ASEN 3113: Heat Conduction Prelab

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1 Question 1

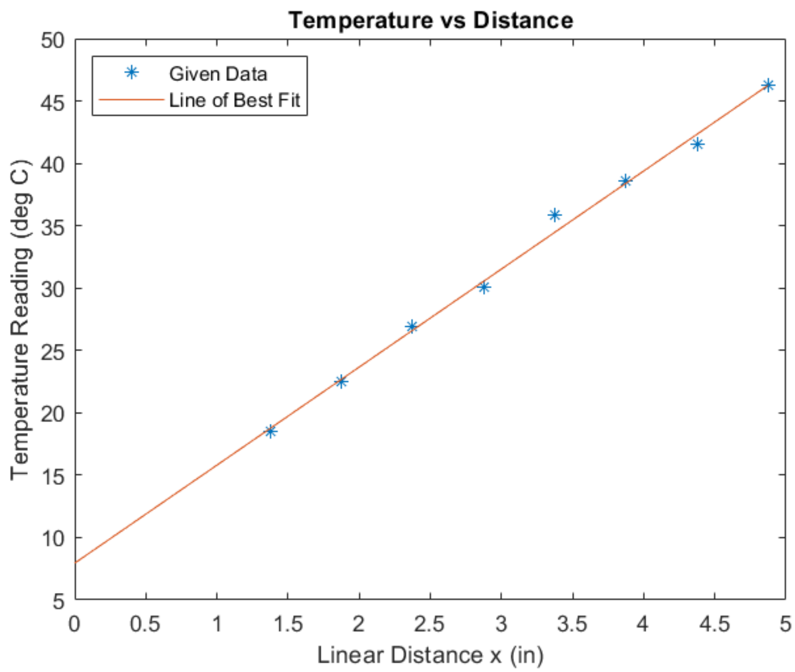
Three assumptions made for teh derivation were:
1)The rod is assumed to only have an axiel dimension, which would introduce some error with our predicted rate of heat loss.
2) Entire cold end of the rod is at constant temperature T_0.
3) Constant initial temperature of the rod

2 Question 2

	Aluminum 7075-T651	Stainless Steel T-303 annealed	brass C360
$\rho(\frac{kg}{m^3})$	2,810	8,000	8,490
$k(W/m-K)$	130	16.2	115
$c_p(\frac{J}{kg \cdot K})$	960	500	380
$\alpha(\frac{m^2}{s})$	$4.819 * 10^{-5}$	$4.05 * 10^{-6}$	$3.565 * 10^{-5}$
Source	www.matweb.com	asm.matweb.com	www.matweb.com

3 Question 3

Using the given data
$T_0 = 7.949$ °C
$H = 7.861$
on the next page is a graph which compares the temperature readings with the calculated best fit line.



4 Question 4

Derived 5equation:

$$b_n = \frac{-8HL(-1)^n}{(2n\pi - \pi)^2}$$

The below pages show the derivation.

$$b_n = -\frac{2H}{L}\int_0^L x\sin(\lambda_n x)\,dx$$

integration by parts:

$$\int u\,dv = uv - \int v\,du$$

$$u = x,\quad dv = \sin(\lambda_n x)\,dx$$
$$du = dx,\quad v = -\frac{\cos(\lambda_n x)}{\lambda_n}$$

$$\int x\sin(\lambda_n x)dx = -\frac{x\cos(\lambda_n x)}{\lambda_n} - \int -\frac{\cos(\lambda_n x)}{\lambda_n}dx$$

$$= \frac{-x\cos(\lambda_n x)}{\lambda_n} + \int \frac{\cos(\lambda_n x)}{\lambda_n}dx$$

let $u = \lambda_n x$, so that $dx = \frac{1}{\lambda_n}du$

then: $\int \frac{\cos(u)}{\lambda_n^2}du = \frac{1}{\lambda_n^2}\int \cos(u)du = \frac{\sin(u)}{\lambda_n^2}$

then $u = \lambda_n x$ so

$$\int \frac{\cos(\lambda_n x)}{\lambda_n}dx = \frac{\sin(\lambda_n x)}{\lambda_n^2}$$

$$\therefore \int_0^L x\sin(\lambda_n x)dx = -\frac{x\cos(\lambda_n x)}{\lambda_n} + \frac{\sin(\lambda_n x)}{\lambda_n^2}\Bigg]_0^L$$

then: $\int_0^L x\sin(\lambda_n x)dx = \frac{\sin(\lambda_n L)}{\lambda_n^2} - \frac{L\cos(\lambda_n L)}{\lambda_n}$

$$\lambda_n = \frac{(2n-1)\pi}{2L}$$

$$\sin(\lambda_n L) = \sin\left[\frac{(2n-1)\pi}{2L} * L\right] = \sin\left(\pi n - \frac{\pi}{2}\right)$$

$$\cos(\lambda_n L) = \cos\left[\frac{(2n-1)\pi}{2L} * L\right] = \cos\left(\pi n - \frac{\pi}{2}\right)$$

using the following identity: $\sin(A-B) = -\cos(A)\sin(B) + \cos(B)\sin(A)$
then $\sin(\pi n - \pi/2) = -\cos(\pi n)\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})\sin(\pi n)$

