UNIVERSITY OF COLORADO - BOULDER

ASEN 2003

DYNAMICS LAB 4

Balanced and Unbalanced Wheel

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Contents

For this experiment, energy methods were used to derive dynamic models for the computation of angular velocity in a system. The two systems included in this lab are a balanced wheel and an unbalanced wheel both with a trailing support and encoder that numerically calculates the true angular velocity of the large cylinder. The first two models were for the balanced wheel, one assuming friction from the shaft is negligible and the other including a constant negative moment from the shaft. For the unbalanced wheel, it is similar to the second model except for the introduction of an extra mass. For the third model the extra mass is treated as a point mass and for the fourth model it is treated as a rigid body. These derivations then were used to determine the error associated with deriving the equation versus using the data that was given within the data for both the balanced and unbalanced wheel.

I. Nomenclature

m_c	=	Mass of Cylinder, [kg]
M_t	=	Mass of Trailing Supports, [kg]
m_m	=	Mass of Extra Mass, [kg]
R	=	Radius of Cylinder, [m]
k	=	Radius of Gyration of Wheel, [m]
I	=	Moment of Inertia, [kg*m^2]
β	=	Slope of Ramp, [degrees]
r_m	=	Radius to Extra Mass, [m]
$r_a m$	=	Radius of the Extra Mass, [m]
g	=	Gravitational Acceleration, [m/s^2]
ω	=	Angular Velocity [rad/s]
α	=	Angular Acceleration [rad/s^2]
θ	=	Angular Position [deg]

II. Model

A. Model 1

Model one describes the balanced wheel system while ignoring friction from its components. Assumptions for this model include, the wheel rolls without slipping and the trailing apparatus translates but does not rotate. Because there is no friction or other external forces acting on the system, there are no moments acting on the system. Using conservation of energy our derivation can be seen as,

model 1

$$\cancel{KE_1} + PE_1 = KE_2 + \cancel{PE_2}$$

$$m_c g h + m_t g h = \frac{1}{2} m_c v^2 + \frac{1}{2} m_t v^2 + \frac{1}{2} I \omega^2$$

$\rightarrow h = R\theta \sin\beta$

$\rightarrow v = \omega r$

$$g r \theta \sin\beta (m_c + m_t) = \frac{1}{2}\left(m_c \omega^2 R^2 + m_t \omega^2 R^2 + I \omega^2\right)$$

$$\omega = \sqrt{\frac{2(gR\theta \sin\beta)(m_c + m_t)}{r^2(m_c + m_t) + I}}$$

$$\omega = \sqrt{\frac{2(gR\theta \sin\beta)(m_c + m_t)}{R^2(m_c + m_t) + mk^2}}$$

This model can then be used to calculate the derivation for the second model.

B. Model 2

Model two describes the same system as model one with the introduction of a constant unknown negative moment applied to the shaft of the wheel. The moment was estimated by comparing the angular velocity computed to the actual angular velocity recorded by the encoder.

model 2

$$\cancel{KE_1} + PE_1 + U_{12} = KE_2 + \cancel{PE_2}$$

* only adding (U_{12}) which adds to the numerator of model 1.

$U_{12} = -M\theta$

* negative work through distance of θ

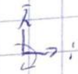
$$\therefore \omega = \sqrt{\frac{2(gR\theta \sin\beta)(m_c + m_t) - M\theta}{R^2(m_c + m_t) + mk^2}}$$

The moment within the equation is multiplied by θ which further approximates the value of ω because it makes it more accurate and accounts for the friction that is felt with the ramp.

C. Model 3

Model three describes the unbalanced wheel apparatus which has a point mass located a distance r from the center of the cylinder. The negative moment due to friction from part 2 is also included in this model. Initially the point mass is located such that a line passing through the offset mass and center of the larger cylinder is perpendicular to the ramp.

First an equation to calculate the magnitude of velocity for the offset mass was derived to be in terms of θ and ω:



$\vec{V}_{om} = \vec{V}_c + \omega \times \vec{r}_{om/c}$

V = velocity of point
ω = angular velocity of cylinder
r_{om} = radius to offset mass
R = radius of cylinder

$\omega = -\omega \hat{k}$

$\vec{r}_{om/c} = r_{om}\sin(\theta)\hat{i} + r_{om}\cos(\theta)\hat{j}$

$\vec{V}_c = R\omega\hat{i}$

$\vec{V}_{om} = R\omega\hat{i} - \omega\hat{k} \times \langle r_{om}\sin(\theta)\hat{i}, r_{om}\cos(\theta)\hat{j}, 0 \rangle$

$$= R\omega\hat{i} - \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ r_{om}\sin(\theta) & r_{om}\cos(\theta) & 0 \end{vmatrix}$$

$\vec{V}_{om} = R\omega\hat{i} + \omega\vec{r}_{om}\cos(\theta)\hat{i} - \omega\vec{r}_{om}\sin(\theta)\hat{j}$

$$|\vec{V}_{om}| = \sqrt{(R\omega + \omega\vec{r}_{om}\cos(\theta))^2 + (\omega\vec{r}_{om}\sin(\theta))^2}$$

$$= \sqrt{R^2\omega^2 + 2R\omega^2 r_{om}\cos(\theta) + \omega^2 r_{om}^2\cos^2(\theta) + \omega^2 r_{om}^2\sin^2(\theta)}$$

