UNIVERSITY OF COLORADO - BOULDER

ASEN 3200 - ORBITAL MECHANICS / ATTITUDE CONTROL

LABORATORY O-3

Sun-Synchronous Orbits and Orbit Transfers

Authors:
Shawn Stone [a]
Selmo Almeida [b]
Anand Trehan [c]
Connor O'Reilly [d]

[a] 109008271
[b] 103088483
[c] 108897226
[d] 107054811

Professor:
DR. KATE DAVIS

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Ann and H.J. Smead
Aerospace Engineering Sciences
UNIVERSITY OF COLORADO BOULDER

This lab first develops a simulation for a sun-synchronous orbit for a satellite orbiting Earth. Then, it calculates the various ΔVs required for a satellite to transfer from the Earth to Mars via either an Hohmann or Bi-elliptic transfer method. These simulations were performed and rendered in STK. The total ΔV required for the Hohmann transfer was found to be 5.5914 km/s, and its time of flight was estimated to be 258.8233 days. By contrast, the Bi-Elliptic Δv required was found to be 11.42 km/s and its associated time of flight was found to be 1020 days. Thus, it was determined that the bi-elliptic method was less efficient in terms of propellant expenditure and the duration of the transfer.

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Introduction

The goal for this lab was to first simulate a sun-synchronous orbit for a satellite about Earth, and then design a simulation for a satellites transfer from Earth to Mars about the Sun. Again, calculations for the semi-major axis (a), eccentricity (e), inclination (i), ascending node (Ω), argument of periapsis (ω), and the true anomaly (θ) were used in order to find angular rates and required ΔVs. This also required an understanding of Hohmann Transfers, and the application of changing velocity at certain positions (periapsis and apoapsis) in order to enter and exit transfer orbits resulting in the desired final orbit.

I. Sun Synchronous Orbits

Question 1

Mission Requirements:

- *Req.1: An orbital period of 1.62 hours.*
- *Req. 2: A perigee altitude between* 500 km *and* 600 km.
- *Req. 3: A perigee that regresses over time.*
- *Req. 4: After one orbital period, the ascending node should possess a longitude of zero to within a tolerance of 0.1 degrees (i.e. its location on the ground track should lie on the Greenwich meridian).*
- *Req. 5: An initial argument of perigee equal to 0 degrees.*

Given requirements 1-5, a sun synchronous orbit was designed as follows. Based on the given orbital period and gravitational constant of the Earth, the semi-major axis was fully determined using equation 1 below and found to be 7002.79 [km].

$$a = \left(\mu\left(\frac{\mathbb{P}}{2\pi}\right)^2\right)^{1/3} \tag{1}$$

Then, using the semi-major axis and perigee altitude constraint, an eccentricity of 0.01068 was found using equation 2 below.

$$e = \left(1 - \frac{r_p}{a}\right) \tag{2}$$

Substituting the known values of J2, R, e, a, and $\dot{\Omega}$ (from the rotation rate of the Earth around the Sun: 360deg/year), the inclination of the orbit was solved using equation 3 below and found to be 97.889 [deg].

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{\left(1-e^2\right)^2 a^{7/2}}\right]cos(i) \tag{3}$$

Lastly, based on requirement 4, a longitude of the ascending node - which constrains the satellite in the same way as setting a RAAN - was found using the period, $\dot{\Omega}$, and rotation rate of Earth about its own axis (roughly 15.04 deg per hour), This was found to be 24.2982 [deg] using equation 4 below.

$$\Delta\lambda = \left(\dot{\omega}_{Earth} - \dot{\Omega}\right)\mathbb{P} \tag{4}$$

The only remaining orbital elements were true anomaly and argument of perigee, the latter of which was declared to be zero from requirement 5 and the former of which was simply set as zero arbitrarily.

Question 2

Figure 1 below is a screenshot of the "Initial State" properties tab from STK, which verifies the orbital elements calculated in question 1.

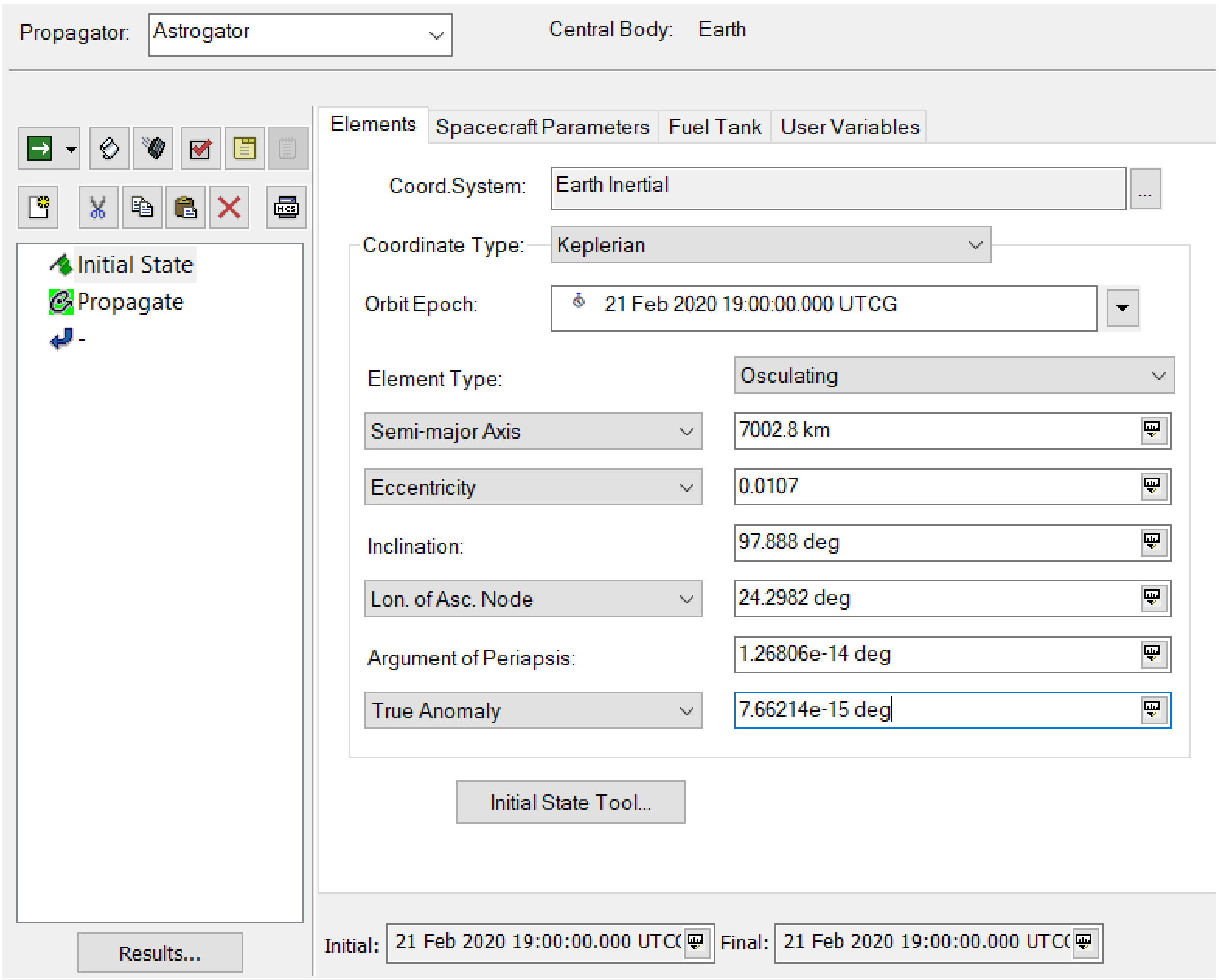


Fig. 1 Initial State Screenshot

After propagating the initial state for one orbital period and adding Earth's North Pole vector to the 3D graphics, the orbit can be seen below in figure 2.