$\cos(\frac{\pi}{2}) = 0$

$\sin(\pi/2) = 1$

$\sin(\pi n - \pi/2) = -\cos(\pi n)$

for $n = 1, 2, 3, 4, \ldots$ $-\cos(\pi n)$ will alternate between 1 and -1
$\therefore$ we can represent it as $(-1)^n$

using the following identity: $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

then $\cos(\pi n - \pi/2) = \cos(\pi n)\cos(\pi/2) + \sin(\pi n)\sin(\pi/2)$

~~$\therefore \cos(\pi n - \pi/2) = \sin(\pi/2)$~~

$\therefore \cos(\pi n - \pi/2) = \sin(\pi n)$ for $n = 1, 2, 3, 4, \ldots$ $\sin(\pi n) = 0$

~~Then $b_n = \frac{-2H}{L}\Big[(-1$~~

Then $$b_n = -\frac{2H}{L}\left[\frac{(-1)^n}{\lambda_n^2}\right]$$

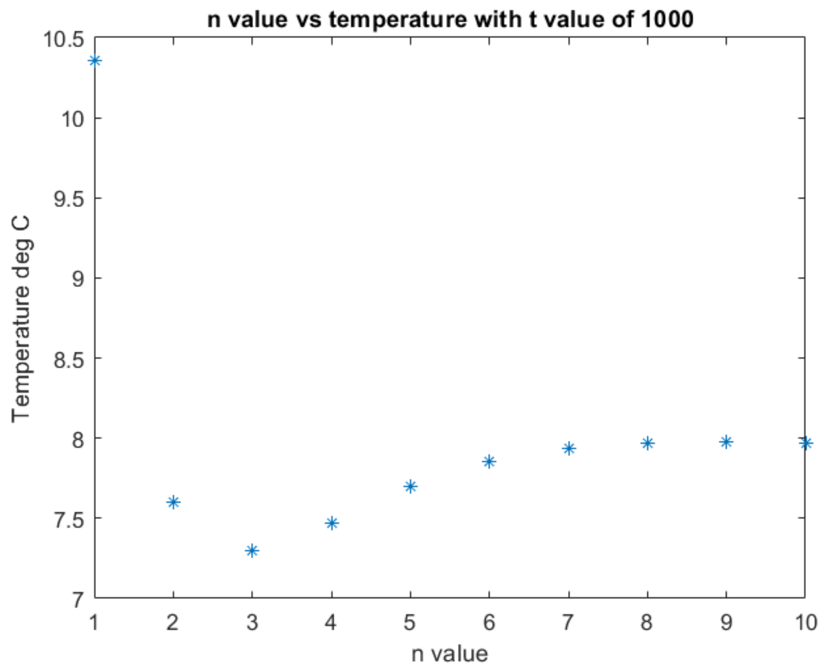
$$\lambda_n^2 = \frac{((2n-1)\pi)^2}{(2L)^2} = \frac{((2n-1)\pi)^2}{4L^2}$$

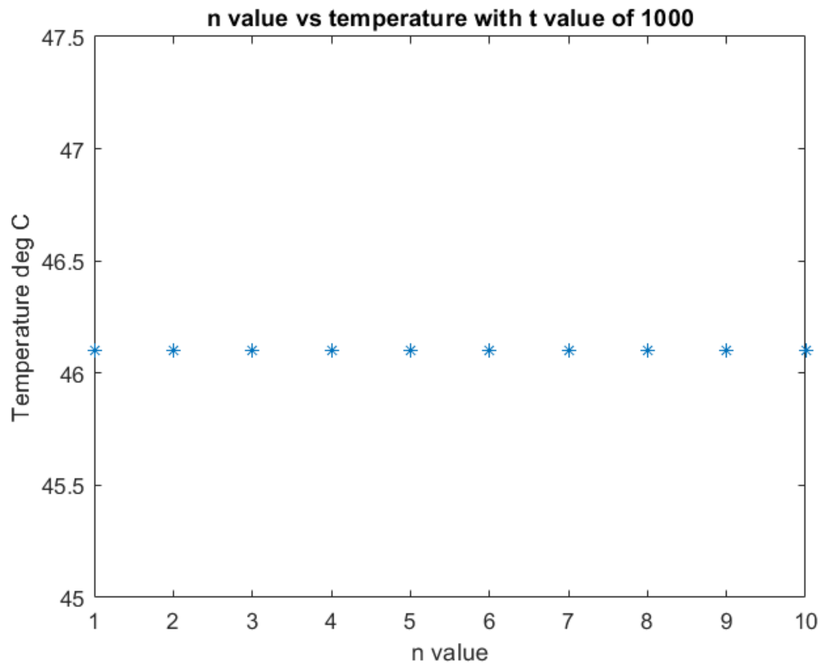
$$b_n = -\frac{2H}{L}\left[(-1)^n \times \frac{4L^2}{(2n\pi-\pi)^2}\right]$$

