University of Colorado - Boulder

ASEN 3200 - Orbital Mechanics/Attitude Dynamics and Control

Section 011, Group 8

Lab A2: Spinning Spacecraft

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The objective of this lab was to learn to characterize the motion and rotation of spinning spacecraft by both qualitative observation of several animated spacecraft simulations, and through physical experimentation and analysis of torques applied to a spinning object, in this case, a bicycle wheel. The first part of this lab involved the high level analysis of five spinning "spacecraft", where the angular velocity, angular momentum, and any external torques or extraneous effect on the kinetic energy were explored. The second part involved the detailed, quantitative study of the effect of external torques on a spinning bicycle wheel. The results of both parts are detailed below, the overall average error of the model from the data was $0.1033\frac{rad}{s}$**. This lab taught the value of qualitative studies on the understanding of fundamental concepts of spacecraft dynamics.**

Introduction

This lab investigates the rotating motion of rigid bodies in a variety of environments, encompassing motion with and without external torque, and examining the result of different scenarios on the behavior of a spinning body. The results of these analyses could inform the placement of reaction control wheels or control moment gyros on board a spacecraft. They could also be used to predict the resulting behavior of a spacecraft given a torque, possibly exerted by a cold thruster or internal angular momentum control device.

I. Spacecraft Animations

A. Preliminary Questions

1. For the angular momentum of a rotating object to move with respect to the inertial frame, there must be some external torque or moment applied to the object.

2. Oblate means that a circular or spherical object is flattened near its poles. Prolate means essentially the opposite, where an object is lengthened or "sharpened" near its poles. Spin near the major or minor axis is when the angular momentum vector of a spacecraft is closely or completely lined up with one of its principle axes. If a spacecraft is spinning near or about the major or minor axis the rotation will be stable, barring any drastic effects due to energy dissipation. The axis a spacecraft is rotating about can usually be fairly easily characterized by observation because spacecraft are usually "long and skinny" in order to fit aboard the rockets that carry them into space. Spacecraft generally spin about their minor or "long" axis, as it provides the most stability. Rotation about the major and intermediate axes is much less stable in spacecraft as the moments of inertia about those axes are usually much more similar to each other than to that of the minor axis.

3. When viewed relative to the body frame, the angular velocity vector traces a cone, and if there is no energy dissipation, it will continue to describe the same cone forever. In cases where energy dissipation needs to be considered, the cone will become progressively smaller until it is aligned with the angular momentum vector and is rotating about one of its stable axes.

4. In order to change the angular momentum vector of a spacecraft, thrust must be applied in order to create a torquing moment to bring the angular momentum vector to the desired orientation. However, this will generate precession due to the fact that the spacecraft is already rotating. This precession will obey the following equation.

$$\vec{M}_G = \frac{d\vec{H}_G}{dt} + \vec{\omega}_{A/I} \times \vec{H}_G \tag{1}$$

B. Analysis

1. Case A

In Case A, there is a torque acting on the object, however its precession effect is minimal. However, the torque does indeed affect the kinetic energy of the object. No energy is dissipated, but energy is in fact added to the object. The initial rotation was not about any of the principal axes, but the end rotation is about the minor axis, and due to the effect of spin axis on kinetic energy according to the following equation, the rotational energy is maximized when the object is spinning about its minor axis.

$$\vec{H}_G = \frac{1}{2} I \omega^2 \tag{2}$$

2. Case B

In Case B, there is indeed a torque present, and it does affect the angular momentum vector of the body. The torque is acting on the body about the z-axis, and, because of the body's angular momentum pointing in the x-direction, the torque causes gyroscopic precession of the body about the y-axis. In this case, energy is also added to the system because the axis of rotation changes to be closer to the minor axis, which, as described above, means a higher rotational kinetic energy.

3. Case C

Case C is an interesting one. There is in fact an external torque being applied to the object, however it does not affect the direction of the angular momentum vector H_G. This is because the torque vector is in line with the angular momentum of the body. There is no energy being dissipated by the body, and energy is conversely being added to the system due to the external torque. This increases the kinetic energy of the object by increasing its angular velocity ω.

