UNIVERSITY OF COLORADO - BOULDER

ASEN 3200 - ORBITAL MECHANICS / ATTITUDE CONTROL

LABORATORY O-2

Mars Orbiter and NEO's in STK

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Presented in this lab are the results of two different simple, two-body problems between a satellite and its orbiting body (Mars for Part 1 and the Sun for Part 2). Matlab and STK were used to perform calculations and generate the 2D and 3D graphics presented (in addition to hand calculations shown in the appendix). For the Martian satellite simulation in Part 1, the orbit of the satellite was determined with the position and velocity vectors, which produced the Keplerian orbital parameters: [a (semi-major axis), e (eccentricity), i (inclination), Ω (ascending node), ω (argument of periapsis), θ (true anomaly)]. In Part 2, the path of the satellite orbiting the Sun was analyzed against the orbit of the Earth. Using a satellite to represent an asteroid/other small cosmic body we were able to model the behavior of a Near Earth Object (NEO). The orbit of this object was modeled with the Keplerian orbital parameters [a, e, i, Ω, ω, θ].

Contents

Introduction

The goal of this lab is to model the behavior of a satellite/NEO with respect to a cosmic body other than the Earth. With a fundamental understanding of two-body problems, calculations for the semi-major axis (a), eccentricity (e), inclination (i), ascending node (Ω), argument of periapsis (ω), and the true anomaly (θ) are determined with given position and velocity vectors. With these calculated parameters, a scenario of a satellite in an areocentric orbit (orbit about Mars) and heliocentric orbit (orbit about the Sun) are modeled.

I. Mars Orbiter

Question 1

The first step in finding the Keplerian Orbital elements was finding the velocity in the z direction. Knowing that $\vec{h} = \vec{r} \times \vec{v}$, it was possible to use a variable placeholder for V_z, and do the cross product with the given r vector: $[-3424.7, -47.5, 1172]$ km. This produces the following angular momentum vector:

$$\vec{h} = \left[\left(3906.3 - \frac{95 * V_z}{2}\right), (3424.7 * V_z + 498.1), (11435)\right] km^2/s \tag{1}$$

Setting the first element of this vector equal to the known $\vec{h}_x = 3948.694$, V_z can be solved as -0.8925 km/s. Finally, this V_z can be used to solve for angular momentum. Next, eccentricity was calculated to be 0.0498 using the following equation:

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{|r|} \tag{2}$$

Next, in order to find the semi-major axis of the orbit, it was necessary to find the specific energy of the system:

$$\epsilon = \frac{V^2}{2} - \frac{\mu}{r} \tag{3}$$

ϵ was calculated to be -5.7934 KJ/kg, which resulted in a semi-major axis of 3696.275 km. Using the momentum vector calculated above, the following equation was used to find the inclination angle, 25.0035 degrees:

$$i = \cos^{-1}\left(\frac{h_z}{h}\right) \tag{4}$$

in order to find Ω and ω, it was necessary to find $\vec{n} = \vec{z} \times \vec{h}$, where $\vec{z} = [0, 0, 1]$. This resulted in $\vec{n} = [3.5560, 3.9487, 0]$. The following equations were then used to calculate ω, Ω, and θ respectively:

$$\Omega = cos^{-1}\left(\frac{\vec{n}}{n}\right) \tag{5}$$

$$\omega = cos^{-1}\left(\frac{\vec{n_x} \cdot \vec{e}}{n * e}\right) \tag{6}$$

$$\theta = cos^{-1}\left(\frac{\vec{e} \cdot \vec{r}}{e * r}\right) \tag{7}$$

These were calculated to be $\Omega = 47.9925°$, $\omega = 62.4913°$, and $\theta = 67.5151°$. Finally, period was calculated using the computed semi-major axis, and using μ=398600 $\frac{m^3}{s^2}$. This was found to be $6.8259 * 10^3$ seconds, or 1.8961 hours.

Question 2

After propagating the orbit and displaying necessary vectors (angular momentum, eccentricity, and the line of nodes), the grease pencil tool in STK was used to draw inclination, right ascension of the ascending node, and argument of periapsis. The vectors were verified visually and are seen below in figure 1.

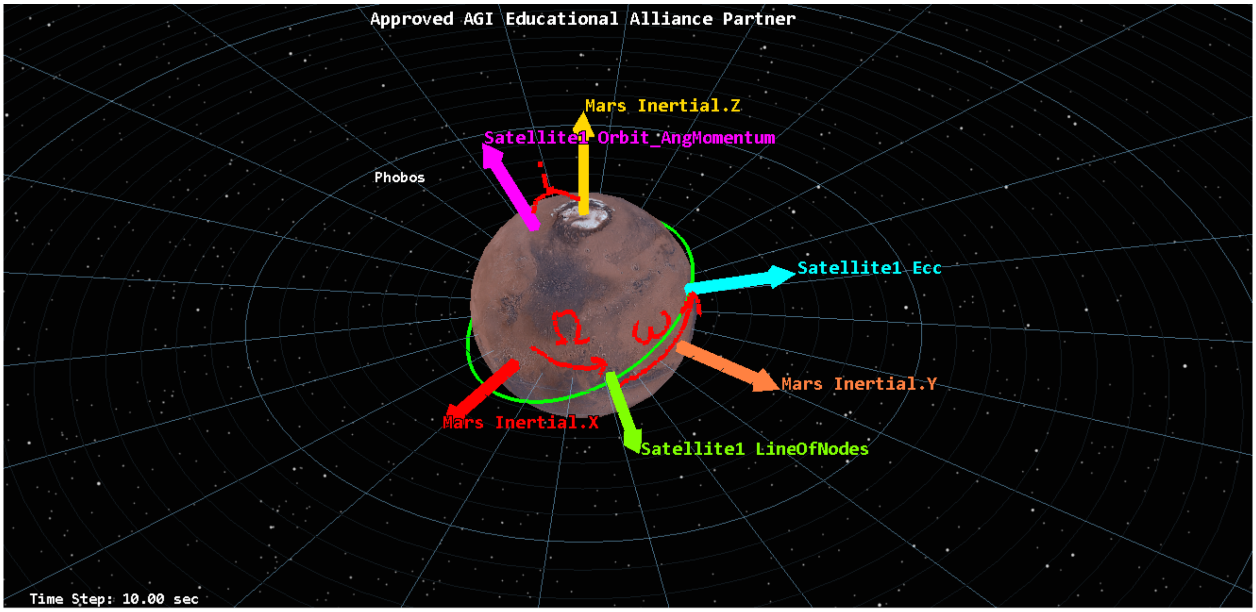


Fig. 1 3D screenshot of Mars orbit with orbital vectors

Question 3

Parameter Set Type: Cartesian

X:	-3422.9939601453265823 km	Vx:	-0.4251244567623955 km/sec
Y:	-47.4784928955733747 km	Vy:	-3.3329973505357899 km/sec
Z:	1171.4147658164449695 km	Vz:	-0.8929550618400890 km/sec

