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## Housekeeping

```
clear all; close all; clc

%{
    CSCI HW8 main script
    Author: Connor O'Reilly
    Date: 11/5/2021
    email: coor1752@colorado.edu
%}
```

## Downloading and Storing data

```
mat = readmatrix('C:\Users\corei\OneDrive\Documents\Classes
\CSCI_3656\mat1-2.txt');

%create cell array containing A matrices
%get size of A
size_mat = size(mat);

%initialize size
A_cell = cell(1 , size_mat(2));

%fill cell

for i = 1:size_mat(2)

    A_cell(i) = mat2cell( mat(:, 1 : i), size_mat(1), i );

end
```

## Part 1

```
% obtain size, rank and condition number for all A_k
```

---

```

%could make more effiecient but low on time lol

%initialize
size_ak = zeros(26 , 2);
rank_ak = zeros(26 , 1);
cond_ak = rank_ak;

for i = 40:65
    curr = cell2mat( A_cell(i) );
    size_ak(i-39, :) = size( curr );
    rank_ak(i-39) = rank(curr);
    cond_ak(i-39) = cond(curr);
end

```

## Part 2 Initialization

```

%Generate 100 random vectors for each  $b_i \in \mathbb{R}^m$ 

%initilize
b_cell = cell(26, 100);

%heavy i know
j = 1;
cnt = 1;
max_a = 100 * max(mat , [] , 'all');
min_a = 100 * min(mat , [] , 'all');

for i = 1:2600
    %create random vector and store into array, lets get creative
    bmat = rand( size_mat(1) , 1);
    b_cell(j , cnt) = mat2cell( bmat , size_mat(1) , 1);
    if(cnt == 100)
        j = j + 1;
        cnt = 0;
    end
    cnt = cnt + 1;
end

```

## Part 2 a

```

%Using built-in equation solver, linsolve compute the least-
squares
%minimizer given A_k and b_i

%gonna need another cell array?

%initialize
x_true = cell(26,100);
j = 1;
cnt = 1;
for i = 1:2600
    x_true(j,cnt) = mat2cell(linsolve( A_cell{j+39} , b_cell{ j,
cnt } ), j + 39 , 1 );

```

---

```

        if(cnt == 100)
            j = j + 1;
            cnt = 0;
        end
        cnt = cnt + 1;
    end
end

```

## Part 2 b

```

%using normal equation solver located in function section least
squares
%minimizer is computed using normal equations

% i know too many for loops in this code

%initialize
x_ne = cell(26,100);
err_ki_NE = zeros(26,100);
j = 1;
cnt = 1;
for i = 1:2600
    %least squares minimizer
    x_ne( j , cnt) = mat2cell(method_NormEq_Cholsky( A_cell{j
+39} , b_cell{ j, cnt } ), j + 39 , 1 ) ;

    %relative error
    err_ki_NE(j,cnt) = norm(x_ne{j,cnt} - x_true{j,cnt}) /
norm(x_true{j,cnt});
    if(cnt == 100)
        j = j + 1;
        cnt = 0;
    end
    cnt = cnt + 1;
end
end

```

## Part 2 c

```

%using normal equation solver located in function section least
squares
%minimizer is computed using normal equations

% i know too many for loops in this code

%initialize
x_qr_mine = cell(26,100);
x_qr_matlab = cell(26,100);
err_ki_QR_bad = zeros(26,100);
err_ki_QR_good = err_ki_QR_bad;
j = 1;
cnt = 1;
for i = 1:2600
    x_qr_mine( j , cnt) = mat2cell(method_ThinQR( A_cell{j+39} ,
b_cell{ j, cnt } , 1 ), j + 39, 1 ) ;

```

---

```

        x_qr_matlab( j , cnt) = mat2cell(method_ThinQR( A_cell{j+39} ,
b_cell{ j, cnt }, 0 ), j + 39, 1 ) ;
        %relative error
        err_ki_QR_bad(j,cnt) = norm(x_qr_mine{j,cnt}-x_true{j,cnt}) /
norm(x_true{j,cnt});
        err_ki_QR_good(j,cnt) = norm(x_qr_matlab{j,cnt}-x_true{j,cnt}) /
norm(x_true{j,cnt});
        if(cnt == 100)
            j = j + 1;
            cnt = 0;
        end
        cnt = cnt + 1;
    end
end

