UNIVERSITY OF COLORADO - BOULDER

ASEN 2003 - INTRODUCTION TO DYNAMICS AND SYSTEMS

LAB 01

# Roller Coaster Design Laboratory

*Author:*
Geoffrey LORD

*Author:*
Jake MILES-COLTHUP

*Author:*
Connor O'REILLY

*Professor:*
Jay MCMAHON

*Professor:*
Bobby HODGKINSON

*Professor:*
Josh MELLIN

28 January 2020

Smead Aerospace
UNIVERSITY OF COLORADO **BOULDER**

# Laboratory 3: *Roller Coaster Design Laboratory*

Geoffrey Lord*, Jake Miles-Colthup†, Connor O'Reilly‡
ASEN 2003 Section 302 Group 17

## I. Abstract

**Amusement parks are a common attraction across the world and demand is high for exciting roller coasters. However, when designing a roller coaster, it is crucial to keep in mind the effects that roller coasters can have on the body as G-forces move blood around and compress the body making it experience more than its own weight. This makes designing exciting roller coasters difficult due to the desire to blend safety with adrenaline. To combat these limitations imposed by G-forces, the team developed a computational tool through a variety of free body diagrams to analyze the G-forces experienced relative to the position on the roller coaster. This computational tool was used to develop a roller coaster track that would meet safety requirements as well as offering users an exciting ride through a variety of track elements. The final design encompasses multiple hills and valleys, a banked turn, a loop, a zero gravity section, and almost maximum G-forces for a safe yet fun ride. The implications of using free body diagrams to develop a computational tool could lead to the easier design of more complex and exciting roller coasters that still meet safety requirements.**

## II. Introduction

The intention of this laboratory experiment was to make use of particle dynamics to design an enjoyable yet safe roller coaster track [1]. In order to execute on this task multiple track elements were analyzed. The design of Group 17's roller coaster track included a zero gravity section, loop, hill, valley, and banked turn before the final breaking section. Throughout all of these elements it was required that the rider would not feel forces greater than those shown in Table 1.

| Direction | G Forces |
|---|---|
| Forward | < 5 [G] |
| Backward | < 4 [G] |
| Up | < 6 [G] |
| Down | < 1 [G] |
| Lateral | < 3 [G] |

Table 1: Direction and G-Force

Conservation of energy was used to calculate the potential energy and kinetic energy at all points on the track. In order to ease the complication of these calculations friction was assumed to be nonexistent. The starting height of the mass-less roller coaster was 125 meters. Additionally, the acceleration due to gravity was assumed to be 9.81 $m/s^2$. Therefore, at the starting point the train has a velocity of $0 m/s$ thus, the total energy of the system will never exceed 1226.25 $N$.

The track was designed by use of a piece wise function comprised of circles and lines. This track begins with a steep drop causing a section of zero gravity before leveling off and entering a loop. After the loop

*108078550
†108740179
‡107054811

section of the track is complete the train will move over a hill and down a valley before entering the banked turn section at the peak of another hill. Next the train will descend to ground level through a valley. Finally, the train will enter a straightaway section where the train will brake and come to a stop at a height of 0m. The track, with all elements labeled can be referenced in Figure 1. Additionally, the roller coaster was designed with twists at certain locations such that the train would always be on the concave side of the hills, valleys, and loops. This ensures that the G-force directed down through the rider is minimized.

## III. Roller Coaster Design

The subsections below represent a breakdown of each track element. All transitions between two track elements have been made at a point where the slope has a value of one or zero to minimize changes in acceleration and to simplify the design of the roller coaster track. Figure 1 identifies each track element that was used in the creation of the roller coaster.

