Lab 04: Frequency Domain and the Spectrum Analyzer

- Assigned: Friday, February 5, 2021
- Prelab Quiz: complete before Tuesday, February 9, 2021 at 8:30 am (Canvas)
- Report Due: Monday, February 15, 2021 at 5:00 pm MT (Canvas)

1 Objectives

- Understand the decibel (dB) representation of a signal and frequency response magnitude
- Understand time/frequency domain transformation of signals using the Fourier series and transform
- Use Matlab (`fft()` command) to compute and identify the frequency content of a sampled signal
- Understand the frequency response of a circuit, and its computation using element impedances
- Use an RC low-pass filter to alter the spectrum of signals

2 Reading

- Scherz and Monk, Practical Electronics for Inventors:
    - Section 2.27.2 (Complex impedances)
    - Section 2.31 (Decibel)
    - Section 2.33 (Two port networks and filters (low-pass, high-pass, transfer function))
    - Section 2.35 & 2.36 (Fourier series, Fourier transform)
- Matlab FFT: http://www.mathworks.com/access/helpdesk/help/techdoc/ref/fft.html
- Optional: Horowitz & Hill, The Art of Electronics: sections 1.16 – 1.19

3 Background

This lab will investigate the time/frequency domain transformation and representation of signals, together with the frequency response characterization of systems. These two topics combined lead to an understanding of circuits as filters, or frequency selective devices. This is one of the fundamental ideas used to describe the behavior of circuits, particularly for those that are used for sensor signal conditioning, and those used for radio communication. Filtering is also used to understand the behavior of other systems having dynamics, e.g. in mechanical, hydraulic, pneumatic, and thermal systems, and is a basic design requirement in feedback control of these systems.

The time domain representation of a signal is the normal view seen on an oscilloscope where the horizontal axis represents time and the vertical axis is the signal amplitude (voltage). The general method for computing the frequency content of a signal is termed Fourier analysis. The main idea is that any signal can be expressed as the weighted sum of a (possibly infinite) number of sinusoids. The magnitude of each component can be plotted versus the frequency of each component. This is a frequency domain representation of the signal, called the magnitude spectrum. An instrument that displays it in this form is called a spectrum analyzer. A common method for determining frequency content of signals is based upon a numerical technique called the Fast Fourier Transform (FFT). We will use the FFT function of MATLAB to study the frequency content of waveforms produced by the function generator feature of the DS212 2 Channel Oscilloscope.

Linear circuits have the property that sinusoidal inputs produce sinusoidal outputs at the same frequency, but with an amplitude and phase shift that varies with the excitation frequency. The relative input-output amplitude change and phase shift, plotted as a function of the excitation frequency, is called the frequency response of the circuit. The frequency response can be computed from the impedance of circuit elements, combined using node and mesh equations just as for circuits containing only resistors. The frequency response makes it easy to understand how an individual sinusoid input is processed by the circuit.

Linearity of the circuit (superposition) also implies that a sum of sinusoids on the input produces a sum of sinusoids on the output, with each sine-wave component being processed according to the circuit's frequency response at the corresponding frequency. Thus the circuit acts as a filter, amplifying or attenuating various frequency components differently. We will examine the low-pass type of filter, which passes low frequencies un-attenuated, but blocks higher frequencies. Also possible are high-pass filters, which pass high frequencies and attenuate lower frequencies. Finally, there exist bandpass filters which are designed to pass only a selected frequency range as determined by the design of the filter.

Much of the analysis of filters and signals in general are described using a particular logarithmic amplitude scale, called the decibel scale. This concept is introduced in this lab.

3.1 Fourier Series

Every continuous[1] periodic function $v(t)$ has a trigonometric Fourier series representation

$$v(t) = a_0 + \sum_{i=0}^{\infty} [a_i \cos(\omega_i t) + b_i \sin(\omega_i t)] \tag{1}$$

where a_0 is the DC offset (average value), and ω_i is the radian frequency of the *i*th harmonic component. If the period of the signal $v(t)$ is T seconds, then $\omega_i = i2\pi/T$ rad/sec. The Fourier coefficients a_i and b_i are real numbers. The peak amplitude of the *i*th harmonic is given by

$$A_i = \sqrt{a_i^2 + b_i^2} \tag{2}$$

A plot of A_i versus i (or ω_i) is called the amplitude spectrum of $v(t)$.