↓ trig identity

$$= \sqrt{R^2\omega^2 + 2Rr_{om}\omega^2\cos(\theta) + \omega^2 r_{om}^2}$$

$$\boxed{|\vec{V}_{om}| = \omega\sqrt{R^2 + 2Rr_{om}\cos(\theta) + r_{om}^2}}$$

Using the Principle of work and energy an equation to compute $\omega(\theta)$ was derived:

PE_{m1}	=	Initial Potential Energy of the offset mass
U_{12}	=	moment acting on system
KE_{c2}	=	Final Kinetic Energy of cylinder
KE_{t2}	=	Final Kinetic Energy of trailing apparatus
KE_{m2}	=	Final Kinetic Energy of the offset mass
PE_{c2}	=	Final Potential Energy of the cylinder
PE_{t2}	=	Final Potential Energy of trailing apparatus
PE_{m2}	=	Final Potential Energy of the offset mass

model 3

$$\cancel{KE_{c1}} + \cancel{KE_{t1}} + \cancel{KE_{m1}} + \cancel{PE_{c1}} + \cancel{PE_{t1}} + PE_{m1} + U_{12} = KE_{c2} + KE_{t2} + KE_{m2} + PE_{c2} + PE_{t2} + PE_{m2}$$

$$PE_{m1} = m_m g (r_m \cos\beta)$$

$$U_{12} = -M\theta$$

$$KE_{c2} = \tfrac{1}{2} m_c v^2 + \tfrac{1}{2} I \omega^2 \Rightarrow \tfrac{1}{2} m_c \omega^2 R^2 + \tfrac{1}{2} I \omega^2$$

$$KE_{t2} = \tfrac{1}{2} m v^2 \Rightarrow \tfrac{1}{2} m_t R^2 \omega^2$$

$$KE_{m2} = \tfrac{1}{2} m_m v^2 \Rightarrow \tfrac{1}{2} m_m \left(R^2 + 2 R r \cos\theta + r^2\right) \omega^2$$

$$PE_{c2} = -m_c g R\theta \sin\beta$$

$$PE_{t2} = -m_t g R\theta \sin\beta$$

$$PE_{m2} = -m_m g \left(R\theta \sin\beta + r_m \cos(\beta + \theta)\right)$$

$$\Rightarrow PE_{m1} + U_{12} - PE_{c2} - PE_{t2} - PE_{m2} = KE_{c2} + KE_{t2} + KE_{m2}$$

$$\Rightarrow PE_{m1} + U_{12} - PE_{c2} - PE_{t2} - PE_{m2} = \tfrac{1}{2}\omega^2 \left[(m_c R^2 + I) + (m_t R^2) + (m_m r_m^2 + I)\right]$$

$$\Rightarrow \omega^2 = \left[\frac{2\left[PE_{m1} + U_{12} - PE_{c2} - PE_{t2} - PE_{m2}\right]}{\left[(m_c R^2 + I) + (m_t R^2) + m_m\left(R^2 + 2 R r_m \cos\theta + r_m^2\right)\right]}\right]$$

$$\Rightarrow \omega = \left[\frac{2\left[PE_{m1} + U_{12} - PE_{c2} - PE_{t2} - PE_{m2}\right]}{\left[(m_c R^2 + m_c K^2) + (m_t R^2) + m_m\left(R^2 + 2 R r \cos\theta + r^2\right)\right]}\right]^{\frac{1}{2}}$$

D. Model 4

Model four describes the same system as model three but the offset mass is now treated as a rigid body. This change only affected the Kinetic Energy of the offset mass with the introduction of the offset mass rotational kinetic energy, the derivation for $\omega(\theta)$ is shown below:

model 4

added mass (m_m)

KE changes to

$$KE_{m2} = \frac{1}{2} m_m v^2 + \frac{1}{2} I_m \omega^2$$

$$= \frac{1}{2}\left(m_m R^2 + 2 R r_m \cos\theta + r^2 + I_m \right)$$

where $I_m = \frac{1}{2} m_m r_{am}^2$

$$\Rightarrow \omega = \left[\frac{2\left[PE_{m1} + U_{12} - PE_{c2} - PE_{t2} - PE_{m2}\right]}{\left(m_c R^2 + m_c k^2\right) + \left(m_t R^2\right) + m_m\left(R^2 + 2 R r_m \cos\theta + r_m^2 + r_{am}^2\right)} \right]^{\frac{1}{2}}$$

III. Results Analysis

Data collected for this experiment was provided to us in four files for each setup. The Data was read in the matlab script Lab4.m. The time, Angle of rotation and Angular velocity for each experiment were collected and used to determine the accuracy of our derived models. We had to extract this data to make accurate plots of the angular velocity versus the angle of rotation for each of these data sets. Figure One demonstrates all of the plots for the data sets balanced one, balanced two, unbalanced one, and unbalanced two. We decided to keep the x-axis and y-axis the same for all of the graphs with the x parameters being from 0 to 15 radians and the y axis being from 0 to 10 radians per second.