4. Case D

Case D depicts an interesting phenomenon. The object begins spinning around its intermediate axis, where the moment of inertia is less than that of the major axis, but greater than that of the minor axis. This type of rotation is inherently unstable, and leads to the "tumbling" shown in the animation. In this case, as with the "T bar" experiment done in zero g environments, almost no energy is lost internally. What little is lost it most likely due to aerodynamic drag, but if this was in space, the object could theoretically continue to rotate for an indefinite period of time.

5. Case E

Finally, we have Case E. This case appears to be the most straight forward of the five. There are no external torques acting on the system, and the angular momentum vector is constant in the inertial frame. There is, however, energy being dissipated by the system. This is recognizable by the fact that the body is initially rotating about its minor axis, where rotational kinetic energy is highest. The rotation axis proceeds to move away from the minor axis towards the major axis. This increases the moment of inertia and decreases the angular velocity, which leads to an overall decrease in the rotational kinetic energy of the body due to the ω^2 term in equation 2.

II. Bicycle Wheel Precession

A. Model for Precession

Using modified Euler moment Equations an estimated percession rate $\left[\dot{\phi}\right]$ can be computed for the bicycle wheel experiment. Initially the angular momentum of the bicycle wheel was derived using the following expression

$$\left[\vec{H}_G\right]_B = [I_G]_B \, [\vec{\omega_w}]_B$$

where the angular momentum of the wheel in the body frame is equal to the product of the moment of inertia of the wheel and the angular velocity of the wheel both represented in the body frame. The moment of inertia matrix of the wheel was originally modeled after the moment of inertia of a hoop and the resulting angular momentum for the wheel is shown below.

$$\vec{H}_G = \begin{bmatrix} MR_w^2 & 0 & 0 \\ 0 & \frac{1}{2}MR_w^2 & 0 \\ 0 & 0 & \frac{1}{2}MR_w^2 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{3}$$

The results with this I matrix were compared to those yielded by the I matrix for a disk, and it was found that I_{disk} in fact yielded significantly more accurate results.

This led the team to implement the moment of inertia using the radius of gyration for the wheel about the x (or b_1) axis, k_1. k_1 was experimentally determined to be $k \approx 0.75 * R_{wheel}$. This was achieved by testing various scaling factors, starting at 0.8 and going down to 0.73 - which showed that 0.75 was clearly a reasonable scaling factor for I_x, allowing the experimental results to match the data very closely. As the value of k_1 is positive and less than the radius of the wheel, it is physically realistic. Values were not determined for I_y or I_z as these are unnecessary to determine the precession rate of the wheel. The final angular momentum function is shown below.

$$\vec{H_G} = \begin{bmatrix} M(0.75\ R_{wheel})^2 & 0 & 0 \\ 0 & f(M, k_2) & 0 \\ 0 & 0 & f(M, k_2) \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{4}$$

Here, we present a figure describing the body frame (wheel frame) and inertial frame (gimbal frame) of the system.