$$\boxed{b_n = \frac{-8HL(-1)^n}{(2\pi n - \pi)^2}}$$

5 Question 5

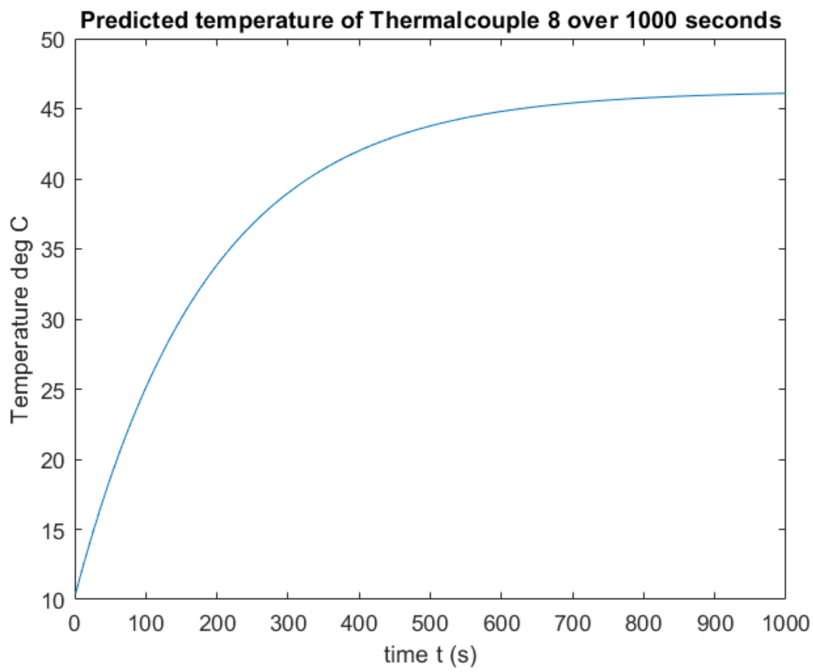
Below are the graphs representing the value of the temperature reading at the last thermocouple with changing values of n and t. The higher the t value the less terms are needed to converge because the pipe would be closer to a steady state.

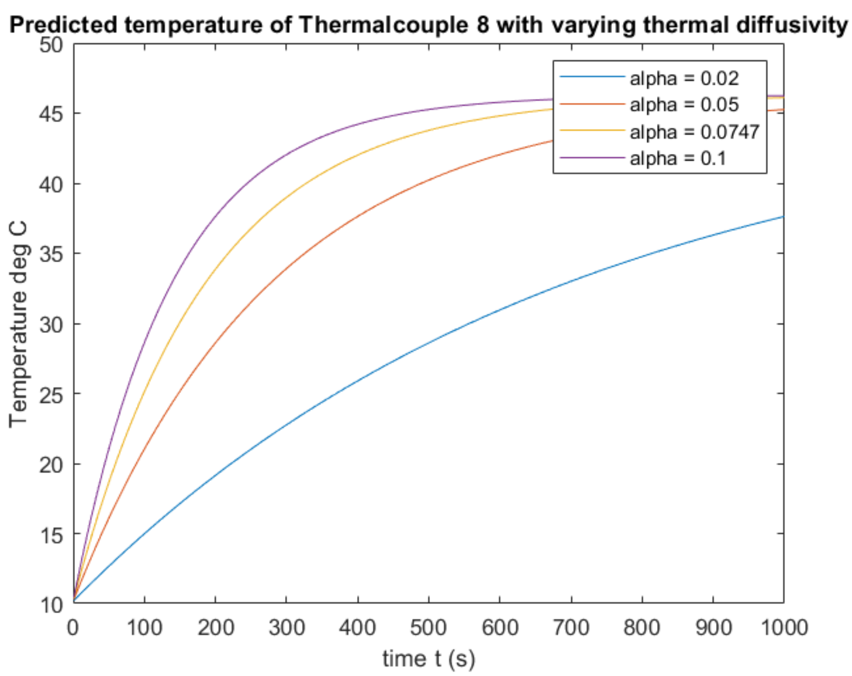




6 Question 6

The overall shape of the transient model becomes more linear as the thermal diffusivity decreases in value. In addition, the higher the thermal diffusivity the sooner the rod approaches its steeady state. This is shown in the second graph on the next page.


