# University of Colorado - Boulder

ASEN 3128: Lab 5

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# Lab 5: Twist? Control

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# I. Problem 1

Motor thrust is determined in the AeroForcesAndMoments\_BodyState\_WindCoeffs.m MATLAB function that was provided in the lab5\_to\_share code folder. The equation that relates the throttle to thrust is

$$T_{prop} = \rho \pi R_{prop}^2 C_{prop} (V_a + \delta_t (k_m - V_a)) (\delta_t (k_m - V_a))$$

where  $T_{prop}$  is the thrust produced by the propeller,  $\delta_t$  is the throttle command,  $k_m$  is the motor constant in [m/s],  $R_{prop}$  is the radius of the propeller,  $C_{prop}$  is a unitless coefficient that describes the efficiency of the propeller and  $V_a$  is the velocity of the air entering the propeller.

# II. Problem 2

A.

To update *CalculateTrimFromStaticStability.m* static stability analysis was used to calculate the angle of attack and elevator angle for trim based on the trim definition and aircraft parameters. At equilibrium

$$C_{L_{trim}} = \frac{W}{\frac{1}{2}\rho V_a^2 S}$$

$$C_{mtrim} = 0$$

. where  $C_{L_q}\hat{q} = C_{m_q}\hat{q} = 0$ . Using these definitions for trim and the provided aircraft parameters the angle of attack and elevator angle for trim were computed using the following matrix equation.

$$\begin{bmatrix} \alpha_{trim} \\ \delta_{e_{trim}} \end{bmatrix} = \begin{bmatrix} C_{L_{trim}} \\ -C_{m_0} \end{bmatrix} \begin{bmatrix} C_{L_{\alpha}} & C_{L_{\delta_e}} \\ C_{m_{\alpha}} & C_{m_{\delta_e}} \end{bmatrix}^{-1}$$
(1)

В.

Using variables  $trim\_de\ finition$  and  $trim\_variables$  the candidate trim condition and the sum of the magnitudes of the total force and total moment acting on the aircraft are were computed. Initially the function CalculateStateTrim.m was created to calculate the state and control input vectors from the input vectors  $trim\_de\ finition$  and  $trim\_variables$ . Assuming no background wind values for  $u_0^E$  and  $w_0^E$  are computed using the following equations and the given value

for  $\theta_0$ .

$$u_0^E = V_0 cos(\theta_0)$$

$$w_0^E = V_0 sin(\theta_0)$$

where  $u_0^E$  and  $w_0^E$  is the velocity of the aircraft in inertial coordinates,  $V_0$  is the airspeed of the aircraft and  $\theta_0$  is the provided pitch angle at trim. The function then returns both the aircraft state and control input vectors for trim. Following, using the provided AeroForcesAndMoments.m function the total force and moment acting on the aircraft were calculated, normalized and summed to obtain the aerodynamic cost.

# III. Problem 3

A.

The Tempest unmanned aircraft is at trim at height h of 600 [m] with an airspeed V of 19 [m/s], Using the provided MATLAB function CalculateTrimVariables.m and designed MATLAB function CalculateTrimFromStaticStability.m values for  $\alpha_0$ ,  $\delta_{e_0}$  and  $\delta_{t_0}$  were computed. CalculateTrimFromStaticStability.m was used to find conditions for trim from static stability, which were then used in CalculateTrimVariables.m in order to optimize the values. Those found in CalculateTrimFromStaticStability.m are

$$\begin{pmatrix}
\alpha_0 \\
\delta_{e_0} \\
\delta_{t_0}
\end{pmatrix} = \begin{pmatrix}
0.0691 \\
-0.0900 \\
0.0918
\end{pmatrix} [rad]$$
(2)

When these values were used in CalculateTrimVariables.m, the values for trim are found to be the following

$$x_{0} = \begin{pmatrix} 0 \\ 0 \\ -600[m] \\ 0 \\ 0.0691[rad] \\ 0 \\ 19.00[m/s] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{u}_{0} = \begin{pmatrix} -0.0900 \\ 0 \\ 0 \\ 0.0918 \end{pmatrix} [rad]$$

# B.

The results from part a are verified when plotted against static stability analysis for the same trim conditions. Both have been plotted in the following seven figures, with calculated trim variables plotted in blue and static stability analysis plotted in red. The results appear similar, however static stability analysis neglects any tail deflection. Both plots tend toward stability as the flight path decreases, but the flight from stability analysis decreases slightly in altitude.