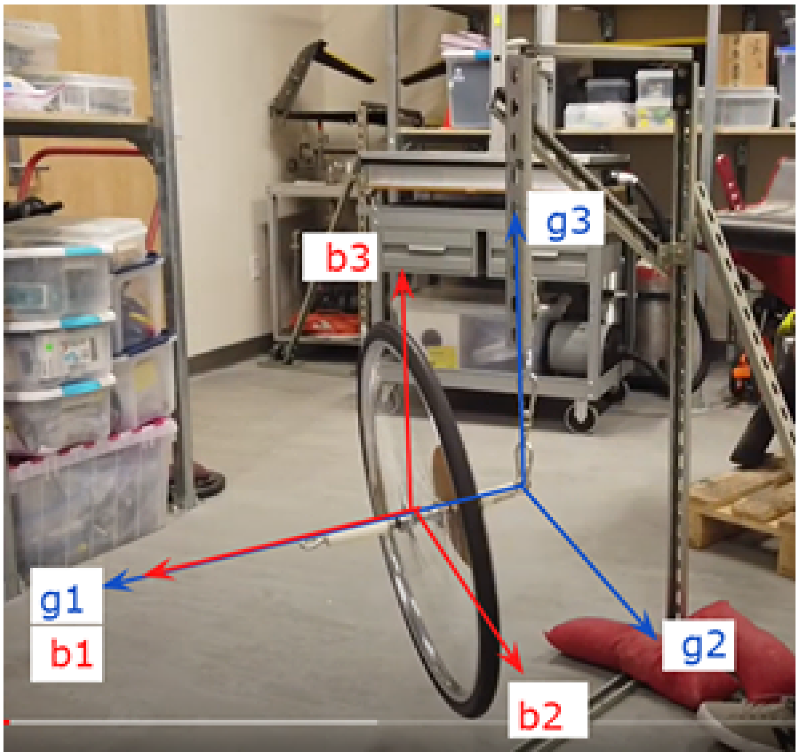

Fig. 1 Axes of Body (b) and Gimbal (g) frames

Assuming the wheel will only spin about its axle ($\hat{b_1}$ axis) the above equation simplifies so that there is only angular momentum about the axle.

$$\vec{H_G} = \omega_1\ m_{wheel}\ R_w^2 \quad \hat{b_1}$$

where ω_1 is equal to the wheel speed, m_{wheel} is the mass of the wheel, and R_w is the radius of the wheel.

We then define a co-moving gimbal frame that is spinning with the body. Its origin is located at the connection between the axle and the chain. The gimbal $\hat{g_1}$ is lined up with the body $\hat{b_1}$ axis and the other $\hat{g_2}$ and $\hat{g_3}$ are parallel to the body $\hat{b_2}$ and $\hat{b_3}$ axes respectively. This gimbal frame is shown below,

Looking at the diagram above it was determined that there is only a moment acting about the $\hat{g_2}$ axis and is equal to the product of the distance from the origin to the center of the wheel and the weight of the wheel. The external moment matrix for the gimbal frame is shown below.

$$\vec{M}_G = \begin{bmatrix} \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ r_{axel}\ m_{wheel}\ g \\ 0 \end{bmatrix} \quad [Nm] \tag{5}$$

We know the moments acting on the wheel in the gimbal frame are equal to the time rate of change of the angular momentum of the wheel in the gimbal frame. Using the transport theorem we can relate the two.

$$\vec{M}_g = {}^{G}\left[\frac{dH_G}{dt}\right] + \omega_G \times \vec{H}_G$$

The spin rate of the gimbal frame is represented as $\omega_G = [0\ 0\ \dot{\Theta}]^T$ where $\dot{\Theta}$ is the precession rate. It was assumed that the angular momentum remains constant in the gimbal frame. We are then left with the following expression

$$\begin{bmatrix} 0 \\ r_{axel}\ m_{wheel}\ g \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\Phi}\Theta_1\ m_{wheel}\ R_w^2 \\ 0 \end{bmatrix} \tag{6}$$

using algebra we can solve for the predicted precession rate

$$\dot{\Theta} = \frac{r_{axel}\ m_{wheel}\ g}{\omega_1 R_w^2}$$

B. Experiment

Initially the wheel diameter and axle length were measured, and an expression to compute the percession rate as a function of spin rate was derived as shown above. Following this, the wheel was spun using a handheld motor while also holding the spin axis horizontal. Four trials where run each with a different initial wheel spin speed, and the precession rate was measured using a stopwatch by taking the time it took for the wheel to complete a full rotation. All collected data was entered manually into the *ASEN_3200_Lab2_main.m* MATLAB script and the experimentally recorded precession rates where then plotted against our predicted precession rates for each trial. Error analysis of these results was then implemented in our main MATLAB script.

C. Results

Presented below are graphs of the angular precession rate and angular period against the spin rate of the wheel. These show striking similarity between the model estimate (shown as a blue line) and the data from each trial.