The Fourier coefficients are obtained by correlating the function $v(t)$ with the $\sin()$ and $\cos()$ harmonic functions over one period:

$$a_0 = \frac{1}{T}\int_0^T v(t)dt \ ; \qquad a_i = \frac{2}{T}\int_0^T v(t)\cos(\omega_i t)dt \ ; \qquad b_i = \frac{2}{T}\int_0^T v(t)\sin(\omega_i t)dt \tag{3}$$

Alternatively, $v(t)$ can be represented by the complex Fourier series

$$v(t) = c_0 + \sum_{i=1}^{\infty} c_i e^{-j\omega_i t} \tag{4}$$

where the complex Fourier coefficients are given by

$$c_i = \frac{1}{T}\int_0^T v(t)e^{-j\omega_i t}dt \tag{5}$$

[1]Technically, signals satisfying the Dirichlet conditions, or Lipschitz continuity. All physical signals are included. Complex coefficient equations taken from: https://www.math24.net/complex-form-fourier-series/

Note the similarity with the Laplace transform

$$V(s) = \int_0^\infty v(t)e^{-st}dt \tag{6}$$

when $s = j\omega_i$. Indeed, the Fourier series is a special case of the Laplace transform. This is why the function $V(s)$ is called the frequency domain representation of $v(t)$, since $V(s)$ is similar to the coefficients a_i, b_i, and c_i, which represent the amplitude of the various sinusoid frequency components in the signal $v(t)$.

The Fast Fourier Transform (FFT) is a numerical technique for performing the integrations to obtain c_i, which takes into account the symmetry in the trigonometric functions to reduce computations. The magnitude spectrum is obtained by

$$c_0 = \frac{a_0}{2}\ ; \qquad c_i = \frac{a_i - jb_i}{2}\ ; \qquad c_i = \frac{a_i + jb_i}{2} \tag{7}$$

Only the harmonics for positive i up to half the signal sampling frequency, called the Nyquist frequency, are used.

4 Prelab (25 pts)

Prelab problems can be found in the lab module on Canvas. Students may collaborate with lab partners to discuss the prelab problems. However, each student must complete a prelab individually, have prelabs worked out in a bound lab book or an electronic notebook such as a tablet, and enter prelab answers into the Canvas.

5 Experiment

For all parts of the experiment, draw a schematic diagram before you connect the circuit, and record all your measurements in your lab book. Include instrument settings.

1. **Signal Frequency Content**

   (a) Download the test data from the Lab 04 Canvas module named `lab04_section_01_signal1.mat` and load that into Matlab. Plot x as a function of t for the first 100 samples generating a time domain plot (`plot(t(1:100),x(1:100);)` – be sure to label your plot!

   Use the `fft()` function within Matlab to generate a 2nd plot illustrating the magnitude spectrum of the signal. Search the documentation for `fft()` online to resolve the complex output of the `fft()` function to a frequency spectrum.

   Use the entire vector with the `fft()` and again be sure to label your plot.

   (b) Download the test data from the Lab 04 Canvas module named `lab04_section_01_signal2.mat` and load that into Matlab and create time and magnitude spectrum plots as was done in (a)

   (c) Download the test data from the Lab 04 Canvas module named `lab04_section_01_signal3.mat` and load that into Matlab and create time and magnitude spectrum plots as was done in (a).

2. **Time and Frequency Domain**

   (a) Set up the function generator feature of the DS212 2-channel oscilloscope to produce a sine wave at 500 Hz while leaving the 50% duty option as-is. (Hint: Press the down the "S" button until you arrive at Page 3; there you will find the `WaveOutOption`).

(b) Connect the output to channel A of the oscilloscope and set the adjustments on the scope to view two cycles on the display. Make sure to change channel A to DC coupling to view the complete output signal, if you do not remember how to do this refer to lab 3. Sketch the result and note the time and voltage axis division values. NOTE: Get as close as possible to displaying two cycles of the sinusoidal signal.

(c) Use the measure page to determine the V_{PP}, V_{RMS}, and frequency of the input signal.

(d) This signal was sampled in real time and recorded for use in MATLAB. Download the file named `lab04_section02_signalSine.mat`. Once loaded you will find three variables in the workspace: Fs, the sampling frequency; t, the time vector (similar to that in the prelab example); and x, the signal of interest.