```

## Part 3

```

%for each of QR and Normal equations compute the average error over
all the
%bi

%initialize
errk_avg_NE = zeros(24,1);
errk_avg_QR_mine = errk_avg_NE;
errk_avg_QR_matlab = errk_avg_NE;
%excluding k = 64 and 65 due to the matrices not being sym pos def

for i = 1:26
    errk_avg_NE(i) = mean(err_ki_NE(i,:));
    errk_avg_QR_mine(i) = mean(err_ki_QR_bad(i,:));
    errk_avg_QR_matlab(i) = mean(err_ki_QR_good(i,:));
end

```

## Part 4

```

%plotting done in plotting section

```

## Display

```

%Part 1
fprintf('\n-----\n')
fprintf('Problem 1: ')
fprintf('\n-----\n\n')

fprintf('          A_k          |          Size (M x N)          |          Rank          |
          Condition Number [ K ]          \n')
fprintf('-----\n')
for i = 1 : 26

```

```

        fprintf(' A_%i | %i x %i | %i\n', (i+39), size_ak(i,1), size_ak(i,2),
rank_ak(i), cond_ak(i));

fprintf('-----\n')
end

%explanation
fprintf('\n-----\n')
fprintf('Discussion: ')
fprintf('\n-----\n\n')

%1: the relationship between the error using QR versus the normal
equations
fprintf('1:\n Average error increases for both QR and the normal
equations as k increases. Least squares error using normal equations
increases\n more drastically than the error using thin qr even with
the for k = 64 and k = 65 being ignored\n\n');
%2: What is the relationship between the errors and the condition
number of Ak?
fprintf('2:\nas stated before, as the condition number of A_k
increases the least squares error also increases \n\n')
%3: Suppose your matrix A is ill-conditioned. Which method is more
favorable?
fprintf('3:\n looking at the matlab implemmentation of thin QR vs
NE thin QR is more favorable for ill-conditioned matrices.\n as the
condition number increases there is noticable round off error using
cholesky factorization. \nFor this homeowrk, cholesky factorization
could not be used for matrices A_64 and A_65 due to them not being
sym pos definite. ');

```

-----  
Problem 1:  
-----

A_k Number [ K ]	Size (M x N)	Rank	Condition
A_40 74.8767	101 x 40	40	
A_41 103.8004	101 x 41	41	
A_42 152.2856	101 x 42	42	
A_43 217.5604	101 x 43	43	

---

A_44	/	101 x 44	/	44	/
328.8920	/				
<hr/>					
A_45	/	101 x 45	/	45	/
483.7805	/				
<hr/>					
A_46	/	101 x 46	/	46	/
753.0465	/				
<hr/>					
A_47	/	101 x 47	/	47	/
1140.0742	/				
<hr/>					
A_48	/	101 x 48	/	48	/
1826.7931	/				
<hr/>					
A_49	/	101 x 49	/	49	/
2846.4223	/				
<hr/>					
A_50	/	101 x 50	/	50	/
4695.0874	/				
<hr/>					
A_51	/	101 x 51	/	51	/
7530.5483	/				
<hr/>					
A_52	/	101 x 52	/	52	/
12789.3765	/				
<hr/>					
A_53	/	101 x 53	/	53	/
21122.7169	/				
<hr/>					
A_54	/	101 x 54	/	54	/
36949.4832	/				
<hr/>					
A_55	/	101 x 55	/	55	/
62868.3337	/				
<hr/>					
A_56	/	101 x 56	/	56	/
113329.3575	/				
<hr/>					
A_57	/	101 x 57	/	57	/
198770.6821	/				
<hr/>					
A_58	/	101 x 58	/	58	/
369475.9187	/				
<hr/>					
A_59	/	101 x 59	/	59	/
668493.2863	/				
<hr/>					
A_60	/	101 x 60	/	60	/
1282274.0197	/				
<hr/>					
A_61	/	101 x 61	/	61	/
2395303.2393	/				
<hr/>					

---

---

A_62	/	101 x 62	/	62	/
4745459.9263	/				
-----					
A_63	/	101 x 63	/	63	/
9161533.4668	/				
-----					
A_64	/	101 x 64	/	64	/
18765737.9716	/				
-----					
A_65	/	101 x 65	/	65	/
37486287.3234	/				
-----					

-----  
Discussion:  
-----

- 1:  
Average error increases for both QR and the normal equations as  $k$  increases. Least squares error using normal equations increases more drastically than the error using thin qr even with the for  $k = 64$  and  $k = 65$  being ignored
- 2:  
as stated before, as the condition number of  $A_k$  increases the least squares error also increases
- 3:  
looking at the matlab implementation of thin QR vs NE thin QR is more favorable for ill-conditioned matrices.  
as the condition number increases there is noticeable round off error using cholesky factorization.  
For this homework, cholesky factorization could not be used for matrices  $A_{64}$  and  $A_{65}$  due to them not being sym pos definite.