Other Elliptic Orbit Parameters :

Ecc. Anom:	64.90239539771576 deg	Mean Anom:	62.31845833882772 deg
Long Peri:	110.4838 deg	Arg. Lat:	130.0064 deg
True Long:	177.9989 deg	Vert FPA:	87.41460900326398 deg
Ang. Mom:	12566.33798343302 km^2/sec	p:	3687.1080901490022370 km
C3:	-11.58690185145855 km^2/sec^2	Energy:	-5.793450925729277 km^2/sec^2
Vel. RA:	262.7311617021826 deg	Vel. Decl:	-14.88292884145941 deg
Rad. Peri:	3512.2005049999997937 km	Vel. Peri:	3.5779101920700351 km/sec
Rad. Apo:	3880.3494950000053905 km	Vel. Apo:	3.23845519575628 km/sec
Mean Mot.:	0.05276452363164747 deg/sec		
Period:	6822.766040932789 sec	Period:	113.7127673488798 min
Period:	1.895212789147997 hr	Period:	0.0789671995478332 day

Fig. 2 Cartesian State Components and Orbital Elements

The given position vector is really close to the value for the position vector STK found using the orbital elements. Each element of the vector was within 2 km. The velocity was also extremely close to the given values in the x and y directions, as well as the calculated value in the z. Each STK velocity value was within a hundredth of the given/calculated. This is likely due to rounding errors in the calculations for the orbital elements that STK took in. Generally, these discrepancies are very small, so its likely that the rounding was the only cause for any error.

Question 4

As shown below in figure 3, the groundtrack does not form a closed loop during one full orbit. This is because Mars is rotating as the satellite orbits overhead, creating the illusion that the orbit isn't fully completed. If Mars and the satellite were rotating in opposite directions, the groundtrack would show what looks like more than a full orbit along the groundtrack.

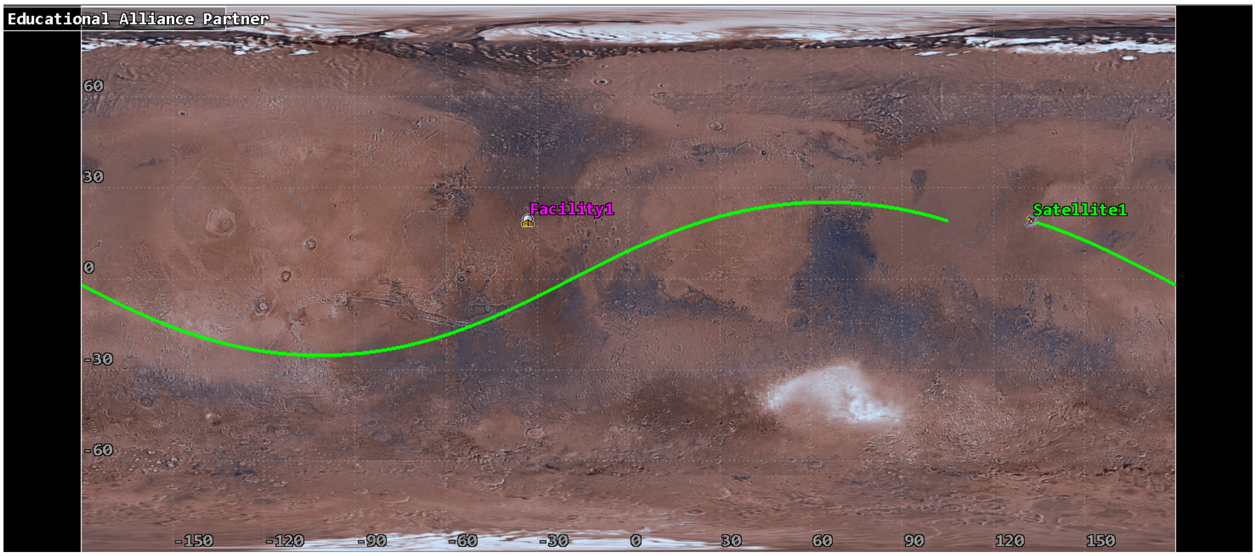


Fig. 3 2D groundtrack of Mars orbiter

Question 5

As shown in figure 3, the unmodified orbit does not pass directly over the Martian habitat within the first orbit, therefore, at least one of the orbital parameters must be changed do get an overhead picture. Keeping all other orbital elements fixed, the right ascension of the ascending node (Ω) was modified until the result shown in figure 4 was found, which occured at $\Omega = 352deg$. This orbital element was chosen over the others because it is analogous to the phase angle of a sinusoudal plot, which shifts the starting position laterally (if we wanted to reach a habitat at a higher latitude, we would need to make the orbit more polar, therefore changing the inclination as well).

