Lab 3

University of Colorado Boulder
Department of Aerospace Engineering Sciences

ASEN 3112 - Section 014 - Group 14

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I. Results

Question 1: Experimental Results

After the experimental results for the simulation were collected, they were parsed and processed for ease of access and interpretation. After some examination, it was noticed that the displacement of each channel linearly increased with time in addition to the effects experienced by the vibrations. In order to account for this, the initial values were first zeroed for each data set were zeroed to account for the start of the experiment and data collection period. Next, a linear regression of the data was computed modeling the gradual increase of the displacement data. The linear regression values were then subtracted from each corresponding channel displacement data to better display the effects of the vibrations. The results are seen in Figures 1 and 2 below.

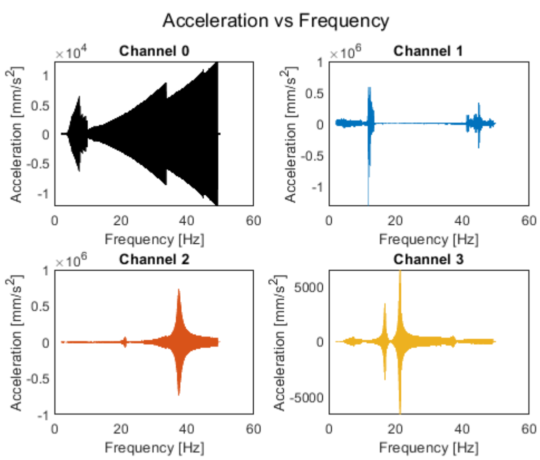

Fig. 1 Acceleration Data

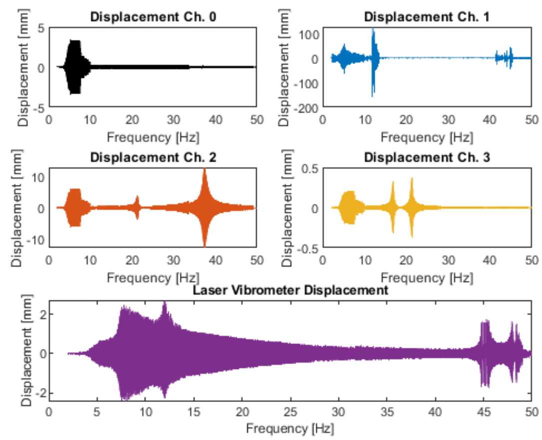

Fig. 2 Displacement Data

Since the shaker table was known to have increased its frequency linearly with time, implementing a Fast Fourier Transform (FFT) on the data would give results independent of any startup, shutdown, or other unknown transients. This FFT was preformed on the acceleration and displacement data in MATLAB, and produces amplitudes vs frequency for each corresponding data set. As the focus of this lab is to investigate resonance and modes, the shaker table's data is plotted as its dimensional amplitude whereas all other data sets were normalized by dividing by the shaker data. This was done frequency-by-frequency, so for any given frequency, the non-shaker amplitude is divided by the shaker amplitude at the same frequency, resulting in a value that represents how much more or less the component vibrates compared to the forcing input.

The results of this analysis are seen in the figures below for each channel on the top two rows along with the vibrometer displacement on the bottom. In comparison, the vibrometer attempts to provide a full picture of the system rather than looking at individual parts of the structure as done by the four channels on the center fuselage, tail, nose and wing respectively.