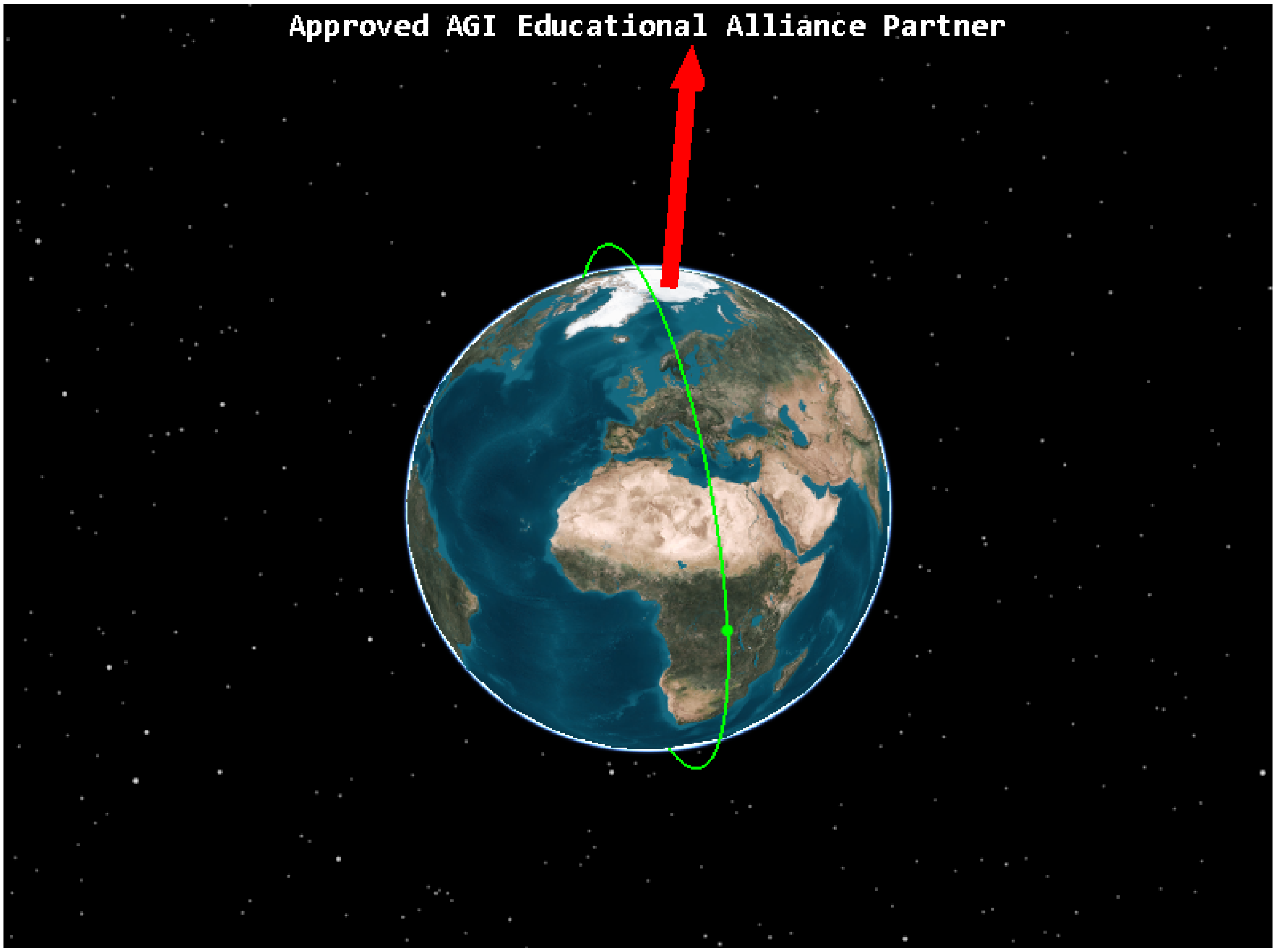


Fig. 2 3D orbit

Question 3

Figure 3 below shows the groundtrack of the satellite for one orbit. As can be seen, the longitude of the ascending node seems to satisfy requirement 4 and posses a longitude of zero at the end of the orbit (lying on the prime meridian).

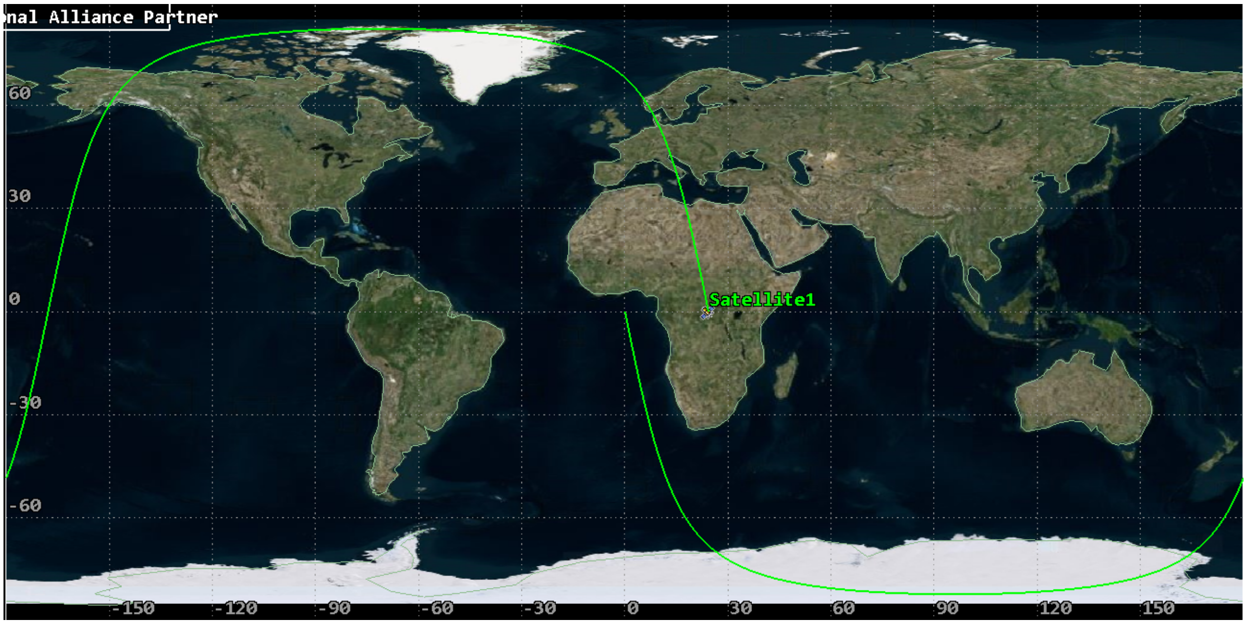


Fig. 3 Groundtrack of Sattelite Orbit

Question 4

In order to verify the mission requirements (aside from visual inspection of the groundtrack and 3D orbit), the summary feature of the Astrogator propagator was used to compare the orbital and state characteristics to what is expected. Shown below (figure 4 and 5) are two screenshots of the summary text that show the perigee altitude between 6878 - 6978 [km], period of roughly 1.62 [hr], and a longitude within 0.1 deg of zero after the propagation of one orbit. Therefore satisfying mission requirements 1, 2, and 4.

```
Geocentric Parameters:
  Latitude:          0.266718840967164 deg
 Longitude:         -0.041581224647396 deg
```

