day of the winter solstice. For example, if the replacement process takes two weeks, it should be started one week before the solstice and completed one week after. This timing of replacement will ensure minimal energy losses over the year.

VIII. Conclusion

The aim of this report was to present *Solar Power Inc* with an analysis of solar panel efficiency for the equatorial island of Suluclac. Efficiency was dependent on the local atmospheric conditions, i.e. absorption rates and cloudiness, and the relative angles between the panels and incoming light throughout a day and year.

Figure 1 relates the cloud cover to the angle of the sun proportional to the time of day, and the angle of the sun to the equatorial axis throughout the year. Using the plot, it is evident that the maximum cloud cover occurs at noon, and the minimum occurs at dusk and dawn. Using the entire plot, it can be seen that the cloud cover is minimal at dusk and dawn throughout the year.

Figure 2 depicts the energy absorbed relative to angles s and u. Figure 3 represents the function W as a contour graph. Using the gradient vector, critical points of the energy function were found. The point $s = 0.3240$ rads and $u = 0.3205$ rads corresponded to a local maximum on the domain, producing a energy output of 2.1325 $\frac{kWh}{m^2 day}$. This value of s corresponds to some time during the Summer. The minimum amount of energy produced was found to be on the boundary of the function W. It occurred at $s = -0.4$ rads and $u = 1.57$ rads, corresponding to a energy output of 0.463 $\frac{kWh}{m^2 day}$.Logically, this occurs at dusk on the winter solstice.

Finally, the solar panels need to be replaced yearly; this process can take up to a week, and no energy can be produced during this time.Hence, to maximize yearly energy production, *Solar Power Inc* should perform maintenance/replacements when the amount of sunlight penetrating the atmosphere is a minimum. Using Figures 2 and 3, as well as the attached Mathematica code, this minimum occurs during the winter solstice (December 21st). As the process is cyclic, the replacement process should be timed such that it is halfway through completion on the day of the winter solstice.