University of Colorado Boulder

ASEN 3200 Orbits Lab 1

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Lab Time: 12:50am

Due: November 3^{rd}, 2020

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Presented in this lab are the results of two different simple, two-body problems between a satellite and its orbiting body (the Earth for Part 1 and the Moon for Part 2). Matlab and STK were used to perform calculations and generate the 2D and 3D graphics presented (in addition to hand calculations shown in the appendix). For the Earth satellite simulation, it was found, as expected, that the eccentricity vector and angular momentum vector remained constant throughout the orbital period. In addition, the desired orbit did not intersect with the orbit of the Moon. As for the Lunar orbit, parameters such as semi-major axis, eccentricity, and true anomaly of the initial satellite were found to be 3998 [km], 0.5398, and 70.47 [deg] respectively. The required observation window was found to be only 9.22 minutes, stretching a latitude of -55 to -30 [deg] and longitude of 160 to 175 [deg] over the lunar surface.

Contents

I. Introduction **2**

II. Earth Satellite Simulation **2**
- Question 1 . . . 2
- Question 2 . . . 3
- Question 3 . . . 3
- Question 4 . . . 3

III. Lunar Surface Observation **6**
- Question 5 . . . 6
- Question 6 . . . 7
- Question 7 . . . 7
- Question 8 . . . 8
- Question 9 . . . 9
- Question 10 . . . 10
- Question 11 . . . 11
- Question 12 . . . 11
- Question 13 . . . 11
- Question 14 . . . 11

IV. Conclusion and Recommendations **12**

Acknowledgements **13**
- Team Member Participation Table . . . 13

References **13**

Appendix A **14**
- Matlab Code . . . 14
- Derivations . . . 17
  - Question 5 . . . 17
  - Question 8 . . . 19
  - Question 9 . . . 22

I. Introduction

The goal of this lab is to become accustomed to the STK environment and to begin applying a basic understanding of two body problems to two different orbital scenarios. Firstly, a scenario of a satellite in an orbit about the Earth will be explored, which includes calculation of parameters such as eccentricity and angular momentum. Secondly, a scenario of a satellite in a polar orbit about the Moon will be explored, including the calculation of an observation window close to the surface. The methods applied to solving these problems can also be applied to real world problems when searching for a rough measure or first estimate of basic orbits, although real problems will need a more thorough analysis for better accuracy.

II. Earth Satellite Simulation

Question 1

Considering the conic equations of motion, it was expected that the angular momentum and eccentricity vectors would remain constant. After running the STK software, this was determined to be true. The of the angular momentum for a given obit of a body that is bounded to another body (such as a satellite bounded to Earth) remains unchanged, this vector can be seen below in Fig. 1.

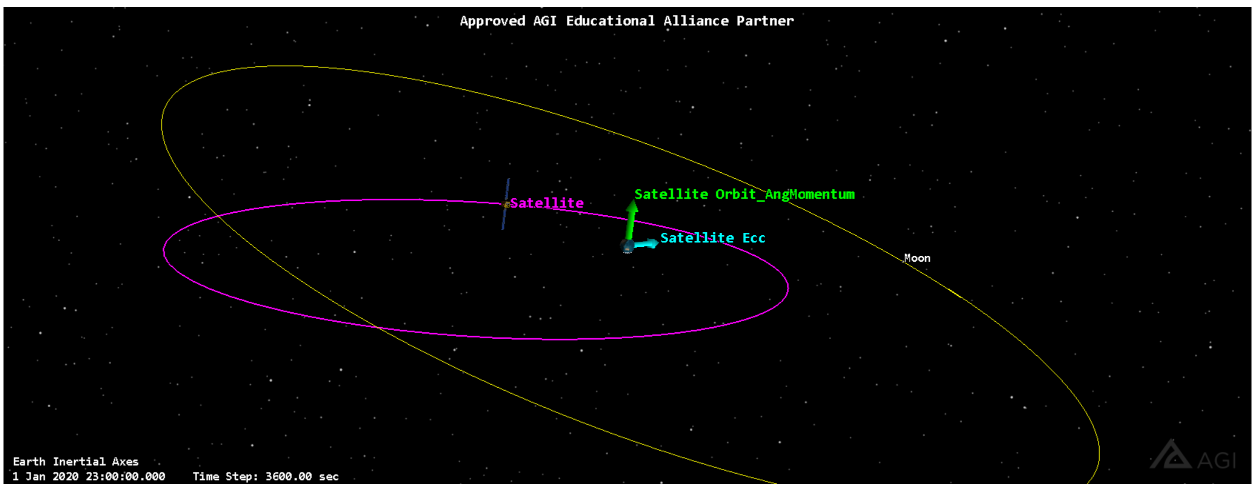


Fig. 1 Earth Scenario 3D View

If this value was to change over time, the satellite may have been disturbed by the gravity of another massive body (i.e. the Moon.). This perturbation would most likely negatively affect the orbit of the satellite and, without correction, would cause the satellite to gain or lose energy, further causing the angular momentum to vary. Furthermore, the eccentricity follows the same behavior. If eccentricity were to change over time, this would cause the shape of the orbit to change. An additional figure showing the perspective looking down (roughly) onto the orbital plane can be seen below in Fig. 2.

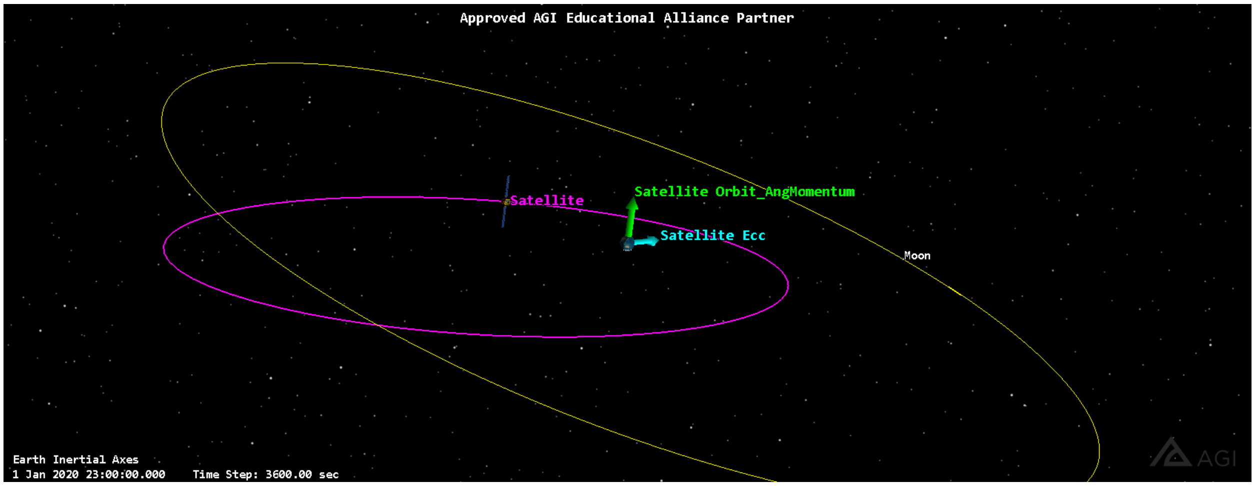


Fig. 2 Earth Scenario Looking Down Onto Orbital Plane

Question 2

The two significant perturbations acting on this spacecraft are the Sun and the Moon's gravitational effects. Being that we desire to orbit the Earth, the Moon's gravitational effect will be the most immediate and, possibly, the largest source of external disturbance. Otherwise, the sun is a very large object also having a very high mass. Due to this, the gravitational effect due to the Sun would also greatly affect the performance of the satellite.

