Lab 4

University of Colorado Boulder
Department of Aerospace Engineering Sciences

ASEN 3112 - Section 014 - Group 14

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I. Results

Question 1: Buckling Load

The critical predicted buckling load, P_{crit} is the load at which the beam buckles and is no longer straight. It can be found analytically using Equation 1 where the Young's modulus is $E = 10000000$ psi, I is the second moment of inertia of the beam and L is the length of the beam with an additional 0.4in to account for fixtures used in the experimental setup.

$$P_{crit} = \frac{\pi^2 E I}{L^2} \tag{1}$$

The second moment of area for the hollow square can be found using $I_{xx} = I_{yy} = \frac{s_{out}^4}{12} - \frac{s_{in}^4}{12}$ where the outer side length is $s_{out} = 0.25$in and the inner side length is $s_{in} = 0.125$in. The second moment of area for the rectangle can be found using $I_{yy} = \frac{s_1^3 s_2}{12}$ where the side lengths are $s_1 = 0.125$in and $s_2 = 1$in. The analytical values were compared to the experimental values found in Figure 1. These force vs. displacements plots were used to identify the load at which the curve transitioned from the initial linear elastic response to buckling. This point is denoted with a black marker on each corresponding plot. This value was identified via visual inspection of where the curve became nonlinear.

The average of the two experimental critical loads was tabulated and compared to the analytically found values. For the square hollow case there is an error of 21.91% and for the rectangular solid there is an error of 32.14%. Theory assumes that the beam is homogeneous, prismatic and an elastic column pinned in an ideal hinge configuration. The material used could have had slight imperfections causing a smaller critical load. It is expected that the choice to neglect friction and stiffness in the hinges will have the largest effect to increase P_{crit} in the experiment over theory. Experimentally, there could have been an error in alignment in the buckling direction to the ruler and in the visual inspection of the experimental results to identify P_{crit}. Discrepancies could also be attributed to tolerance in the Arduino measurements or error associated with the load cell calibration and output.

Specimen	Length [in]	I [$lb * in^2$]	P_{crit} [lb]	Exp P_{crit} Average [lb]	Error
Square Hollow	10.65	3.0518e-04	265.5526	207.3510	21.9172%
Rectangular Solid	11.15	1.6276e-04	129.2108	87.6703	32.1494%

Table 1 Experimental and Analytical Values

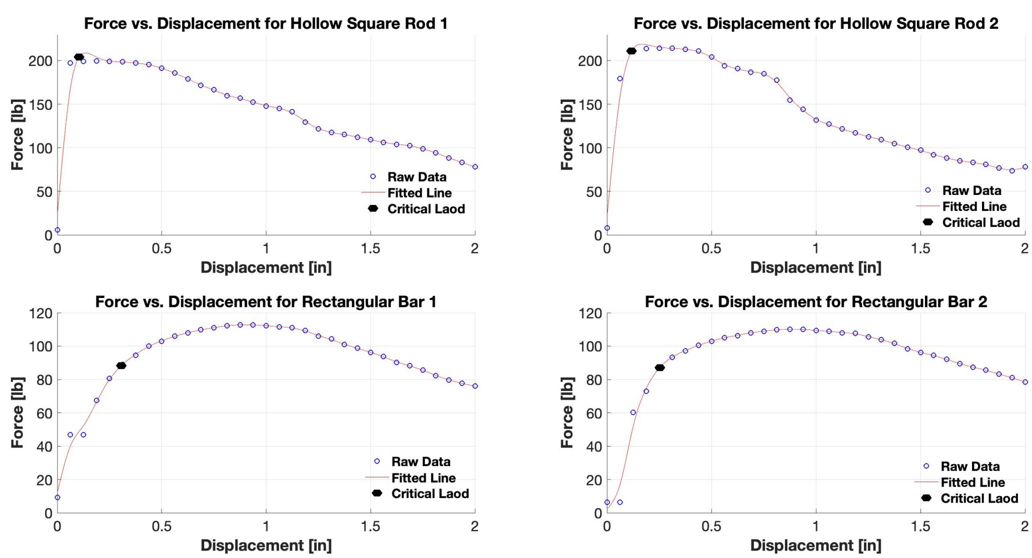

Fig. 1 Experimental Force vs. Displacements

Question 2: Post-Buckling Behavior

After calculating the critical buckling load for each case, a relationship between the load and the lateral deflection was approximated using the following equation:

$$P = P_{crit}(1 + \frac{\pi^2}{8L^2}\delta^2) \tag{2}$$

Using a range of horizontal deflections corresponding to those found experimentally, a vector of loads was created correlating to their respective displacements. For loads above the respective critical buckling loads, the predicted data follows the trend established by Equation 2. For values of loads below the corresponding critical buckling loads, it was assumed the load continuously increased while the horizontal deflection was maintained at zero and hence the specimen held its rigid structure up to the given load. The results of this analysis are shown below in Figure 2.