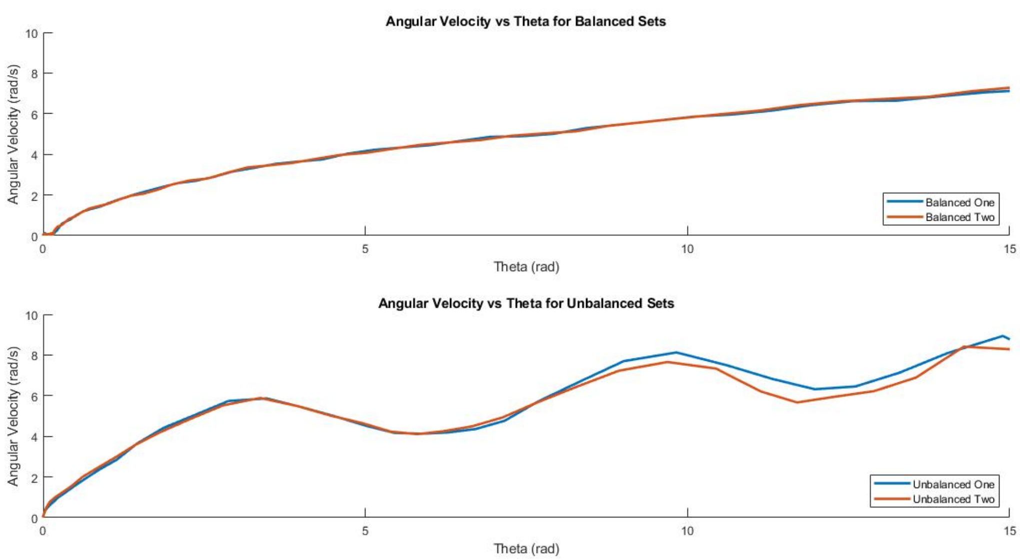


Figure 1: Plots for Angular velocity vs Angle of Rotation for each data set

We then went on to work with both of our derivations for Model One and Model Two. Below, Figure Two demonstrates two separate plots for the comparison between the experimental data for Balanced One and Model One for both of the balanced data sets. We decided to only plot balanced one for the comparison in this case because the experimental data would overlap for both sets, so this route makes it easier to read the plot. However, model one for both of the data sets overlaps anyway, which goes to show how close both of the data sets are to one another. The second plot shows the experimental data taken from the balanced one data set versus our derivation for model two. We plotted this with only the data from balanced one, but with the model being recalculated five times with five different moments. The values for moments that we used were 0.5, 1, 1.5, 2, and 2.5. Based off of the graph, we determined that the most accurate moment coefficient we could have used was 1.5, which is then what we used for the rest of model two's calculations.

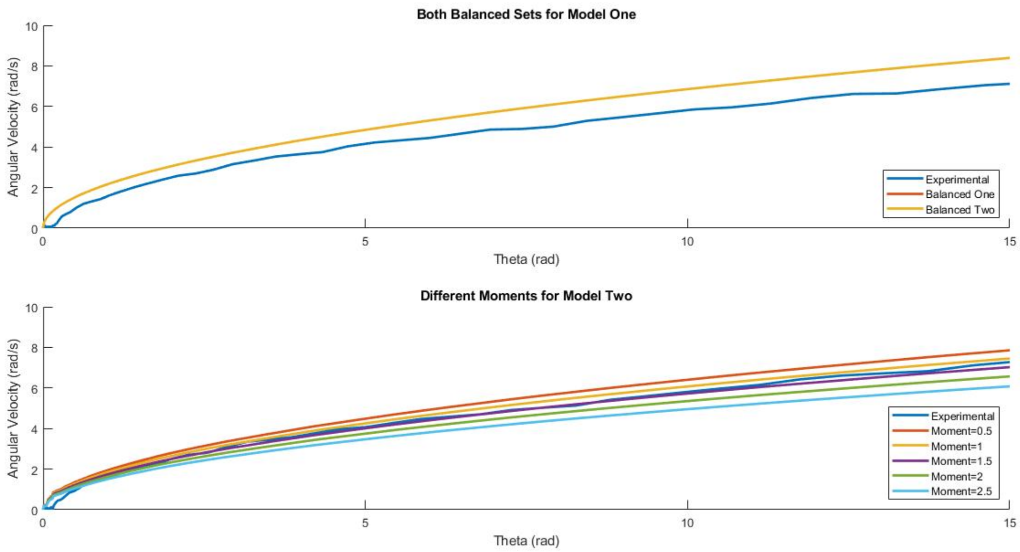


Figure 2: Plots for Model One and Model Two with Balanced One as the basis for the Experimental Data

Next, we made an additional subplot which can be found in Figure Three. In the top graph, we have the data for the balanced one set being plotted. The blue line with the markers is the experimental data extracted from the data file, the dashed line is our derivation for model one, and the solid line is our derivation for model two. The solid line for model two also uses our moment of 1.5, since it was the most accurate moment coefficient. Based off of the results of this plot, we determined that model two was the most accurate model for comparing to the original data for balanced one. The plot below this one is then the exact same thing, but with the data set for balanced two instead of balanced one. We came to the same conclusion based off of this graph, which is that model two is much more accurate than model one.