Fig. 4 Longitude Verification

```
Ecc. Anom:        0.4583571848470591 deg       Mean Anom:        0.4534528622769051 deg
Long Peri:         100.1015586534094 deg        Arg. Lat:        0.2894524258643673 deg
True Long:          100.564846680869 deg        Vert FPA:         89.99509539667515 deg
 Ang. Mom:         52829.91036619279 km^2/sec          p:     7001.9978372250852772 km
       C3:        -56.92015562918343 km^2/sec^2   Energy:        -28.46007781459171 km^2/sec^2
  Vel. RA:         8.202872468509653 deg       Vel. Decl:         82.10692454544846 deg
Rad. Peri:     6927.8703349742809223 km        Vel. Peri:         7.625707152671340 km/sec
 Rad. Apo:     7077.7288097910331999 km         Vel. Apo:          7.464246199022221 km/sec
Mean Mot.:       0.06172825148041949 deg/sec
   Period:         5832.013565363888 sec          Period:         97.20022608939813 min
   Period:         1.620003768156635 hr           Period:       0.06750015700652648 day
```

Fig. 5 Orbital Element Verification

Question 5

Shown below in figure 6 is the same orbit propagated over 100 orbital periods in order to visualize the effect of J2 perturbation.

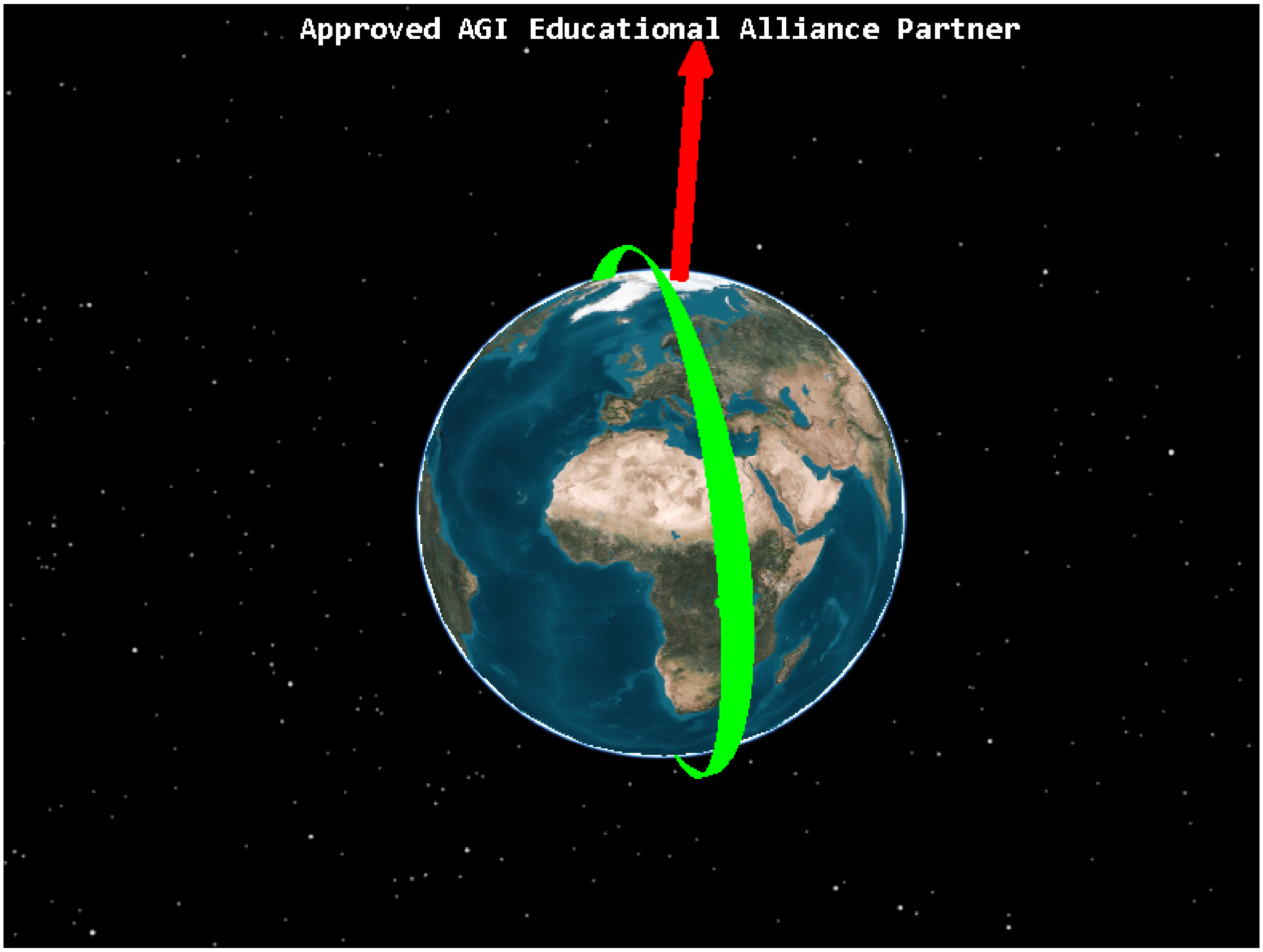


Fig. 6 100 Orbital Periods

Question 6

To more accurately observe the effect of J2 on the orbital elements, graphs of the semimajor axis, right ascension of the ascending node, and argument of periapsis were plotted over time, shown below in figures 7 through 9.