Question 3

From the STK model of the satellite orbit and the orbit of the Moon, it was noticed that **the orbit of the satellite does not intersect the Moon's orbit**. The figure below shows this and was best seen from a narrow side view of both orbits.

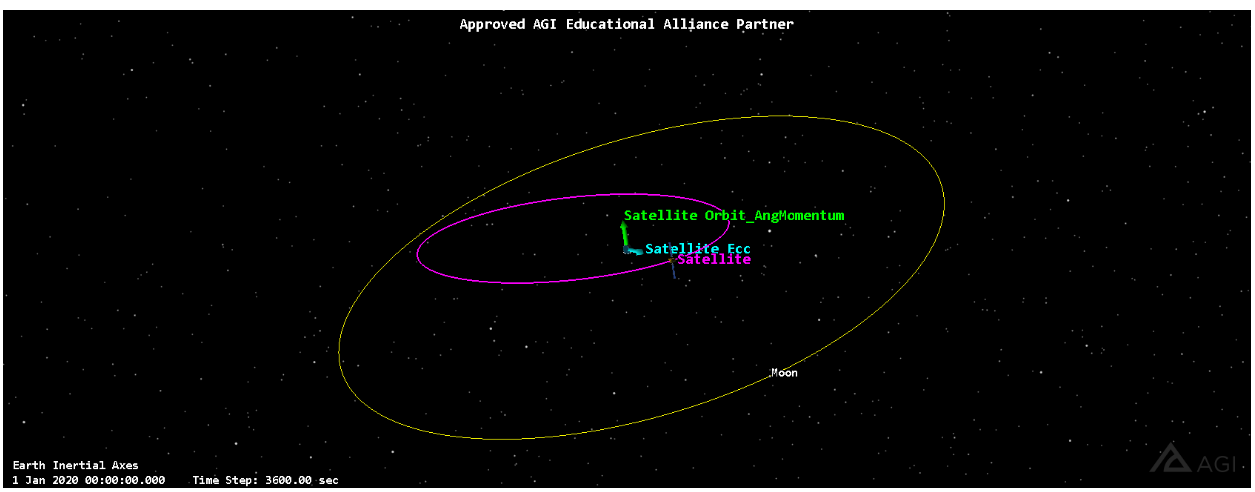


Fig. 3 Non-Intersecting orbits of satellite and Moon.

Question 4

A MATLAB script utilizing ODE45 was written to simulate the path of the satellite around Earth. The value of the gravitational parameter for this MATLAB simulation was 398600. This is roughly what STK uses as well. The initial conditions were found by calculating the radius and velocity at perigee, along with the associated inclination angle using the following equations:

$$r_p = a(1 - 3) \tag{1}$$

$$v_p = \sqrt{2 * \mu * a * (1 - e^2)} \tag{2}$$

These were fed into the ODE, along with a time vector of 0 to the length of the period, found using:

$$T = \left(2 * \pi * \sqrt{\frac{a^3}{\mu}}\right) \tag{3}$$

After the ODE call output the position and velocity vectors a DCM rotation about the y axis was utilized to incorporate the inclination angle.The resulting orbit matches up really nicely with the orbit modeled in STK. The following figure illustrates its orbit and starting position:

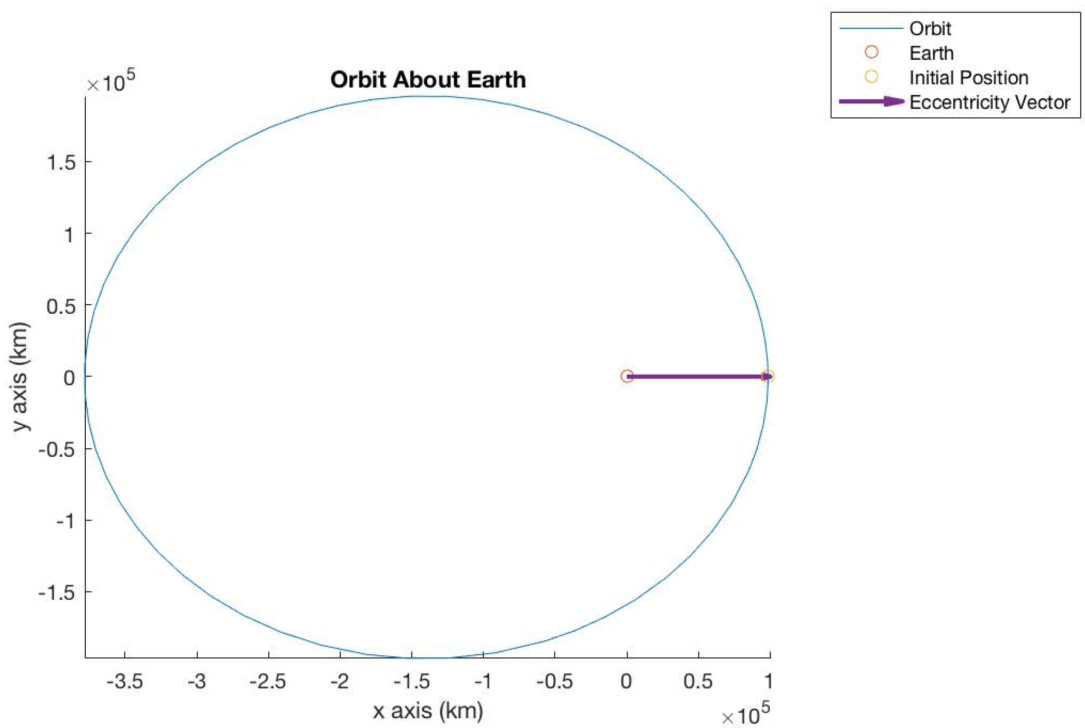


Fig. 4 Graph of the Orbit in the Orbit Plane

Next, eccentricity was found using the following equation:

$$e = \frac{\vec{v} \times \vec{h}}{\mu_{earth}} - \frac{\vec{r}}{|\vec{r}|} \tag{4}$$

For one period, eccentricity was constant, as expected, and very close to the given value of e = .587. Slight variations can be ignored, as they are likely due to rounding in ode45, as well as the relatively small scale of the graph.

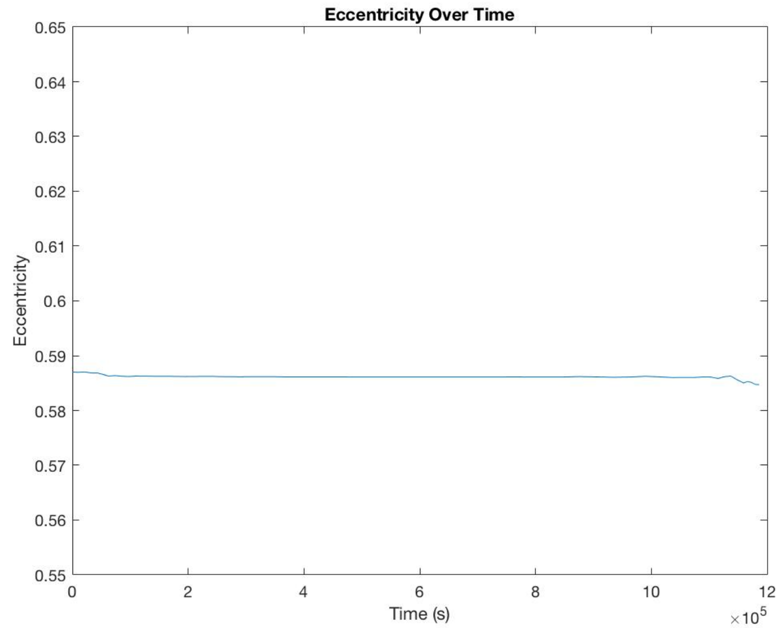


Fig. 5 Eccentricity

Finally, specific angular momentum was found using the following equation:

$$|\vec{h}| = \vec{r} \times \vec{v} \tag{5}$$

This too was constant, as expected and hovered around $2.55 * 10^5 \frac{km^2}{s}$:

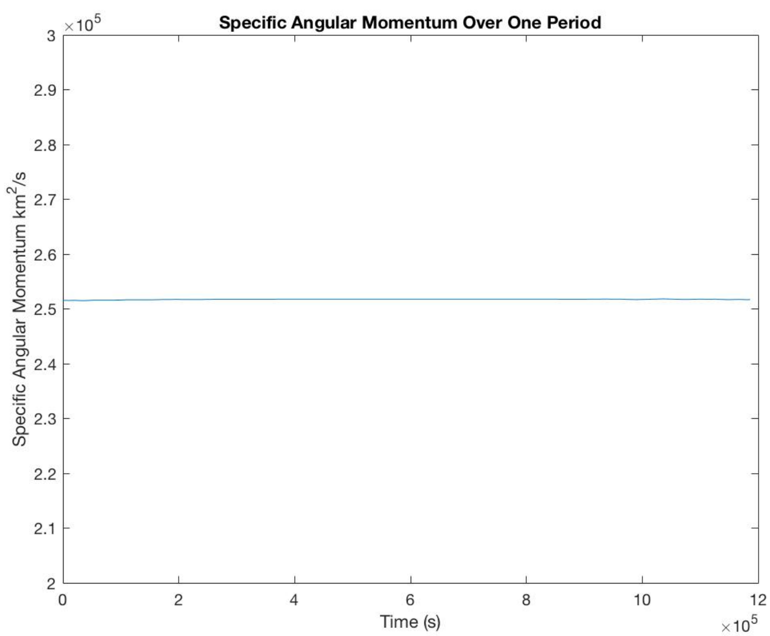


Fig. 6 Angular Momentum

This plots confirm that the model works, as trends were consistent with expecations, and with the STK model. Of course, it is important to note that this simulation relies on ode45, which has an inherent potential for rounding errors within reason. Nevertheless, this can be considered a viable way of simulating the satellites orbit.

III. Lunar Surface Observation

Question 5

Orbital Period (T)	Periapsis Radius (r_p)	Initial Location of S/C (r)
6.3 hours	1840 km	2400 km

Table 1 Given values for question 5.

To convert the 6.3 hour orbital period (T) to the quantity for the semi-major axis (a) the equation below (6) was used.

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{6}$$

Knowing this relationship, isolating for the semi-major axis was simple enough. The result is as shown below.

$$a = \left[\mu\left(\frac{T}{2\pi}\right)^2\right]^{1/3} \tag{7}$$

After converting 6.3 hours to 22680 seconds, a was determined to be **3998 km**. With this the eccentricity of the orbit can be determined by using equation 8 and isolating for eccentricity, e.

$$r_p = a(1-e) \tag{8}$$

With slight manipulations of the above equation eccentricity was determined to be:

$$e = 1 - \frac{r_p}{a} \tag{9}$$

Using the given value in table 1 for r_p and the calculated semi-major axis, the value for eccentricity is **0.5398**.

Lastly, the true anomaly (θ) was desired given a position (r) of 2400 kilometers away from the Moon. Here, two equations were used to determined this value. They can be seen below.

$$r = \frac{p}{1 + ecos\theta} \tag{10}$$

$$\frac{p}{1+e} = a(1-e) \tag{11}$$

By taking the equation immediately above (11) and rearranging values to isolate for the semi-parameter (p), we get:

$$p = a(1-e^2) \tag{12}$$

This can then be inserted into equation 10 by substituting the semi-parameter in terms of the semi-major axis and the eccentricity. This produces the equation below:

$$r = \frac{a(1-e^2)}{1 + ecos\theta} \tag{13}$$

From here, the equation above can be simplified to solve for true anomaly. The equation determined to solve for the true anomaly is as seen below:

$$\theta = cos^{-1}\left(\frac{1}{e}\left[\frac{a(1-e^2)}{r} - 1\right]\right) \tag{14}$$

The above equation and given values for a, e, and r results in a true anomaly of **70.47**°.

Semi-Major Axis (a)	Eccentricity (e)	True Anomaly (θ)
3998 km	0.5398	70.47°

Table 2 Final desired values for question 5.

For full derivations of the values calculated above look to Appendix A: Derivations. A figure looking down on the orbital plane in STK can be seen below (Fig. 7), which visually corroborates the results presented.

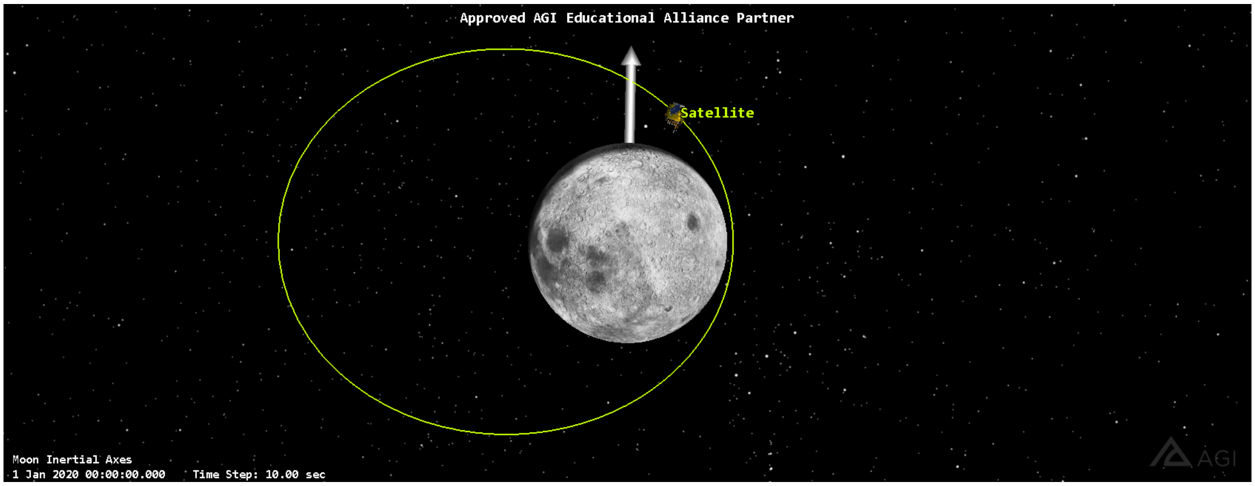


Fig. 7 Lunar polar orbit, as seen looking down on orbital plane

Question 6

The Moon's north pole vector can be seen below in Fig. 8, represented by the white arrow.

Question 7

Shown below (Fig. 8) is a 3D screenshot of the satellite's polar orbit about the Moon, shown with the Earth in view.