## Plotting

```
%average error versus k for both QR and Normal Equations using
semilogy

%my implementation of thin QR
figure(1)
semilogy(40:65, errk_avg_NE, '-o', 40:65, errk_avg_QR_mine, '-*')
hold on;
title('Average Error versus k for both QR and NE')
ylabel('Least Squares Error')
xlabel('Number of columns')
legend('NE' , 'my own implementation of thin QR');
grid on;
hold off;

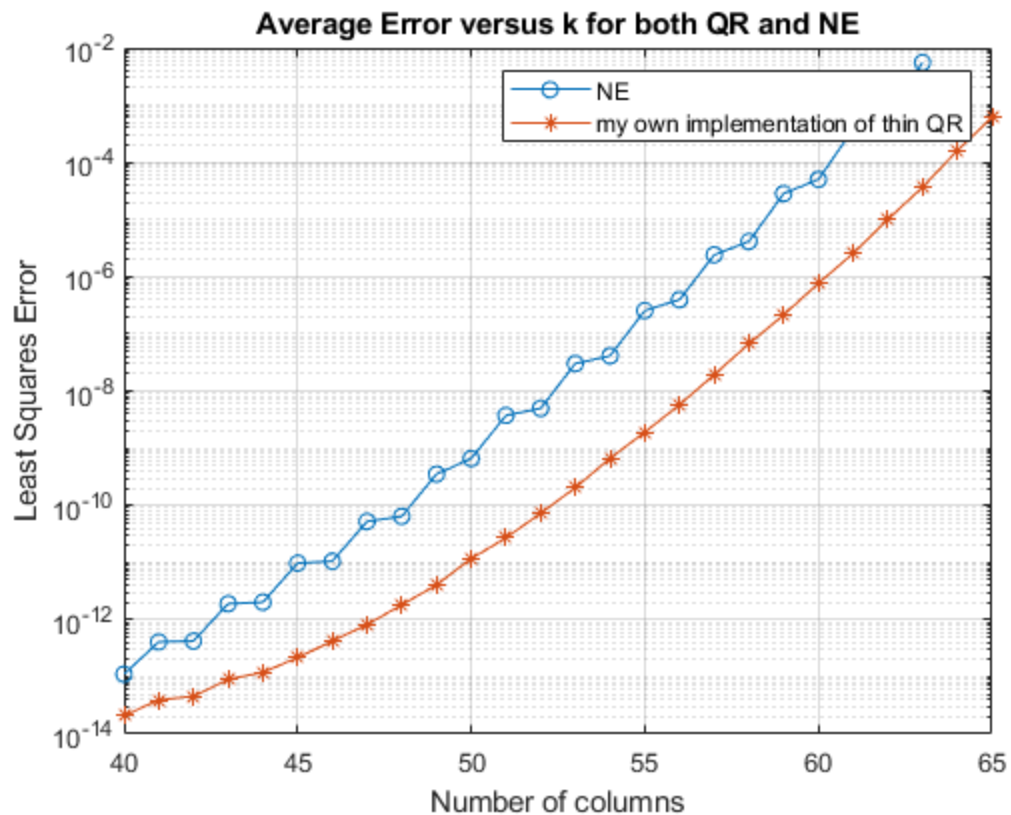
%matlab implementation of thin QR
figure(2)
```

```