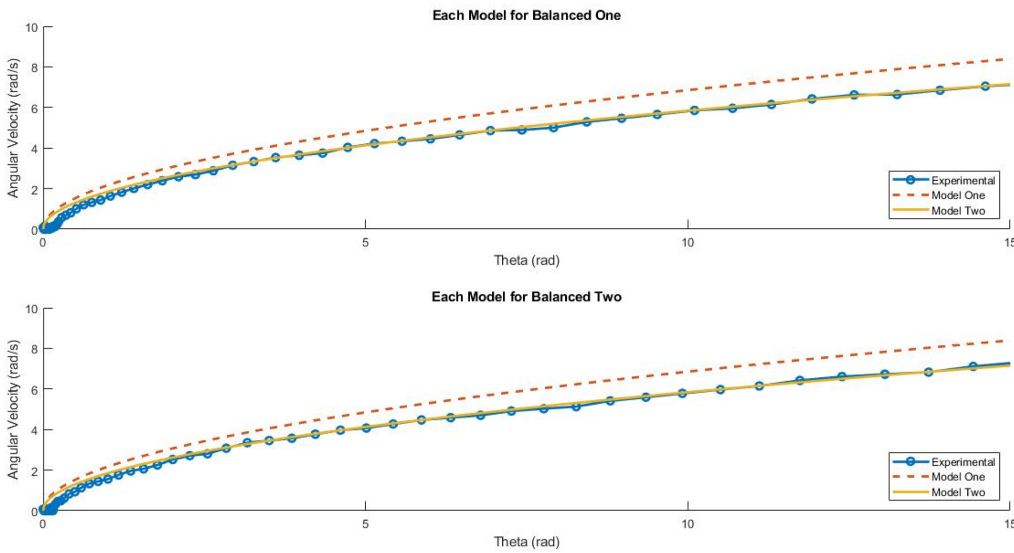


Figure 3: Plots for Angular Velocity vs Angle of Rotation for each Balanced Set, with both Model One and Model Two included

Following this section, we had to make another subplot which contained two individual graphs for unbalanced one and unbalanced two data. The top graph is the data for unbalanced one with both Model Three and Model Four being graphed with it. For model three, we used a moment of 0.75 within the derivation and for model four, we used a moment of 1. Because of this change in moment and the assumption that there is the extra radius of the mass in the denominator for model four, model four was less accurate based off of the experimental data for unbalanced one. The data set for unbalanced two uses the same moments, which means that again, model three is more accurate. Figure Four below shows both of these plots with their comparison for each model.

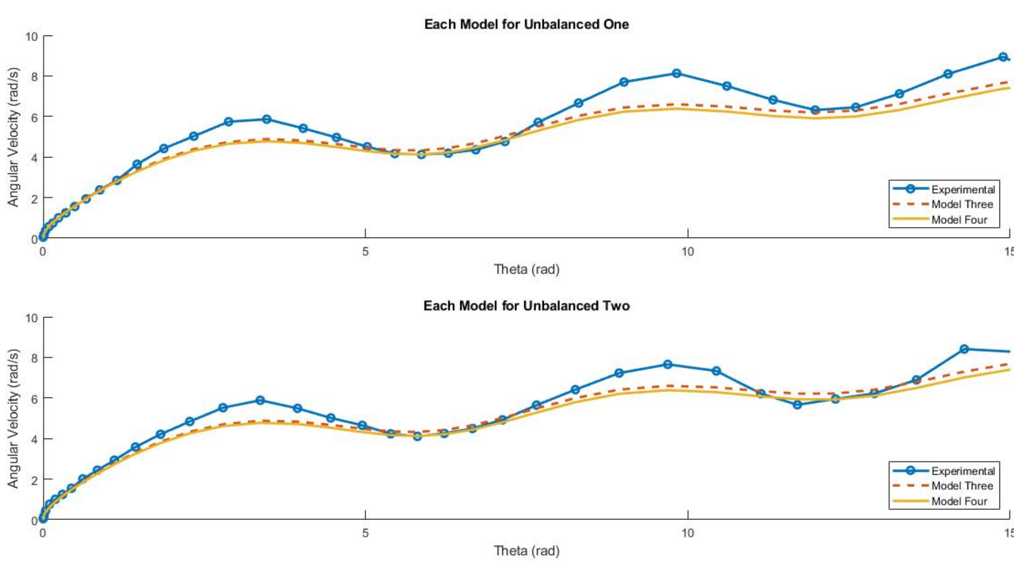


Figure 4: Plots for Angular velocity vs Angle of Rotation for Unbalanced Data set with Model Three and Model Four