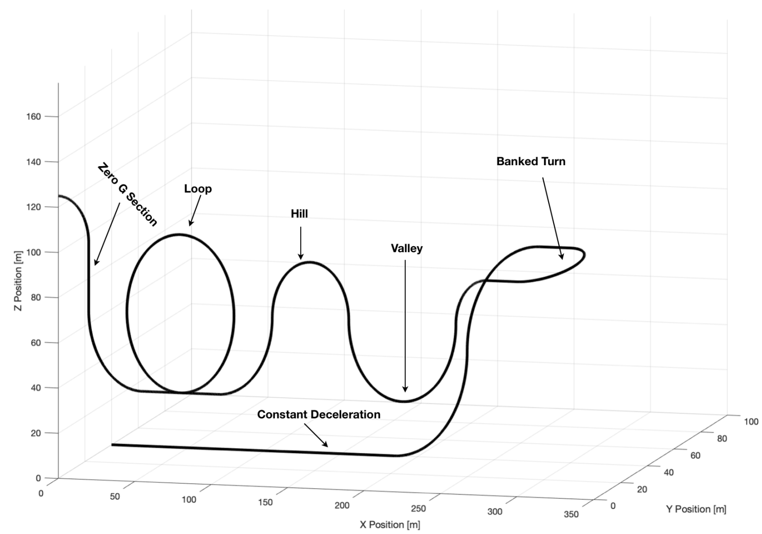


Figure 1: Track Elements of Roller Coaster

### A. Flat Section

Flat sections are used three times throughout the roller coaster track. They are used twice at the beginning and end of the loop and then used once again at the end of the track during the breaking section. The derivation for the G-forces in the flat section referenced in Figure 2 is as follows.

$$\Sigma F_y = 0 = N - mg$$
$$N = mg$$
$$\text{G-force} = 1$$

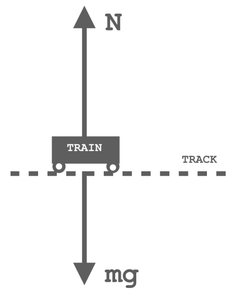


Figure 2: Flat Track FBD

### B. Loop

There is one loop section in the roller coaster and the derivation for G-forces throughout the loop is as follows with theta as referenced in Figure 3.

$$\Sigma F_r = -m\frac{v^2}{r} = -N - mg\sin\theta$$
$$N = m\frac{v^2}{r} - mg\sin\theta$$
$$\text{G-force} = \frac{v^2}{g \cdot r} - mg\sin\theta$$

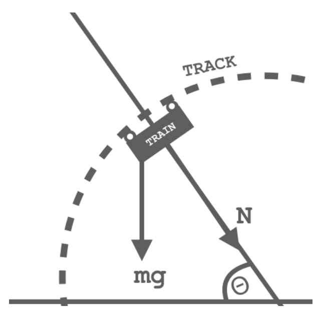


Figure 3: Loop Track FBD

### C. Valley

Throughout the track, there are several valley elements which are composed of quarter circles. Due to the design of the track, since the train is always on the concave side of the element, these elements can be treated similar to the loop above with the following derivation. Again theta is referenced as shown in Figure 3.

$$\Sigma F_r = -m\frac{v^2}{r} = -N - mg\sin\theta$$
$$N = m\frac{v^2}{r} - mg\sin\theta$$
$$\text{G-force} = \frac{v^2}{g \cdot r} - mg\sin\theta$$

### D. Hill

There are also multiple hill elements throughout the track which, much like the valley elements, are composed of quarter circles and can be fundamentally based on the loop free body diagram and derivation seen below. Theta is referenced from Figure 3.

$$\Sigma F_r = -m\frac{v^2}{r} = -N - mg\sin\theta$$
$$N = m\frac{v^2}{r} - mg\sin\theta$$
$$\text{G-force} = \frac{v^2}{g \cdot r} - mg\sin\theta$$

### E. Banked Turn

The free body diagram that illustrates the forces acting on the train throughout the banked turn can be seen below in Figure 4.