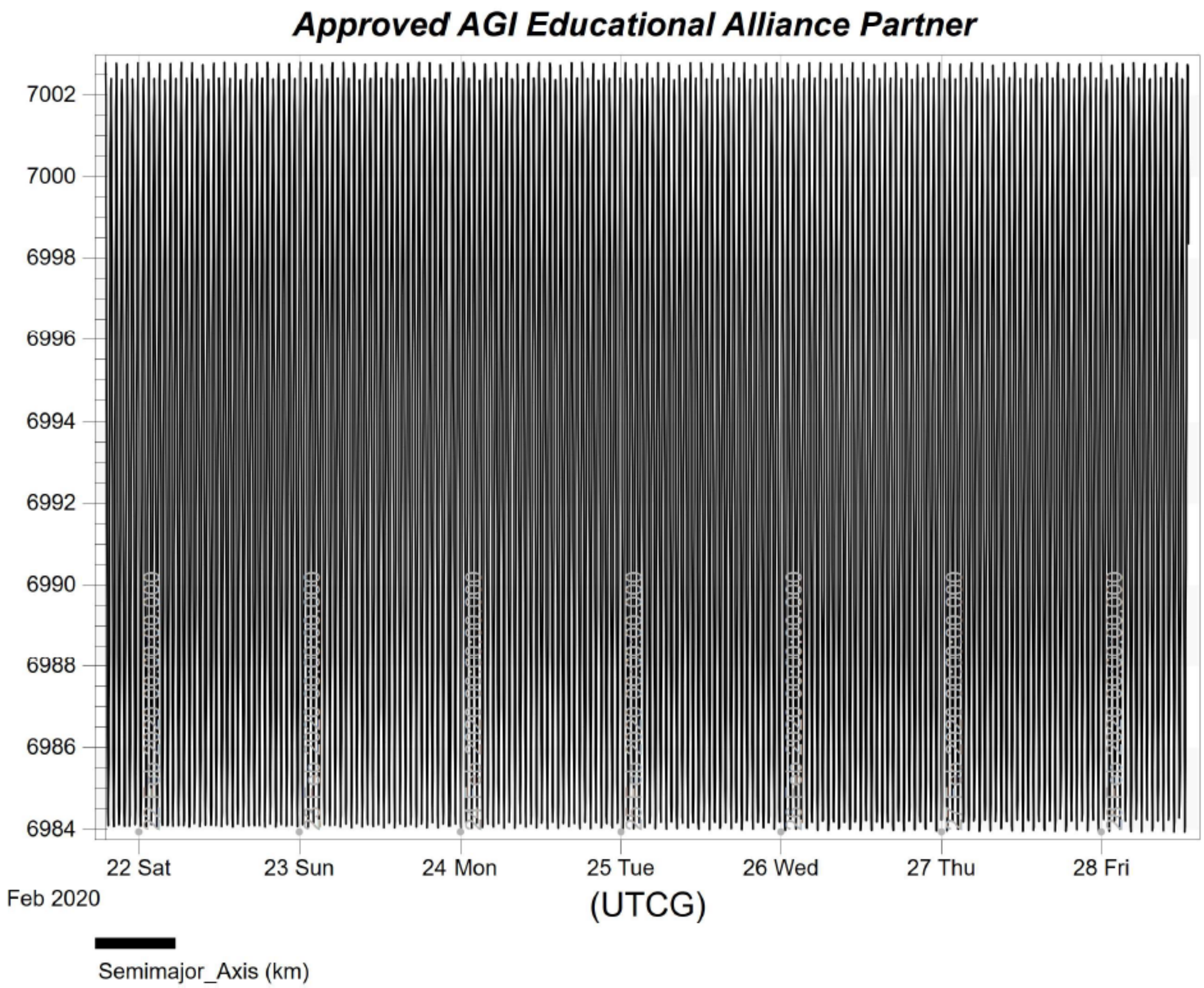


Fig. 7 Semimajor Axis over 100 Orbits

While the semimajor axis does appear to oscillate rapidly by approximately 18 km, there is no long term degredation or change in the semimajor axis due to the effect of J2. This is expected, as only the right ascension of the ascending node (RAAN) and argument of periapsis are expected to change due to the effect of J2 according to lecture information and Curtis' textbook.

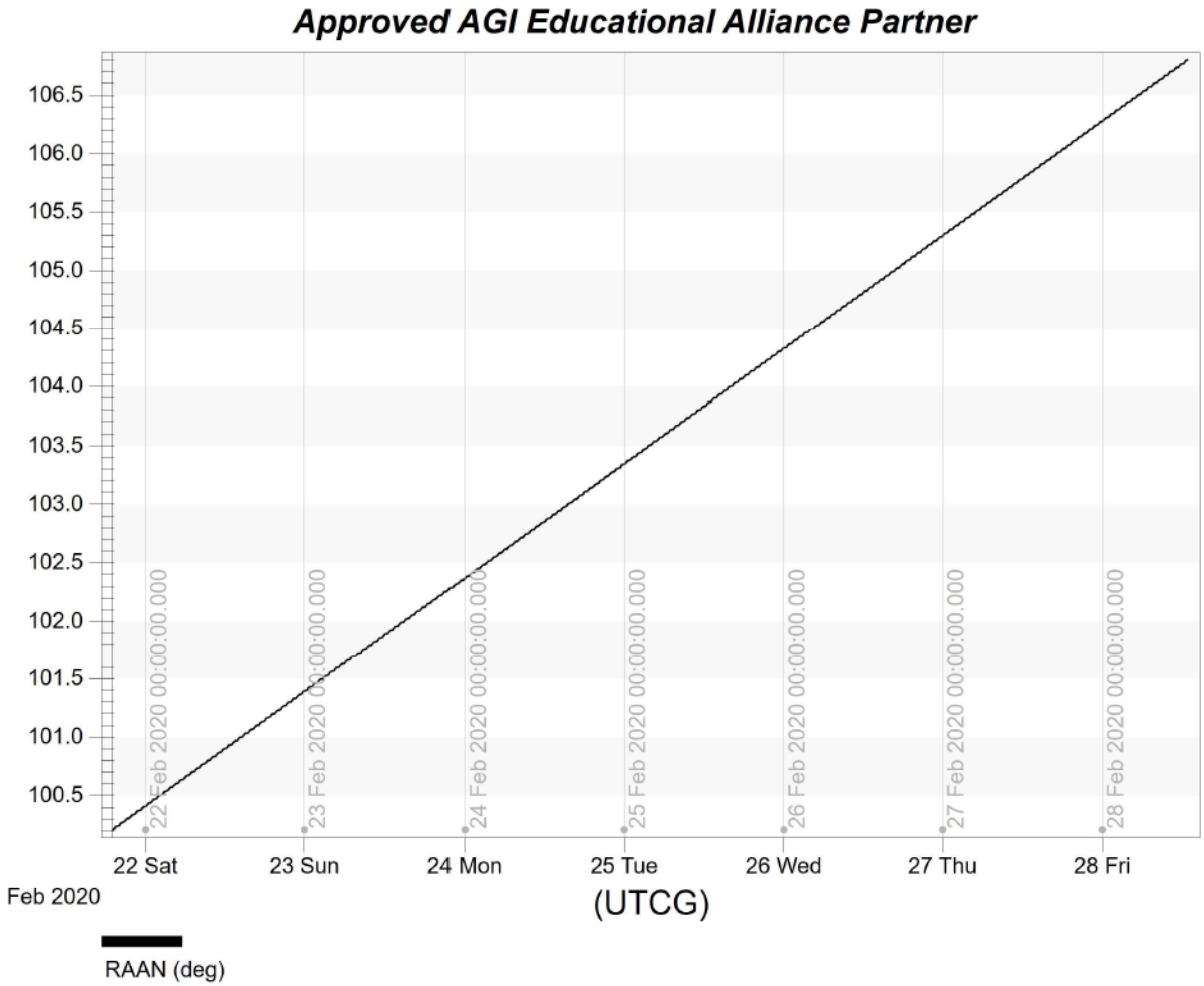


Fig. 8 RAAN over 100 Orbits

Retrograde orbits, defined as orbits with an inclination between 90 and 180 deg, see an eastward shift in their RAAN's due to the effect of J2. As expected, because of the 97.889 deg inclination, the RAAN of this satellite's orbit increases from approximately 100 deg to over 107 deg, corresponding to an eastward shift across the ground, seen above in figure 8.