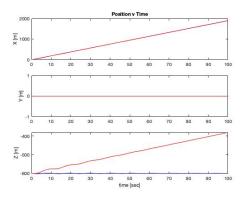


Fig. 1 Position vs. Time for 3 a and b

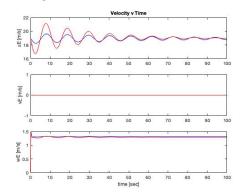


Fig. 3 Velocity vs. Time for 3 a and b

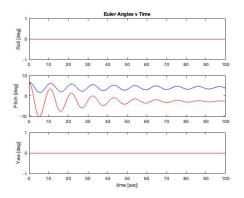


Fig. 2 Euler Angles vs. Time for 3 a and b

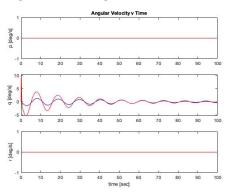


Fig. 4 Angular Velocity vs. Time for 3 a and b

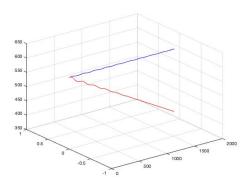


Fig. 5 3D Positional Plot for 3 a and b

# IV. Problem 4

# A.

As done in Problem 3, the aircraft is simulated without background wind at trim conditions. The aircraft is simulated at a velocity of 21 m/s at an altitude of 2000m. The same method of using the MATLAB functions CalculateTrimVariables.m and CalculateTrimFromStaticStability.m, to find values for  $\alpha_0$ ,  $\delta_{e_0}$  and  $\delta_{t_0}$ .

#### B.

A background wind of [10; 10; 0] m/s is added to the simulation done in part A, with identical initial conditions and control inputs. The trim conditions are designed for flight without background wind, meaning flight is unstable with wind. The aircraft shifts in the x and y direction, and falls in altitude. Both the lateral and longitudinal dynamics oscillate about a different value. The wavering of the aircraft remains constant as the aircraft continues, and does not settle towards any value.

# C.

In order for the aircraft to be in trim under these conditions, the initial state vector must be adjusted to negate the winds affects. This is done by adding the wind velocity, in body coordinates, to the aircraft state vector as found in part 4 a. This will have the effect of negating the lateral effects of the wind, and allowing for trim flight. The result are included the following vectors for initial state and control surfaces.

$$x_{0} = \begin{pmatrix} 0 \\ 0 \\ -2000[m] \\ 0 \\ 0.130[rad] \\ 0 \\ 28.9[m/s] \\ 10 \\ 1.30 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{u}_0 = \begin{pmatrix} -0.7854 \\ 0 \\ 0 \\ 0.0825 \end{pmatrix} [rad]$$

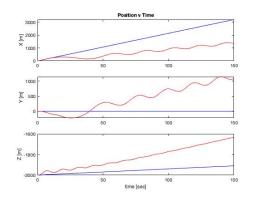


Fig. 6 Position vs. Time for 4 a and b

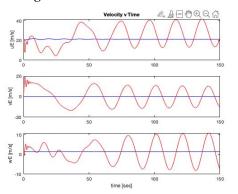


Fig. 8 Velocity vs. Time for 4 a and b

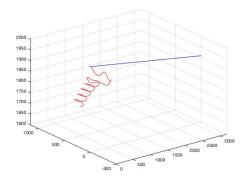


Fig. 10 3D Positional Plot for 4 a and b

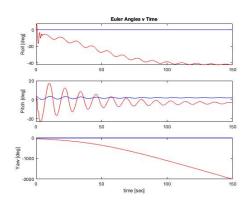


Fig. 7 Euler Angles vs. Time for 4 a and b

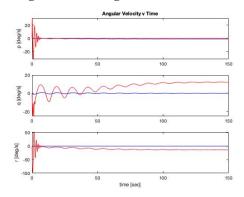


Fig. 9 Angular Velocity vs. Time for 4 a and b

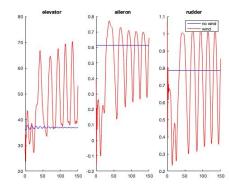


Fig. 11 Wind Angles vs. Time for 4 a and b

D.