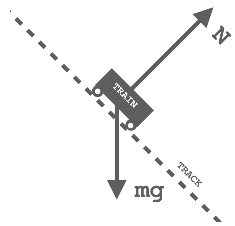


Figure 4: Banked Turn FBD

To calculate position, a value of theta varied from one to 180, the position of the cart was represented as vectors using the following equations.

$$x = x_0 + r * sin(\theta)$$
$$z = z_0 + r - (r * cos(\theta))$$

Because the banked turn does not have any change in height the height was the same as the initial height.

$$y = y_0 * ones(1, length(x))$$

To calculate the G's through the banked turn the forces in the normal and lateral direction were found. The following equations were provided in the Roller Coaster Lab Intro slides.

$$\sum F_z = Ncos(\theta) - Lsin(\theta) = mg$$
$$\sum F_r = -Nsin(\theta) - Lcos(\theta) = \frac{-mv^2}{r}$$

Where $r$ represents the radius of the turn, $L$ and $N$ represent the lateral and normal force, $\theta$ represents the bank angle, $v$ represents the velocity of the cart and $g$ represents the gravitational acceleration.
Selecting a value for the bank angle and the radius of the turn allowed the above system of equations to be solved using matrices.

$$A = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ -sin(\theta) & -cos(\theta) \end{bmatrix} B = \begin{bmatrix} g \\ \frac{-v^2}{r} \end{bmatrix} X = \begin{bmatrix} N \\ L \end{bmatrix}$$
$$X = A^{-1} * B$$

To calculate the magnitude of G's the following equation was used

$$G_{magnitude} = \sqrt{(\frac{N}{g})^2 + (\frac{L}{g})^2}$$

The following values were calculated using the above equations and matlab

| | |
|---|---|
| Lateral G's | 1.626 |
| Vertical G's | 3.03 |
| Magnitude of G's | 3.44 |

### F. Zero Gravity

There is one zero gravity section on the track which occurs right at the beginning. The effect of zero gravity is achieved through mimicking free fall conditions as is referenced in Figure 5. The derivation for G-forces is as follows.

$$\Sigma F_x = 0 = N$$
$$N = 0$$
$$\text{G-force} = 0$$

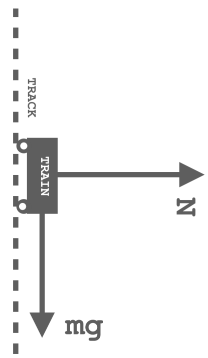


Figure 5: Zero-G FBD

### G. Constant Deceleration

The roller coaster track ends with a flat section where the breaking mechanism is applied. This section is 185 meters long thus allowing the train to break at a constant deceleration of 6.62837 $\frac{m}{s^2}$. During this section of deceleration there is a constant G force acting in downward with a value of one G. In addition there is a second constant G force acting in the backwards direction with a value of 0.67567 Gs. The derivations for the computed values is shown below and is based off of the free body diagram shown in Figure 6.

Using Newton's second law, the only force acting upon the cart in the x direction is the force of the breaking mechanism.

$$\sum F_x = ma_x$$

Mass is negligible so the Backward force is equal to the acceleration.

$$F_x = a_x$$

Because there are no other forces acting upon the cart, the acceleration will be equal to the backward Gs the rider experiences multiplied by gravitational acceleration.

$$a_x = G_{backward} * 9.81$$

Acceleration can be represented as the following equation

$$a = \frac{dv}{dt}$$

This was simplified further to get rid of the $dt$ term

$$v = \frac{ds}{dt}$$
$$dt = \frac{ds}{v}$$
$$a = v * \frac{dv}{ds}$$

Lastly the following integral was solved

$$\int_{x_0}^{x_f} a_x dx = \int_{v_0}^{0} v dv$$
$$a_x(x_f - x_0) = \frac{0^2}{2} - \frac{v_0^2}{2}$$
$$x_f = x_0 - \frac{v_0^2}{2 * a_x}$$
$$x_f = x_0 - \frac{v_0^2}{2 * 9.81 * G_{backward}}$$

The last equation was used to solve for the backward G's and the acceleration of this track element.

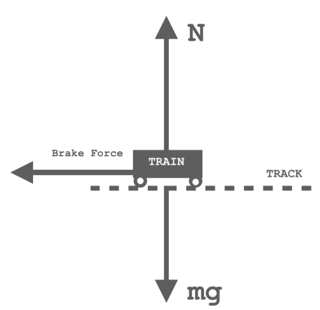


Figure 6: Flat Track FBD

# IV. Performance Analysis

## A. G-Force

The graph above shows the Lateral G-Forces experienced along the path. Before and after the banked turn the graph shows a gradient of colors. This error is caused by an abrupt change from 0 G to 1.626 G which the matlab function *colormap* did not account for. The only section where lateral G's occurred is during the banked turn section with a value of 1.626 G.