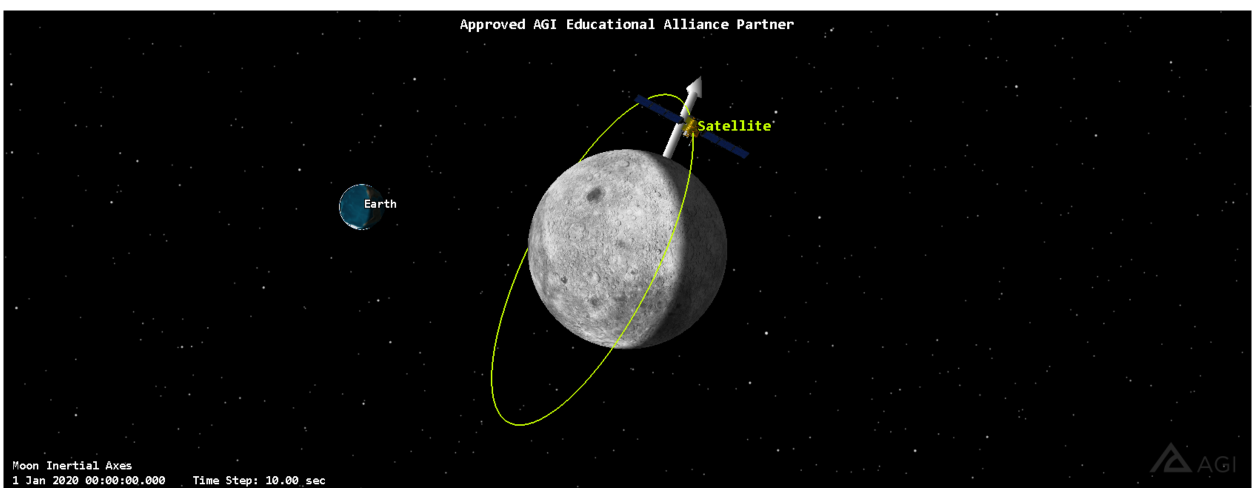


Fig. 8 3D view

Question 8

To determine the first opportunity of the spacecrafts observation window given its initial conditions, Kepler's equation was implemented. The steps below outline the process in solving for the time it took for the spacecraft to reach the first observation window. The initial conditions of the spacecraft are unchanged from those previously mentioned in Question 5 (tables 1 and 2). It is important to mention that the first opportunity at which observations can be made occurs at an altitude of 500 kilometers, implying that we are only interested at this specific position. Thus, for this particular problem, the altitude of 200 kilometers has no relevance to our time calculation. Knowing this, we can begin to solve for time given the current position of the spacecraft at 2400 kilometers to the desired position of where the altitude is 500 kilometers (r_{500} = 2237* kilometers).

Kepler's equation,

$$n(t - t_p) = E - esinE, \tag{15}$$

can be used to solve the duration of time it will take the spacecraft to reach the first observation window. From the initial conditions (both given and solved for in Question 4), the mean motion (n) and eccentricity (e) are known. $t - t_p$, which is equal to Δt, is what we are solving for and the eccentric anomaly (E) is unknown. Thus, E must be determined before the time of flight can be solved.

To solve for E, the true anomaly of the orbit must first be determined. Using equation 14 in Question 5, the true anomaly can be found. Note, there is a slight deviation in the value produced by this equation and what is expected. The expected value will reside somewhere between 180° to 360°. The value this is produced lies between 0° to 180°. To adjust this, a simple difference is taken using the relationship below where θ represents the true anomaly and θ' is the angle neighboring θ across the radius vector.

$$\theta = 360 - \theta' \tag{16}$$

Implementing equation 14 and equation 16 the true anomaly at the first observation window was determined to be 5.229 rad or 299.58°. From here the true anomaly can be translated to the eccentric anomaly (E). Looking to the figure below, The eccentric anomaly can be determined and isolated in terms of the semi-major axis (a), eccentricity (e), radius of interest (r), and the true anomaly (θ).

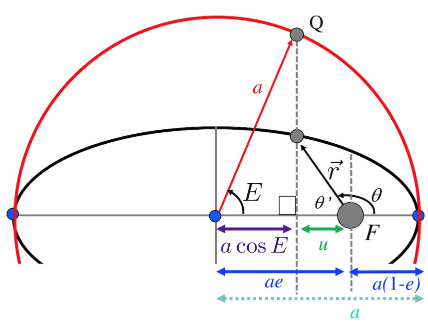


Fig. 9 Visual representation for eccentric anomaly calculation

From the image above, the relationship

$$acosE = ae - rcos\theta$$

was determined. This can be further simplified to find the value for E at the true anomaly of 299.58°. Thus, the final form in solving for E is:

$$E = cos^{-1}\left(e - \frac{r}{a}cos\theta\right) \tag{17}$$

After inserting all of the previously found values, the initial eccentric anomaly E_1 was determined to be 70.177°, or 1.225 rad. The final eccentric anomaly, E_2, was found to be 74.717°. Note that E_2 is similar to the true anomaly, where it must also be adjusted to account for a position beyond apoapsis and heading towards periapsis. Therefore, using

*Moon radius is 1738.1 kilometers, given by NASA Moon fact sheet at: https://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html equation 16 and substituting θ for E produces the value of 285.28°, or 4.979 rad. From here, Kepler's equation can be used to determine the duration of flight from the given initial state to its desired position of an altitude of 500 kilometers.

Recall that Kepler's equation was determined using integration as a function of eccentric anomaly (E). Thus, the work above found the starting position (E_1) and ending position (E_2) to implement into Kepler's equation to satisfy the definite integral. Using Kepler's equation and inserting the boundary conditions of $E_1 = 1.225rad$ and $E_2 = 4.979rad$ the equation is written as follows:

$$n(t - t_p) = E - esinE|_{E_1}^{E_2} \tag{18}$$

The equation above produces a time of **4.796 hours**. For full derivation, look to Appendix A: Derivations - Question 8.

Question 9

For solving this problem, the implementation of the conic equation was first used to solve for the true anomaly at altitudes of 500 and 200 kilometers. Respectively, this give position radii of r_{500} = 2237 kilometers and r_{200} = 1937 kilometers. To save time and space, it can be recalled that in Question 5, the true anomaly at r_{500} was initially found to be **299.58° or 5.229 rad**. Similar to how this value was determined, the true anomaly at r_{200} can be solved for. We use the conic equation and isolate θ as shown in equation 14. Again, knowing the values of eccentricity (e), semi-major axis (a), and the desired position ($r = r_{200}$) the anomaly at an altitude of 200 kilometers was 31.02°. Of course, this needs to be adjusted due to this angle representing the position produced at periapsis and rotating clockwise. Therefore equation 16 must be implemented to give the true anomaly about a counter-clockwise rotation starting at periapsis. Thus, after the aforementioned equation was used, the true anomaly at r_{200} was determined to be **320.98° or 5.742 rad**. **It is important to mention that there are 2 windows of observation in the given satellite orbit.** Due to periapsis of the orbit being 1840 kilometers, this means that this altitude is approximately 103 kilometers, which is too close for the spacecraft to make observations. Thus, data can be captured just after leaving periapsis (0°-180°) as well as just before reaching periapsis (180°-360°).

r_{500}	r_{200}
299.58°	320.98°
5.229 rad	5.7418 rad

Table 3 True anomalies.