Initially to get the yaw angle needed satisfy the requirement of the problem statement the aircraft wind triangle needed to be examined. Knowing that the air relative velocity was solely in the direction of the longitudinal x axis of the aircraft the crab angle could be approximated using the law of sines. After solving for the crab angle the yaw angle could be computed by the addition on crab angle and the euler angle psi. Below is the equation used to determine the yaw angle,

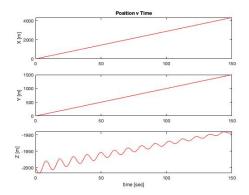


Fig. 12 Position vs. Time for 4 c

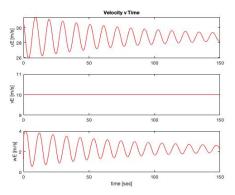


Fig. 14 Velocity vs. Time for 4 c

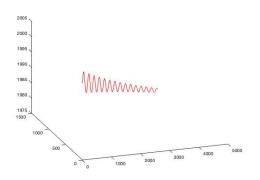


Fig. 16 3D Positional Plot for 4 c

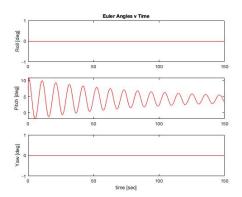


Fig. 13 Euler Angles vs. Time for 4 c

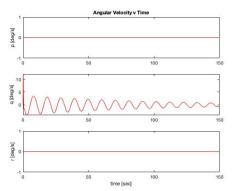


Fig. 15 Angular Velocity vs. Time for 4 c

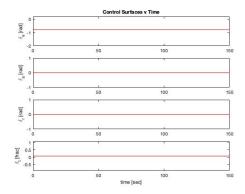


Fig. 17 Control Surfaces vs. Time for 4 c

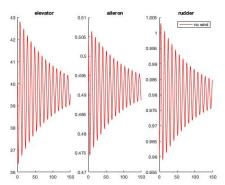


Fig. 18 Wind Angles vs. Time for 4 c

$$\psi = \chi - asin\left(sin(\chi_w - \chi) * \frac{|W^E|}{|V|}\right)$$

where  $\psi$  is the yaw angle,  $\chi$  is the course angle,  $\chi_w$  is the wind angle,  $W^E$  is the wind velocity in inertial coordinates and V is the velocity of the aircraft in body coordinates. The yaw angle was determine to be  $\psi = 1.2224$  [rad].

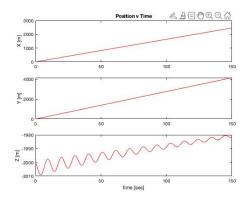


Fig. 19 Position vs. Time for 4 d

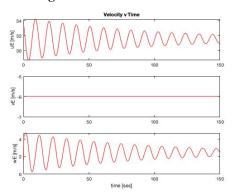


Fig. 21 Velocity vs. Time for 4 d

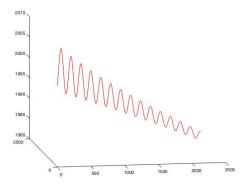


Fig. 23 3D Positional Plot for 4 d

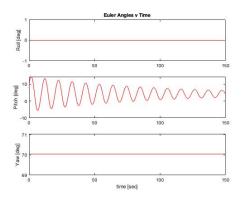


Fig. 20 Euler Angles vs. Time for 4 d

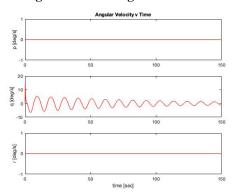


Fig. 22 Angular Velocity vs. Time for 4 d

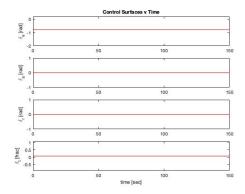


Fig. 24 Control Surfaces vs. Time for 4 d

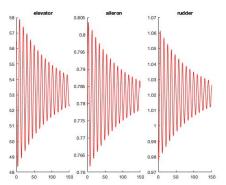


Fig. 25 Wind Angles vs. Time for 4 d