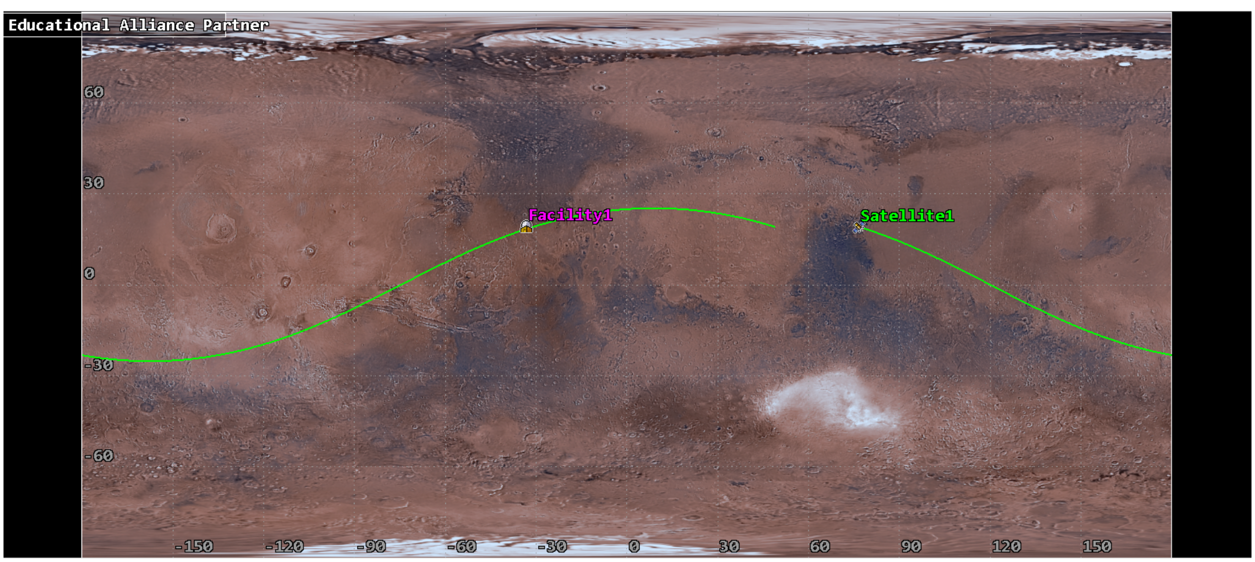


Fig. 4 2D groundtrack of Mars orbiter, with Ω = 352 deg

Question 6

Observing figure 5, the satellite passes directly overhead twice during a full (Earth) day's orbit, shown below.

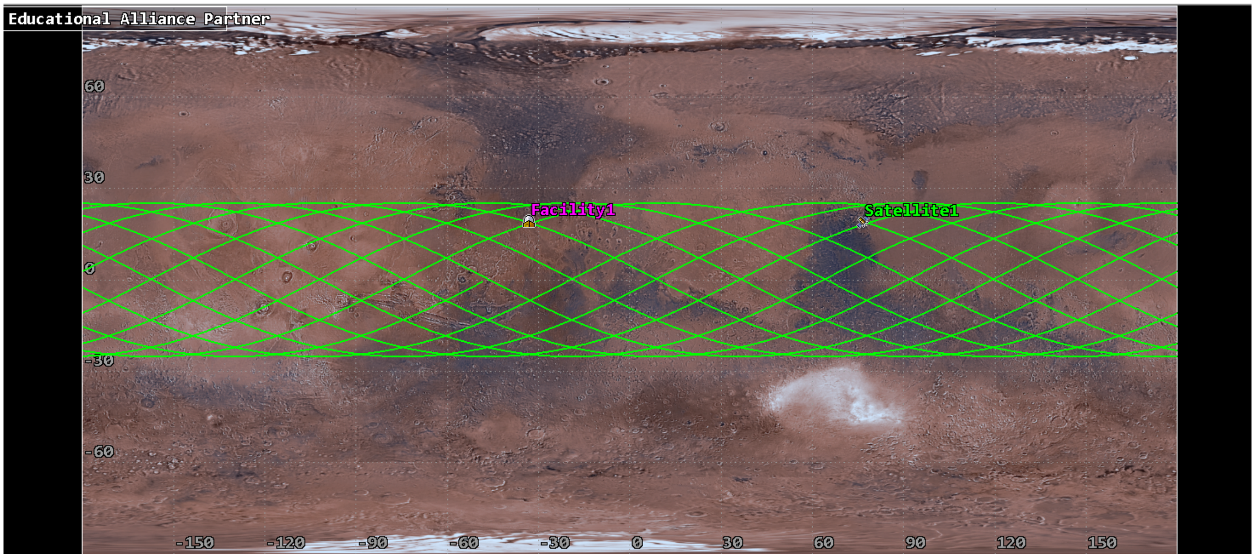


Fig. 5 2D groundtrack of Mars orbiter propagated for one full day

II. Near Earth Objects

Theory - Gibbs method

Three astrometric observations of a NEO were made at three separate times $[t_1, t_2, t_3]$ and three different geocentric position vectors $[r_1, r_2, r_3]$ were obtained for each respective time. Using the conservation of angular momentum, the unit vector normal to the plane of position vectors r_2 and r_3 must be perpendicular to the unit vector in the direction of r_1. To check this the following dot product must be equal to zero:

$$\hat{u}_{r_1} \cdot \hat{C}_{23} = 0$$

Where $\hat{u}_{r_1}$ represents the unit vector in the direction of r_1 and $\hat{C}_{23}$ is the unit normal vector normal to the plane of position vectors r_2 and r_3

$$\hat{u}_{r_1} = \frac{r_1}{|\vec{r_1}|}$$

$$\hat{C}_{23} = \frac{(r_2 \times r_3)}{||r_2 \times r_3||}$$

After verifying that the dot product is zero, we begin to derive an equation to find the velocity for any of the observed position vectors. Initially the following equation is used in the derivation,

$$\vec{v} \times \vec{h} = \mu \left(\frac{\vec{r}}{|\vec{r}|} + \vec{e} \right) \tag{8}$$

where $\vec{e}$ and $\vec{h}$ are the eccentricity and angular momentum vector for the NEO. Applying the identity $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$, and realizing $\vec{h} \cdot \vec{h} = h^2$ and $\vec{v} \cdot \vec{h} = 0$ due to the vector properties we can rewrite equation (8) as

$$\vec{v} = \frac{\mu}{|\vec{h}|^2} \left(\frac{\vec{h} \times \vec{r}}{|\vec{r}|} + \vec{h} \times \vec{e} \right) \tag{9}$$