Because our derivations for each model are not exact compared to the omega and theta extracted straight from the four data sets, there is residual differences between the data derivations and the actual data. Below in Figure Five is a subplot with four different graphs for each of the data sets and their error difference between each of the models for their set. The balanced sets only use model one and model two, and the unbalanced sets only use model three and model four; therefore, that is all they have on their own graph. For each graph we went with consistent axis parameters just like before with the x axis spanning from 0 to 15 radians still, and the y axis spanning from -2 to 2 radians per second now. This is because the maximum differential we experience between our derivations and the actual data will always be in between -2 and 2. The equation we were given to use to calculate the residual difference for each model was $Residual = Observed(Experimental) - Predicted(Model)$. For the first graph, we have the residual difference for the balanced one set. Model one in this case is larger than two because two was more accurate, and both are negative because our derivations make it so that the model is greater than the experimental. The next graph contains the balanced two data set. We assumed that the moment for this set was the 1.5 that we determined from Figure Two above. It has the same trend as the plot for balanced one, because they both use the same model. Then for the unbalanced data sets, it is a little different. For unbalanced one, model four has a slightly larger residual difference than model three, which demonstrates that model three would be more accurate. This stays consistent for both data sets as they stay pretty consistent for their residual difference. Both data sets also sway in between positive and negative values since it acts as a wave and it oscillates between being greater than or less than the experimental data. The largest sources of error we determined were having small variations in our derivations compared to the original model. For the balanced data sets, model two is much more accurate and lies almost exactly on top of the experimental data. This is due to the fact that we have a moment that makes it so the derivation is correct to the data set. For the unbalanced data however, we could never make a derivation that reached the peaks exactly to line up with the experimental data. Therefore, our moment coefficient would only decrease the size of the peaks of the model when we would increase it, but if we would decrease the moment, the peaks would not be large enough to reach the experimental data. Therefore, our largest source of error was in the conceptualization of the derivation and not being accurate enough for model three and model four.

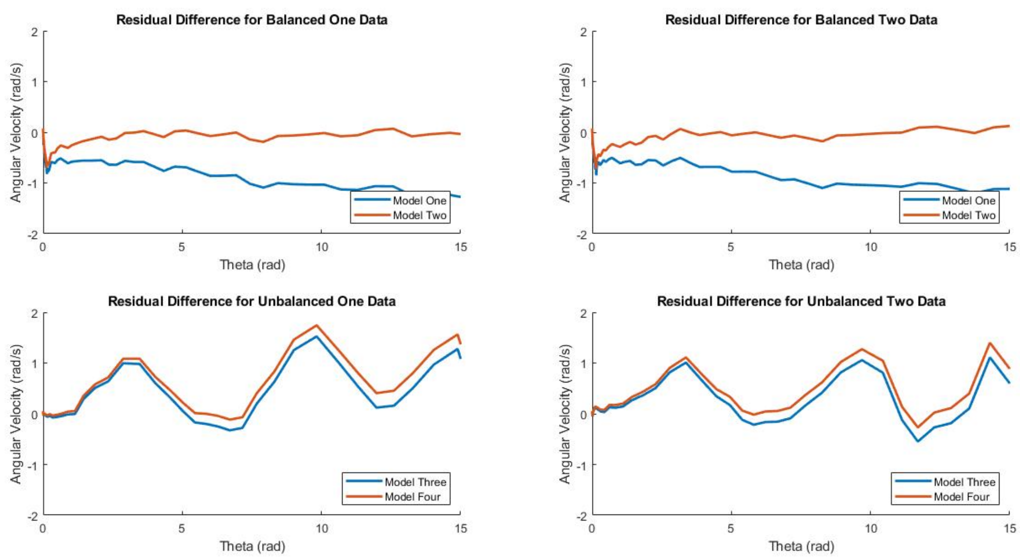


Figure 5: Plots for the Residual Difference between all of the Data Sets

We then had to do a few more calculations based off of the residual data. For Table One below, we assumed that for our calculations, we used the residual data that had the highest values, so model one for both balanced one and two, and model four for both unbalanced one and two. We have the standard deviation for each of the sets, which is where 68% of the data falls in between. It is the largest for balanced two, which makes sense because we felt the most off balanced about this data set. Next was the mean of all of the residuals, which again was highest for balanced two. Then, we calculated the uncertainty which follows the equation uncertainty equals sigma over the square root of N where sigma is the standard deviation and N is the number of observations. The largest value for this one is unbalanced two, because it has a small amount of observations. Then is the observations which is just based off of how many data points there are for each residual. Last is the number of occurrences when the residual is greater than three times the standard deviation, which is zero for every data set. All of this data can be seen below in Table One.

	Balanced One	**Balanced Two**	**Unbalanced One**	**Unbalanced Two**
Standard Deviation (rad/s)	3.8478	3.8559	0.9814	2.7494
Mean (rad/s)	-3.1097	-3.1905	0.2994	-0.9320
Uncertainty	0.3101	0.3107	0.1592	0.4098
Number of Observations	154	154	38	45
Residuals greater than three STD	0	0	0	0

Table 1: All of the Residual data for each of the Data sets

IV. Conclusions and Recommendations

Matlab was used throughout this lab to compare our derived models of both the balanced and unbalanced wheel apparatus to the collected experimental data. Model 2 revealed the importance of including the moment due to friction acting on the system to our derivations. This is shown in the residual plots where the residual for model two remains minimal while for model one it continues to increase as theta increases. Model four conveys the relative unimportance in treating the offset mass as a rigid body, as the difference between the residual plots for model three and four is minimal. A noticeable problem in our derivations is that our models do not account for side to side movement of the wheel apparatus. This could be the reason for major some error in our models calculations. To improve this experiment a system that could prevent the wheel from moving side to side could be installed and accounted for in the frictional moment coefficient. However, we believe that our derivations were done to the best of our ability and that everything matched what we were hoping. This lab helped us to understand the fundamental concepts we have been going over in lecture, and helped us understand the experimental importance of the dynamics equations.

V. References

ASEN 2003 LAB 4: BALANCED AND UNBALANCED WHEEL, University of Colorado at Boulder, Boulder, CO, 2020.