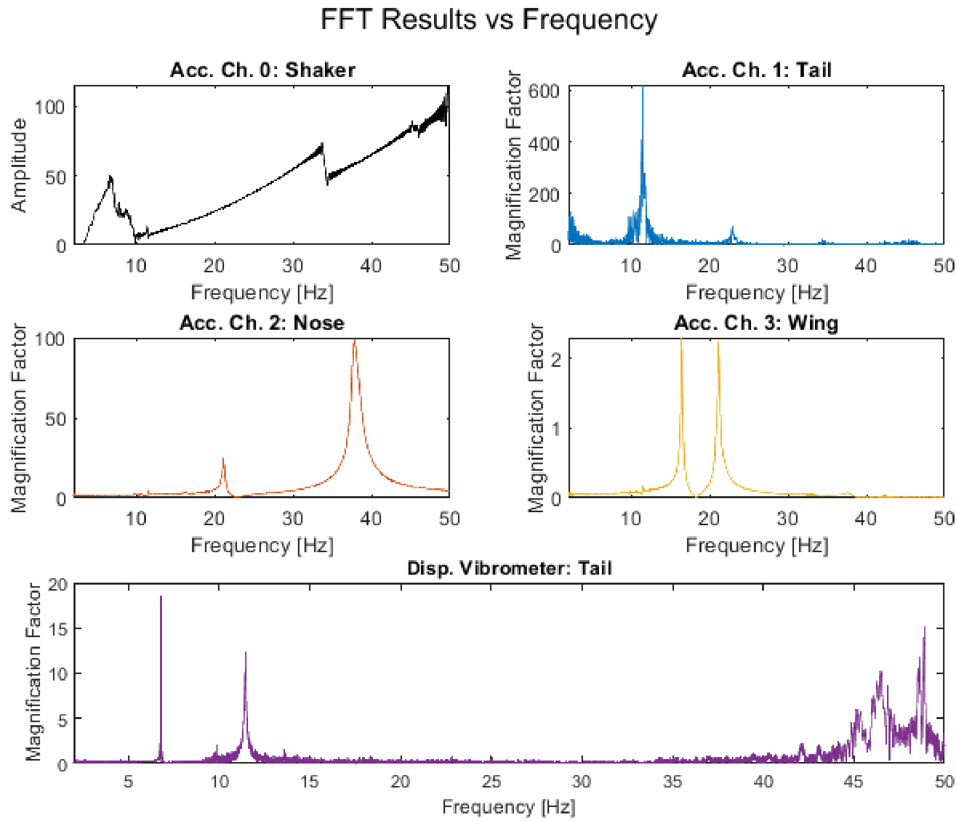

Fig. 3 Response of System due to Excitation Frequency

The results from the FFT analysis produced damped frequencies rather than the desired natural frequency that is sought by in the analysis. With this in mind, a logarithmic decrement function was created to find the damping ratio of each data set and use that data to find the corresponding natural frequency given the previously calculated damped frequencies. Based on this analysis along with Figure 3 the greatest magnification of vibrations occurred at following resonant frequencies: 42.7 rad/s, 71.8 rad/s, 132.8 rad/s, 237.5 rad/s, and 312.1 rad/s. The results are further summarized and displayed below in Table 1.

Mode	**Sensor**	**"Damped" Natural Freq [rad/s]**	**Damping Ratio**	**Natural Frequency [rad/s]**
1	**Tail**	**42.7460**	**0.01145**	**42.7488**
2	**Tail**	**71.6892**	**0.04883**	**71.7748**
3	**Wing**	**132.7781**	**0.01757**	**132.7986**
4	**Nose**	**237.4844**	**0.004203**	**237.4865**
5	**Fuselage**	**312.0923**	**0.007086**	**312.1001**

Table 1 Describing values of Each mode determined from experimental data

In order to look at possible trends between parts of the aircraft model more closely, a non-physical plotting approach is taken below. In order to look more closely at when the amplification peaks occur, instead of looking at how big they are, the tail, nose, wingtip, and vibrometer plots are all scaled by their own maximums. Again, no physical meaning is held in the heights of the peaks in Figure 3, as the point of the plot is not only to visualize each response on top of each-other, but instead to more clearly see when peaks line up on the frequency axis.

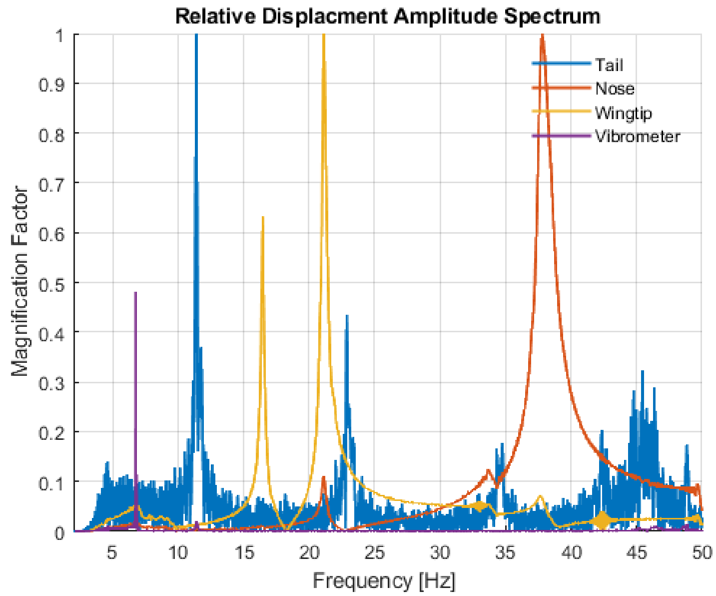

Fig. 4 Collection of Responses Scaled to Have a Maximum Response of 1

This technique allows a clearer picture of what parts of the aircraft model are involved in each mode. From the above plot to get qualitative data, and Figure 3 for quantitative data, it can be seen that Mode 1 consists of the wings vibrating and tail vibrating horizontally, with the tail resonating much more strongly than the wings. Mode 2 and 3 consists of a tail only and wing only response, respectively. The fourth mode shows responses in the nose and wings, with a slightly delayed response in the tail that may not actually be associated with the mode. Finally, the last mode in our frequency sweep was a strong nose resonance.

Question 2: FEM Results - Resonant Frequencies

Two FEM models are used to predict the natural frequency of this airplane shaker system. One has two elements, and the other has four. For each model, we have a master stiffness matrix $\hat{K}$ and a mass matrix $\hat{M}$, which are given. These matrices were reduced by removing the first and second rows and columns of $\hat{K}$ and $\hat{M}$, which contain only zeros. In the two-element model this results in:

$$\hat{K}_2 = \begin{bmatrix} 184 & 0 & -92 & 276 \\ 0 & 2208 & -276 & 552 \\ -92 & -276 & 92 & -276 \\ 276 & 0 & -276 & 1104 \end{bmatrix} \tag{1}$$

$$\hat{M}_2 = \begin{bmatrix} 0.000143 & 0 & 0.000022 & -0.000029 \\ 0 & 0.000185 & 0.000029 & -0.000039 \\ 0.000022 & 0.000029 & 0.000355 & 0.000076 \\ -0.000029 & -0.000039 & 0.000076 & 0.005885 \end{bmatrix} \tag{2}$$