From here, Kepler's equation can be used to determine the eccentric anomalies at the respective altitudes. Again, recall that the eccentric anomaly at the position of r_{500} was found to be 285.28°, or 4.979 rad. To further find the eccentric anomaly at r_{200} equation 17 in question 8 was used. Following the steps previously mentioned, the eccentric anomaly at this position was found to be 272.93° or 4.7637 rad. Implementing the integral behavior discussed just prior to equation 17 in question 8, the time of observation was found to be **9.22 minutes**. This observation period can be seen below in STK (Fig. 10).

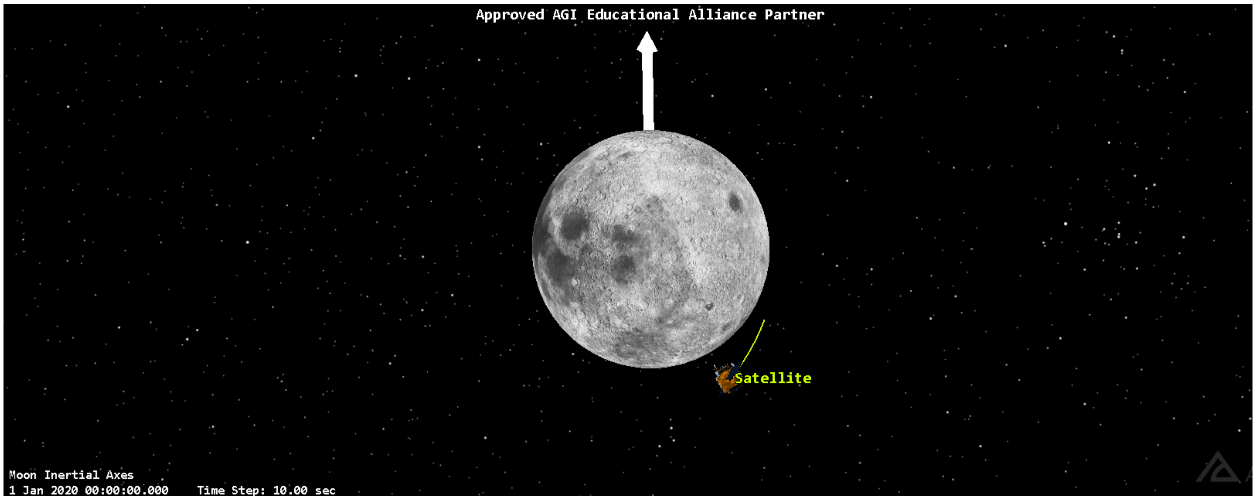


Fig. 10 First Observation Window

If we look at the entire orbit, the window of observation would be double this value to account for the symmetry and dynamical behavior of the spacecraft. With that mentioned, it was determined that the spacecraft's entire window of observation is approximately 2.44% of 6.3 hour orbit. (Look to Appendix A: Derivation - Question 9 for full derivation)

Question 10

The range of latitude covered by the satellite was found to be approximately -55 to -30 deg with a corresponding longitude range of 160 to 175 deg. Comparing the ground track shown below (Fig. 11) with the US Geological Survey map of the moon (ref [2]), "Leibnitz" was one of the craters nearby (found at approximately 180 longitude by -40 latitude).

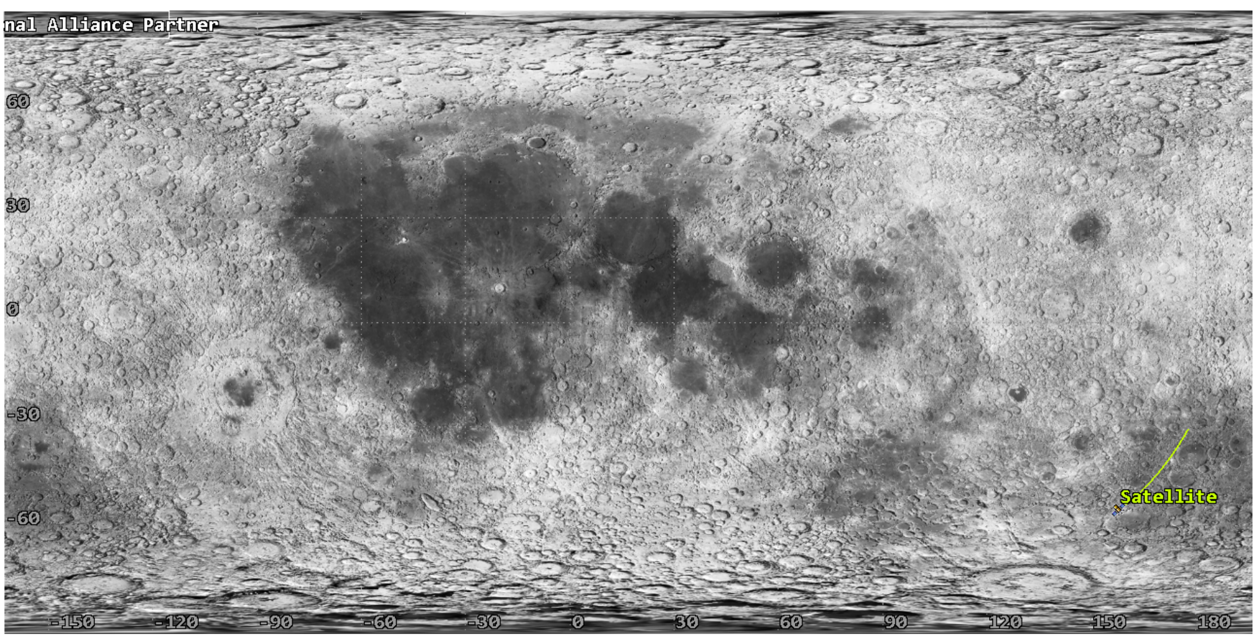


Fig. 11 Groundtrack of observation window

Question 11

A function was developed that solves Kepler's equation to find the eccentric anomaly. First, a tolerance was set. It was determined to be $1 * 10^{-16}$, as this is the smallest error MATLAB can compute without additional toolboxes. Next, it sets an initial guess based on the mean anomaly. Essentially, if the satellite is approaching apogee, then the mean anomaly is added to half the eccentricity. If the satellite has past apogee, then half the eccentricity is subtracted by the mean anomaly. Next, the function iterates through values of E, recalculating until the error is lower than the tolerance. Finally, an E value of 0.4678 was outputted.

Question 12

After using the above function to find E, altitude was calculated. First, the semi-major axis of the satellite's orbit was found using the following equation:

$$a_{moon} = \left(\frac{T^2}{2\pi}\mu_{moon}\right)^{\frac{1}{3}} \tag{19}$$

Next, the eccentricity of the moon was found using:

$$e_{moon} = -\left(\frac{R_p}{a-1}\right) \tag{20}$$

Finally, M for this orbit was found, using the given time past perigee in seconds:

$$M = \sqrt{\frac{\mu_{moon}}{a^3}} * t \tag{21}$$

After finding E using the Kepler function, the radius of orbit at that time could be found:

$$r = a_{moon}(1 - e_{moon} * \cos E) \tag{22}$$

Thus, altitude could be found by subtracting the radius of the moon:

$$altitude = r - r_{moon} \tag{23}$$

This was found to be 334.8075 km.

Question 13

The two significant perturbations acting on a satellite in a lunar orbit would be the gravity of both the Earth and the Sun. The contribution of the Earth's gravity is significant because of its relative distance, whereas the Sun's gravity is significant because of its mass.

Question 14

The figure below shows a 100 orbit cycle of the satellite, including a perturbation due to the Earth's gravity. Due to an overlaying of the orbit on top of itself, the variation can be seen by the increase in the thickness of the orbit track, shown below (Fig. 12).