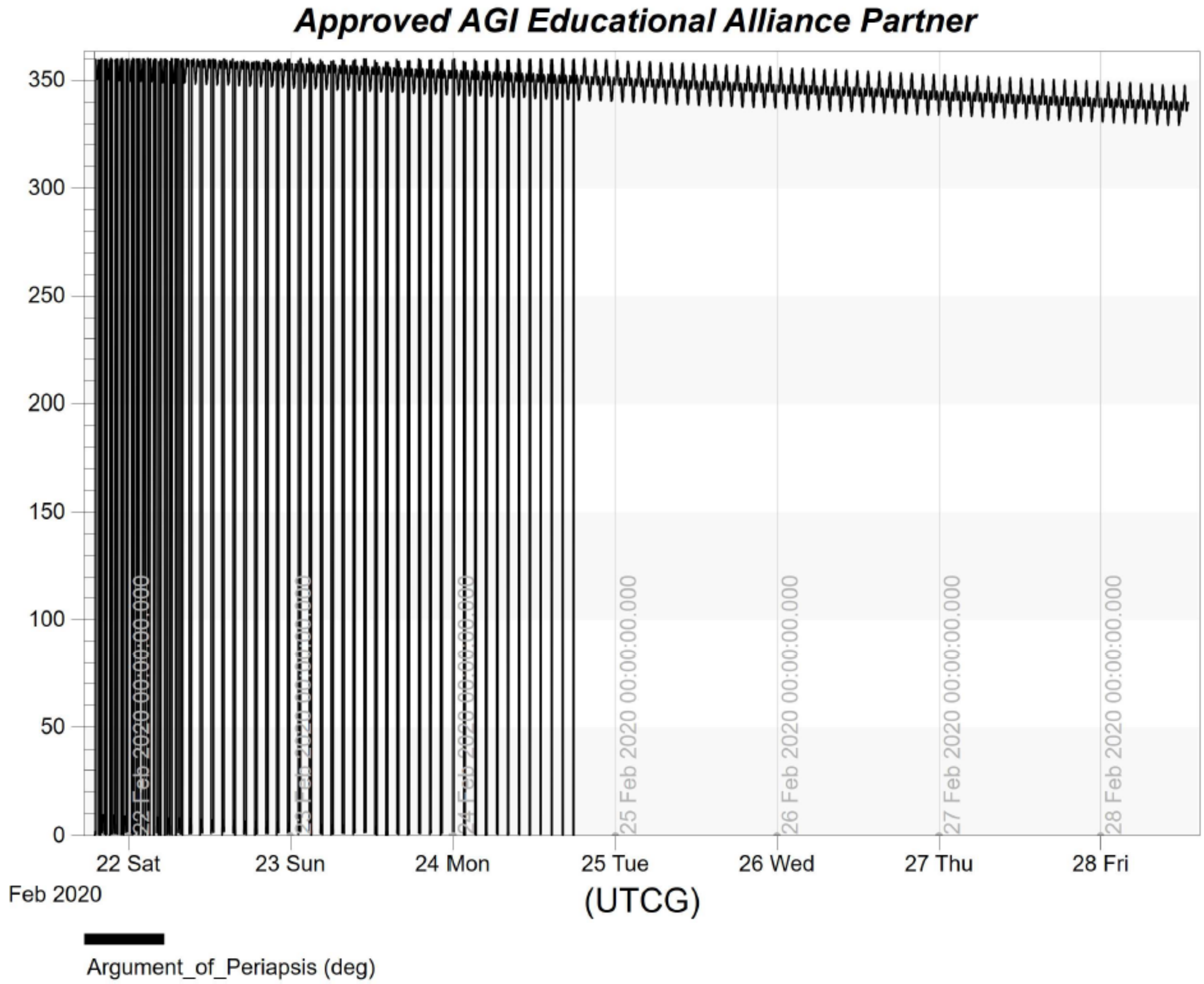


Fig. 9 Argument of Periapsis over 100 Orbits

Similar to the change in RAAN, the argument of periapsis sees a change over time due to the efffect of J2 and the inclination of its orbit. Because the orbit has an inclination between 63.4 and 116.6 deg, the argument of periapsis regresses over time, which is seen above in figure 9.

II. Bound for Mars

Question 7

Next, an Hohmann transfer to Mars from Earth was calculated. The following relationship was used to find the speed of each planet:

$$V_{circ} = \sqrt{\frac{\mu_{sun}}{a_{planet}}} \tag{5}$$

In addition, the following equation was utilized in order to find the velocity required to enter the transfer orbit, and the velocity the spacecraft would have when it reached Mars:

$$V_{transfer} = \sqrt{\frac{2 * \mu_{sun}}{a_{planet}} - \frac{\mu_{sun}}{a_{transfer}}} \tag{6}$$

The initial velocity (the velocity of Earth about the sun) was found to be 29.7846 km/s. The velocity required to enter the transfer orbit was found to be 32.7281 km/s. The following expression found the total ΔV_1 required:

$$\Delta V_1 = |V_{circ_e} - V_{transfer_{earth}}| = 2.9435 km/s \tag{7}$$

As the speed of Earth was found to be lower than that of the velocity required to enter the transfer orbit, this change velocity is a positive one. Next, it was necessary to calculate the delta V required to match the orbit of Mars. A similar process was followed, with Mars' velocity found as 24.1316 km/s and the velocity in transfer orbit at Mars being 21.4836 km/s. Thus:

$$\Delta V_2 = |V_{circ_m} - V_{transfer_{mars}}| = 2.6479 km/s \tag{8}$$

In this case, the spacecraft is traveling in the transfer orbit at a slower speed than Mars' when it arrives there, so it is again required to increase the velocity of the spacecraft. Adding the two calculated delta Vs provides the following:

$$\Delta V_{total} = 2.9435 + 2.6479 = 5.5914 km/s \tag{9}$$

Some assumptions in these calculations were that the orbits are co-planar, that the planets orbits are circular, instantaneous impulsive manuever, and that the only perturbating body is the sun. Given that the ΔV required to get to the moon from eath is roughly 6.1 km/s, this total ΔV is relatively small. The following diagram illustrates the transfer, along with the ΔVs:

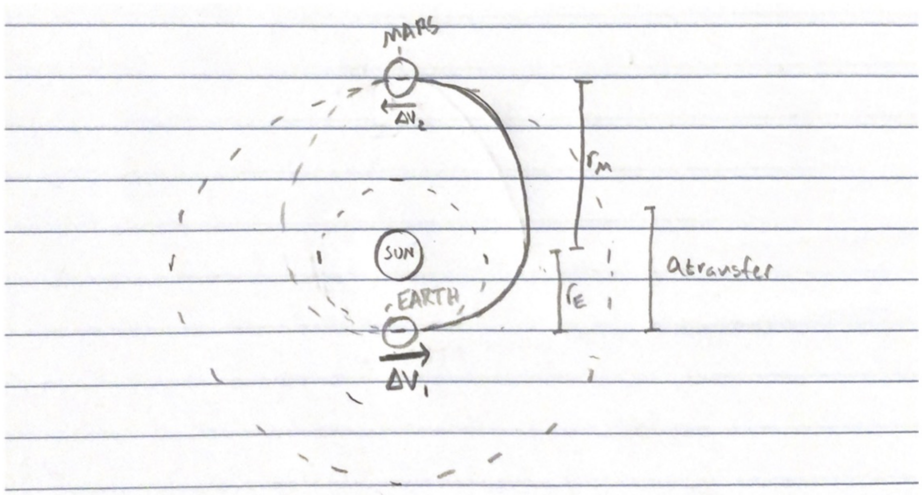


Fig. 10 Hohmann Transfer

The calculations for this can be found in the MATLAB code in the Appendix. The following expression calculated the TOF for the orbit:

$$TOF = \pi\sqrt{\frac{a_{transfer}^3}{\mu}} \tag{10}$$

This was found to be $2.2362 * 10^7$ seconds or 258.8233 days.