Backward G's occurred only in the deceleration section of the track with a value of 0.67567 G.

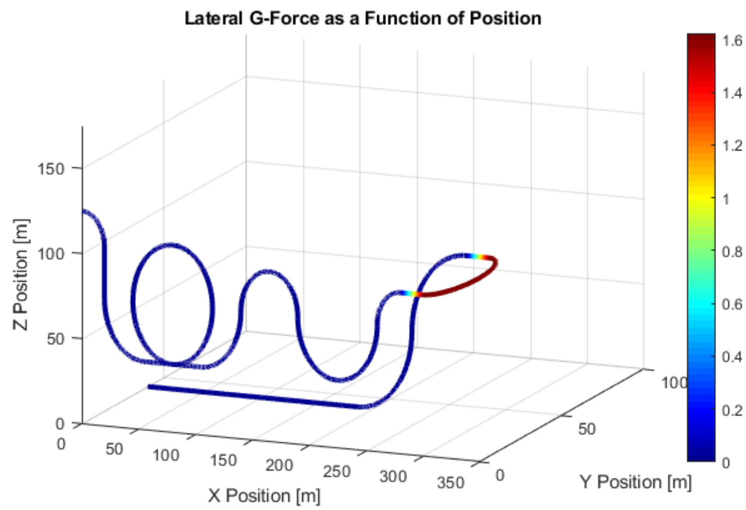


Figure 7: Lateral G-Force vs. Position

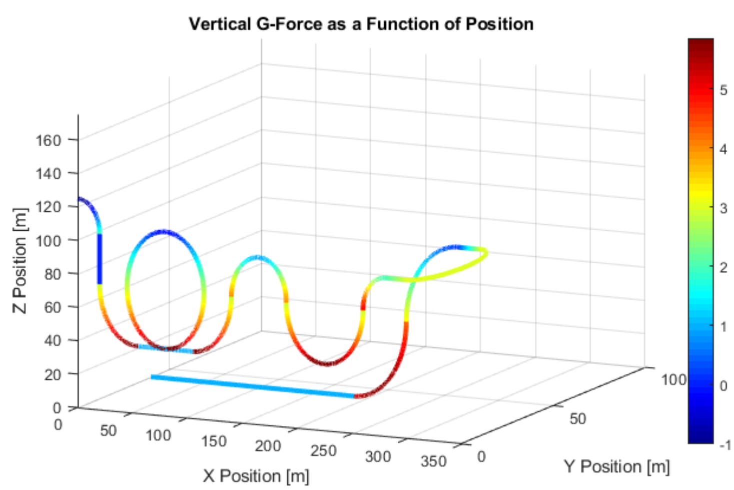


Figure 8: Vertical G-Force vs. Position

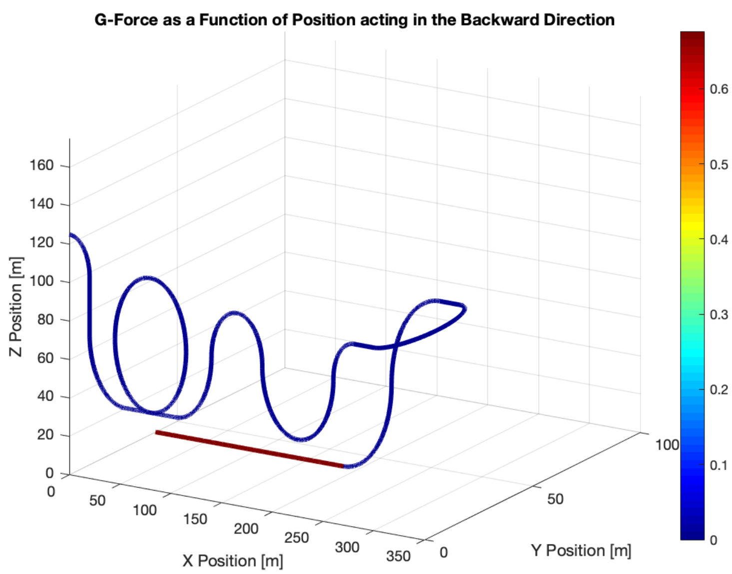


Figure 9: Front to Back G-Force vs. Position

## B. Speed

The team used conservation of energy principles to determine the kinetic energy and thus the velocity at any vertical height on the roller coaster. This equation was derived as follows.