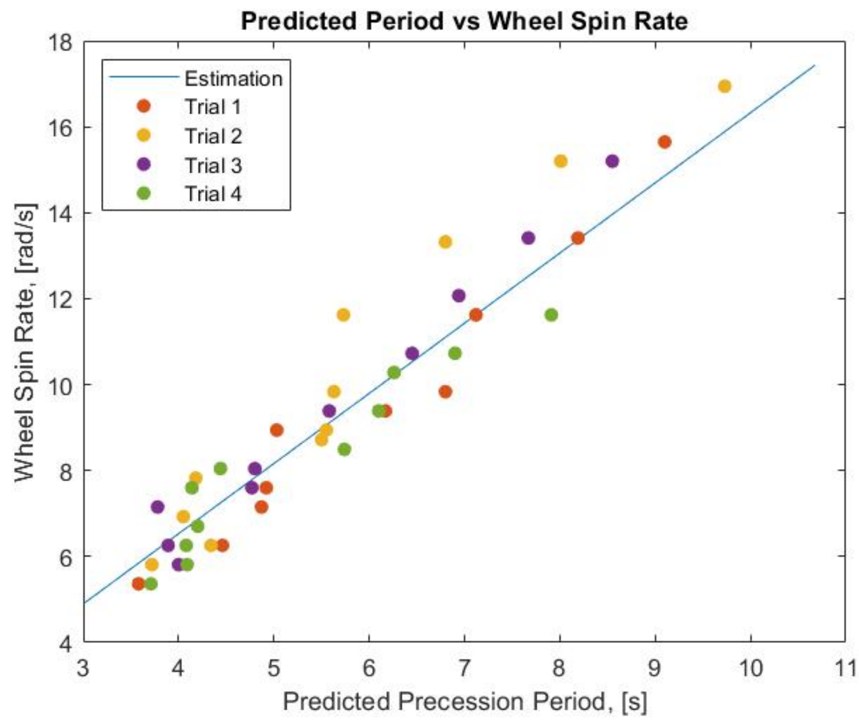

Fig. 2 Precession Period vs Spin Rate

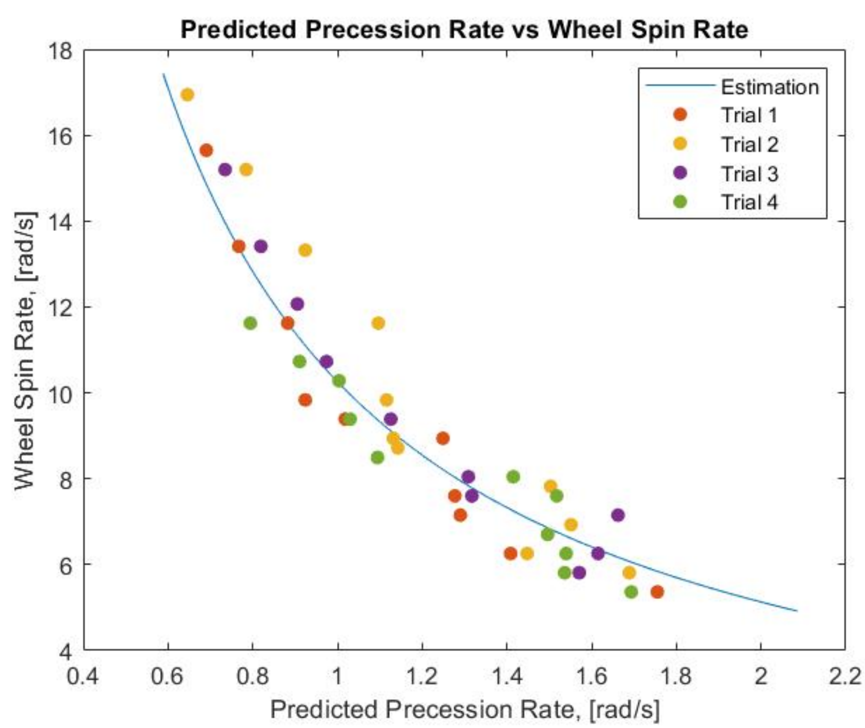

Fig. 3 Precession Rate vs Spin Rate

D. Analysis

The results shown above are impressive. The estimation line in each graph visually appears very similar to a line of best fit for the data provided, showing that this is indeed an accurate model to represent the precession rate of a wheel in this configuration. Though there is variation and error in the data, it is small - and its' magnitude and sources will be discussed below.