VI. Acknowledgements

Thank you to our Lab instructors Bobby Hodgkinson and Josh Mellin, along with all the TA's for help and support throughout the lab and everything they have done for us.

VII. Appendices

A. Team Participation

	Derivations	Code	Write Up
Benjamin Lodwick	1	2	1
Connor O'Reilly	1	1	2
Nathan Portman	2	1	1

B. MatLab Code

```
%% Lab 4 Code

%% housekeeping
clear all;
clc;

%% General Constants

M = 11.7;           %[kg] Mass of cylinder
M_0 = 0.7;          %[kg] Mass of trailing supports
m = 3.4;            %[kg] Mass of extra mass
R = 0.235;          %[m] Radius of cylinder
K = 0.203;          %[m] Radius of gyration of wheel
I = M*K^(2);        %[kg/m^2] Moment of Intertia
Beta = 5.5;         %[deg] Slope of ramp
r = 0.178;          %[m] Radius to extra mass
r_m = 0.019;        %[m] Radius of the extra mass
g = 9.81;           %[m/s^2] Gravitational acceleration

%% Read in data files

[b1_time, b1_theta, b1_omega] = Readfile('balanced_1');         %Read data for bal one
[b2_time, b2_theta, b2_omega] = Readfile('balanced_2');         %Read data for bal two
[ub1_time, ub1_theta, ub1_omega] = Readfile('unbalanced_1');    %Read data for unbal one
[ub2_time, ub2_theta, ub2_omega] = Readfile('unbalanced_2');    %Read data for unbal two

%% Model 1

[w1_b1] = Model_one(g, R, b1_theta, Beta, M, M_0, K);       %Run Model_one with bal one data
[w1_b2] = Model_one(g, R, b2_theta, Beta, M, M_0, K);       %Run Model_one with bal two data
Resid_1_1 = b1_omega - w1_b1;                               %Compute residual for model one ...
    balanced one
Resid_1_2 = b2_omega - w1_b2;                               %Compute residual for model one ...
    balanced two

%% Model 2

Moment = 1.5;
[w2_b1] = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %Run Model_two with bal ...
    one data
[w2_b2] = real(Model_two(g, R, b2_theta, Beta, M, M_0, K, Moment));   %Run Model_two with bal ...
    two data
Resid_2_1 = b1_omega - w2_b1;                                        %Compute residual for ...
    model two balanced one
Resid_2_2 = b2_omega - w2_b2;                                        %Compute residual for ...
    model two balanced two

Moment = 0.5;                                                       %Change moment
M1_w2 = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %New model with moment
Moment = 1;                                                         %Change moment
M2_w2 = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %New model with moment
Moment = 1.5;                                                       %Change moment
```

```
M3_w2 = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %New model with moment
Moment = 2;                                                          %Change moment
M4_w2 = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %New model with moment
Moment = 2.5;                                                          %Change moment
M5_w2 = real(Model_two(g, R, b1_theta, Beta, M, M_0, K, Moment));   %New model with moment

%% Model 3

Moment = .75;
[w3_ub1] = real(Model_three(m, g, R, Beta, M, ub1_theta, M_0, Moment, K, r));   %Run ...
    Model_three with ubal one data
w3_ub1 = w3_ub1(:,1);
[w3_ub2] = real(Model_three(m, g, R, Beta, M, ub2_theta, M_0, Moment, K, r));   %Run ...
    Model_three with ubal two data
w3_ub2 = w3_ub2(:,1);
Resid_3_1 = ub1_omega - w3_ub1;                                     %Compute residual for ...
    model three unbalanced one
Resid_3_2 = ub2_omega - w3_ub2;                                     %Compute residual for ...
    model three unbalanced two

%% Model 4

Moment = 1;
[w4_ub1] = real(Model_four(m, g, R, Beta, M, ub1_theta, M_0, Moment, K, r, r_m));   %Run ...
    Model_four with ubal one data
w4_ub1 = w4_ub1(:,1);
[w4_ub2] = real(Model_four(m, g, R, Beta, M, ub2_theta, M_0, Moment, K, r, r_m));   %Run ...
    Model_four with ubal two data
w4_ub2 = w4_ub2(:,1);