(e) Use the MATLAB code you developed for Section 1 (*Signal Frequency Content*) to view the amplitude spectrum of the signal sampled from the DS212 2-Channel oscilloscope and sketch the result, noting the frequency and amplitude of all significant peaks.

(f) Zoom in on the relevant parts of the FFT. Then use the MATLAB cursor tool to measure the frequency and amplitude of the spectral peak.

(g) Modify the function generator settings to produce a square wave with 50% duty cycle rather than a sine wave, keeping all other settings the same.

(h) Use the measure page to determine the V_{PP}, V_{RMS}, and frequency of the input signal.

(i) Repeat d through f, this time using the file named `lab04_section02_signalSquare.mat`.

3. **Low-Pass Filter Circuit**

(a) On your protoboard, construct the low-pass filter circuit you designed in the Prelab, part 4.

(b) Set up the oscilloscope wave generator to produce a 50 Hz sine wave. NOTE: The signal automatically has a 1 V DC offset and 1 V amplitude (2 V_{PP}).

(c) Connect the output of your filter circuit to channel A of the scope. Remember to have the channel set to DC coupling or you will not see the DC offset.

(d) Sketch the results in the time domain and compare the filtered output (ch A) to the waveform from the function generator. Note the amplitude and frequency of the waveforms. Describe your results.

(e) Repeat a) through d) for 200 Hz and 5 kHz input sine waves.

(f) Samples were taken for the output signal of your circuit at 50 Hz, 200 Hz, and 5 kHz input sine waves (same amplitude and offset as in 3b above) from the function generator. For the files named `lab04_section03_signal50.mat`, `lab04_section03_signal200.mat`, and `lab04_section03_signal5k.mat` corresponding to the 50 Hz, 200 Hz and 5 kHz input sine waves respectively, use the code developed in Section 1: Signal Frequency Content to record the output frequency and amplitude measured in the FFT plots.

(g) Generate the frequency response of this RC circuit starting by taking measurements starting from 100 Hz and using each increment possible with the oscilloscope (i.e 100 Hz, 200 Hz, 500 Hz, 1 kHz, 2 kHz, 5 kHz, and 10 kHz) up to 10 kHz. Calculate the attenuation of the output signal relative to the input signal and plot the results versus frequency (Tip: use a log scale for frequency).

6 Analysis

1. **Decibels**

   (a) What is the rule (i.e. the equation) for calculating the amplitude of a signal output from a circuit in dBV, based on the input amplitude in dBV and the frequency response magnitude in dB?

2. **Signal Frequency Content**

   (a) Estimate the amplitude and the frequency of the periodic waveform using the time domain plot for the variable `x` in `lab04_section01_signal1.mat`. How confident are you in the estimate and why? Estimate the amplitude and the frequency of the periodic waveform using the magnitude spectrum plot for the variable `x` in `lab04_section01_signal1.mat`. How confident are you in the estimate and why?

   (b) Repeat (a) using your plots for `lab04_section01_signal2.mat`.

   (c) Repeat (a) using your plots for `lab04_section01_signal3.mat` (refer to: Mathworks Technical Note #1703 available at: http://www.mathworks.com/support/tech-notes/1700/1703.html)

3. **Time and Frequency Domain**

   (a) Compare the sine wave results of part 2e) with the expected values from the prelab question 4. How accurate are the amplitudes and frequencies of the measured spectral lines?

   (b) Compare the square wave results of part 2e) with prelab question 2b), as in part a).

4. **Low-Pass Filter**

   (a) Using the bode plot you've created, estimate the -3 dB point of your circuit and the cutoff frequency. Add these to your plot.

   (b) Estimate the roll off slope after the -3 dB point; that is, the attenuation per increase in frequency. Show this in your plot.

   (c) Design a low-pass filter that will pass a 100 Hz sine wave with minimal attenuation but will attenuate an 8 kHz sine wave by 30 dB.

7 Grading Template

______/25	**Prelab - (25 pts)**	
	1. Decibels	______/05
	2. Signal Frequency Content	______/05
	3. Time and Frequency	______/10
	4. Low Pass Filter	______/05
______/25	**Experiment - (25 pts)**	
	1. Signal Frequency Content	______/09
	2. Time and Frequency	______/06
	3. Low Pass Filter	______/10
______/25	**Analysis - (25 pts)**	
	1. Decibels	______/03
	2. Signal Frequency Content	______/09
	3. Time and Frequency Domain	______/04
	4. Low Pass Filter	______/09
______/75	**Total**	