Few assumptions were made to simplify the process of developing an expression to relate wheel speed and precession rate. To simplify the derivation of angular momentum of the wheel, it was assumed that the wheel only rotated about its axle. In addition it was assumed that the co-moving frame remained aligned with the body frame of the wheel while in the experiments the wheel would tip as its speed decreased. To help matrix operations in our error analysis it was assumed that the wheel continued to spin at the final measured wheel speed if no more wheel speed values where given.

Error analysis for each trial was completed by taking the difference between the predicted spin rate and the true spin rate for each recorded wheel speed.

Trial 1													
Given Speed, MPH	35	30	26	22	21	20	17	16	14	12	12	12	12
Error Between Real & Estimated Values, [rad/s]	-0.0348	-0.0022	0.0002	0.1191	0.0744	-0.1017	0.0728	0.1441	0.2304	0.1573	0.1324	0.1474	0.1375
Trial 2													
Given Speed, MPH	37.9	34	29.8	26	22	20	19.5	17.5	15.5	14	13		
Error Between Real & Estimated Values, [rad/s]	-0.0403	-0.1095	-0.1539	-0.2139	-0.0729	0.0153	0.0344	-0.1918	-0.0709	0.1914	0.0762		
Trial 3													
Given Speed, MPH	34	30	27	24	21	18	17	16	14	13			
Error Between Real & Estimated Values, [rad/s]	-0.0599	-0.0542	-0.0554	-0.0180	-0.0332	-0.0341	0.0327	-0.2279	0.0240	0.1945			
Trial 4													
Given Speed, MPH	26	24	23	21	19	18	17	15	14	13	12	12	12
Error Between Real & Estimated Values, [rad/s]	0.0883	0.0456	-0.0060	0.0627	0.1132	-0.1402	-0.1678	0.0339	0.0992	0.2290	0.2188	0.2632	0.2096

It was observed that the magnitude of computed error for wheel speed would increase with time as the wheel precessed freely and then would decrease when the wheel precession was stopped so that the axle could be brought back to the horizontal. Assuming that the wheel does not drop or rotate while precessing allowed us to derive our model easily, but contributed to a misrepresentation of the true angular velocity of the wheel related to the gimbal frame, and the relationship between the gimbal and body fixed frames. Creating a model that accounts for the drop in the wheel over time would be difficult but would lead to more accurate results.

Furthermore, providing a measurement of the wheel's rotation speed once for every rotation would increase the accuracy of the prediction. Assuming that the wheel continued at its final speed allowed us to calculate error for rotations at which no speed was provided, but this assumption is clearly false due to the non-negligible effects of friction causing the wheel to slow down over time.

Another potential source of error is assuming that the wheel does not rotate about its $\hat{b_2}$ or $\hat{b_3}$ axis. In the video the wheel appears to only be rotating about its axle but there could be some wobble about its other axis that can not be easily measured or observed. Accounting for the minor angular velocity of that theoretical wobble could increase the accuracy in the computation for the angular momentum of the wheel in the gimbal frame.

Conclusions & Recommendations

Overall this lab greatly increased understanding of both the analytical nature of spacecraft rotations, and the underlying concepts that are fundamental to the processes involved. The spinning spacecraft animations were an excellent qualitative representation of the dynamics of a spacecraft when rotating with and without external torques. They are a great tool for help in visualizing the dynamics of motion that apply to this sort of situation. A list of improvements to this section of the lab would most notably include changing the format of the .mp4 files used to generate the GIFs of the animations. They were unplayable on many computers, and inconvenient workarounds had to be employed. Dynamic equations and MATLAB computations were used together to predict the precession rate of the bicycle wheel experiment. The predicted values computed using our model where then compared to the measured precession rate. Through error analysis it was observed that as the wheel orientation changed, the accuracy of our model decreased. Other assumptions that contributed to our models inaccuracy include assuming the wheel speed is constant through a precession and that the wheel does not wobble about its other axis. Creating a new model to characterize the motion of the wheel would be difficult but could lead to more accurate predictions. To improve this lab it would be nice to have the wheel speed for each full rotation just so that we would not have to interpolate wheel speed values for some rotations.