Resid_4_1 = ub1_omega - w4_ub1;                                     %Compute residual for ...
    model four unbalanced one
Resid_4_2 = ub2_omega - w4_ub2;                                     %Compute residual for ...
    model four unbalanced two

%% Residual data RESULTS 15

sig1 = std(Resid_1_1);      %Standard deviation of balanced one
sig2 = std(Resid_1_2);      %Standard deviation of balanced two
sig3 = std(Resid_4_1);      %Standard deviation of unbalanced one
sig4 = std(Resid_4_2);      %Standard deviation of unbalanced two
mean1 = mean(Resid_1_1);    %Mean for balanced one
mean2 = mean(Resid_1_2);    %Mean for balanced two
mean3 = mean(Resid_4_1);    %Mean for unbalanced one
mean4 = mean(Resid_4_2);    %Mean for unbalanced two
N1 = length(Resid_1_1);     %N number of observations
N2 = length(Resid_1_2);     %N number of observations
N3 = length(Resid_4_1);     %N number of observations
N4 = length(Resid_4_2);     %N number of observations
uncer1 = sig1 / sqrt(N1);   %Uncertainty from equation given
uncer2 = sig2 / sqrt(N2);   %Uncertainty from equation given
uncer3 = sig3 / sqrt(N3);   %Uncertainty from equation given
uncer4 = sig4 / sqrt(N4);   %Uncertainty from equation given
occur1 = 0;                 %Set initial value
occur2 = 0;                 %Set initial value
occur3 = 0;                 %Set initial value
occur4 = 0;                 %Set initial value
for i = 1:N1
    if Resid_1_1(i) > 3*sig1
        occur1 = occur1 + 1;    %Find if the residual value is greater than 3sig
    end
end
for i = 1:N2
    if Resid_1_2(i) > 3*sig2
        occur2 = occur2 + 1;    %Find if the residual value is greater than 3sig
    end
end
for i = 1:N3
    if Resid_4_1(i) > 3*sig3
        occur3 = occur3 + 1;    %Find if the residual value is greater than 3sig
    end
```

```
end
for i = 1:N4
    if Resid_4_2(i) > 3*sig4
        occur4 = occur4 + 1;    %Find if the residual value is greater than 3sig
    end
end

%% Plots for w vs theta RESULTS 10

figure
subplot(2,1,1)
hold on
plot(b1_theta, b1_omega,'LineWidth',2)
plot(b2_theta, b2_omega,'LineWidth',2)
xlim([0 15])
ylim([0 10])
title('Angular Velocity vs Theta for Balanced Sets')
xlabel('Theta (rad)')
ylabel('Angular Velocity (rad/s)')
legend('Balanced One','Balanced Two','Location','southeast')
hold off

subplot(2,1,2)
hold on
plot(ub1_theta, ub1_omega,'LineWidth',2)
plot(ub2_theta, ub2_omega,'LineWidth',2)
xlim([0 15])
ylim([0 10])
title('Angular Velocity vs Theta for Unbalanced Sets')
xlabel('Theta (rad)')
ylabel('Angular Velocity (rad/s)')
legend('Unbalanced One','Unbalanced Two','Location','southeast')
hold off

%% Plots for Model One and Moments for Model Two RESULTS 11

figure
subplot(2,1,1)
hold on
plot(b1_theta, b1_omega,'LineWidth',2)  %Plot the experimental data
plot(b1_theta, w1_b1,'LineWidth',2)     %Plot data for Model_one
plot(b2_theta, w1_b2,'LineWIdth',2)     %Plot data for Model_two
xlim([0 15])
ylim([0 10])
title('Both Balanced Sets for Model One')    %Title for Balanced ONE data
xlabel('Theta (rad)')                   %Label x axis
ylabel('Angular Velocity (rad/s)')      %Label y axis
legend('Experimental','Balanced One','Balanced Two','Location','southeast')  %Create the ...
    legend for all plots
hold off

subplot(2,1,2)
hold on
plot(b2_theta, b2_omega,'LineWidth',2)  %Plot the experimental data
plot(b2_theta, M1_w2,'LineWidth',2)     %Plot each Moment
plot(b2_theta, M2_w2,'LineWidth',2)     %Plot each Moment
plot(b2_theta, M3_w2,'LineWidth',2)     %Plot each Moment
plot(b2_theta, M4_w2,'LineWidth',2)     %Plot each Moment
plot(b2_theta, M5_w2,'LineWidth',2)     %Plot each Moment
xlim([0 15])
ylim([0 10])
title('Different Moments for Model Two')    %Title for Balanced TWO data
xlabel('Theta (rad)')                   %Label x axis
ylabel('Angular Velocity (rad/s)')      %Label y axis
legend('Experimental','Moment=0.5','Moment=1','Moment=1.5','Moment=2','Moment=2.5','Location','southeast') ...
    %Create the legend for all plots
hold off

%% Plots for Both Data RESULTS 12