and in the four-element model,

$$\hat{K}_4 = \begin{bmatrix} 1472 & 0 & -736 & 1104 & 0 & 0 & 0 & 0 \\ 0 & 4416 & -1104 & 1104 & 0 & 0 & 0 & 0 \\ -736 & -1104 & 1472 & 0 & -736 & 1104 & 0 & 0 \\ 1104 & 1104 & 0 & 4416 & -1104 & 1104 & 0 & 0 \\ 0 & 0 & -736 & -1104 & 1472 & 0 & -736 & 1104 \\ 0 & 0 & 1104 & 1104 & 0 & 4416 & -1104 & 1104 \\ 0 & 0 & 0 & 0 & -736 & -1104 & 736 & -1104 \\ 0 & 0 & 0 & 0 & 1104 & 1104 & -1104 & 2208 \end{bmatrix} \tag{3}$$

$$\hat{M}_4 = \begin{bmatrix} 0.000072 & 0 & 0.000011 & -0.000007 & 0 & 0 & 0 & 0 \\ 0 & 0.000023 & 0.000007 & -0.000005 & 0 & 0 & 0 & 0 \\ 0.000011 & 0.000007 & 0.000072 & -0.000005 & 0.000011 & -0.000007 & 0 & 0 \\ -0.000007 & -0.000005 & 0 & 0.000023 & 0.000007 & -0.000005 & 0 & 0 \\ 0 & 0 & 0.000011 & 0.000007 & 0.000072 & 0 & 0.000011 & -0.000007 \\ 0 & 0 & -0.000007 & -0.000005 & 0 & 0.000023 & 0.000007 & -0.000005 \\ 0 & 0 & 0 & 0 & 0.000011 & 0.000007 & 0.000319 & 0.000125 \\ 0 & 0 & 0 & 0 & -0.000007 & -0.000005 & 0.000125 & 0.005804 \end{bmatrix} \tag{4}$$

Then, Equation 5 was used to solve for the resonant frequency, ω. We do not need to know $\mathbf{U}$ to find ω, since we can rearrange this to Equation 6 and find the ω values that cause a determinant of zero for the matrix $[\hat{K} - \omega^2 \hat{M}]$.

$$\hat{K}\mathbf{U} = \omega^2 \hat{M}\mathbf{U} \tag{5}$$

$$[\hat{K} - \omega^2 \hat{M}]\mathbf{U} = 0 \tag{6}$$

The first three resonant frequencies of this system with the two-element model are 12.0285 Hz, 51.0263 Hz, and 202.5140 Hz. With the four-element model the frequencies are 12.0294 Hz, 51.0824 Hz, and 203.3665 Hz. Between the two FEM models, there is 0.0075% error for the first resonant frequency, 0.11% error for the second, and 0.42% error. Since these errors are small and accuracy of the models do not increase significantly when we double the number of elements, we can be confident that adding more elements to the model would not be worth the extra computation cost, as it is already about as accurate as it will get and we can accurately use either of these models to go forward with error analysis.

Converting units, the resonant frequencies from the FEM model become 75.6 rad/s, 320.6 rad/s, and 1272.4 rad/s. The third of these is far above any frequencies measured during the experiment, but the first two align with two of the resonant frequencies identified earlier.

The second and fifth natural frequencies identified were 71.7748 rad/s and 312.1 rad/s. Compared to the FEM model frequencies that line up with these, they have, respectively, 5.06%, and 2.65% error. These errors are reasonable, and there are several reasons why they still occur.

Although the FEM models are accurate to the movement of the fuselage element of the airplane model, it does not take into account the other elements of the wing or tail, which likely have a small effect on how resonant frequencies show up in experiment. There may also be manufacturing imperfections that affect the properties of the material used in the experiment, including imperfections in the metal throughout, and the affect of the fastening of the wing and tail on the strength of the metal.

Question 3: FEM Results - Mode Shapes

Figures 5 and 6 display the shapes of the first three modes provided by the 2-element and 4-element FE models respectively. The resonant frequencies between each mode were very similar and it is expected that the mode shapes are also similar. The y-axis can be multiplied by an arbitrary constant and the mode shape will not change because it gives information about *relative* displacement. The mode shapes can be compared to the experimental data. Note that the experimental measurements span the entire plane while the FE model solely models the fuselage and tail so only those experimental displacements can be compared to the FE models. The first two model resonant frequencies are within the 2-50Hz experimental range. The normalized displacement from the center fuselage and tail sensors near the resonant frequencies are plotted at corresponding frequencies. The FE model only considers the back of the plane and is a rough approximation. There are some slight discrepancies between the relative displacement in the FE model and the experimental tail displacement. For the 50 Hz case there was no displacement in the tail sensor at 50 Hz, so visual inspection of Figure 2 to see displacement was used to identify that near 44 Hz corresponded well to the FE model. The modes represent behavior in the entire structure but specific modes at distinct frequencies can be active at a certain part of the structure. For example in the first two mode shapes it can be concluded that the motion is localized in the tail.