Rewritting this equation in the perifocal coordinate system where $\vec{e} = |\vec{e}|\hat{\mathbf{p}}$ and $\vec{h} = |\vec{h}|\hat{\mathbf{w}}$, equation (9) becomes

$$\vec{v} = \frac{\mu}{|\vec{h}|} \left(\frac{\hat{\mathbf{w}} \times \vec{r}}{|\vec{r}|} + e(\hat{\mathbf{w}} \times \hat{\mathbf{p}}) \right) \tag{10}$$

observing that $\hat{\mathbf{w}} \times \hat{\mathbf{p}} = \hat{\mathbf{q}}$ equation (9) becomes

$$\vec{v} = \frac{\mu}{|\vec{h}|}\left(\frac{\hat{\mathbf{w}} \times \vec{r}}{|\vec{r}|} + e\hat{\mathbf{q}}\right) \tag{11}$$

Because vectors $\vec{r_1}, \vec{r_2}$ and $\vec{r_3}$ lie in the same plane scalar factors c_1 and c_2 can be applied so that $\vec{r_2} = c_1\vec{r_1} + c_3\vec{r_3}$. Taking the dot product with the previous equation and the eccentricity vector allows us to describe the orbit using the three observed position vectors so that

$$\vec{r_2} \cdot \vec{e} = c_1\vec{r_1} \cdot \vec{e} + c_3\vec{r_3} \cdot \vec{e} \tag{12}$$

Referencing the orbit equation the relation $\vec{r} \cdot \vec{e} = \frac{|\vec{h}|^2}{\mu} - |\vec{r}|$ is substituted into equation (12) resulting in

$$\frac{|\vec{h}|^2}{\mu} - |\vec{r_2}| = c_1\left(\frac{|\vec{h}|^2}{\mu} - |\vec{r_1}|\right) + c_3\left(\frac{|\vec{h}|^2}{\mu} - |\vec{r_3}|\right) \tag{13}$$

taking the cross product of $\vec{r_2} = c_1\vec{r_1} + c_3\vec{r_3}$ with $\vec{r_1}$ and $\vec{r_2}$ allows us to obtain the following

$$\vec{r_2} \times \vec{r_1} = c_3(\vec{r_3} \times \vec{r_1}) \;\;,\;\; \vec{r_2} \times \vec{r_3} = -c_1(\vec{r_3} \times \vec{r_1}) \tag{14}$$

Multiplying equation (13) by $\vec{r_3} \times \vec{r_1}$ and substituting in equations from (14) returns

$$\frac{|\vec{h}|^2}{\mu}(\vec{r_1} \times \vec{r_2} \; + \; \vec{r_2} \times \vec{r_3} \; + \; \vec{r_3} \times \vec{r_3}) = |\vec{r_1}|(\vec{r_2} \times \vec{r_3}) + |\vec{r_2}|(\vec{r_3} \times \vec{r_1}) + |\vec{r_3}|(\vec{r_1} \times \vec{r_2}) \tag{15}$$

The above equation only has one unknown which is the angular momentum $|\vec{h}|$. The vectors on the left hand and right hand side of equation (15) can be rewritten as

$$\vec{D} = \vec{r_1} \times \vec{r_2} \; + \; \vec{r_2} \times \vec{r_3} \; + \; \vec{r_3} \times \vec{r_3} \tag{16}$$

$$\vec{N} = |\vec{r_1}|(\vec{r_2} \times \vec{r_3}) + |\vec{r_2}|(\vec{r_3} \times \vec{r_1}) + |\vec{r_3}|(\vec{r_1} \times \vec{r_2}) \tag{17}$$

So that equation (15) can be rewritten as

$$\vec{N} = \frac{|\vec{h}|^2}{\mu}\vec{D} \tag{18}$$

the angular momentum of the NEO can then be determined by taking the magnitude of both $\vec{N}$ and $\vec{D}$ and solving for $|\vec{h}|$.

$$|\vec{h}| = \sqrt{\mu\frac{|\vec{N}|}{|\vec{D}|}} \tag{19}$$

Because all the obtained position vectors lie in the same plane, cross products $\vec{r_1} \times \vec{r_2}, r_3 \vec{\times} \vec{r_1}$ and $\vec{r_2} \times \vec{r_3}$ will be normal to the orbital plane. Allowing us to represent the orbit normal unit vector $\hat{\mathbf{W}}$ as

$$\hat{\mathbf{w}} = \frac{\vec{D}}{|\vec{D}|} \tag{20}$$

Lastly an expression for $\hat{\mathbf{q}}$ in terms of $\vec{r_1}, \vec{r_2}$ and $\vec{r_3}$ needs to be derived. Its noticed that

$$\hat{\mathbf{q}} = \hat{\mathbf{w}} \times \hat{\mathbf{p}} = \frac{\vec{D}}{|\vec{D}|} \times \frac{\vec{e}}{|\vec{e}|} \tag{21}$$

Substituting in equation (16), applying the identity $(A \times B) \times C = B(A \cdot C) - C(A \cdot B)$ to equation (21) the following equations are obtained

$$(\vec{r_2} \times \vec{r_3}) \times \vec{e} = \vec{r_3}(\vec{r_2} \cdot \vec{e}) - \vec{r_2}(\vec{r_3} \cdot \vec{e})$$
$$(\vec{r_3} \times \vec{r_1}) \times \vec{e} = \vec{r_1}(\vec{r_3} \cdot \vec{e}) - \vec{r_3}(\vec{r_1} \cdot \vec{e})$$
$$(\vec{r_1} \times \vec{r_2}) \times \vec{e} = \vec{r_2}(\vec{r_1} \cdot \vec{e}) - \vec{r_1}(\vec{r_2} \cdot \vec{e})$$

Applying the relation $\vec{r} \cdot \vec{e} = \frac{|\vec{h}|^2}{\mu} - |\vec{r}|$, summing all resulting three equations and subbing into equation (21) produces

$$\hat{\mathbf{q}} = \frac{\vec{S}}{|\vec{D}|\ |\vec{e}|} \tag{22}$$

where

$$\vec{S} = \vec{r}_1(|\vec{r}_2| - |\vec{r}_3|) + \vec{r}_2(|\vec{r}_3| - |\vec{r}_1|) + \vec{r}_3(|\vec{r}_1| - |\vec{r}_2|)$$

Lastly substituting equations (19), (20) and (22) into equation (11) produces an expression to compute the velocity of the observed NEO using only three position vectors.

$$\vec{v} = \sqrt{\frac{\mu}{|\vec{N}|\ |\vec{D}|}\left(\frac{\vec{D} \times \vec{r}}{|\vec{r}|} + \vec{S}\right)} \tag{23}$$

The above theory was applied to a matlab script *gibbs.m* and was used to compute the Sun-centered ICRF velocity vector at the second observation time.