References

Axelrad, P., "ASEN 3200 Lab A-2: Spinning Spacecraft" Sept. 15, 2020.

Curtis, H., *Orbital Mechanics for Engineering Students*, 3rd ed., Elsevier, 2014.

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Team Member Participation

2 - Lead, 1 - Participated/Contributed, 0 - Did not Contribute

Name	Plan	Model	Experiment	Results	Code	Report
Tomaz Remec	1	1	1	2	0	2
Hunter Daboll	1	2	1	1	2	1
Connor O'Reilly	2	1	1	2	1	2

Group members have reviewed and approved this table.
Initials: WHD, TJR, CTO

Appendix: MatLab Code, Main Script

```
%% Housekeeping
%{
ASEN 3200 Lab 2 Main/Driver Script
Authors: W. Daboll, C. O'Reilly
Last Edited: 9/24/2020
%}
clear all; close all; clc;


%% Define Constant Parameters
Diameter = 25.5 * 0.0254; %in to m
Radius = 12.75 * 0.0254; %in to m
AxleLength  = 7.5 * 0.0254; %in to m
Mass = 2.59; %kg
Thickness = 1.625 * 0.0254 ; %in to m
%% Calculate MOI
I_hoop = [Mass*Radius^2 0.5*Mass*Radius^2 0.5*Mass*Radius^2];
I_disk = [0.5*Mass*Radius^2 0.25*Mass*Radius^2 0.25*Mass*Radius^2];

scaling_factor = 0.75;
k = Radius * scaling_factor;
Ig = [Mass*k^2];
%% Calculate Moment about y axis
M = Mass*9.81*AxleLength;
%% Create theoretical spin rate vector
speed = linspace(11,39,160).* 0.44704; % MPH to m/s
%% Calculate estimated precession rate based on w_spin
w_spin = (speed / Radius); % rad per second

w_est_prec = M ./ (Ig(1)* w_spin); % Omega precession - estimate - trial 1
T_est_prec = 2*pi./w_est_prec;



%% Create vectors representing period and speed
measured_speed = [35 30 26 22 21 20 17 16 14 12 0;
         37.9 34 29.8 26 22 20 19.5 17.5 15.5 14 13;
         34 30 27 24 21 18 17 16 14 13 0;
         26 24 23 21 19 18 17 15 14 13 12 ] .* 0.44704;
     % ^ MPH to m/s, each row is a different experiment,
     %each column an entry within the given experiment. Here, a zero
     %denotes no value (There were only 10 data points taken in tests 1 and
     %3, the zeros maintain array dimensions)
pickUp = [6 9; 4 8; 6 9; 7 0];
    % ^ Indicates the index of a row in the speed array immediately after
    % Bobby picks the wheel back up, again each row denotes a new experiment.
    % Here, a zero denotes no value(it was only pickec up once in the final experiment)
period = [9.1 8.19 7.12 6.80 6.17 5.03 4.92 4.87 4.46 3.58 3.53 3.56 3.54;
```

```
    9.73 8.01 6.80 5.73 5.63 5.55 5.5 4.18 4.05 4.34 3.72 0 0;
    8.55 7.67 6.94 6.45 5.58 4.80 4.77 3.78 3.89 4.00 0 0 0;
    7.91 6.90 6.26 6.10 5.74 4.44 4.14 4.20 4.08 4.09 3.71 3.81 3.69] ;
    % ^ seconds. 1st index = released, consecutive indeces = 1 rotation each,
    % rows and columns as denoted for speed
%% Calculate true precession rate based on precession period
wt_prec_1 = 2*pi./nonzeros(period(1,:)); % Omega precession - true - trial 1
Period_true_prec_1 = nonzeros(period(1,:));