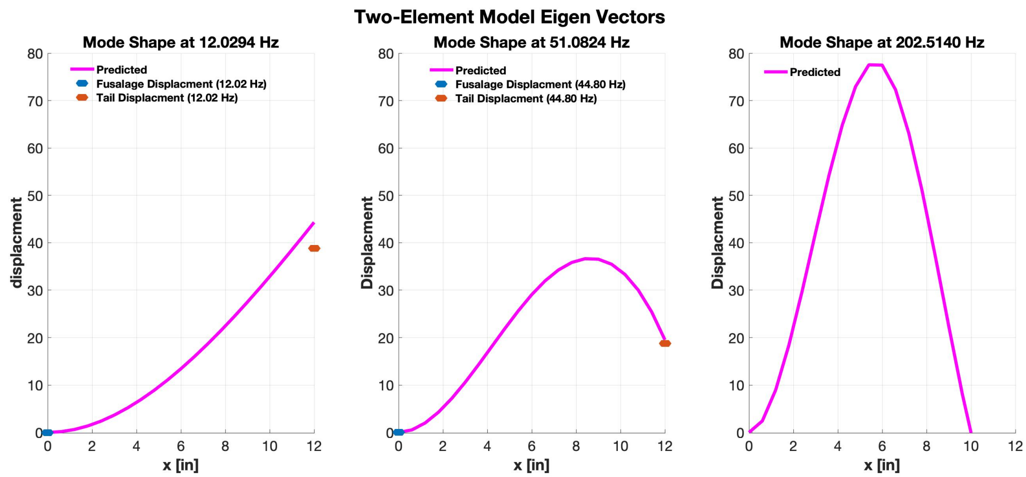

Fig. 5 Two Element Model Mode Shapes Compared with Experimental Fuselage and Tail Displacements

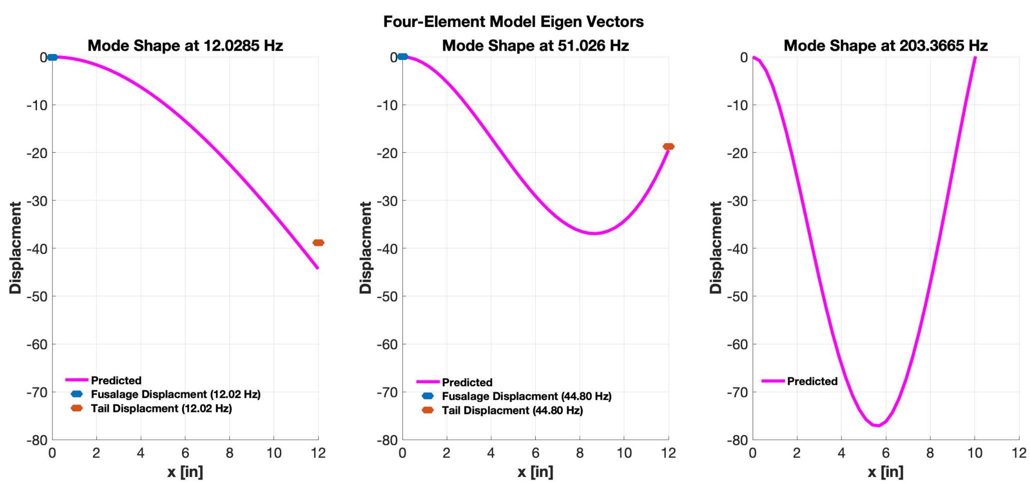

Fig. 6 Four Element Model Mode Shapes Compared with Experimental Fuselage and Tail Displacements

References

[1] Hussein, M. *ASEN 3112 Structures Lab 3 Description*. ASEN 3112. Spring 2021.

[2] Hussein, M. *ASEN 3112 Lab 3 Data Labels Airplane*. ASEN 3112. Spring 2021.

[3] Hussein, M. *ASEN 3112 Structures Textbook*. ASEN 3112. Spring 2021.