Question 8

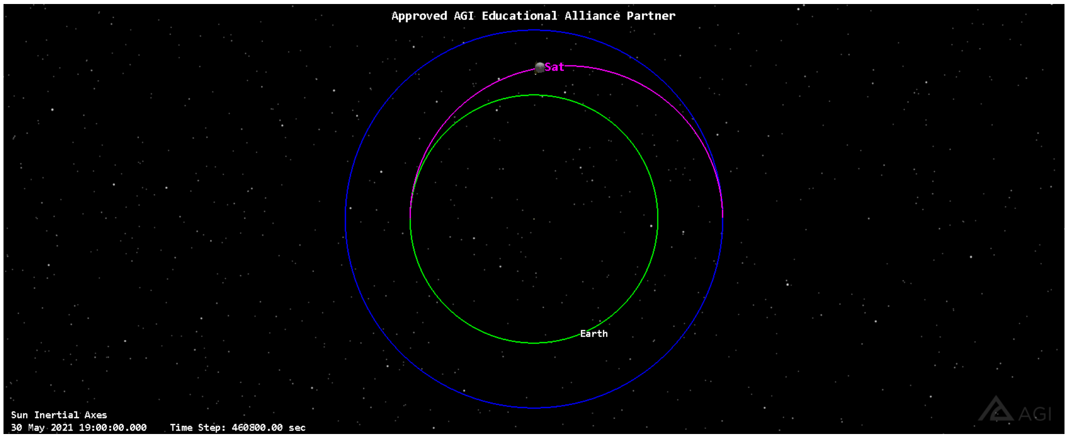


Fig. 11 heliocentric transfer looking down the angular momentum vector of elliptical transfer orbit

Question 9

```
Other Hyperbolic Orbit Parameters :
 Ecc. Anom:        -0.7074920689996679 deg        Mean Anom:         -1942076.298904553 deg
 Long Peri:           181.0248248366287 deg        Arg. Lat:            180.3171069706139 deg
 True Long:           180.3173504883243 deg        Vert FPA:            90.70747409057401 deg
  Ang. Mom:           20334955127.70909 km^2/sec           p:     1.0374057752917525e+15 km
        C3:           2895.054756371031 km^2/sec^2   Energy:            1447.527378185516 km^2/sec^2
    Vel. RA:          271.1169374866884 deg        Vel. Decl:          -23.43353294295343 deg
 Rad. Peri:     3.7793287306336558e+08 km          Vel. Peri:         53.8057326500403619 km/sec
  Rad. Apo:    -3.7793314842979217e+08 km         Excess Vel:            53.80571304583772 km/sec
               Time Past Periapsis:           -86735.37444633908 sec
         Time Past Descending Node:            38883.88141132772 sec
     Beta Angle (Orbit plane to Sun):        0.000156858562778982 deg
Mean Sidereal Greenwich Hour Angle:             236.545719049583 deg
```

Fig. 12 Heliocentric transfer looking down the angular momentum vector of elliptical transfer orbit

The highlighted orbital parameters above are the radius of Periapsis and Apoapsis at the end of the final propagate segment. Both values are equal and opposite of each other which verifies the the final orbit is circular with a radius around 3.779 $[km]$. Although the final orbit is circular, the simulated orbit greater than the desired radius $(1.5234[AU])$ by 1.003 $[AU]$. Assuming that both Earth's and Mars' orbits are circular and coplanar is not a reasonable assumption. Its known that both orbits are not actually perfectly circular and the Mars orbit is actually inclined around 1.85 degrees to that of Earth's. These two assumptions may not only cause for the large difference between the desired and the simulated orbital elements other factors could include the gravitational fields of Earth and Mars, solar radiation and the assumption that all impulsive manuevers are instantaneous.

Question 10

A bi-elliptic transfer to design a heliocentric trajectory for a spacecraft from Earth to Mars was first determined by assuming that the intermediate radius of the bi-elliptic transfer was 2.7 AU (approximately 4.0391×10^8 km). The basic idea of the Bi-elliptic transfer is Δv_1 t raise apoapsis to r_B, where r_1 is equal to $r_{p,t1}$ and r_B greater than r_2. Furthermore, there must be a Δv_2 to raise periapsis of the transfer orbit to that of the final orbit, where $r_B = r_{a,t1} = r_{a,t2}$, and a Δv_3 to allow for an injection into the final orbit where $r_2 = r_{p,t2}$. Also, due to the circular orbit assumption, it can be stated that $r_1 = a_1$ and $r_2 = a_2$.

To begin, the first transfer arc must connect to orbit 1 at periapsis. This has the desired apoapsis radius, r_B, and from the geometry of the orbit we know:

- $r_{p,t1} = a_1$

- $r_{a,t1} = r_B$
- $a_{t,1} = \frac{1}{2}(r_{p,t1} + r_{a,t1})$

Also, there must be similar relationship for the second transfer arc that must connect to the first transfer arc at apoapsis. In this state, the periapsis radius is equal to that of orbit 2. The geometric quantities in this state are as follows:

- $r_{p,t2} = a_2$
- $r_{a,t2} = r_B$
- $a_{t,2} = \frac{1}{2}(r_{p,t2} + r_{a,t2})$

Given the above equations, the quantities produced can be seen in the table below.

Orbit	$r_{p,t\#}$	$r_{a,t\#}$	$a_{t\#}$
1	1.495E8	4.039E8	2.768E8
2	2.279E8	4.0391E8	3.159E8

Table 1 Bi-elliptic orbital quantities (km).

With the values above, 6 key velocities can be calculated to determine the total Δv as well as the time of flight (TOF) for this transfer. Of the 6 velocities, 2 are associated with velocity at both the first and second circular orbits, and the four remaining pertain to the velocities within both the first and second transfer arcs. The equations below were used to find $\Delta v_1, \Delta v_2$ and Δv_3. For the circular orbits, the general equation

$$v_{\#} = \sqrt{\frac{\mu}{r_{\#}}}\hat{\theta} \tag{11}$$

was used and gave the results:

$\vec{v}_1$	$\vec{v}_2$
29.7846	24.1314

Table 2 Circular orbit velocities (km/s).

Furthermore, the equations for the first and second transfer arcs were:

$$v_{p,t\#} = \sqrt{\frac{2\mu}{r_{p,t\#}} - \frac{\mu}{a_{t\#}}}\hat{\theta} \tag{12}$$

$$v_{a,t\#} = \sqrt{\frac{2\mu}{r_{a,t\#}} - \frac{\mu}{a_{t\#}}}\hat{\theta} \tag{13}$$

Using these equations, the followed values were determined:

$\vec{v}_{p,t1}$	$\vec{v}_{a,t1}$	$\vec{v}_{p,t2}$	$\vec{v}_{a,t2}$
35.98	13.33	27.29	15.40

Table 3 Transfer arc velocities (km/s).

Given the quantities in tables 2 and 3 Δv_1, Δv_2, and Δv_3 can be calculated with the equations below.

$$\Delta v_1 = v_{p,t1} - v_1 \tag{14}$$

$$\Delta v_2 = v_{a,t2} - v_{a,t1} \tag{15}$$

$$\Delta v_3 = v_2 - v_{p,t2} \tag{16}$$

From the equations above, the total Δv can be expressed as the sum of the absolute values of these three terms. Thus, the final values for each individual Δvs and Δv_{total} are listed in the table below.

Δv_1	Δv_2	Δv_3	Δv_{total}
6.20	2.07	-3.16	11.42

Table 4 Transfer arc velocities (km/s).

Considering the time of flight (TOF), geometry shows that for this transfer the sum of half the orbital period of the elliptical orbits for both transfer arcs is all that is necessary. Note that the limiting case is when r_B approaches infinity, producing a bi-parabolic transfer. The equation below found that the **TOF for this transfer is 1020 days**. It is important to point out that all but one of the transfer orbit velocities were found speed up the spacecraft. This is to be expected due to the spacecraft's final desired position to be in a smaller orbit. Thus, when it is close to reaching this position it must slow down.

$$TOF = \frac{P_{t1}}{2} + \frac{P_{t2}}{2} = \pi\sqrt{\frac{a_{t1}^3}{\mu}} + \pi\sqrt{\frac{a_{t2}^3}{\mu}} \tag{17}$$