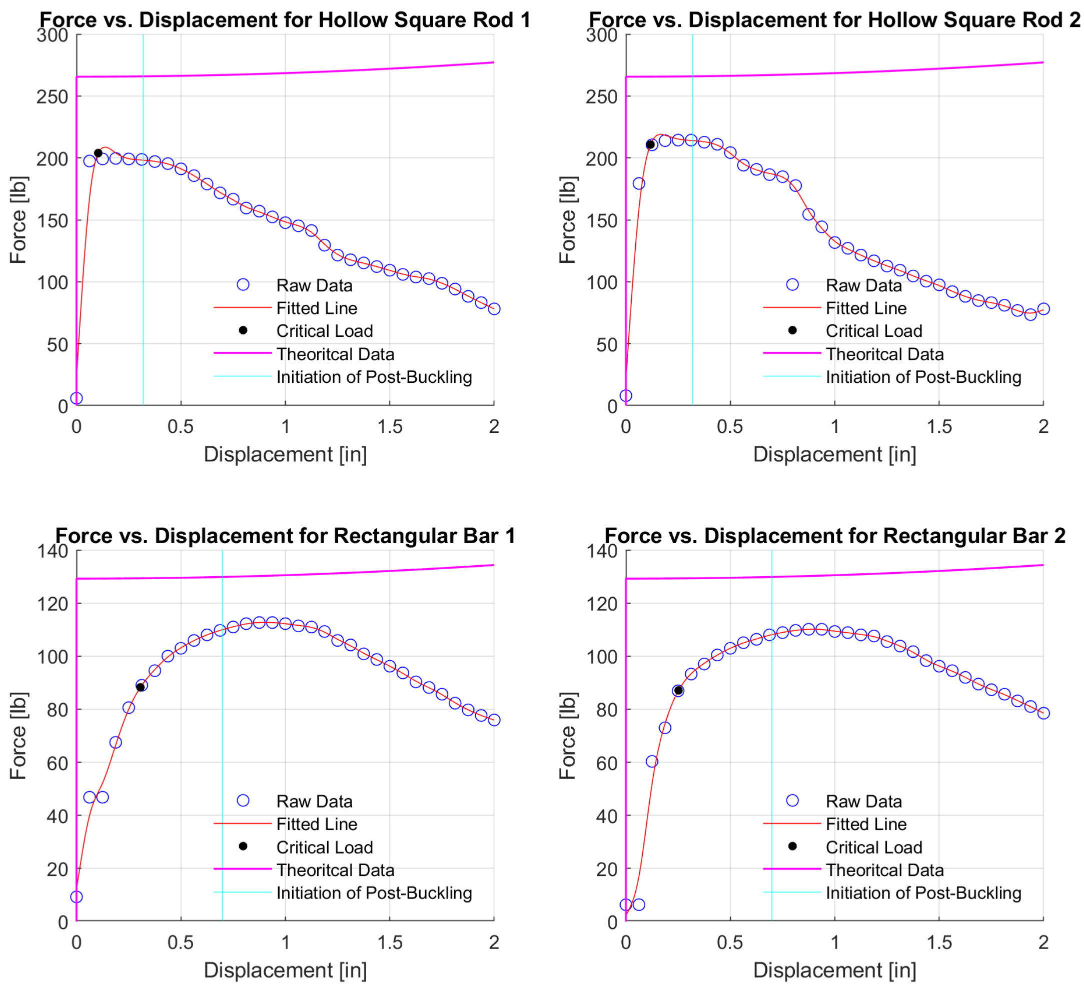

Fig. 2 Predicted and Experimentally Measured Applied Load vs Lateral Displacement

Next, the lateral deflection corresponding to the initiation of the plastic post-buckling was thoroughly examined. To arrive at the desired results, Hooke's law was employed as evident with Equation 3.