```

```
figure
subplot(2,1,1)
hold on
plot(b1_theta, b1_omega,'-o','LineWidth',2)  %Plot the experimental data
plot(b1_theta, w1_b1,'--','LineWidth',2)     %Plot data for Model_one
plot(b1_theta, w2_b1,'LineWidth',2)     %Plot data for Model_two
xlim([0 15])
ylim([0 10])
title('Each Model for Balanced One')    %Title for Balanced ONE data
xlabel('Theta (rad)')                   %Label x axis
ylabel('Angular Velocity (rad/s)')      %Label y axis
legend('Experimental','Model One','Model Two','Location','southeast')  %Create the legend for ...
    all plots
hold off

subplot(2,1,2)
hold on
plot(b2_theta, b2_omega,'-o','LineWidth',2)  %Plot the experimental data
plot(b2_theta, w1_b2,'--','LineWidth',2)     %Plot data for Model_one
plot(b2_theta, w2_b2,'LineWidth',2)     %Plot data for Model_two
xlim([0 15])
ylim([0 10])
title('Each Model for Balanced Two')    %Title for Balanced ONE data
xlabel('Theta (rad)')                   %Label x axis
ylabel('Angular Velocity (rad/s)')      %Label y axis
legend('Experimental','Model One','Model Two','Location','southeast')  %Create the legend for ...
    all plots
hold off

%% Plots for Model 3 and 4 RESULTS 13

figure
subplot(2,1,1)
hold on
plot(ub1_theta, ub1_omega,'-o','LineWidth',2)  %Plot the experimental data
plot(ub1_theta, w3_ub1,'--','LineWidth',2)     %Plot data for Model_three
plot(ub1_theta, w4_ub1,'LineWIdth',2)     %Plot data for Model_four
xlim([0 15])
ylim([0 10])
title('Each Model for Unbalanced One')    %Title for UNBalanced ONE data
xlabel('Theta (rad)')                     %Label x axis
ylabel('Angular Velocity (rad/s)')        %Label y axis
legend('Experimental','Model Three','Model Four','Location','southeast')  %Create the legend ...
    for all plots
hold off

subplot(2,1,2)
hold on
plot(ub2_theta, ub2_omega,'-o','LineWidth',2)  %Plot the experimental data
plot(ub2_theta, w3_ub2,'--','LineWidth',2)     %Plot data for Model_three
plot(ub2_theta, w4_ub2,'LineWIdth',2)     %Plot data for Model_four
xlim([0 15])
ylim([0 10])
title('Each Model for Unbalanced Two')    %Title for UNBalanced TWO data
xlabel('Theta (rad)')                     %Label x axis
ylabel('Angular Velocity (rad/s)')        %Label y axis
legend('Experimental','Model Three','Model Four','Location','southeast')  %Create the legend ...
    for all plots
hold off

%% Plots for Residual

figure
subplot(2,2,1)                                    %Make a subplot for residual
hold on
plot(b1_theta, Resid_1_1,'LineWidth',2)           %Plot resid for model one
plot(b1_theta, Resid_2_1,'LineWidth',2)           %Plot resid for model 2
xlim([0 15])                                      %Set xlim
ylim([-2 2])
title('Residual Difference for Balanced One Data')  %Title
```

```
xlabel('Theta (rad)')                                   %Axis
ylabel('Angular Velocity (rad/s)')
legend('Model One','Model Two','Location','southeast')
hold off

subplot(2,2,2)                                          %Make a subplot for residual
hold on
plot(b2_theta, Resid_1_2,'LineWidth',2)                 %Plot resid for model one
plot(b2_theta, Resid_2_2,'LineWidth',2)                 %Plot resid for model 2
xlim([0 15])                                            %Set xlim
ylim([-2 2])
title('Residual Difference for Balanced Two Data')  %Title
xlabel('Theta (rad)')                                   %Axis
ylabel('Angular Velocity (rad/s)')
legend('Model One','Model Two','Location','southeast')
hold off

subplot(2,2,3)                                          %Make a subplot for residual
hold on
plot(ub1_theta, Resid_3_1,'LineWidth',2)                 %Plot resid for model three
plot(ub1_theta, Resid_4_1,'LineWidth',2)                 %Plot resid for model 4
xlim([0 15])                                            %Set xlim
ylim([-2 2])
title('Residual Difference for Unbalanced One Data')  %Title
xlabel('Theta (rad)')                                   %Axis
ylabel('Angular Velocity (rad/s)')
legend('Model Three','Model Four','Location','southeast')
hold off

subplot(2,2,4)                                          %Make a subplot for residual
hold on
plot(ub2_theta, Resid_3_2,'LineWidth',2)                 %Plot resid for model three
plot(ub2_theta, Resid_4_2,'LineWidth',2)                 %Plot resid for model 4
xlim([0 15])                                            %Set xlim
ylim([-2 2])
title('Residual Difference for Unbalanced Two Data')  %Title
xlabel('Theta (rad)')                                   %Axis
ylabel('Angular Velocity (rad/s)')
legend('Model Three','Model Four','Location','southeast')
hold off

%% Functions

function [time, theta, omega] = Readfile(filename)
txt = readtable(filename);      % Reads in the data for balanced one
txt = table2array(txt);         %Converts to an array
time = txt(:,1);                %[s] The time for data balanced one
theta = txt(:,2);               %[rad] The theta angle for balanced one
omega = txt(:,3);               %[rad/s] The omega angular velocity for balanced one
end

function [w] = Model_one(g, R, theta, Beta, M, M_0, K)  %Solve w for our derivation
w = sqrt((2*g*R*theta*sind(Beta)*(M+M_0))/(M*(R.^(2)+K.^(2))+M_0*R.^(2)));
end

function [w] = Model_two(g, R, theta, Beta, M, M_0, K, Moment)  %Solve w for our derivation
w = sqrt((2.*g.*R.*theta.*sind(Beta).*(M+M_0)-Moment*theta)./(M.*(R.^(2)+K.^(2))+M_0.*R.^(2)));
end

function [w] = Model_three(m, g, R, Beta, M, theta, M_0, Moment, K, r)  %Solve w for our ...
    derivation
w = ...
    sqrt((2*((m*g*(R*theta*sind(Beta)+r*cosd(Beta)))+(M*g*(R*theta*sind(Beta)))+(M_0*g*(R*theta*sind(Beta)))-(m*g
end

function [w] = Model_four(m, g, R, Beta, M, theta, M_0, Moment, K, r, r_m)  %Solve w for our ...
    derivation
w = ...
    sqrt((2*((m*g*(R*theta*sind(Beta)+r*cosd(Beta)))+(M*g*(R*theta*sind(Beta)))+(M_0*g*(R*theta*sind(Beta)))-(m*g
end
```