Question 11

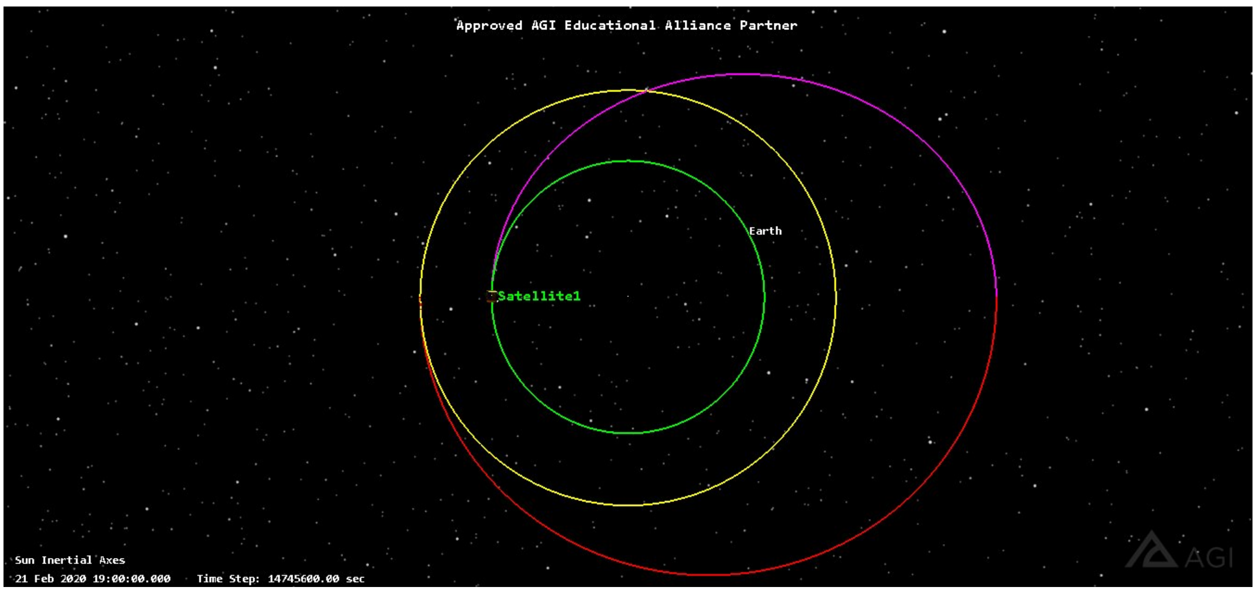


Fig. 13 Hohmann Transfer.

As it can be seen above, implementing the additional Δv from the initial Hohmann transfer was successful. Here, the green and yellow circular orbits represents that of Earth and Mars, respectively. The purple line represents the transfer orbit from earth to apoapsis, followed by the red line that brings the satellite back to its desired position of Mars. This picture is looking down (normal) to the orbit plane.

Question 12

```
 Ecc. Anom:          19.46987006577436 deg       Mean Anom:          14013507.97
731508 deg
 Long Peri:           304.084958000016 deg        Arg. Lat:          323.1914242
734576 deg
 True Long:          323.1906139625504 deg        Vert FPA:          70.89437059
726301 deg
  Ang. Mom:          7954757021.985785 km^2/sec          p:     1.58750850954168
22e+14 km
        C3:          1251.808729012842 km^2/sec^2   Energy:           625.904364
506421 km^2/sec^2
   Vel. RA:          31.83384416414948 deg       Vel. Decl:          12.87952726
216375 deg
 Rad. Peri:     2.2483158170913655e+08 km        Vel. Peri:        35.3809592118
460188 km/sec
  Rad. Apo:    -2.2483221854834729e+08 km       Excess Vel:          35.38090910
382098 km/sec
```

Fig. 14 Mars' Radius.

```
 Ecc. Anom:          68.98042201354876 deg       Mean Anom:          3960100.722
302204 deg
 Long Peri:          266.0414212900847 deg        Arg. Lat:          322.6553111
161357 deg
 True Long:          322.6428053211579 deg        Vert FPA:          33.39966553
843955 deg
  Ang. Mom:          1128900940.816287 km^2/sec          p:     3.19723011188861
67e+12 km
        C3:          258.9433453273803 km^2/sec^2   Energy:          129.4716726
636902 km^2/sec^2
   Vel. RA:          356.3671795723513 deg       Vel. Decl:         -1.568376804
677588 deg
 Rad. Peri:     7.0152625343440309e+07 km        Vel. Peri:        16.0920697591
803759 km/sec
  Rad. Apo:    -7.0155704012567684e+07 km       Excess Vel:          16.09171666
813023 km/sec
```

Fig. 15 Final Transfer Radius.

Looking to the above highlighted values, it can be immediately seen that the Mars' orbit radius determined by STK12 and that of the orbit given in the lab document (2.289×10^8) are very close to the same. This discrepancy is most likely due to rounding issues in our calculations, being that we did not include any outside forces/elements to interact with our spacecraft other than the Sun. Therefore, this was strictly a two-body problem.

As for the bi-elliptic transfer orbit from apoapsis to periapsis, it did appear that there may be the possibility of the spacecraft interacting with Jupiter (the planet following Mars). Depending on how we interpret Jupiter's orbit in circular form, the spacecraft may potentially collide with this massive planet. This is based on Jupiter's proximity to the Sun being 7.79×10^8 km [*] from the Sun. Thus, this question may need more detail as too what resides between Jupiter and Mars such as the proximity of Jupiter's moons in relation to the Sun, as well as how we want to model Jupiter into this system (Jupiter circular orbit approximation).

[*] https://www.universetoday.com/44615/distance-from-the-sun-to-jupiter/: :text=The%20distance%20from%20the%20Sun%20to%20Jupiter%20is, Jupiter%20and%20the%20rest%20of%20the%20Solar%20

Question 13

Looking at the results provided in questions 7 and 10, it can be immediately seen that the TOF of the Hohmann transfer is significantly less than that of the Bi-elliptic transfer. By design, the Hohmann transfer is one of, if not the most efficient means of transferring a spacecraft or something similar to another celestial body. Therefore, the Hohmann transfer's total Δv should be significantly smaller than that of a Bi-elliptic orbit transfer. As the results have shown above, this is proven to be the case. The Bi-elliptic transfer's Δv_{total} is 11.4217 km/s, which is nearly double that of the Hohmann transfer (5.5916 km/s). Thus, the cost of performing a Bi-elliptic orbit transfer comes at a greater cost than the Hohmann transfer.

Question 14

Five different considerations in the mission design process that may impact the range of acceptable transfer times, total ΔV and/or transfer geometry can include:

- Mass of the spacecraft - due to the expending of fuel (i.e. reducing mass), the thrust force acting on the spacecraft over time will vary.
- Interaction between other celestial bodies - due to many spheres of gravitational influence, the spacecraft is likely to be pulled slightly toward objects with large masses, such as other planets and large asteroids.
- Interaction with the Sun - this may be negligible at a far enough position, but solar pressure/drag can reduced the effectiveness of the burn required to reach another celestial body from Earth.
- Telemetry - if the spacecraft is to interact with undesired celestial bodies, it may need to be told by ground communication on Earth to reorient itself into a position that allows it to continue on a path that brings it to a desired orbit.
- Instrumentation (mass), in this case, the mass of the spacecraft may have a negative impact of the effective burn. If the spacecraft has too much mass, the cost of a burn may be larger due to it needed to propel more mass.

Conclusion and Recommendations

In the first part of this lab, STK was utilized to simulate a Sun-synchronous orbit, after finding the rotation rate of the Earth around the Sun. In the second part, the ΔV values required for an Hohmann transfer between Earth and Mars were derived and implemented in STK. Then, the ΔVs for a Bi-Elliptic transfer were found, and this transfer was simulated in STK. The Bi-Elliptic was found to be significantly less efficient, and the TOF was found to be significantly longer by almost a factor of 4.

In regards to a possible augmentation or advancement beyond this lab, our group would be curious to see how these approximations and orbits would look for planets with orbits farther from the sun. These orbits are more elliptical, so more difficult to model and accurately predict, and it would be interesting to see how well the same processes described in this lab would perform in less ideal circumstances.