$$\sigma = E\epsilon \tag{3}$$

The strain at every point of the beam (ϵ) can be calculated using the following relationship:

$$\epsilon = \kappa(x)y \tag{4}$$

Where, the curvature of the beam (κ) can be approximated by:

$$\kappa(x) = v(x)" = \frac{d^2v}{dx^2} \tag{5}$$

Given that the buckled beam deforms according to the corresponding mode shape, which is determined by the geometry and boundary conditions. In the case pinned-pinned boundary conditions, the mode shape corresponds to a sine as seen by Equation 6.

$$v(x) = \delta sin\left(\frac{pi * x}{L}\right) \tag{6}$$

Where x corresponds to half the length of the specimen assuming that the maximum deflection occurs at that point.
By combining the process outlined above, the initiation of plastic post-buckling can be predicted using the following rearranged equation:

$$\delta = \left(\frac{1}{\sigma}E * \frac{t}{2} * \left(\frac{\pi^2}{L^2}\right) * sin\left(\frac{\pi * x}{L}\right)\right)^{-1} \tag{7}$$

By using Equation 7, for the aluminum square hollow cross section specimen, the initiation of plastic post-buckling was found to occur at a displacement of 0.3218 inches. Similarly, for the aluminum rectangular solid cross section, the initiation of the post-buckling occurred at a lateral deflection of 0.7054 inches. These results are displayed on Figure 2 for reference and ease of visualization.

Question 3: Design Study

The buckling load and yielding loads for each beam is plotted for a simply supported case in Figure 3 and a fixed-fixed case in Figure 4 versus beam length. As we would expect, a shorter beam is more resistant to buckling, and it can also be seen that the effect of lowering P_{crit} exhibits asymptotic behavior for large lengths. The yielding load does not vary with length, as this is only dependent on the cross-sectional area of the beams and the Young's Modulus.

If we only look about failure by yielding, the solid rectangular beam (beam 2) seems much stronger, since it has a greater cross-sectional area than the hollow square beam. However, for any length longer than about 6 inches in the simply supported case or 9 inches in the fixed-fixed case, failure by buckling will occur in the solid beam before failure by yielding. In this case, the hollow square beam will actually be stronger, and is also a more efficient choice in terms of material cost and weight. This same relationship of the hollow beam being stronger also holds for all lengths of the simply supported case that were explored, meaning that in most cases, the hollow beam is the better choice.