wt_prec_2 = 2*pi./nonzeros(period(2,:)); % Omega precession - true - trial 2
Period_true_prec_2 = nonzeros(period(2,:));
wt_prec_3 = 2*pi./nonzeros(period(3,:)); % Omega precession - true - trial 3
Period_true_prec_3 = nonzeros(period(3,:));
wt_prec_4 = 2*pi./nonzeros(period(4,:)); % Omega precession - true - trial 4
Period_true_prec_4 = nonzeros(period(4,:));

%% Calculate error
measured_speed_1 = [35 30 26 22 21 20 17 16 14 12 12 12 12] .* 0.44704;
measured_speed_2 = [37.9 34 29.8 26 22 20 19.5 17.5 15.5 14 13 13 13] .* 0.44704;
measured_speed_3 = [34 30 27 24 21 18 17 16 14 13 13 13 13] .* 0.44704;
measured_speed_4 = [26 24 23 21 19 18 17 15 14 13 12 12 12] .* 0.44704;
w_est_prec_1 = M ./ (Ig(1)* (measured_speed_1 / Radius)); % One to one
%values for estimated precession at the given measured speeds,
% to make it easy to calculate the error
w_est_prec_2 = M ./ (Ig(1)* (measured_speed_2 / Radius));
w_est_prec_3 = M ./ (Ig(1)* (measured_speed_3 / Radius));
w_est_prec_4 = M ./ (Ig(1)* (measured_speed_4 / Radius));

error1 = zeros(1,length(wt_prec_1));
error2 = zeros(1,length(wt_prec_2));
error3 = zeros(1,length(wt_prec_3));
error4 = zeros(1,length(wt_prec_4));
for k = 1:length(wt_prec_1)
    error1(k) = w_est_prec_1(k) - wt_prec_1(k);
end
for k = 1:length(wt_prec_2)
    error2(k) = w_est_prec_2(k) - wt_prec_2(k);
end
for k = 1:length(wt_prec_3)
    error3(k) = w_est_prec_3(k) - wt_prec_3(k);
end
for k = 1:length(wt_prec_4)
    error4(k) = w_est_prec_4(k) - wt_prec_4(k);
end

fprintf("Error 1\n");
disp(error1);
fprintf("Error 2\n");
disp(error2);
fprintf("Error 3\n");
disp(error3);
fprintf("Error 4\n");
disp(error4);
%% Plotting
%Predicted Precession Rate vs Wheel Spin Rate
figure();
plot(w_est_prec,speed);
hold on;
scatter(wt_prec_1(1:10),nonzeros(measured_speed(1,:)),'filled')
scatter(wt_prec_2,nonzeros(measured_speed(2,:)),'filled')
scatter(wt_prec_3,nonzeros(measured_speed(3,:)),'filled')
scatter(wt_prec_4(1:11),nonzeros(measured_speed(4,:)),'filled')
hold off;
title("Predicted Precession Rate vs Wheel Spin Rate");
ylabel("Wheel Spin Rate, [rad/s]")
legend("Estimation","Trial 1","Trial 2","Trial 3","Trial 4");
xlabel("Predicted Precession Rate, [rad/s]")
```

```
%Predicted Period vs Wheel Spin Rate
figure();
plot(T_est_prec,speed);
hold on;
scatter(Period_true_prec_1(1:10),nonzeros(measured_speed(1,:)),'filled')
scatter(Period_true_prec_2,nonzeros(measured_speed(2,:)),'filled')
scatter(Period_true_prec_3,nonzeros(measured_speed(3,:)),'filled')
scatter(Period_true_prec_4(1:11),nonzeros(measured_speed(4,:)),'filled')
hold off;
title("Predicted Period vs Wheel Spin Rate");
ylabel("Wheel Spin Rate, [rad/s]")
legend("Estimation","Trial 1","Trial 2","Trial 3","Trial 4",'Location','northwest');
xlabel("Predicted Precession Period, [s]")

mean(abs(error1))
mean(abs(error2))
mean(abs(error3))
mean(abs(error4))
```