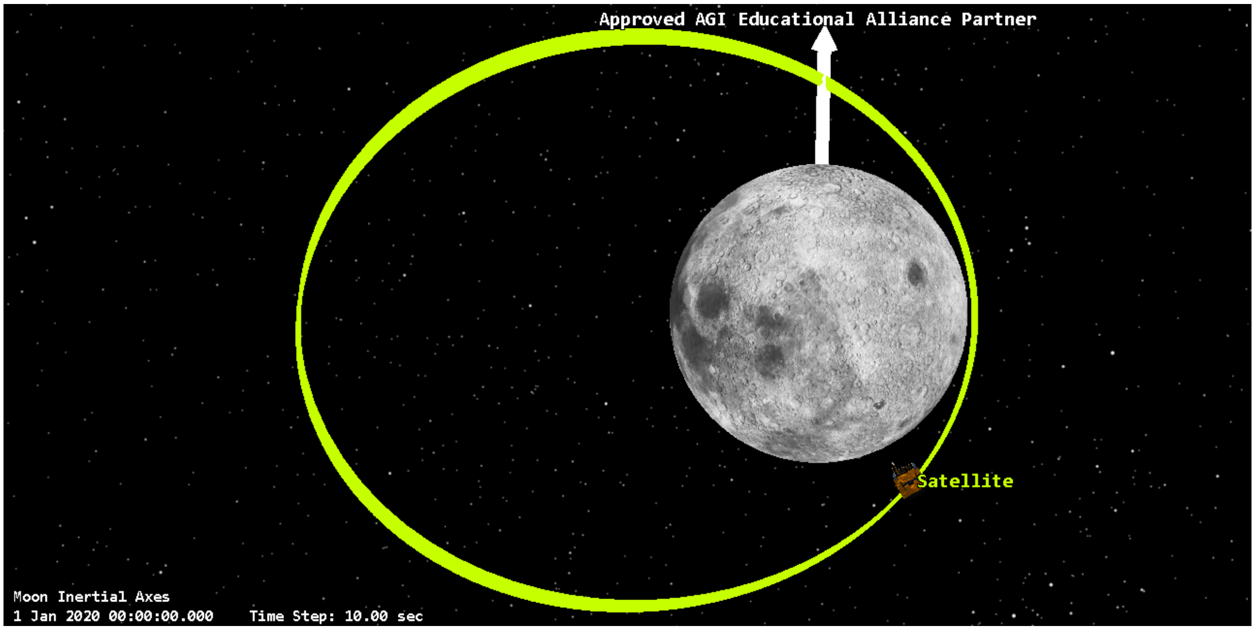


Fig. 12   Lunar orbit with Earth's gravity acting as a perturbation

IV. Conclusion and Recommendations

In this lab, the orbital behavior about two different bodies was analyzed. Of course, the orbits in themselves are based upon the specific instrumentation of the spacecraft, but for 2 equal spacecrafts with one set of operating conditions, the behavior of the spacecraft will be different when orbiting 2 different bodies. Unfortunately, a 3-body analysis was looked at at a high level, so not many details could be extracted in this particular orbit. Although this was not looked into further detail, the implications of the 3-body orbit had a long-term effect on the spacecraft orbit. Thus, the perturbations of the third body is are hard to detect over a short time range, but over a large amount of time becomes clear.

We would like to explore the behavior of the motion of galaxies in relation those that are neighboring, such as The Milky Way and Andromeda. It would be interesting to see the scale at which this computer simulation and our ability to model larger, more complex systems with the tools we have provided. As for recommendations, it would be nice to have been instructed to create Matlab functions to solve what was done by hand (questions 5,8,9). This wasn't necessarily taxing, but it would prove to be a useful tool in later work.

Acknowledgements

Thanks to Professor Davis and the TA's for their assistance on this lab!

Team Member Participation Table

Name	Plan	Model	Experiment	Results	Report	Code	Initials
Anand Trehan	1	1	1	1	1	2	AT
Selmo Almeida	1	1	1	2	2	1	SA
Shawn Stone	1	1	2	1	1	1	SS
Connor O'Reilly	1	1	0	0	0	0	CT

2 - Lead
1 - Contributor

References

[1] Smead Department of Aerospace Engineering. "ASEN 3200 LAB O-1" Oct. 2020. University of Colorado at Boulder. Lab Document and Supplemental Resources.

[2] Hare, T.M., Hayward, R.K., Blue, J.S., Archinal, B.A., Robinson, M.S., Speyerer, E.J., Wagner, R.V., Smith, D.E., Zuber, M.T., Neumann, G.A., and Mazarico, E., 2015, Image mosaic and topographic map of the moon: U.S. Geological Survey Scientific Investigations Map 3316, 2 sheets, pubs.usgs.gov/sim/3316/downloads/sim3316$_sheet1_lo_res.pdf$

Appendix A

Matlab Code

```
%% Anand Trehan

clear
clc

%% Question 4

% parameters
mu = 398600;
e = .587;
Re = 6378;
a = 242200;%37.9735*Re;
h = sqrt(mu*a*(1-e^2));
T = 2*pi*sqrt(a^3/mu);
inc = 10; % degrees

% initial conditions
rPerigee = a*(1-e);

vPerigee = sqrt(2*mu/rPerigee-mu/a);


IC = [rPerigee 0 0 0 vPerigee 0];
tvec = [0 T];

% ode45 call
[t,x] = ode45(@(t,y) odeFun(y,mu),tvec,IC);

% incorporate inclination
pos = x(:,1:3);
vel = x(:,4:6);

C2mat = [cosd(inc) 0 sind(inc);
         0         1 0        ;
         -sind(inc) 0 cosd(inc)];

finalPos = pos*C2mat;
finalVel = vel*C2mat;

xPos1 = finalPos(:,1);
yPos2 = finalPos(:,2);
zPos3 = finalPos(:,3);

% plot
figure(1)
plot3(xPos1,yPos2,zPos3)
hold on
plot3(0,0,0,'o')
plot3(xPos1(1),yPos2(1),zPos3(1),'o')
quiver3(0,0,0, xPos1(1), yPos2(1), zPos3(1),1.02,'LineWidth',2)
```

```
legend('Orbit','Earth','Initial Position','Eccentricity Vector')
xlabel('x axis (km)')
ylabel('y axis (km)')
zlabel('z axis (km)')
title('Orbit About Earth')
axis equal
hold off

% finding momentum

% for i = 1:size(t)
%     hCalc(i) = norm(finalPos(i,:))*norm(finalVel(i,:));
% end

hCalc = cross(finalPos,finalVel);

hMag = vecnorm(hCalc,2,2);

figure(2)
plot(t,hMag)
ylim([2e5 3e5])
xlabel('Time (s)')
ylabel('Specific Angular Momentum km^2/s')
title('Specific Angular Momentum Over One Period')
ecc = [0 0 0];

for i = 1:length(t)
    temp = cross(finalVel(i,:),hCalc(i,:))/mu - finalPos(i,:)/norm(finalPos(i,:));
    ecc = [ecc;temp];
end
ecc = vecnorm(ecc(2:110,:),2,2);

figure(3)
plot(t,ecc)
ylim([.55 .65])
xlabel('Time (s)')
ylabel('Eccentricity')
title('Eccentricity Over Time')


% Question 11

% parameters
mu_moon = 4904;
rMoon = 1737;
Rp = 1840;
T = 6.3*3600;

% calculated values
a_moon = ((T^2/(2*pi)^2)*mu_moon)^(1/3);
e_moon = -(Rp/a_moon-1);
M = sqrt(mu_moon/a_moon^3)*13.5*60;

E = keplers(e_moon, M);
```

```
r = a_moon*e_moon^2-a_moon*e_moon*cos(E)+a_moon*(1-e_moon^2);
% r = a_moon*(1-e_moon*cos(E));
alt = r - rMoon;
%% odeFun
function [out] = odeFun(y,mu)
xPos = y(1);
yPos = y(2);
zPos = y(3);
velocityX = y(4);
velocityY = y(5);
velocityZ = y(6);
rNorm = sqrt(xPos^2+yPos^2+zPos^2);

posVec = [xPos yPos zPos];
dVdt = -mu*posVec/rNorm^3;
dVxdt = dVdt(1);
dVydt = dVdt(2);
dVzdt = dVdt(3);

out = [velocityX; velocityY; velocityZ; dVxdt; dVydt; dVzdt];
end

%% keplers
function E = keplers(e, M)
%error tolerance:
error = 1.e-16;
%starting value for E:
if M < pi
E = M + e/2;
end
if M > pi
E = M - e/2;
end

% iteration
temp = 1;
while abs(temp) > error
temp = (E  - e*sin(E) - M)/(1 - e*cos(E));
E = E - temp; end
end
```