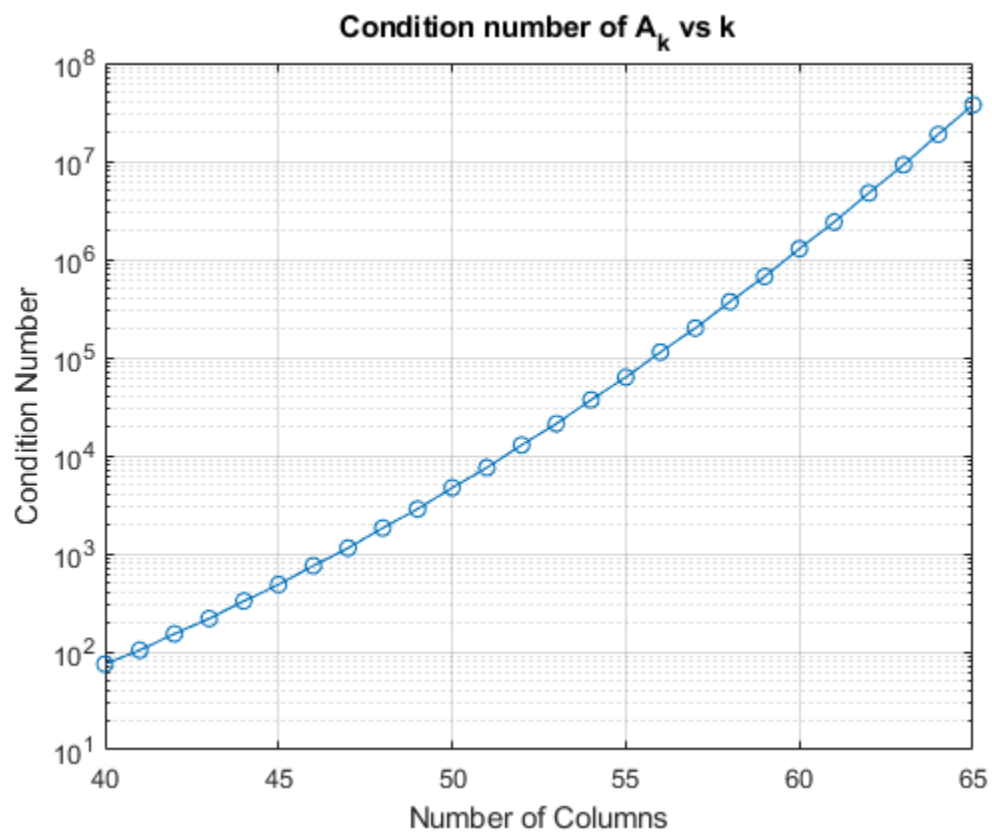
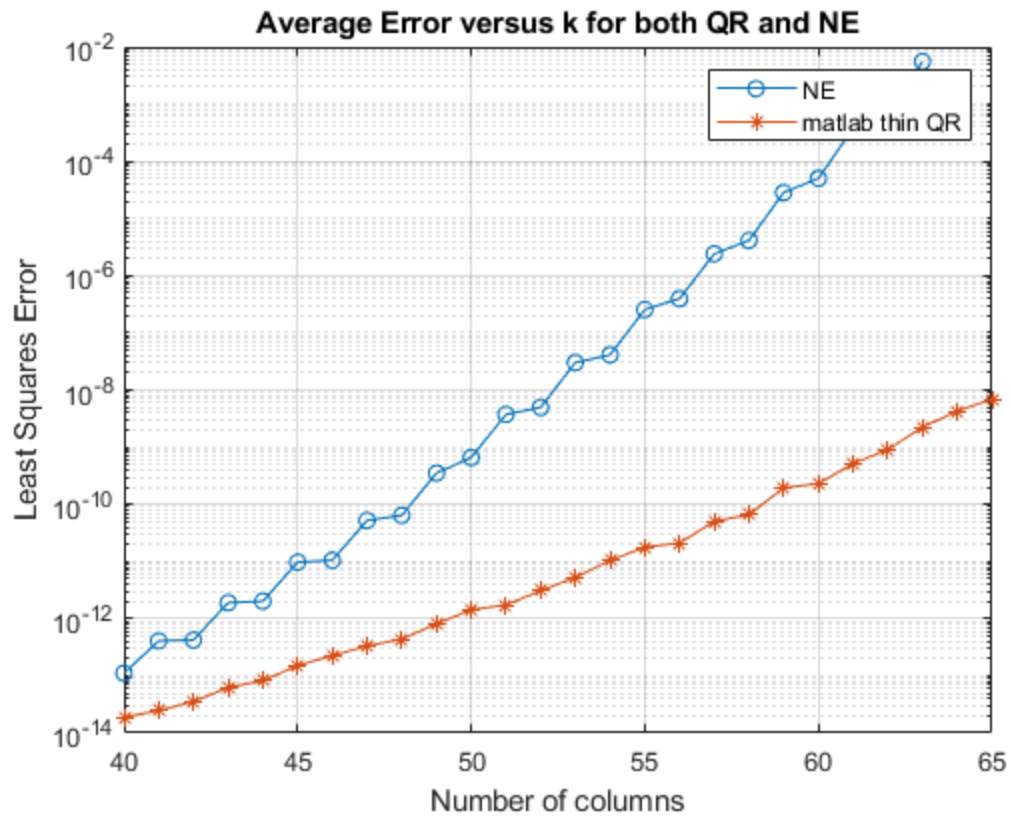
semilogy(40:65, errk_avg_NE, '-o', 40:65, errk_avg_QR_matlab, '-*')
hold on;
title('Average Error versus k for both QR and NE')
ylabel('Least Squares Error')
xlabel('Number of columns')
legend('NE' , 'matlab thin QR');
grid on;
hold off;

%condition number of A_k vs k
figure(3)
semilogy(40:65, cond_ak, '-o');
hold on;
grid on;
title('Condition number of A_k vs k')
ylabel('Condition Number')
xlabel('Number of Columns')
hold off;