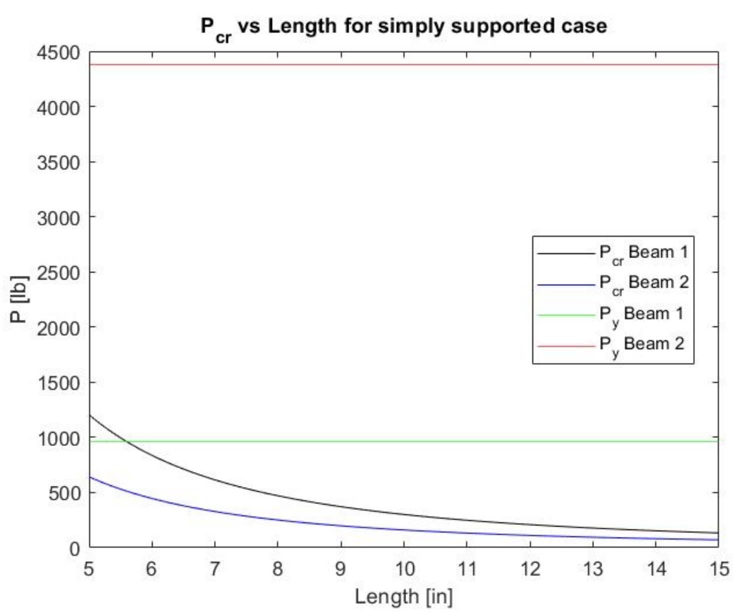

Fig. 3 Simply Supported Case

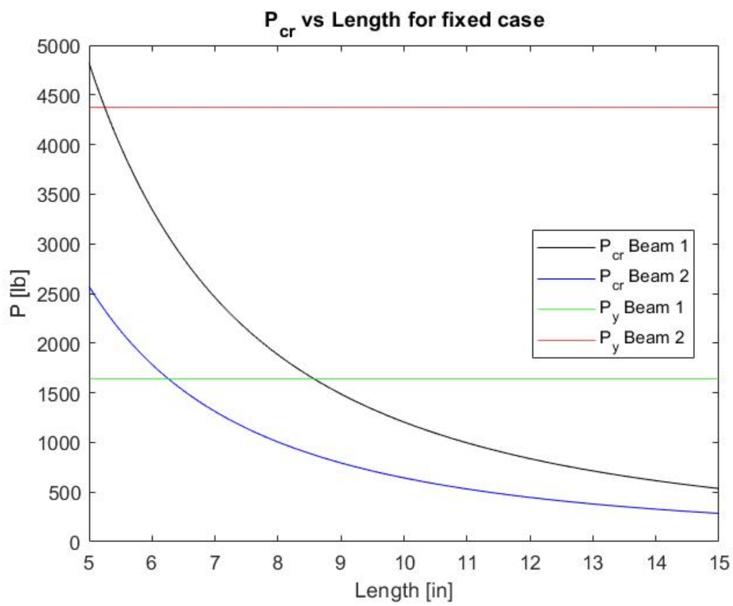

Fig. 4 Fixed-Fixed Case

References

[1] Hussein, M. *ASEN 3112 Structures Lab 4 Description*. ASEN 3112. Spring 2021.

[2] Hussein, M. *ASEN 3112 Structures Textbook*. ASEN 3112. Spring 2021.

II. Appendix: Code

```
%% Housekeeping
clear all
close all
clc

%% Import data
load('square_rod_1.mat')
sq1_data = data;
load('square_rod_2.mat')
sq2_data = data;
load('thinbar_ 1.mat')
bar1_data = data;
load('thinbar_2.mat')
bar2_data = data;

%% Knowns
% Rectangular rod material properties
L_bar = 11.15;
% Square rod material properties
L_sq = 10.65;

%% Analyze Sqaure 1 Data
% figure
% plot(sq1_data(:,3),sq1_data(:,2),'.')
% xlabel('Displacement Bin [in.]')
% ylabel('Voltage [mV]')
% title('Hollow Square Rod 1: Raw Data (Voltage vs. Displacement)')

% pull the first peice of data from each bin
counter = 1;
for i = 0:1/16:2
   temp = find(sq1_data(:,3) == i);
   mV_read(counter) = sq1_data(temp(2),2)*1000;

   counter = counter+1;

end

% convert to kg
lb_sq1 = mV_read/2.37;

% generate a fit structure
sq1_fit = fit([0:1/16:2]',lb_sq1', 'smoothingspline');


figure
```

```matlab
subplot(2,2,1)
hold on
legend boxoff
plot(sq1_fit, [0:1/16:2],lb_sq1, 'bo')
plot(0.104, 203.918,'k.','MarkerSize',15);
title('Force vs. Displacement for Hollow Square Rod 1');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Laod');
ylim([0,230])
grid on

%% Analyze Sqaure 2 Data
% figure
% plot(sq2_data(:,3),sq2_data(:,2),'.')
% xlabel('Displacement Bin [in.]')
% ylabel('Voltage [mV]')
% title('Hollow Square Rod 2: Raw Data (Voltage vs. Displacement)')


% pull the first peice of data from each bin
counter = 1;
for i = 0:1/16:2
   try
   temp = find(sq2_data(:,3) == i);
   mV_read(counter) = sq2_data(temp(2),2)*1000;

   counter = counter+1;

   catch
       fprintf('Square 2 Data:\n')
       fprintf('Displacement Bin Missing, Skipping Bin: %0.4f\n',i)
   end
end

% convert to kg
lb_sq2 = mV_read/2.37;

% generate a fit structure
sq2_fit = fit([0:1/16:2]',lb_sq2', 'smoothingspline');


subplot(2,2,2)
hold on
legend boxoff
plot(sq2_fit, [0:1/16:2],lb_sq2, 'bo')
plot(0.118, 210.784,'k.','MarkerSize',15);
title('Force vs. Displacement for Hollow Square Rod 2');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Laod');
ylim([0,230])
grid on
```

```
%% Analyze Bar 1 Data
% figure
% plot(bar1_data(:,3),bar1_data(:,2),'.')
% xlabel('Displacement Bin [in.]')
% ylabel('Voltage [mV]')
% title('Rectangular Bar 1: Raw Data (Voltage vs. Displacement)')

% pull the first peice of data from each bin
counter = 1;
for i = 0:1/16:2
   try
   temp = find(bar1_data(:,3) == i);
   mV_read(counter) = bar1_data(temp(2),2)*1000;

   counter = counter+1;

   catch
       fprintf('Bar 1 Data:\n')
       fprintf('Displacement Bin Missing, Skipping Bin: %0.4f\n',i)
   end
end

% convert to kg
lb_bar1 = mV_read/2.37;

% generate a fit structure
bar1_fit = fit([0:1/16:2]',lb_bar1', 'smoothingspline');

subplot(2,2,3)
hold on
legend boxoff
plot(bar1_fit, [0:1/16:2],lb_bar1, 'bo')
plot(0.306, 88.2457,'k.','MarkerSize',15);
title('Force vs. Displacement for Rectangular Bar 1');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Laod');
ylim([0,120])
grid on

%% Analyze Bar 2 Data
% subplot(2,2,4)
% plot(bar2_data(:,3),bar2_data(:,2),'.')
% xlabel('Displacement Bin [in.]')
% ylabel('Voltage [mV]')
% title('Rectangular Bar 2: Raw Data (Voltage vs. Displacement)')

% pull the first peice of data from each bin
counter = 1;
for i = 0:1/16:2
   try
   temp = find(bar2_data(:,3) == i);
   mV_read(counter) = bar2_data(temp(2),2)*1000;