Derivations

Question 5

a. Orbital Period of 6.3 hrs
b. Periapsis radius of r_p=1840 km
c. Polar orbit (Hint: this constraint sets the inclination, an angle which orients the orbital plane, and should be selected to ensure that the spacecraft passes over the poles. You can use the 2D and/or 3D graphics windows to find a feasible value using guess and check with visual verification)
d. Right Asc. Of Asc. Node = 0 deg
e. Argument of Periapsis = 0 deg
f. Initial location of spacecraft such that r = 2400 km, moving away from periapsis.

Question 5: Show your working to convert information in (a), (b), (f) in the above question to the following quantities: a, e, θ*.

Period $\mathbb{P} = \dfrac{2\pi ab}{h}$

$\mu_{moon} = 4.905\times10^{12}\ m^3/s^2$

Make substitutions for b and h

$b = a\sqrt{(1-e^2)}$

$h = \sqrt{\mu a\,(1-e^2)}$

a = unknown

$\mathbb{P} = 2\pi\sqrt{\dfrac{a^3}{\mu}}$

$$\underbrace{P = 6.3\text{hrs} = 378\text{min} = 22680\,s}_{\text{Reference to the moon}}$$

$$\therefore \quad 22680 = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\left(\frac{22680}{2\pi}\right)^2 = \frac{a^3}{\mu}$$

$$a^3 = \mu\left(\frac{22680}{2\pi}\right)^2$$

$$a = \left[\mu\left(\frac{22680}{2\pi}\right)^2\right]^{1/3}$$

$$a = \left[\left(4.905\times10^{12}\,\frac{m^3}{s^2}\right)\left(\frac{22680\text{ sec}}{2\pi}\right)^2\right]^{1/3}$$

$$\boxed{a = 3998114\,m = 3998\text{ km}}$$

Fig. 13 Page 1/2: Question 5 hand calculations.

b. Periapsis radius of r_p=1840 km $\longrightarrow$ convert to e

$$r_p = \frac{p}{1+e} = a(1-e) \qquad \text{where } a = 3998 \text{ km (above)}$$

$$1-e = r_p/a \rightarrow e = 1 - r_p/a \longrightarrow e = 1 - \frac{1840}{3998}$$

$$\boxed{e = 0.5398}$$

f. Initial location of spacecraft such that r = 2400 km, moving away from periapsis.

$$r = \frac{p}{1+e\cos\theta} \qquad \underbrace{r_p = \frac{p}{1+e} = a(1-e)}$$

$$p = a(1-e)(1+e)$$

$$p = a(1-e^2)$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \rightarrow r(1+re\cos\theta) = a(1-e^2)$$

$$1+e\cos\theta = \frac{a(1-e^2)}{r} \rightarrow \cos\theta = \frac{1}{e}\left[\frac{a(1-e^2)}{r} - 1\right]$$

$$\theta = \cos^{-1}\left(\frac{1}{e}\left[\frac{a(1-e^2)}{r} - 1\right]\right) = \cos^{-1}\left(\frac{1}{.5398}\left[\frac{3998(1-.5398^2)}{2400} - 1\right]\right)$$

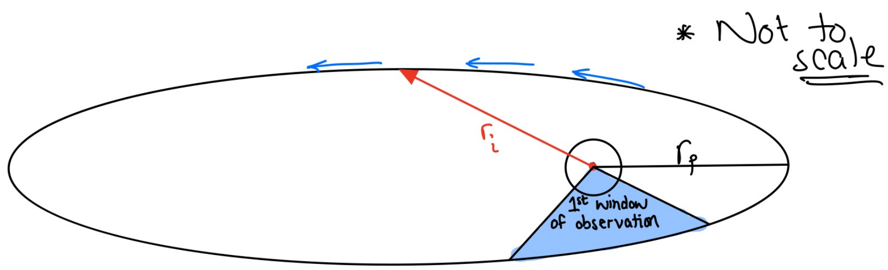
$$\boxed{\theta = 70.47^\circ}$$

Fig. 14 Page 2/2: Question 5 hand calculations.

Question 8: The spacecraft is designed with equipment that is only able to clearly make surface observations between altitudes of 200 and 500 km. From the specified initial condition, calculate the time in hours that the spacecraft must wait until its first opportunity to observe the surface, subject to the equipment constraints. Use Kepler's equation and include your calculations in your lab report.

Initial Conditions:

a. Orbital Period of 6.3 hrs
b. Periapsis radius of r_p=1840 km r_{moon} = 1738.1 km ← Google
c. Polar orbit (Hint: this constraint sets the inclination, an angle which orients the orbital plane, and should be selected to ensure that the spacecraft passes over the poles. You can use the 2D and/or 3D graphics windows to find a feasible value using guess and check with visual verification)
d. Right Asc. Of Asc. Node = 0 deg
e. Argument of Periapsis = 0 deg
f. Initial location of spacecraft such that r = 2400 km, moving away from periapsis.



$r_p - r_{moon}$ = 1840 − 1737 = 103 km ∴ at periapsis the satellite is too close to study the lunar surface

$$n = \frac{2\pi}{P} \rightarrow n = \frac{2\pi}{6.3\,hr} = \frac{2\pi}{22680\,s} \longrightarrow n = 2.77 \times 10^{-4}\ s^{-1}$$

Altitude 500 km: r = 1737 + 500 = 2237 km

Altitude 200 km: r = 1737 + 200 = 1937 km

Technically, the problem asks for the time it takes to reach the start of observation, NOT the duration ∴ the 200 km is extraneous information.

Fig. 15 Page 1/3: Question 8 hand calculations.