```







---

# Functions

```
function [x] = method_NormEq_Cholesky(A,b)
%{
    Purpose: Matlab implementation of method to solve least squares
    problem using normal
    equations with cholesky
    Inputs:
        A: LHS matrix for  $Ax = b$ 
        b: RHS matrix for  $Ax = b$ 
    output:
        x: variable matrix x in  $Ax = b$  (x) which minimizes
         $\|b-Ax\|_2^2$ 
%}

%form  $A^T A \in \mathbb{R}^{n \times n}$  and  $A^T b \in \mathbb{R}^n$ 
ata = A'*A;
atb = A'*b;

%Solve normal eqns,  $(A^T A)x = A^T b$  for x
%first lets double check
%get size to make sure  $A = n \times n$ 
size_A = size(ata);
%check rank
r_ata = rank(ata);

if( (size_A(1) == size_A(2) ) && ( r_ata == size_A(1) ) )
    %matrix is sym, pos def, compute and solve cholesky
    factorization (
        %determine variable vector using cholesky factorization
        R = chol(ata);
        x = R\(R'\atb);
    else
        %cant use error func for this homework so itll break, if using
        %outside of hw uncomment
        %error('Matrix is not Symmetric Positive Definite')

        %if this error occurs create nan array maybe later could be
        used as a flag in
        %error calc
        siz_atb = size(atb);
        warning('Error not accounted for due to A_%i' x A is not pos
        def', siz_atb(1))
        x = NaN(siz_atb);
    end
end

function [ x ] = method_ThinQR(A, b, bool)
%{
    Purpose: Matlab implementation to solve the linear least-squares
```

---

---

```

    problem using a method based on Thin QR factorization

    Input:
        A: LHS matrix,  $Ax = b$ 
        b: RHS matrix,  $Ax = b$ 
        bool: boolean value, 1 if you want to use my janky code which
seems
        wrong, or built in qr function that works
    Output:
        x: variable matrix x in  $Ax = b$  (x) which minimizes
         $\|b - Ax\|_2^2$ 
    %}

%alright so because i am trying to do both the homeowkrs this weekend
im
%assuming the grant-schmidt orthogonalization and least squares is
similar
%to that of the thin QR decomp in lecture. Sauer's numerical analysis
and
%https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-
spring-2010/related-resources/MIT18_06S10_gramschmidtmat.pdf
%were used as a reference

%compute the thin QR decomposition

if bool
    %obtain size of A
    size_A = size(A);

    %initialize r matrix. upper right traingular n x n
    r = zeros(size_A(2),size_A(2));
    %initialize q tall matrixx orthonormal columns, m x n
    q = zeros( size_A );

    %Grant-Schmidt orthogonalization
    for j = 1 : size_A(2)

        y = A(:,j);

        for i = 1 : j-1

            r(i,j) = q(:, i)' * A(:, j);
            y = y - r(i,j) * q(:, i);

        end
        r(j , j )= norm(y);
        q(:,j) = y/r(j , j);

    end
else

    % if this doesnt work just use qr matlab
    [q,r] = qr(A, 0);

```

---







[illegible]

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