```

```
    counter = counter+1;

    catch
        fprintf('Bar 2 Data:\n')
        fprintf('Displacement Bin Missing, Skipping Bin: %0.4f\n',i)
    end
end

% convert to kg
lb_bar2 = mV_read/2.37;

% generate a fit structure
bar2_fit = fit([0:1/16:2]',lb_bar2', 'smoothingspline');


subplot(2,2,4)
hold on
legend boxoff
plot(bar2_fit, [0:1/16:2],lb_bar2, 'bo')
plot(0.252, 87.095,'kd','MarkerSize',15);
title('Force vs. Displacement for Rectangular Bar 2');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Load');
ylim([0,120])
grid on

%% Question 1

e = 10000000;

% hollow rod geometry and area moment of inertia
so = 0.25;
si = 0.125;
i = (so^4)/12 - (si^4)/12;

% rectangularr rod geometry and area moment of ineretia
s1 = 0.125;
s2 = 1;
iy = (s1^3*s2)/12;

% solve for p_crit
p_hollow = ((pi^2*e*i)/(10.25 + 0.4)^2);
p_r_y = (pi^2*e*iy)/(10.75 + 0.4)^2;

% Experiminetal values
h1 = 203.918;
h2 = 210.784;
r1 = 88.2457;
r2 = 87.095;

h_avg = (h1+h2)/2
r_avg = (r1+r2)/2