Acknowledgements

Thanks to Professor Davis and the TA's for their assistance on this lab!

Team Member Participation Table

Name	Plan	Model	Experiment	Results	Report	Code	Initials
Anand Trehan	1	2	1	1	1	1	AT
Selmo Almeida	1	1	1	2 - Part 2	2	1	SA
Shawn Stone	1	1	2	2 - Part 1	1	1	SS
Connor O'Reilly	2	1	1	1	1	2	CO

2 - Lead
1 - Contributor

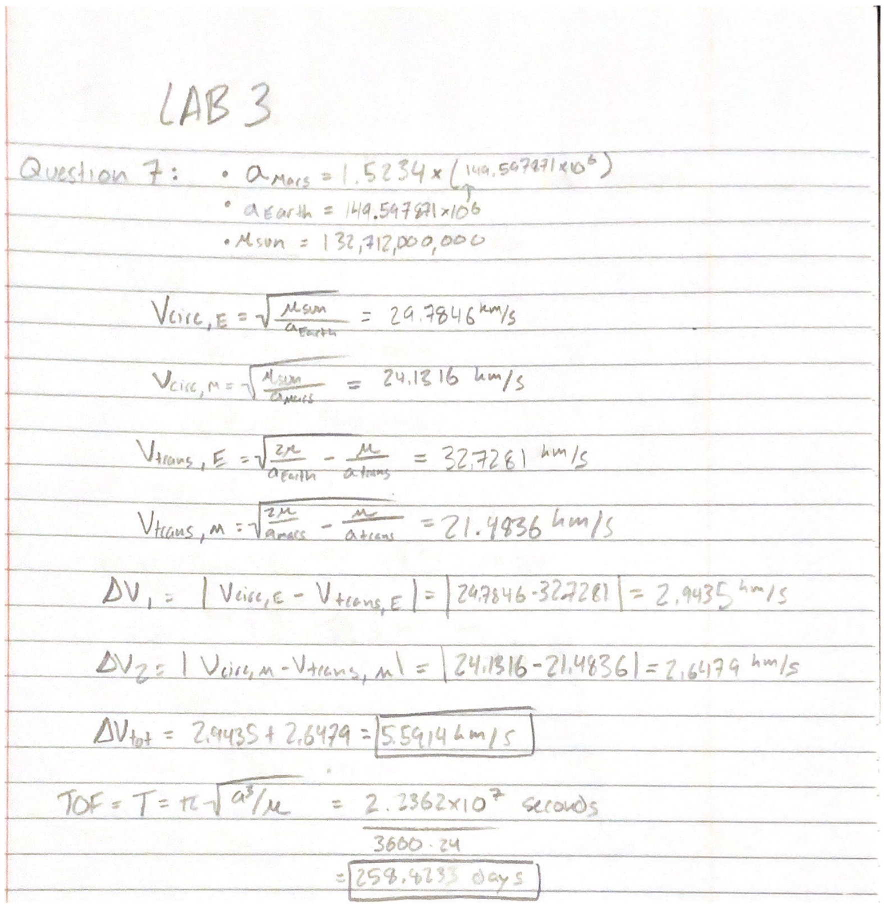
References

Smead Department of Aerospace Engineering. "ASEN 3200 LAB O-3" Oct. 2020. University of Colorado at Boulder. Lab Document and Supplemental Resources.

Curtis, H., *Orbital Mechanics for Engineering Students*, 3rd ed., Elsevier, 2014.

Paul Chodas, Shakeh Khudikyan, Alan Chamberlin, `https://cneos.jpl.nasa.gov/about/neo_groups.html`

Appendix A: Derivations


LAB 3

Question 7: • $a_{Mars} = 1.5234 \times (149.597871 \times 10^6)$

• $a_{Earth} = 149.597871 \times 10^6$

• $\mu_{sun} = 132,712,000,000$

$$V_{circ,E} = \sqrt{\frac{\mu_{sun}}{a_{Earth}}} = 29.7846 \text{ km/s}$$

$$V_{circ,M} = \sqrt{\frac{\mu_{sun}}{a_{Mars}}} = 24.1316 \text{ km/s}$$

$$V_{trans,E} = \sqrt{\frac{2\mu}{a_{Earth}} - \frac{\mu}{a_{trans}}} = 32.7281 \text{ km/s}$$

$$V_{trans,M} = \sqrt{\frac{2\mu}{a_{Mars}} - \frac{\mu}{a_{trans}}} = 21.4836 \text{ km/s}$$

$$\Delta V_1 = |V_{circ,E} - V_{trans,E}| = |29.7846 - 32.7281| = 2.9435 \text{ km/s}$$

$$\Delta V_2 = |V_{circ,M} - V_{trans,M}| = |24.1316 - 21.4836| = 2.6479 \text{ km/s}$$

$$\Delta V_{tot} = 2.9435 + 2.6479 = \boxed{5.5914 \text{ km/s}}$$

$$TOF = T = \pi\sqrt{a^3/\mu} = \frac{2.2362 \times 10^7 \text{ seconds}}{3600 \cdot 24} = \boxed{258.8233 \text{ days}}$$

Fig. 16 Hohmann Transfer Derivations

Appendix B: Matlab Code

Questions 7 10

```
%% ASEN 3200 (Orbits) Lab 3
clear all
close all
clc

%% Question 7
R_E = 149597871;
R_M = 2.279E8;
a_trans = .5*(R_E+R_M);
mu_sun = 132712E6;
v_E = sqrt(mu_sun/R_E);
v_M = sqrt(mu_sun/R_M);
delta_v1 = sqrt((2*mu_sun/R_E)-(mu_sun/a_trans)) - v_E;
delta_v2 = v_M - sqrt((2*mu_sun/R_M)-(mu_sun/a_trans));

delta_v_total = abs(delta_v1) + abs(delta_v2);

TOF = pi*sqrt((a_trans^3)/mu_sun)*(1/3600)*(1/24);

%% Question 10
r_B = 4.0391E8;
r_pt1 = R_E;
r_at1 = r_B;
a_t1 = .5*(r_pt1+r_at1);

r_pt2 = R_M;
r_at2 = r_B;
a_t2 = .5*(r_pt2+r_at2);

% Velocity on CIrcular Orbit 1
q10_v1 =  sqrt(mu_sun/r_pt1);

% Velocities on Transfer Arc 1
q10_v_pt1 = sqrt((2*mu_sun/r_pt1)-(mu_sun/a_t1));
q10_v_at1 = sqrt((2*mu_sun/r_at1)-(mu_sun/a_t1));

% Velocity on CIrcular Orbit 1
q10_v2 =  sqrt(mu_sun/r_pt2);

% Velocities on Transfer Arc 2
q10_v_pt2 = sqrt((2*mu_sun/r_pt2)-(mu_sun/a_t2));
q10_v_at2 = sqrt((2*mu_sun/r_at2)-(mu_sun/a_t2));

% At periapsis along Transfer Arch 1
q10_delta_v1 = q10_v_pt1 - q10_v1;

% At apoapses Along Transfer Arcs
q10_delta_v2 = q10_v_at2 - q10_v_at1;

% At periapsis along Transfer Arch 1
```

```
q10_delta_v3 = q10_v2 - q10_v_pt2;

delta_v_total2 = abs(q10_delta_v1) + abs(q10_delta_v2) + abs(q10_delta_v3);

% Bielliptic TOF
TOF2 = (pi*sqrt((a_t1^3)/mu_sun) + pi*sqrt((a_t2^3)/mu_sun)) *(1/3600)*(1/24)
```