Theory - Converting Inertial State Vector to Orbital Elements

After implementing Gibbs method into a matlab script, the position and velocity for time two were used to compute orbital elements. Using Algorithm 4.2 as a guide, initially the magnitude of the position and velocity vector were computed. These are denoted as $|\vec{r}_2|$ and $|\vec{v}_2|$. Using these values the radial velocity is computed using the following equation

$$v_r = \frac{\vec{r}_2 \times \vec{v}}{|\vec{v}_2|}$$

Depending on the sign of the radial velocity, if it is positive it is flying away from perigee and negative if it is flying towards perigee. Following the angular momentum vector of the NEO is computed by taking the cross product of the position and velocity vector. $\vec{h} = \vec{r} \times \vec{v}$ and let its magnitude be represented by $|\vec{h}|$. The inclination angle can be determined using equation (4) shown in the mars orbiter section. Following the line of nodes is computed following the same method explained in the mars orbiter section where $\vec{N}$ represents the line of nodes and $\vec{N} = [0, 0, 1] \times \vec{h}$. To compute the Right Ascension of Ascending node $[\Omega]$, a quadrant check is needed where if the $\hat{Y}$ component of the line of nodes is positive it implies that $0 < \Omega < 180°$ and otherwise $180 < \Omega < 360°$. Computing the right ascension of ascending node can be simplified to

$$\Omega = \begin{cases} cos^{-1}\left(\frac{\vec{N}_X}{|\vec{N}|}\right) & (\vec{N}_Y \geq 0) \\ 360° - cos^{-1}\left(\frac{\vec{N}_X}{|\vec{N}|}\right) & (\vec{N}_Y < 0) \end{cases}$$

Following, the eccentricity vector is determined using equation 2 in the mars orbiter section.

$$\vec{e} = \frac{\vec{v}_2 \times \vec{h}}{\mu} - \frac{\vec{r}_2}{|\vec{r}_2|}$$

the magnitude of the eccentricity vector is represented by $|\vec{e}|$ and will be used in the computation for the argument of perigee. Similarly to the computation of the right ascension node a quadrant check is needed. If $\vec{e_Z} \geq 0$ it implies that $0 < \omega < 180°$ otherwise $180 << 360°$. In summary

$$\omega = \begin{cases} cos^{-1}\left(\frac{\vec{N} \cdot \vec{e}}{|\vec{N}|\ |\vec{e}|}\right) & (\vec{e_Z} \geq 0) \\ 360° - cos^{-1}\left(\frac{\vec{N} \cdot \vec{e}}{|\vec{N}|\ |\vec{e}|}\right) & (\vec{e_Z} < 0) \end{cases}$$

The true anomaly $[\theta]$ is computed with the eccentricity vector and the position vector. As stated earlier if the radial velocity $[v_r]$ has zero or positive magnitude the NEO will be traveling away from perigee meaning the true anomaly will be in between 0° and 180°. Otherwise, if the radial velocity is negative the NEO is moving from apogee towards perigee meaning true anomaly is between 180° and 360°. In summary

$$\theta = \begin{cases} cos^{-1}\left(\frac{\vec{e}}{|\vec{e}|} \cdot \frac{\vec{r_2}}{|\vec{r_2}|}\right) & (v_r \geq 0) \\ 360° - cos^{-1}\left(\frac{\vec{e}}{|\vec{e}|} \cdot \frac{\vec{r_2}}{|\vec{r_2}|}\right) & (v_r < 0) \end{cases}$$

Lastly the semi major axis can be computed using values obtained for $|\vec{h}|$, μ and $|\vec{e}|$. Rearranging the following equation to determine the magnitude of angular velocity we are able to compute the semi major axis

$$|\vec{h}| = \sqrt{\mu a(1 - |\vec{e}|)}$$

$$a = \frac{|\vec{h}|^2}{(1 - |\vec{e}|^2)}$$

Matlab Code

The above theory was implemented into the *gibbs.m* matlab script. *gibbs.m* takes the three position vectors in kilometers and the gravitation constant of the planetary body as inputs. After computing the velocity vector at time two, Keplerian orbital elements were computed using the theory explained in the above section. Running the matlab script with the observed position vectors the following orbital elements were returned