$$ME_i = ME_f$$
$$KE_i + PE_i = KE_f + PE_f$$
$$PE_i = KE_f + PE_f$$
$$mgh_i = \frac{1}{2}mv_f{}^2 + mgh_f$$
$$v_f = \sqrt{2g(h_i - h_f)}$$

The team then plotted the position of the roller coaster colored according to velocity as seen in Figure 10.

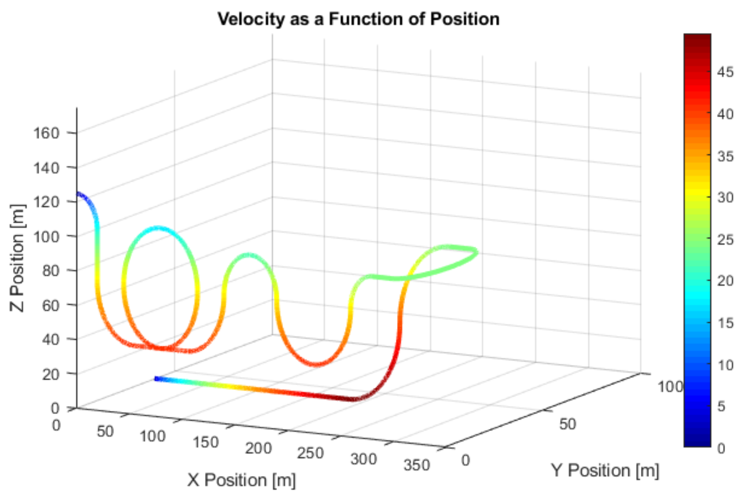


Figure 10: Velocity vs. Position

## C. Overall Design Performance

The design created falls within all of the constraints of the deliverables. The roller coaster track has a length of 1058.30 meters with a maximum velocity of 49.52 meters per second. The maximum G-force experienced during the ride is a upward G-force of magnitude 5.86 G's. This force occurs at the bottom of the last hill right before the section of constant deceleration. The time duration was calculated through the knowledge that a differential track length divided be velocity would equal a differential time. The total duration was calculated to be 45.9 s. All of the performance metrics that were required were successfully achieved. As stated earlier, in some sections of the roller coaster the track was twisted to keep the G forces pointing in the correct direction. In real world applications this may introduce new forces that were not accounted for in our calculations, which could go against the previous safety requirements.

## V. Conclusions

During this design process a model was made to evaluate various safety metrics that must be noted when creating a safe but enjoyable roller coaster ride. Various track elements were used to increase the complexity of the roller coaster design. These elements were all variations of circular and linear functions to ease the process of deriving the G-force equations. These calculations were made by use of principals of particle motion, Newtons laws, and the law of conservation of energy. This process resulted in the development of a roller coaster that comes near to but never surpasses unhealthy G-force levels thus insuring the rider will experience the maximum amount of enjoyment without the possibility of harm or injury. Furthermore, the model created calculated the velocities and G-forces at every point on the track and illustrated those values in clear figures shown above.

Is was these calculations that allowed for validation of the design as well as a high degree of certainty that the track will be safe and enjoyable for the rider. This model was limited in many ways, specifically in the assumption that the track has zero friction acting on it. This limitation in the model is one that must be overcome before production of this design is feasible. Additionally, there was assumed to be no drag which would further distort the accelerations acting on the train. Finally, this model was created assuming particle motion thus, there was assumed to be no mass. The introduction of a mass into the system would significantly increase the forces acting on the train and have potential structural impacts. However, the addition of a mass would not change the G-Forces acting on the system.

Future iterations of this model would include mass, friction, and drag to produce more accurate model that is more realistic to a real world situation. Assuming all of these forces to be acting on the train would greatly optimize the model and increase feasibility of production of this design. The development of this tool and potential future iterations of it would have implications on the design of roller coasters and other similar tracks where G-forces are important to track.