II. Appendix: Code

```
% Maria Callas
% Connor O'Reilly
% Quentin Morton
% Vyacheslav Rychenko
% Preston Tee
% ASEN 3112 - Lab 3
% ASEN3112_Lab3_Main.m
% Edited: 04/17/21

% Inputs: Airplane Data
% Output: Plots

% Housekeeping
clear all;
close all;
clc;

% Read in the Data
airplaneDataStruct = importdata('ASEN 3112_Sp21_Lab_3_Data-Airplane.txt');
airplaneData = airplaneDataStruct.data;

% Parse through the data
time = airplaneData(:,1);
AccCh0 = airplaneData(:,2);
AccCh1 = airplaneData(:,3);
AccCh2 = airplaneData(:,4);
AccCh3 = airplaneData(:,5);
DispCh0 = airplaneData(:,6);
DispCh1 = airplaneData(:,7);
DispCh2 = airplaneData(:,8);
DispCh3 = airplaneData(:,9);
LaserVibro = airplaneData(:,10);

% Rezero Variables
time = time - time(1);
AccCh0 = AccCh0 - AccCh0(1);
AccCh1 = AccCh1 - AccCh1(1);
AccCh2 = AccCh2 - AccCh2(1);
AccCh3 = AccCh3 - AccCh3(1);
DispCh0 = DispCh0 - DispCh0(1);
DispCh1 = DispCh1 - DispCh1(1);
DispCh2 = DispCh2 - DispCh2(1);
DispCh3 = DispCh3 - DispCh3(1);
LaserVibro = LaserVibro - LaserVibro(1);

% Frequency Vector
freq = linspace(2,50,length(time))';

% Plot Acceleration vs Frequency
figure(1)
subplot(2,2,1)
```

```
plot(freq, AccCh0,'k')
title('Channel 0')
xlabel('Frequency [Hz]')
ylabel('Acceleration [mm/s^2]')
subplot(2,2,2)
plot(freq, AccCh1)
title('Channel 1')
xlabel('Frequency [Hz]')
ylabel('Acceleration [mm/s^2]')
subplot(2,2,3)
plot(freq, AccCh2, 'color', '#D95319')
title('Channel 2')
xlabel('Frequency [Hz]')
ylabel('Acceleration [mm/s^2]')
subplot(2,2,4)
plot(freq, AccCh3, 'color', '#EDB120')
title('Channel 3')
xlabel('Frequency [Hz]')
ylabel('Acceleration [mm/s^2]')

sgtitle('Acceleration vs Frequency')

% Linear Regression
Ch0_Poly = polyfit([freq(1),freq(end)],[DispCh0(1),DispCh0(end)],1);
Ch1_Poly = polyfit([freq(1),freq(end)],[DispCh1(1),DispCh1(end)],1);
Ch2_Poly = polyfit([freq(1),freq(end)],[DispCh2(1),DispCh2(end)],1);
Ch3_Poly = polyfit([freq(1),freq(end)],[DispCh3(1),DispCh3(end)],1);

% Adjust for Magnification Factor
DispCh0_Processed = DispCh0 - Ch0_Poly(1).*freq;
DispCh1_Processed = DispCh1 - Ch1_Poly(1).*freq;
DispCh2_Processed = DispCh2 - Ch2_Poly(1).*freq;
DispCh3_Processed = DispCh3 - Ch3_Poly(1).*freq;

% Rezero the Processed Data
DispCh0_Processed = DispCh0_Processed - DispCh0_Processed(1);
DispCh1_Processed = DispCh1_Processed - DispCh1_Processed(1);
DispCh2_Processed = DispCh2_Processed - DispCh2_Processed(1);
DispCh3_Processed = DispCh3_Processed - DispCh3_Processed(1);

% % Plot Displacement vs Frequency
figure
subplot(3,2,1)
plot(freq,DispCh0_Processed,'k')
title('Displacement Ch. 0')
xlabel('Frequency [Hz]')
ylabel('Displacement [mm]')
subplot(3,2,2)
plot(freq,DispCh1_Processed)
title('Displacement Ch. 1')
xlabel('Frequency [Hz]')
ylabel('Displacement [mm]')
subplot(3,2,3)
plot(freq,DispCh2_Processed, 'color', '#D95319')
```

```
title('Displacement Ch. 2')
xlabel('Frequency [Hz]')
ylabel('Displacement [mm]')
subplot(3,2,4)
plot(freq,DispCh3_Processed, 'color', '#EDB120')
title('Displacement Ch. 3')
xlabel('Frequency [Hz]')
ylabel('Displacement [mm]')
subplot(3,2,[5 6])
plot(freq, LaserVibro, 'color', '#7E2F8E')
title('Laser Vibrometer Displacement')
xlabel('Frequency [Hz]')
ylabel('Displacement [mm]')

%
% sgtitle('Displacement vs Frequency')
%
% % Plot Vibrometer
% figure(3)
% plot(freq, LaserVibro)
% title('Laser Vibrometer Displacement')
% xlabel('Frequency [Hz]')
% ylabel('Displacement [mm]')

% Find Index for Resonant Frequency
[~,I0] = max(DispCh0_Processed);
[~,I1] = max(DispCh1_Processed);
[~,I2] = max(DispCh2_Processed);
[~,I3_1] = max(DispCh3_Processed(1:264500));
[~,I3_2] = max(DispCh3_Processed);

% Resonant Frequency
Res_Freq = [freq(I0),freq(I1),freq(I2),freq(I3_1),freq(I3_2)];

%% Question 1: Analyzing Data with FFT

% sampling and nyquist frequencies
t_s = (time(end) - time(1))/length(time);   % time per sample
f_s = 1/t_s;                                % sampling frequency
f_n = f_s/2;                                % nyquist frequncy
n = length(time);                           % number of samples

% Acceleration data
AccCh0 = airplaneData(:,2);
AccCh1 = airplaneData(:,3);
AccCh2 = airplaneData(:,4);
AccCh3 = airplaneData(:,5);

% Perform FFT Analysis on each Acceleration Channel and Vibrometer
[af0,ay0] = fft_analysis(n, AccCh0, f_s);
[af1,ay1] = fft_analysis(n, AccCh1, f_s);
[af2,ay2] = fft_analysis(n, AccCh2, f_s);
[af3,ay3] = fft_analysis(n, AccCh3, f_s);
[d0f,d0y] = fft_analysis(n, DispCh0_Processed, f_s);
```

```
[vf,vy] = fft_analysis(n, LaserVibro, f_s);

% Find the maximum damped frequency of each analysis
[~,index0] = max(ay0);
omegaD_0 = af0(index0);
[~,index1] = max(ay1);
omegaD_1 = af1(index1);
[~,index2] = max(ay2);
omegaD_2 = af2(index2);
[~,index3] = max(ay3);
omegaD_3 = af3(index3);
[~,index_Vibro] = max(vy);
omegaD_Vibro = vf(index_Vibro);

% Find the damping ratio and natural frequency of each analysis
log_dec_func(index0 , omegaD_0, AccCh0, time)
log_dec_func(index1 , omegaD_1, AccCh1, time)
log_dec_func(index2 , omegaD_2, AccCh2, time)
log_dec_func(index3 , omegaD_3, AccCh3, time)
log_dec_func(index_Vibro , omegaD_Vibro, LaserVibro, time)

%% Plot Acceleration vs Frequency
figure(1)
subplot(3,2,1)
plot(af0, ay0, 'k')
hold off
title('Acc. Ch. 0: Shaker')
xlabel('Frequency [Hz]')
ylabel('Amplitude')
xlim([2 50]);
subplot(3,2,2)
plot(af1, ay1./ay0)
title('Acc. Ch. 1: Tail')
xlabel('Frequency [Hz]')
ylabel('Magnification Factor')
xlim([2 50]);
subplot(3,2,3)
plot(af2, ay2./ay0, 'color', '#D95319')
title('Acc. Ch. 2: Nose')
xlabel('Frequency [Hz]')
ylabel('Magnification Factor')
xlim([2 50]);
subplot(3,2,4)
plot(af3, ay3./ay0, 'color', '#EDB120')
title('Acc. Ch. 3: Wing')
xlabel('Frequency [Hz]')
ylabel('Magnification Factor')
xlim([2 50]);
subplot(3,2,[5 6])
plot(vf, vy./d0y, 'color', '#7E2F8E')
title('Disp. Vibrometer: Tail')
xlabel('Frequency [Hz]')
ylabel('Magnification Factor')
xlim([2 50]);
```

```
sgtitle('FFT Results vs Frequency')

figure
grid on
legend boxoff
hold on
title('Relative Displacment Amplitude Spectrum');
xlabel('Frequency [Hz] ');
ylabel('Magnification Factor');
plot(af1, ay1/max(ay1), 'LineWidth', 1.5)
plot(af2, ay2/max(ay2), 'LineWidth', 1.5)
plot(af3, ay3/max(ay3), 'LineWidth', 1.5)
plot(vf,vy, 'LineWidth', 1.5)
legend('Tail', 'Nose', 'Wingtip','Vibrometer');
xlim([2 50]);