Predicted temperature of Thermalcouple 8 over 1000 seconds
Temperature deg C
time t (s)
50
45
40
35
30
25
20
15
10
0
100
200
300
400
500
600
700
800
900
1000


Predicted temperature of Thermalcouple 8 with varying thermal diffusivity
alpha = 0.02
alpha = 0.05
alpha = 0.0747
alpha = 0.1
Temperature deg C
time t (s)
50
45
40
35
30
25
20
15
10
0
100
200
300
400
500
600
700
800
900
1000

7 Matlab Scripts

7.1 Q3 code

```
%% House cleaning
clc; clear all; close all;

%% Given
Tx = [18.53 22.47 26.87 30.05 35.87 38.56 41.50 46.26];
%in diagram thermocouples Th1 to Th8 are 0.5 in apart
xvals = [1.375 1.875 2.375 2.875 3.375 3.875 4.375 4.875];
%% Determine line best fit
ht = polyfit(xvals,Tx,1);
x1 = [ 0 1.375 1.875 2.375 2.875 3.375 3.875 4.375 4.875];
y1 = polyval(ht,x1);
T_0 = y1(1);
figure(1)
plot(xvals,Tx,'*')
hold on
plot(x1,y1)
hold on
title('Temperature vs Distance')
xlabel('Linear Distance x (in)')
ylabel('Temperature Reading (deg C)')
legend('Given Data','Line of Best Fit','Location','northwest')
hold off
fprintf('T_0: %0.3f \n',T_0)
fprintf('H: %0.3f \n',ht(1))
```

7.2 Q5 code

```
clc; clear all;
L = 5.875; %in
H = 7.861;
T_0 = 7.949; %deg C
alph = 0.0747;
lambda =@(n)((2*n-1)*pi)/(2*L);
bn =@(n) (-8*H*L*(-1)^(n-1))/((2*n-1)*pi)^2;

x = 4.875;
t = 1;
%sumation
 yeet = zeros(1,10);
for i = 1:10
    sum = 0;
    for j = 1:i
        sum = sum + bn(j)*sin(lambda(j)*x)* exp(-(lambda(j)^2)*alph*t);
    end
    yeet(i) = sum;
end
temp = zeros(1,10);
for i = 1:10
    temp(i) = T_0 + H*x + yeet(i);
end

figure
plot(1:10,temp,'*')
hold on
xlabel('n value')
ylabel('Temperature deg C')
title('n value vs temperature with t value of 1000')
```

7.3 Q6 code for just varying temperature

```
clc; clear all;
L = 5.875; %in
H = 7.861;
T_0 = 7.949; %deg C
alph = 0.0747;
lambda = pi/(2*L);
bn =@(n) (4*H*L*(-1)^(n-1))/((2*n-1)*pi);

x = 4.875;

%sumation
 yeet = zeros(1,10);

 for t = 1:100
    sum = 0;
    for j = 1:10
        sum = sum + bn(j)*sin(lambda*x)* exp(-(lambda^2)*alph*t);
    end
    yeet(t) = sum;
 end
temp = zeros(1,100);
for i = 1:100
    temp(i) = T_0 + H*x + yeet(i);
end

figure
plot(1:100,temp)
hold on
xlabel('time t (s)')
ylabel('Temperature deg C')
title('Predicted temperature of Thermalcouple 8 over 100 seconds')
```

7.4 Q6 code for varying thermal diffusivity

```
clc; clear all;
L = 5.875; %in
H = 7.861;
T_0 = 7.949; %deg C
alph = [0.02,0.05,0.0747,0.1];
lambda = pi/(2*L);
bn =@(n) (-8*H*L*(-1)^(n-1))/((2*n-1)*pi)^2;

x = 4.875;
emptyCell = cell(1,3,1);
%sumation
 yeet = zeros(1,1000);
 for i = 1:4
    a = alph(i);
    for t = 1:1000
       sum = bn(1)*sin(lambda*x)* exp(-(lambda^2)*a*t);
       yeet(t) = T_0 + H*x + sum;
    end
    emptyCell(1,i,1) = mat2cell(yeet,1,1000);
 end


figure
plot(1:1000,cell2mat(emptyCell(1,1,1)))
hold on
plot(1:1000,cell2mat(emptyCell(1,2,1)))
hold on
plot(1:1000,cell2mat(emptyCell(1,3,1)))
hold on
plot(1:1000,cell2mat(emptyCell(1,4,1)))
xlabel('time t (s)')
ylabel('Temperature deg C')
title('Predicted temperature of Thermalcouple 8 with varying thermal diffusivity')
legend('alpha = 0.02','alpha = 0.05','alpha = 0.0747','alpha = 0.1')
```