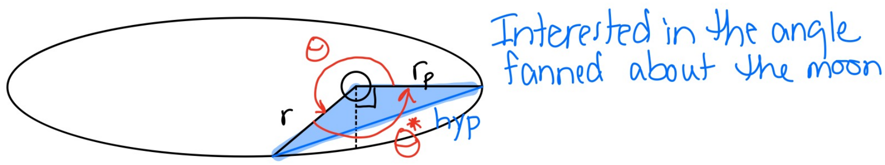
Kepler's Equation: $M = n(t - t_p) = E - e\sin E$

Previously solved for:

$n = 2.77\times10^{-4}\ s^{-1}$

$e = 0.5398$ ... Problem 5

Need E @ $r = 2237$ km



$$r = \frac{p}{1 + e\cos\theta} \qquad p = \frac{b^2}{a} = a(1 - e^2) = \frac{h^2}{m}$$

Solve for θ:

$$1 + e\cos\theta = \frac{p}{r} \longrightarrow \cos\theta^* = \frac{1}{e}\left[\frac{p}{r} - 1\right]$$

$$\therefore\ \theta^* = \cos^{-1}\left[\frac{1}{e}\left(\frac{p}{r} - 1\right)\right]$$

$$\downarrow$$

$$\theta^* = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r} - 1\right)\right]$$

θ @ Alt. 500km:

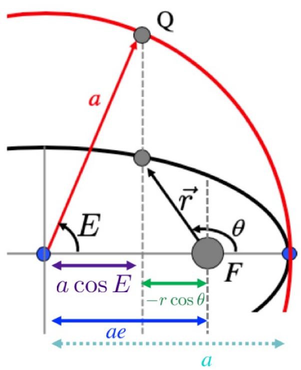
$$\theta_5^* = \cos^{-1}\left[\frac{1}{0.5398}\left(\frac{3998(1 - 0.5398^2)}{2237} - 1\right)\right]$$

$$= \cos^{-1}\left(\frac{1}{0.5398}(0.2664)\right) \longrightarrow \cos^{-1}(0.4936)$$

$\theta_5^* = 1.0546\text{ rad} = 60.42 \quad \therefore\ \theta_5 = 360 - 60.42 : \theta_5 = \sim300^\circ$ / 5.229 rad

Need θ at initial condition to determine E.

Fig. 16 Page 2/3: Question 8 hand calculations.



$$a\cos E = ae - r\cos\theta$$

$$\rightarrow E = \cos^{-1}\left(e - \frac{r}{a}\cos\theta\right)$$

$\theta = 70.47°$ as found in problem 5

$$\therefore E_1^* = \cos^{-1}\left(0.5398 - \frac{2402}{3998}\cos(70.47)\right)$$

$$E_2^* = \cos^{-1}\left(0.5398 - \frac{2237}{3998}\cos(299.58)\right)$$

$$\mathbf{E_1^* = 70.177° = 1.225\ rad}$$

$E_2^* = 74.717 \longrightarrow E_2 = 360 - E_2^*$: $\mathbf{E_2 = 285.28° = 4.979\ rad}$

Keplers Equation: $M = n(t - t_p) = E - e\sin E$

Knowing θ we now have the bounds for E → [

$$n(t - t_p) = E - e\sin E\Big|_{70.177°}^{285.28°}$$

$$= E - e\sin E\Big|_{1.225}^{4.979}$$

$$= \left[4.979 - (0.5398)\sin(4.979)\right] - \left[1.225 - (0.5398)\sin(1.225)\right]$$

$$= \left[4.979 - 0.5398(-.9647) - 1.225 + 0.5398(0.9408)\right]$$

$$= 4.7825$$

$$t - t_p = \frac{1}{n}(4.7825) = \frac{1}{2.77\times10^{-4}}(4.7825) = 17266 \text{ seconds}$$

$$\boxed{\Delta t = 4.796 \text{ hours}}$$ ← makes sense given the period and dynamics

Fig. 17 Page 3/3: Question 8 hand calculations.

Question 9

Question 9: Use the conic equation to calculate the value of the true anomaly at the beginning and end of this first surface observation window. Use Kepler's equation to calculate the time interval corresponding to this observation window. Show your work in your lab report. Verify your results in STK by modifying the true anomaly of the initial state to equal the true anomaly at the beginning of this first surface observation window. Modify the propagate segment duration stopping condition to reflect the time interval you calculated. Run the MCS again and include a screenshot of the portion of the spacecraft trajectory for which an observation window occurs.

$$r = \frac{p}{1 + e\cos\theta} \qquad p = \frac{b^2}{a} = a(1 - e^2) = \frac{h^2}{m}$$

Altitude 500 km: $r_5 = 1737 + 500 = 2237$ km

Altitude 200 km: $r_2 = 1737 + 200 = 1937$ km

Solve for θ:

$$1 + e\cos\theta = \frac{p}{r} \longrightarrow \cos\theta = \frac{1}{e}\left[\frac{p}{r} - 1\right]$$

$$\therefore \theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{p}{r} - 1\right)\right]$$

$$\downarrow$$

$$\theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r} - 1\right)\right]$$

θ @ Alt. 500km:

$$\theta_5^* = \cos^{-1}\left[\frac{1}{0.5398}\left(\frac{3998(1-0.5398^2)}{2237} - 1\right)\right]$$

$$= \cos^{-1}\left(\frac{1}{0.5398}(0.2664)\right) \longrightarrow \cos^{-1}(0.4936)$$

$$\theta_5^* = 1.0546 \text{ rad} = 60.42^\circ$$

$$\boxed{\theta_5 = 5.229 \text{ rad} = 299.58^\circ \text{ (start)}}$$

Fig. 18 Page 1/3: Question 9 hand calculations.

$$\theta_2^* = \cos^{-1}\left[\frac{1}{0.5398}\left(\frac{3998(1-0.5398^2)}{1937} - 1\right)\right]$$

$$= \cos^{-1}\left[\frac{1}{0.5398}(0.4625)\right]$$

$$\theta_2^* = 0.5414 \text{ rad} = 31.02^\circ$$

$$\boxed{\theta_2 = 5.7418 \text{rad} = 328.98^\circ \quad (\text{End})}$$

From Problem 8:

$$E^* = \cos^{-1}\left(e - \frac{r}{a}\cos\theta\right)$$

$$E_5^* = \cos^{-1}\left(0.5398 - \left(\frac{2237}{3998}\right)\cos(299.58^\circ)\right)$$

$$= \cos^{-1}(0.2148)$$

$$E_5^* = 77.594^\circ = 1.3543 \text{ rad}$$ ← Lower half of the orbit

$$E_2^* = \cos^{-1}\left(0.5398 - \left(\frac{1937}{3998}\right)\cos(328.98^\circ)\right)$$

$$= \cos^{-1}(0.05128)$$

$$E_2^* = 87.06^\circ = 1.5195 \text{ rad}$$ ← lower half of the orbit

$$\left.\begin{aligned} E_5 &= 2\pi - E_5^* = 4.9289 \text{ rad} \\ E_2 &= 2\pi - E_2^* = 4.7637 \text{ rad} \end{aligned}\right\} \text{Accounting for position}$$

Fig. 19 Page 2/3: Question 9 hand calculations.

Kepler's Equation: $M = n(t - t_p) = E - e\sin E$

$$n(t-t_p) = E - e\sin E \Big|_{start}^{finish}$$

$$\Delta t = \frac{1}{n}\left(E - e\sin E\right)\Big|_{E_2}^{E_5}$$

$$\Delta t = \frac{1}{2.77\times10^{-4}}\left[4.9289 - 0.5398\sin(4.9289) - \left(4.7637 - 0.5398\sin(4.7637)\right)\right]$$

$$\Delta t = \frac{1}{2.77\times10^{-4}}\left[0.1533\right]$$

$$\Delta t = 553.46 \text{ seconds}$$

$$\boxed{\Delta t = 9.22 \text{ minutes}}$$

Fig. 20 Page 3/3: Question 9 hand calculations.