a	$1.0436 \times 10^8 [km]$
e	0.1957
i	4.6524°
ω	96.3371°
Ω	223.3791°
θ	334.0688°

```
    function [v2, a, e_mag, i, omega, OMEGA, theta] = gibbs(r1 ,r2 ,r3 ,mu)

%{
    Author: Connor O'Reilly
    Co - Authors: Anad Trehan, Shawn Stone, Selmo Almeida

    Purpose:  implements Gibb's method to use the three position vectors
              to compute the Sun-centered ICRF velocity vector at the second
              observation time [t2].
    Inputs:
        r1, r2, r3: three position vectors in Sun-centered ICRF coordinates
                    at t1, t2, t3 respectively
        mu: Gravitational Parameter

    Outputs:
        v_2: Sun-centered ICRF velocity vector at the second observation time

    Last Edited: 11/3/2020 2:42 PM

%}
%% Calculate r1, r2, and r3
r1_mag = norm(r1);
r2_mag = norm(r2);
r3_mag = norm(r3);
```

```
%% Calculate C12, C23, C31
C12 = cross(r1, r2);
C23 = cross(r2, r3);
C31 = cross(r3, r1);

%% Verify that u_r1 . C^23 = 0
C23_hat = C23./norm(C23);
uc = dot((r1/r1_mag),C23_hat);
fprintf('dot(u_r1,C_23) = %f', uc)

%% Calculate N, D, and S using Eqns (5.13), (5.14), and (5.21), respectively
N = r1_mag.*C23 + r2_mag.*C31 + r3_mag.*C12;
D = C12 + C23 + C31;
S = r1.*(r2_mag - r3_mag) + r2.*(r3_mag - r1_mag) + r3.*(r1_mag - r2_mag);

N_mag = norm(N);
D_mag = norm(D);
S_mag = norm(S);

v2 = sqrt(mu/(N_mag * D_mag)) * ( cross(D,r2)/r2_mag + S );


%% Keplerian orbital elements (a, e, i, oemga,?, theta)

%caclulate speed
v2_mag = norm(v2);

%calculate radial velocity
v_r = dot(r2,v2)/r2_mag;

%if v_r > 0  satelite is flying away from perigee
%if v_r < 0 it is flying toward perigee

%calculating specific angular momentum
h = cross(r2,v2);
h_mag = norm(h);

%inclination angle
i = acos(h(3)/h_mag);

%Calculate the node line vector and the the magnitude of the node line
%vector
NL = cross([0 0 1], h);
NL_mag = norm(NL);

% Calculate the right ascension of the ascending node
if (NL(2) >= 0)
    OMEGA = acos(NL(1)/NL_mag);
else
    OMEGA = ( 360 * pi/180) - acos(NL(2)/NL_mag);
end

%calculate eccentricity vector
```

```
e  = (1/mu) * ( (v2_mag^2 - (mu/r2_mag)).*r2 - r2_mag .* v_r .* v2  );
%calculate eccentricity
e_mag = norm(e);

%Caclulate the argument of perigee
if (e(3) >= 0 )
    omega = acos((dot(NL,e))/(NL_mag*e_mag));
else
    omega = (360 * pi/180) - acos((dot(NL,e))/(NL_mag*e_mag));
end

%calculate the true anomaly
if (v_r >= 0)
    theta = acos( (1/e_mag) * ((h_mag^2/(mu*r2_mag))-1) );
else
    theta = (360 * pi/180) - acos( (1/e_mag) * ((h_mag^2/(mu*r2_mag))-1) );
end

%calculate semi-major axis
a = (h_mag^2) / (mu * (1-e_mag^2));


end
```

Question 9

Three vectors indicating the Sun ICRF X,Y and Z directions were added to the 3D graphics window of the NEO orbit and below are two images of the NEO's heliocentric orbit.

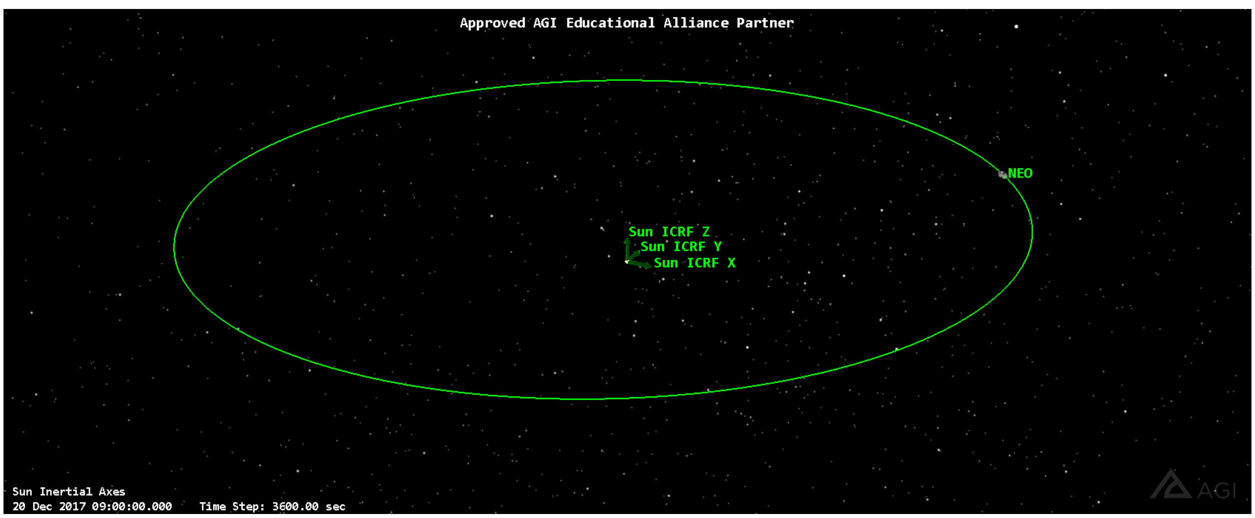


Fig. 6 3D screenshot of NEO orbit with Sun ICRF direction vectors

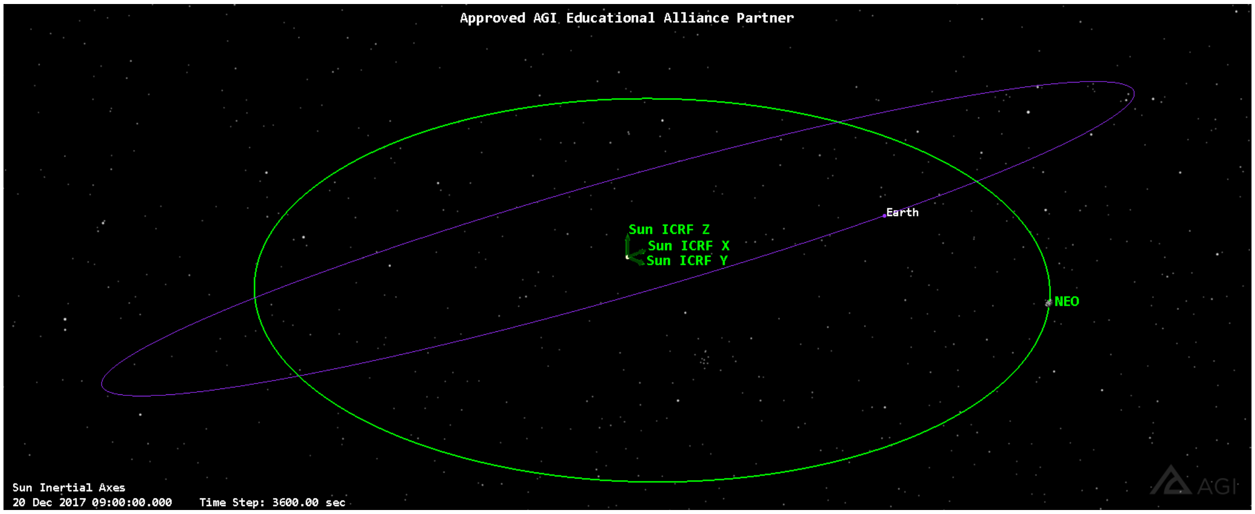


Fig. 7 3D screenshot of NEO orbit with Sun ICRF direction vectors and Earth's orbit