%%  Question 3: Mode Shapes

%% Call provided FE code to get x and v vectors
[x_2, v_2, f2] = Lab_3_FEM_Solve(2);
[x_4, v_4, f4] = Lab_3_FEM_Solve(4);

%% look for experimental displacments correspsonding to FE resonant f
[f2_1_val,f2_1_index]=min(abs(freq-f2(1)));
[f2_2_val,f2_2_index]=min(abs(freq-44.80));

f1_2_fusalage = DispCh0_Processed(f2_1_index);
f1_2_tail = DispCh1_Processed(f2_1_index);

f2_2_fusalage = DispCh0_Processed(f2_2_index);
f2_2_tail = DispCh1_Processed(f2_2_index);

[f4_1_val,f4_1_index]=min(abs(freq-f2(1)));
[f4_2_val,f4_2_index]=min(abs(freq-44.80));

f1_4_fusalage = DispCh0_Processed(f4_1_index);
f1_4_tail = DispCh1_Processed(f4_1_index);

f2_4_fusalage = DispCh0_Processed(f4_2_index);
f2_4_tail = DispCh1_Processed(f4_2_index);

figure
subplot(1,3,1)
grid on
legend boxoff
hold on
title('Mode Shape at 12.0294 Hz');
xlabel('x [in]');
ylabel('displacment');
set(gca,'FontSize',18);
% plot(x_4(:,3),v_4(:,3)./abs((max(abs(v_4(:,3))))),'m', 'LineWidth',4)
plot(x_2(:,3),v_2(:,3),'m', 'LineWidth',4)
plot(0, f1_2_fusalage, 'd', 'MarkerFaceColor', 'k', 'LineWidth',8);
```

```
plot(12, -f1_2_tail, 'd','MarkerFaceColor', 'g', 'LineWidth',8);
legend('Predicted', 'Fusalage Displacment (12.02 Hz)', 'Tail Displacment (12.02 Hz)');
ylim([-0.1 80]);
xlim([0 12]);

subplot(1,3,2)
grid on
legend boxoff
hold on
title('Mode Shape at 51.0824 Hz','fontweight','bold', 'Fontsize', 16);
xlabel('x [in]', 'fontweight','bold', 'Fontsize', 14);
ylabel('Displacment', 'fontweight','bold', 'Fontsize', 14);
set(gca,'FontSize',18);
%plot(x_4(:,4),v_4(:,4)./abs((max(abs(v_4(:,4))))),'m', 'LineWidth',4)
plot(x_2(:,4),v_2(:,4),'m', 'LineWidth',4)
plot(0, f2_2_fusalage, 'd', 'LineWidth',8);
plot(12, -f2_2_tail, 'd', 'LineWidth',8);
legend('Predicted', 'Fusalage Displacment (44.80 Hz)', 'Tail Displacment (44.80 Hz)', 'fontweight','bold', 'Fontsize', 14);
ylim([-0.1 80]);
xlim([0 12]);
sgtitle('Two-Element Model Eigen Vectors','fontweight','bold', 'Fontsize',24);

subplot(1,3,3)
grid on
legend boxoff
hold on
title('Mode Shape at 202.5140 Hz','fontweight','bold', 'Fontsize', 16);
xlabel('x [in]', 'fontweight','bold', 'Fontsize', 14);
ylabel('Displacment', 'fontweight','bold', 'Fontsize', 14);
set(gca,'FontSize',18);
plot(x_2(:,5),v_2(:,5),'m', 'LineWidth',4)
legend('Predicted', 'fontweight','bold', 'Fontsize', 14);
sgtitle('Two-Element Model Eigen Vectors','fontweight','bold', 'Fontsize',24);
ylim([-0.1 80]);
xlim([0 12]);

figure
subplot(1,3,1)
grid on
legend boxoff
hold on
title('Mode Shape at 12.0285 Hz','fontweight','bold', 'Fontsize', 16);
xlabel('x [in]', 'fontweight','bold', 'Fontsize', 14);
ylabel('Displacment', 'fontweight','bold', 'Fontsize', 14);
set(gca,'FontSize',18);
% plot(x_2(:,3),v_2(:,3)./abs((max(abs(v_2(:,3))))),'m','LineWidth',4)
plot(x_4(:,3),v_4(:,3),'m','LineWidth',4)
plot(0, f1_4_fusalage, 'd', 'LineWidth',8);
plot(12,f1_4_tail, 'd', 'LineWidth',8);
legend('Predicted', 'Fusalage Displacment (12.02 Hz)', 'Tail Displacment (12.02 Hz)',...
    'fontweight','bold', 'Fontsize', 14, 'Location', 'southwest');
```

```
ylim([-80 0.1]);
xlim([0 12]);

subplot(1,3,2)
grid on
legend boxoff
hold on
title('Mode Shape at 51.026 Hz','fontweight','bold', 'Fontsize', 16);
xlabel('x [in]', 'fontweight','bold', 'Fontsize', 14);
ylabel('Displacment', 'fontweight','bold', 'Fontsize', 14);
set(gca,'FontSize',18);
% plot(x_2(:,4),v_2(:,4)./abs((max(abs(v_2(:,4))))),'m','LineWidth',4)
plot(x_4(:,4),v_4(:,4),'m','LineWidth',4)
plot(0, f2_4_fusalage, 'd', 'LineWidth',8);
plot(12,f2_4_tail, 'd', 'LineWidth',8);
legend('Predicted', 'Fusalage Displacment (44.80 Hz)', 'Tail Displacment
    (44.80 Hz)',...
     'fontweight','bold', 'Fontsize', 14, 'Location', 'southwest');
ylim([-80 0.1]);
xlim([0 12]);

subplot(1,3,3)
grid on
legend boxoff
hold on
title('Mode Shape at 203.3665 Hz','fontweight','bold', 'Fontsize', 16);
xlabel('x [in]', 'fontweight','bold', 'Fontsize', 14);
ylabel('Displacment', 'fontweight','bold', 'Fontsize', 14);
set(gca,'FontSize',18);
plot(x_4(:,5),v_4(:,5),'m', 'LineWidth',4)
legend('Predicted', 'fontweight','bold', 'Fontsize', 14);
ylim([-80 0.1]);
xlim([0 12]);
sgtitle('Four-Element Model Eigen Vectors','fontweight','bold', 'Fontsize',
    20);


function [x_all, v_all, frq] = Lab_3_FEM_Solve(modelType)
L   = 12;         % in
L_E = 4.5;        % in
L_R = 5;          % in
w   = 1;          % in
h   = 1/8;        % in
h_E = 1/4;        % in
h_R = 0.040;      % in
E   = 10175000; % psi
rho = 0.0002505;% lb-sec^2/in^4

M_T = 1.1310*rho; % in^3 rho
S_T = 0.5655*rho; % in^4 rho
I_T = 23.124*rho; % in^5 rho

A     = w*h;
I_zz = w*h^3/12;
```

```
switch modelType

    case 2
        ne=2;

        c_M2 = rho*A*L/100800;
        c_K2 = 4*E*I_zz/L^3;

        M=c_M2*[19272,1458*L,5928,-642*L,0,0;
            1458*L,172*L^2,642*L,-73*L^2,0,0;
            5928,642*L,38544,0,5928,-642*L;
            -642*L,-73*L^2,0,344*L^2,642*L,-73*L^2;
            0,0,5928,642*L,19272,-1458*L;
            0,0,-642*L,-73*L^2,-1458*L,172*L^2] ...
            +[0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , M_T,  S_T;
            0 , 0 , 0 , 0 , S_T,  I_T];

        K=c_K2*[24,6*L,-24,6*L,0,0;
            6*L,2*L^2,-6*L,L^2,0,0;
            -24,-6*L,48,0,-24,6*L;
            6*L,L^2,0,4*L^2,-6*L,L^2;
            0,0,-24,-6*L,24,-6*L;
            0,0,6*L,L^2,-6*L,2*L^2];
    case 4
        ne=4;

        c_M4=rho*A*L/806400;
        c_K4=8*E*I_zz/L^3;
        M=c_M4*[77088, 2916*L,23712,-1284*L, 0, 0, 0, 0, 0, 0;
            2916*L, 172*L^2, 1284*L, -73*L^2, 0, 0, 0, 0, 0, 0;
            23712, 1284*L, 154176, 0, 23712, -1284*L, 0, 0, 0, 0;
            -1284*L, -73*L^2, 0, 344*L^2, 1284*L, -73*L^2, 0, 0, 0, 0;
            0, 0, 23712, 1284*L, 154176, 0, 23712, -1284*L, 0, 0;
            0, 0, -1284*L, -73*L^2, 0, 344*L^2, 1284*L, -73*L^2, 0, 0;
            0, 0, 0, 0, 23712, 1284*L, 154176, 0, 23712, -1284*L;
            0, 0, 0, 0, -1284*L, -73*L^2, 0, 344*L^2, 1284*L, -73*L^2;
            0, 0, 0, 0, 0, 0, 23712, 1284*L, 77088 , -2916*L;
            0, 0, 0, 0, 0, 0, -1284*L, -73*L^2, -2916*L , 172*L^2] ...
            + [0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 ;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , M_T,  S_T;
            0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , S_T,  I_T];