# References


[1] *ASEN 2003: Roller Coaster Lab Intro.pdf* University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020

[2] *ASEN 2003: Lab1Assignment.pdf* University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020

[3] *ASEN 2003: TechnicalWriting.pdf*. University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020

[4] *ASEN 2003: ASEN2003 Lecture 1.pdf* University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020

[5] *ASEN 2003: ASEN2003 Lecture 2.pdf* University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020

[6] *ASEN 2003: ASEN2003 Lecture 3.pdf* University of Colorado at Boulder, Aerospace Engineering Sciences, Spring 2020


# VI. Acknowledgements


Our group would like to acknowledge the help of the entire teaching team including Professor Jay McMahon, Professor Bobby Hodgkinson, and Professor Josh Mellin. Furthermore, we would like thank the University of Colorado Boulder SMEAD Aerospace program for providing phenomenal facilities to further our education.


# VII. Appendix A: Group Contributions

Our team had a great dynamic throughout the process of this design lab. Each member contributed evenly and was able to provide valuable insight. Jake focused primarily on the formation of the Roller Coaster track and the circular and linear equations that resulted in a phenomenal design. Connor was incredibly helpful in the banked turn and loop calculations and Geoffrey aided in the creation of our free body diagrams and organization of the report. Furthermore, Jake, Connor and Geoffrey all worked on velocity, normal, constant deceleration, and G-force calculations together. Overall, the success of this design laboratory was contingent on the work of every team member included.