Looking at the three dimensional images produced in STK the NEO orbit appears to be circular.This is due to the small eccentricity computed, although it is close to zero the value sill falls between zero and one meaning the NEO orbit is elliptical. One significant perturbation acting on this spacecraft is the Earth's gravitational effect. Looking at figure seven we see that at Earth's orbit does appear to be positioned relatively close to the NEO's orbit which would cause Earth's gravitational effect to be the most immediate and largest source of external disturbance.

Question 10

The NEO's orbit does cross the Earth's heliocentric orbit. Analyzing figure seven it is difficult to determine if the orbits do cross but re orientating the view point in the 3D graphics window its determined the orbits do cross.

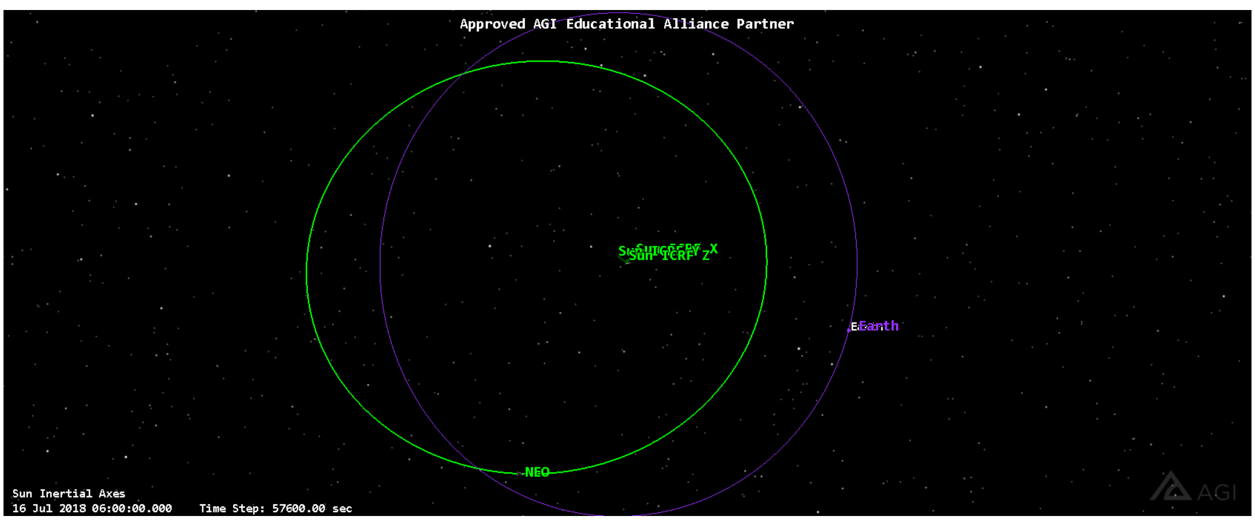


Fig. 8 3D screenshot of NEO orbit with Sun ICRF direction vectors

From this new view its observed the orbit do in fact cross, and assuming that the NEO is an asteroid leaves us to 4 possible groups for the NEO. Utilizing the provided hyperlink it was determined that the the NEO belongs to the Atens NEO group. Atens are defined as Earth-crossing near Earth asteroids that have a semi-major axes smaller than Earth's which was computed earlier to be around 0.6976 $[AU]$

Question 11

Using the same link provided in question 10, potentially hazardous asteroids are near Earth asteroids whose minimum orbit intersection distance with Earth is 0.05 au or less. Using STK a vector from the center of the NEO to the center of Earth was produced into the 3D graphics window and the animation for the first orbit of the NEO was run. Though the magnitude of the vector, which represented the minimum distance between each object, did not dip below 0.6 $[Au]$ which is well above the maximum distance for a potentially hazardous asteroid. Although a close approach did not occur, below is an image taken from the 3D graphics window on 9th Decemeber 2017 at time 18:00:00.000 which is an approximate estimate on the date of the closest distance between the two bodies. Shown is the vector from the center of the NEO body to the center of the Earth body with magnitude 0.63 $[Au]$.