```

```
        K=c_K4*[ 96, 12*L, -96, 12*L, 0, 0, 0, 0, 0, 0;
                  12*L, 2*L^2, -12*L,L^2, 0, 0, 0, 0, 0, 0;
                 -96, -12*L, 192, 0, -96, 12*L, 0, 0, 0, 0;
                  12*L,L^2, 0, 4*L^2, -12*L,L^2, 0, 0, 0, 0;
                  0,  0, -96, -12*L, 192, 0, -96, 12*L, 0, 0;
                  0, 0, 12*L,L^2, 0, 4*L^2, -12*L,L^2, 0, 0;
                  0, 0, 0, 0, -96, -12*L, 192, 0, -96, 12*L;
                  0, 0, 0, 0, 12*L,L^2, 0, 4*L^2, -12*L,L^2;
                  0, 0, 0, 0, 0, 0, -96, -12*L, 96, -12*L;
                  0, 0, 0, 0, 0, 0, 12*L,L^2, -12*L, 2*L^2;];
    otherwise
        error('incorrect model');
end

M_red=M(3:end,3:end);
K_red=K(3:end,3:end);

% [ev_red,el]=eig(M_red\K_red);
[ev_red,el]=eig(K_red,M_red);
omega=real(sqrt(diag(el)));

omega(1:3)/2/pi;

[frq,isx]=sort(1/2/pi*omega);

ev_red=ev_red(:,isx);

ev=0*K;
ev(3:end,3:end)=ev_red;

nsub=10;
scale=1.0;

nv=ne*nsub+1;
Le=L/ne;
dx=Le/nsub;

x_all = 0;
v_all = 0;

for t=3:5 %size(ev,2)

    k=0;
    x=zeros(nv,1);
    v=zeros(nv,1);

    for e=1:ne

        xxi=Le*(e-1);
        vi=ev(2*e-1,t);
        teti=ev(2*e,t);
        vj=ev(2*e+1,t);
        tetj=ev(2*e+2,t);