—— Jake Miles-Colthup

—— Connor O'Reilly

—— Geoffrey Lord

# VIII. Appendix B: MatLab Code

## A. Matlab Code

```
%% Housekeeping
clc; clear all; close all;

%% Path functions
%Quarter circle concave down descending
x_pos_1 = linspace(0,20,1000);
function_1 = 105 + sqrt(400-x_pos_1.^2);
theta_1 = atand((function_1-105)./x_pos_1);
velocity_1 = sqrt(2*9.81*(125-function_1));
g_1 = velocity_1.^2/(9.81*20) - sind(theta_1);

% vertical line
x_pos_2 = 20*ones(1,1000);
function_2 = linspace(105,75,1000);
theta_2 = 90*ones(1,1000);
velocity_2 = sqrt(2*9.81*(125-function_2));
g_2 = zeros(1,1000);

%quarter circle concave up descending
x_pos_3 = linspace(20,55,1000);
function_3 = 75 - sqrt(1225-(x_pos_3-55).^2);
```

```
theta_3 = linspace(360,270,1000);
velocity_3 = sqrt(2*9.81*(125-function_3));
g_3 = velocity_3.^2/(9.81*35) - sind(theta_3);

%flat line
x_pos_4 = linspace(55,80,1000);
function_4 = 40*ones(1,1000);
theta_4 = zeros(1,1000);
velocity_4 = sqrt(2*9.81*(125-function_4));
g_4 = ones(1,1000);

%quarter circle concave up ascending
x_pos_5 = linspace(80,115,1000);
function_5 = 75 - sqrt(1225-(x_pos_5-80).^2);
theta_5 = linspace(270,180,1000);
velocity_5 = sqrt(2*9.81*(125-function_5));
g_5 = velocity_5.^2/(9.81*35) - sind(theta_5);

%half circle concave down
x_pos_6 = linspace(45,115,1000);
function_6 = 75 + sqrt(1225-(x_pos_6-80).^2);
x_pos_6 = flip(x_pos_6);
theta_6 = linspace(180,0,1000);
velocity_6 = sqrt(2*9.81*(125-function_6));
g_6 = velocity_6.^2/(9.81*35) - sind(theta_6);

%quarter circle concave up descending
x_pos_7 = linspace(45,80,1000);
function_7 = 75 - sqrt(1225-(x_pos_7-80).^2);
theta_7 = linspace(360,270,1000);
velocity_7 = sqrt(2*9.81*(125-function_7));
g_7 = velocity_7.^2/(9.81*35) - sind(theta_7);

%flat line
x_pos_8 = linspace(80,105,1000);
function_8 = 40*ones(1,1000);
theta_8 = zeros(1,1000);
velocity_8 = sqrt(2*9.81*(125-function_8));
g_8 = ones(1,1000);

%quarter circle concave up ascending
x_pos_9 = linspace(105,140,1000);
function_9 = 75 - sqrt(1225-(x_pos_9-105).^2);
theta_9 = linspace(270,180,1000);
velocity_9 = sqrt(2*9.81*(125-function_9));
g_9 = velocity_9.^2/(9.81*35) - sind(theta_9);

%half circle concave down
x_pos_10 = linspace(140,190,1000);
function_10 = 75 + sqrt(625 - (x_pos_10-165).^2);
theta_10 = linspace(0,180,1000);
velocity_10 = sqrt(2*9.81*(125-function_10));
g_10 = velocity_10.^2/(9.81*25) - sind(theta_10);

%half circle concave up
x_pos_11 = linspace(190,260,1000);
function_11 = 75 - sqrt(1227 - (x_pos_11-225).^2);
theta_11 = linspace(360,180,1000);
velocity_11 = sqrt(2*9.81*(125-function_11));
g_11 = velocity_11.^2/(9.81*35) - sind(theta_11);

%quarter circle concave down ascending
x_pos_12 = linspace(260,280,1000);
function_12 = 75 + sqrt(400 - (x_pos_12-280).^2);
theta_12 = linspace(0,90,1000);
velocity_12 = sqrt(2*9.81*(125-function_12));
g_12 = velocity_12.^2/(9.81*20) - sind(theta_12);

% banked turn
[x_pos_13, z_pos_13, function_13, N, L, hor_g, vert_g, mag_n, s, velocity_13] = ...
```

```
    banked_turn3(x_pos_12(end), 0, function_12(end), 45, 20);

%quarter circle concave down descending
x_pos_14 = linspace(280,232.5,1000);
function_14 = 47.5 + sqrt(2256.25 - (x_pos_14-280).^2);
z_pos_14 = 40*ones(1,1000);
theta_14 = linspace(90,0,1000);
velocity_14 = sqrt(2*9.81*(125-function_14));
g_14 = velocity_14.^2/(9.81*47.5) - sind(theta_14);

%quarter circle concave up descending
x_pos_15 = linspace(232.5,185,1000);
function_15 = 47.5 - sqrt(2256.25 - (x_pos_15-185).^2);
z_pos_15 = 40*ones(1,1000);
theta_15 =linspace(270,360,1000);
velocity_15 = sqrt(2*9.81*(125-function_15));
g_15 = velocity_15.^2/(9.81*47.5)-sind(theta_15);

%flat line
x_pos_16 = linspace(185,0,1000);
function_16 = zeros(1,1000);
z_pos_16 = 40*ones(1,1000);