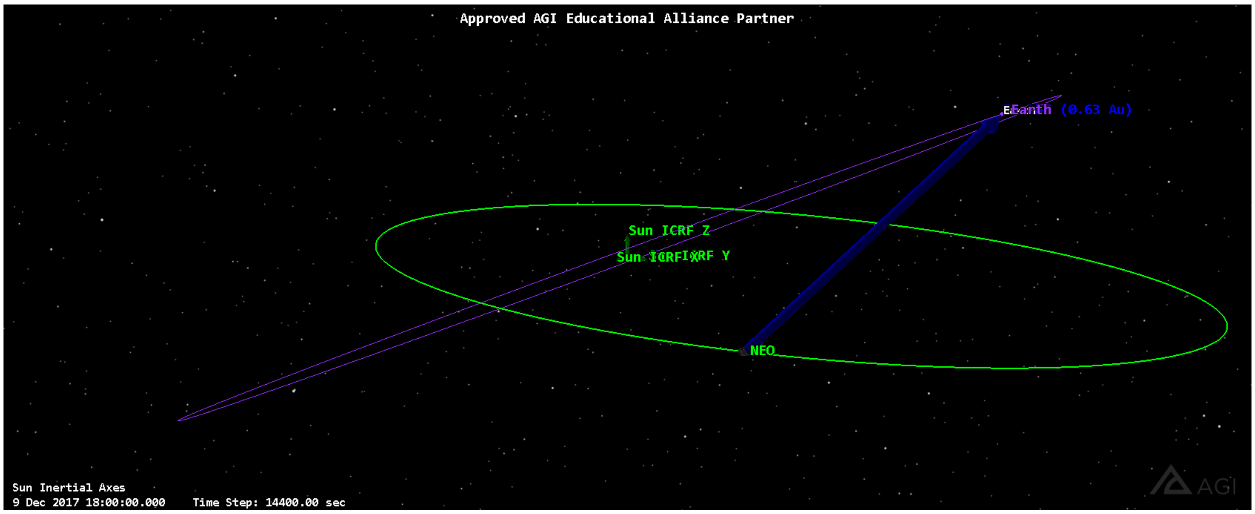


Fig. 9 NEO and Earth orbit with Distance vector

Conclusion and Recommendations

This lab further developed our understanding of orbital mechanics by analyzing elements outside the orbital plane (inclination, right ascension of the ascending node, and argument of periapsis). Also, for orbits about certain objects, like the Sun, some considerations need to made. For example, it is important to determine the possibility/probability of orbit intersections between an asteroid and a planet/other body (such as with Earth). In addition, rather than starting from the Keplerian orbital elements to model satellite orbits, position and velocity information was converted to the orbital parameters using the equations of motion derived in lecture. This is more accurate to a real world scenario, where position and velocity data can be computed from a ground station, which then needs to be analyzed to obtain the orbital parameters. These scenarios better represent the real world behaviors of satellites/asteroids in motion. Thus, this lab has given groups the toolkit to better model/understand object orbits and predict their locations over time.

Beyond the given objectives, it would be interesting to see how the model is affected by adding thrust for changing maneuvers (Martian satellite) as well as changing the mass of the NEO (Sun orbiting object). The instructions were understood to give a basic understanding of the behavior of orbiting objects, but it would be a fun experiment (possibly worth extra credit) if groups could tweak a couple of these conditions to reorient the satellite into a new desired position, such as an initially equatorial orbit to a polar orbit.

Acknowledgements

Thanks to Professor Davis and the TA's for their assistance on this lab!

Team Member Participation Table

Name	Plan	Model	Experiment	Results	Report	Code	Initials
Anand Trehan	1	2	1	1	1	1	AT
Selmo Almeida	1	1	1	1	2	1	SA
Shawn Stone	1	1	2	2	1	1	SS
Connor O'Reilly	2	1	1	1	1	2	CO

2 - Lead
1 - Contributor

References

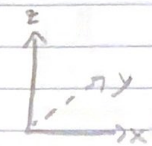
Smead Department of Aerospace Engineering. "ASEN 3200 LAB O-1" Oct. 2020. University of Colorado at Boulder. Lab Document and Supplemental Resources.

Curtis, H., *Orbital Mechanics for Engineering Students*, 3rd ed., Elsevier, 2014.
Paul Chodas, Shakeh Khudikyan, Alan Chamberlin, `https://cneos.jpl.nasa.gov/about/neo_groups.html`

Appendix A: Derivations

LAB 2

Question 1: $\mu = 42828 \ km^3/s^2$



- $\vec{r} = -3424.7\hat{x} - 47.5\hat{y} + 1172\hat{z}) \ km$
- $|\vec{v} \cdot x| = .425 \ km/s$
- $|\vec{v} \cdot y| = 3.33 \ km/s$; $\vec{v} \cdot y = -3.33 \ km/s$
- $\vec{h} = 3,948.694\hat{x} - 3,556.357\hat{y} + 11,394.338\hat{z} \ km^2/s$

$h = r \times v$, setting $\vec{V} \cdot z$ as a variable "S" in matlab and doing the cross product:

$$h = \left[3.9063 \times 10^3 - \frac{955}{2}, \frac{342475}{10} + 498.1, 1.1435 \times 10^4 \right]$$

$$3.9063 \times 10^3 - \frac{95}{2} \cdot S = 3,948.694$$

$$S = \boxed{-.8925 = \vec{V} \cdot z} \ Km/s$$

$$\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \frac{\vec{r}}{r}$$

$$= \frac{[-.425, -3.33, -.8925] \times [3948.694, -3,556.357, 11,394.338]}{\mu} - \frac{[-3424.7, -47.5, 1172]}{\| -3424.7, -47.5, 1172 \|}$$

$$\boxed{e = .0498} = [-.0144, .0439, .0187]$$

★ V_x must be negative, as $V_x \times V_y = V_z$, and since V_y is negative, V_x must also be negative to produce z as positive upward. Also, in order to find the given $\vec{h}$ by crossing $r \times v$, r_x must be negative

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

$$= \frac{\|.425, -3.333, -.8925\|^2}{2} - \frac{42828}{\|-3424.7, -47.5, 1172\|}$$

$$\boxed{\varepsilon = -5.7934 \ kJ/kg}$$

orbital elements
$a = 3696.275$ km
$e = .0498$
$i = 25.0035°$
$\Omega = 47.9925°$
$w = 62.4913°$
$\theta = 67.5151°$
$P = 6.8259 \times 10^3$ s $= 1.8961$ hrs

$$a = \frac{-\mu}{2\varepsilon} = \frac{-42828}{2(-5.7934)}$$

$$\boxed{a = 3696.275 \text{ km}}$$

$$i = \cos^{-1}\left(\frac{h_z}{h}\right)$$

$$= \cos^{-1}\left(\frac{11344}{\|h\|}\right) = \boxed{25.0035° = i}$$

$$\vec{n} = \hat{z} \times \hat{h}$$

$$= [0, 0, 1] \times [h] = [3.5560, 3.9487, 0] = \vec{n}$$

$$\Omega = \cos^{-1}\left(\frac{n_x}{n}\right)$$

$$= \cos^{-1}\left(3.5560/\|n\|\right) = \boxed{47.9925° = \Omega} \qquad n_y > 0; \ \Omega = \Omega$$

$$w = \cos^{-1}\left(\frac{\vec{n} \cdot \vec{e}}{ne}\right)$$

$$\boxed{w = 62.4913°} \qquad e_z = .0187, > 0$$

$$\theta = \cos^{-1}\left(\frac{\vec{e} \cdot \vec{r}}{er}\right) \qquad \vec{r} \cdot \vec{v} = 567.8050, > 0$$

$$\boxed{\theta = 67.5151°}$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} = \boxed{6.8259 \times 10^3 \text{ s} = 1.8961 \text{ hours}}$$