```

```
        if (e==1); ni=0;else; ni=1;end

        for n=ni:nsub

            xk=xxi+dx*n;
            xi=(2*n-nsub)/nsub;
            vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(xi^2-3)*xi+Le*(xi^2-1)*(tetj
                -teti+(teti+tetj)*xi)));

            k=k+1;
            x(k)=xk;
            v(k)=vk;

        end
    end

    x_all(1:length(x),t) = x;
    v_all(1:length(v),t) = v;

end

end
```

```
function [x,y] = fft_analysis(n, signal, f_s)
%{
    Function: Perform an fft analysis on the signal data and return the
    corresponding frequency amplitude data.
    Inputs:
        n: number of samples
        signal: data set
        f_s: sampling frequency
    Ouputs:
        x: frequency
        y: amplitude
%}
    Y = fft(signal);
    P2 = abs(Y/n);
    P1 = P2(1:n/2+1);
    P1(2:end-1) = 2*P1(2:end-1);
    x = f_s*(0:(n/2))/n;
    y = P1;
end
```

```
function [] = log_dec_func(resonant_index , f_d, disp_chan, time)
%{
    Function: Using inputs, natural frequency of specified channel will be
    determined using logarithmic decrement from displacement v time plot
    Inputs:
        resonant_index: channels resonant freq index
        f_d: damping frequency
        disp_channel: displacement channel data
        time: time array
```

```
    Ouptuts:

%}

%determine index at resonant frequency
start = resonant_index-100; %not regerstring first peak decrease by 100

%damping ang freq (just used maximum fft value)
wd = 2*pi*f_d; % damping angular freq [ rad/s ]

%iniital amplitude at resonant frequency
X1 = disp_chan(resonant_index);

%determine values after following cycles
[peaks , locs] = findpeaks(disp_chan(start:floor(0.15*start+start)));

%cut off after peakss have decreased
[~, loc1] = min(abs(peaks - X1 * 0.20));
locs = locs(1:loc1) ;
locs = locs+start;
%use last peak
num_cycles = length(locs);
X_last = disp_chan(locs(end));

%estimate damping ratio
del = (1/num_cycles) * log(X1/X_last);
dr = 1/sqrt( 1 + ( (2*pi) / del)^2);
wn = wd/sqrt(1-dr^2);

figure
plot(time(start:locs(end)), disp_chan(start:locs(end)))
hold on
scatter(  time(locs),  disp_chan( locs ) ) %all peaks after damping period
scatter( time(locs(1)) , disp_chan( locs(1)), 'r' ,'filled' )
scatter( time(locs(end)) , disp_chan( locs(end)), 'r' ,'filled' )
legend('Displacement', 'peaks' , 'X1' , 'Xn')
xlabel('Time [ sec ]')
ylabel('Displacement [ mm ]')
title('Displacement v Time')
fprintf('\n Damped Angular Frequency [rad/sec]: %f\n', wd)
fprintf('Damping Ratio: %f\n', dr)
fprintf('Natural Frequency [rad/s]: %f \n', wn)
end
```

III. Appendix: Participation Report

Group Leader:

Connor O'Reilly: Arranged group meeting, attempted to help debugged Experimental Results Code, log_dec_func participated in writing of report. **Contribution Factor: 100%**

Other Member Contributions:

Quentin Morton: Worked on and completed FEM Results-Resonant frequencies write up section and code. Attended all of the meetings outside of lab time. In addition helped with Experimental Results section. **Contribution**

Factor: 100%

Maria Callas: Worked on and completed FEM Results-Mode Shape write up section, plots and code. Attended all of the meetings outside of lab time and attended office hours on behalf of the group.
Contribution Factor: 100%

Preston Tee: Worked on Experimental Results write up section and code, helped debug and fix FFT code relating to the Experimental Results section. In addition helped with FEM Results - Mode Shapes section. **Contribution Factor: 100%**

Slava Rychenko: Worked on and completed most of the Experimental Results write up section and code. Attended all of the meetings outside of lab time. **Contribution Factor: 100%**