theta_16 = zeros(1,1000);
a = -6.62837;
velocity_16(1) = velocity_15(end);

%velocity calculation based on kinematics
for i = 2:1000
    velocity_16(i) = sqrt(velocity_16(1)^2+2*a*(185-x_pos_16(i)));
end
g_16 = ones(1,1000);


%% Combine all position, velocity, g-force vectors
x_pos = [x_pos_1 x_pos_2 x_pos_3 x_pos_4 x_pos_5 x_pos_6 x_pos_7 x_pos_8 x_pos_9 x_pos_10 ...
    x_pos_11 x_pos_12 x_pos_13 x_pos_14 x_pos_15 x_pos_16];
y_pos = [function_1 function_2 function_3 function_4 function_5 function_6 function_7 ...
    function_8 function_9 function_10 function_11 function_12 function_13 function_14 ...
    function_15 function_16];
z_pos = [zeros(1,12000), z_pos_13 z_pos_14 z_pos_15 z_pos_16];
theta = [theta_1 theta_2 theta_3 theta_4 theta_5 theta_6 theta_7 theta_8 theta_9 theta_10 ...
    theta_11 theta_12 theta_14 theta_15 theta_16];
velocity = [velocity_1 velocity_2 velocity_3 velocity_4 velocity_5 velocity_6 velocity_7 ...
    velocity_8 velocity_9 velocity_10 velocity_11 velocity_12 velocity_13 velocity_14 ...
    velocity_15 velocity_16];
g_force = [g_1 g_2 g_3 g_4 g_5 g_6 g_7 g_8 g_9 g_10 g_11 g_12 vert_g g_14 g_15 g_16];
g_force_hor = [zeros(1,12000) hor_g zeros(1,3000)];



%% Solve for dx, dy, dz, ds, path length, duration
dx = zeros(1,length(x_pos)-1);
dy = zeros(1,length(y_pos)-1);
dz = zeros(1,length(z_pos)-1);
ds = zeros(1,length(x_pos)-1);
dt = zeros(1,length(x_pos)-1);


for i = 1:length(x_pos)-1
    dx(i) = x_pos(i+1) - x_pos(i);
    dy(i) = y_pos(i+1) - y_pos(i);
    dz(i) = z_pos(i+1) - z_pos(i);
    ds(i) = sqrt(dx(i)^2 + dy(i)^2 + dz(i)^2);
    dt(i) = ds(i)/velocity(i);
end
dt(1) = 0;
path_length = sum(ds);
duration = sum(dt);

 %% Plotting
```

```
%velocity against position
figure(1)
hold on
xx = [x_pos; x_pos];
yy = [y_pos; y_pos];
vvelocity = [velocity; velocity];
zz = [z_pos; z_pos];
surf(xx, zz, yy, vvelocity, 'EdgeColor', 'interp', 'LineWidth', 3)
colormap('jet')
grid on
xlim([0 350])
ylim([0 100])
zlim([0 175])
xlabel('X Position [m]')
ylabel('Y Position [m]')
zlabel('Z Position [m]')
title('Velocity as a Function of Position')

% vertical g force against position
figure(2)
hold on
xx = [x_pos; x_pos];
yy = [y_pos; y_pos];
ggforce = [g_force; g_force];
zz = [z_pos; z_pos];
surf(xx, zz, yy, ggforce, 'EdgeColor', 'interp', 'LineWidth', 3)
colormap('jet')
grid on
xlim([0 350])
ylim([0 100])
zlim([0 175])
xlabel('X Position [m]')
ylabel('Y Position [m]')
zlabel('Z Position [m]')
title('Vertical G-Force as a Function of Position')

%lateral g force against position
figure(3)
hold on
xx = [x_pos; x_pos];
yy = [y_pos; y_pos];
ggforceh = [g_force_hor; g_force_hor];
zz = [z_pos; z_pos];
surf(xx, zz, yy, ggforceh, 'EdgeColor', 'interp', 'LineWidth', 3)
colormap('jet')
grid on
xlim([0 350])
ylim([0 100])
zlim([0 175])
xlabel('X Position [m]')
ylabel('Y Position [m]')
zlabel('Z Position [m]')
title('Lateral G-Force as a Function of Position')

function [x_vals, z_vals, y_vals, N, L, hor_g, vert_g, mag_n, s, vel] = banked_turn(x_0, z_0, ...
    y_0, theta, r)
% Banked_turn is used for the ASEN 2003 rollercoaster lab
%   Function will create a blah blah blah


%% position
phi = (0:180); %theta around turn
z_vals = z_0 + r - (r * cosd(phi) );
x_vals = x_0 + r * sind(phi);
y_vals = y_0 * ones(1,length(x_vals));

%velocity
vel = sqrt(2 * 9.81 * (125-y_0) );

```

```
%solving equations
a = [cosd(theta) -sind(theta) ; -sind(theta) -cos(theta)];
b = [ 9.81; -vel^2/r];
a_inv = inv(a);
% variable mat: x = [N; L]
X = a_inv * b;

N = X(1);
L = X(2);

vert_g = N * ones(1, length(phi)) / 9.81;
hor_g = L * ones(1, length(phi)) / 9.81;

mag_n = sqrt(vert_g.^2 + hor_g.^2);

s = pi * r; %track length
vel = sqrt(2 * 9.81 * (125 - (y_0*ones(1,length(x_vals)))));
end
```