```

```
h_error = (abs(h_avg - p_hollow)/p_hollow)*100

r_error = (abs(r_avg - p_r_y )/p_r_y )*100

%% Question 2
delta = 0:0.01:2;
for i = 1:length(delta)
    P_sq(i) = p_hollow*(1+pi^2/(8*L_sq^2)*delta(i)^2);
    P_rect(i) = p_r_y*(1+pi^2/(8*L_bar^2)*delta(i)^2);
end

E = 10000000; % psi
sigma = 35000; % psi

delta_crit_sq = ((1/sigma)*E*(so/2)*(pi^2/L_sq^2)*sin(pi*(L_sq/2)/L_sq))^(-1);
delta_crit_rect = ((1/sigma)*E*(s1/2)*(pi^2/L_bar^2)*sin(pi*(L_bar/2)/L_bar))
    ^(-1);

initSq = find(delta_crit_sq > delta,1,'last');
initRect = find(delta_crit_rect > delta,1,'last');

figure
subplot(2,2,1)
hold on
legend boxoff
plot(sq1_fit, [0:1/16:2],lb_sq1, 'bo')
plot(0.104, 203.918,'k.','MarkerSize',15);
plot([0,0,delta],[0,p_hollow,P_sq],'m','LineWidth',1)
xline(delta(initSq),'c')
title('Force vs. Displacement for Hollow Square Rod 1');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Load','Theoritcal Data','
    Initiation of Post-Buckling','Location','SouthEast');
grid on

subplot(2,2,2)
hold on
legend boxoff
plot(sq2_fit, [0:1/16:2],lb_sq2, 'bo')
plot(0.118, 210.784,'k.','MarkerSize',15);
plot([0,0,delta],[0,p_hollow,P_sq],'m','LineWidth',1)
xline(delta(initSq),'c')
title('Force vs. Displacement for Hollow Square Rod 2');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Load','Theoritcal Data','
    Initiation of Post-Buckling','Location','SouthEast');
grid on

subplot(2,2,3)
hold on
legend boxoff
plot(bar1_fit, [0:1/16:2],lb_bar1, 'bo')
```

```matlab
plot(0.306, 88.2457,'k.','MarkerSize',15);
plot([0,0,delta],[0,p_r_y,P_rect],'m','LineWidth',1)
xline(delta(initRect),'c')
title('Force vs. Displacement for Rectangular Bar 1');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Load','Theoritcal Data','
    Initiation of Post-Buckling','Location','SouthEast');
grid on

subplot(2,2,4)
hold on
legend boxoff
plot(bar2_fit, [0:1/16:2],lb_bar2, 'bo')
plot(0.252, 87.095,'k.','MarkerSize',15);
plot([0,0,delta],[0,p_r_y,P_rect],'m','LineWidth',1)
xline(delta(initRect),'c')
title('Force vs. Displacement for Rectangular Bar 2');
ylabel('Force [lb]');
xlabel('Displacement [in]');
legend('Raw Data', 'Fitted Line', 'Critical Load','Theoritcal Data','
    Initiation of Post-Buckling','Location','SouthEast');
grid on
```

```matlab
%% Lab 4 Part 3
%Quentin H. Morton {109209382}
%April 26, 2021

%% housekeeping
clear
clc
close all

%% Define things
E = 10000000; %[psi]
I_1 = 3.0518*10^-4; %[lb*in^2]
I_2 = 1.6276*10^-4; %[lb*in^2]
%Calculate EI for each beam
EI_1 = E*I_1;
EI_2 = E*I_2;
%Make vector of lengths to use for graphs
L = linspace(5, 15, 100); %[in]

%% Critical loads P_cr

%pinned-pinned
Pcr_1_p = pi^2*EI_1./(L.^2);
Pcr_2_p = pi^2*EI_2./(L.^2);
%fixed-fixed
k_star = 0.5;
Pcr_1_f = (pi^2)*EI_1./(k_star^2.*L.^2);
Pcr_2_f = (pi^2)*EI_2./(k_star^2.*L.^2);

```

```
%% Yielding loads
%Yield Strength
sig_y = 35000; %[psi]
%CS Area of beams
A_1 = (0.25^2) - (0.25-2*(0.0625))^2; %[in^2]
A_2 = 0.125; % [in^2]
%And...math
Py_1 = sig_y*A_1;
Py_2 = sig_y*A_2;


%% Make figures!
figure (1)
%Has buckling and yielding loads for simply supported case
plot(L, Pcr_1_p, 'k')
hold on
plot(L, Pcr_2_p, 'b');
yline(Py_1, 'g');
yline(Py_2, 'r');
hold off
xlabel('Length [in]')
ylabel('P [lb]')
title('P_{cr} vs Length for simply supported case')
legend('P_{cr} Beam 1', 'P_{cr} Beam 2', 'P_y Beam 1', 'P_y Beam 2','Location'
    , 'Best');

figure (2)
%Has buckling and yielding loads for fixed-fixed case
plot(L, Pcr_1_f, 'k');
hold on
plot(L, Pcr_2_f, 'b');
yline(Py_1, 'g');
yline(Py_2, 'r');
hold off
xlabel('Length [in]')
ylabel('P [lb]')
title('P_{cr} vs Length for fixed case')
legend('P_{cr} Beam 1', 'P_{cr} Beam 2','P_y Beam 1', 'P_y Beam 2', 'Location'
    , 'Best');
```

III. Appendix: Participation Report

Group Leader: Preston Tee

Other Member Contributions:

Connor O'Reilly: Participated in report writing and proofing, compiled, formatted, and organized code. **Contribution Factor: 100%**

Quentin Morton: Lead question 3 analysis, code, and report section, participated in report writing and proofing. **Contribution Factor: 100%**

Maria Callas: Lead question 1 analysis, code, and report section, participated in report writing and proofing. **Contribution Factor: 100%**

Preston Tee: Team lead, provided assistance to other members' analysis, lead integration of report sections and proofing.**Contribution Factor: 100%**

Slava Rychenko: Lead question 2 analysis, code, and report section, participated in report writing and proofing. **Contribution Factor: 100%**