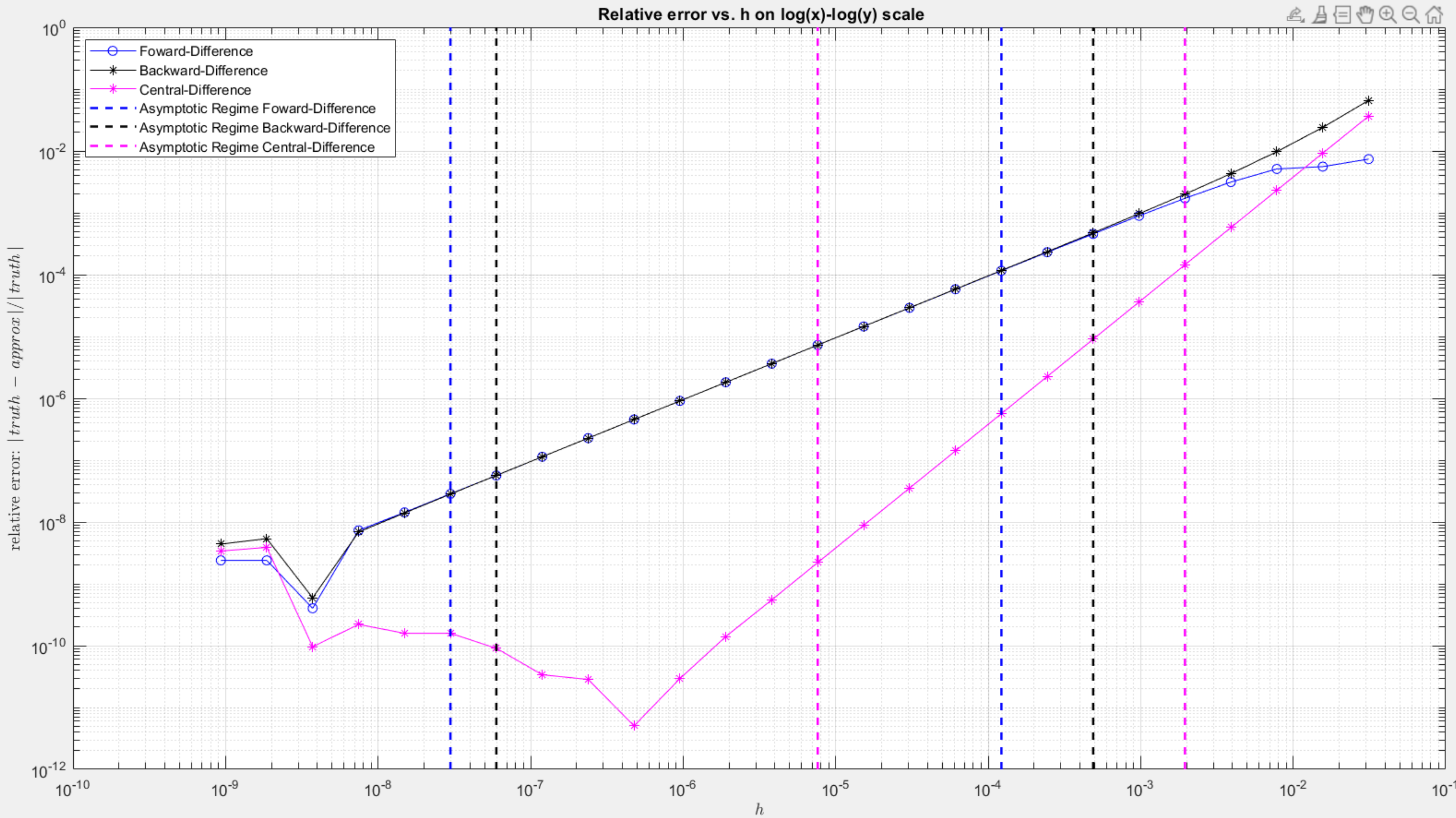
Part 1:

| Method | Estimated Convergence Rate | Theory Convergence Rate |
| --- | --- | --- |
| Forward Difference | 0.99875 | 1 |
| Backward Difference | 1.00245 | 1 |
| Central Difference | 2.00005 | 2 |

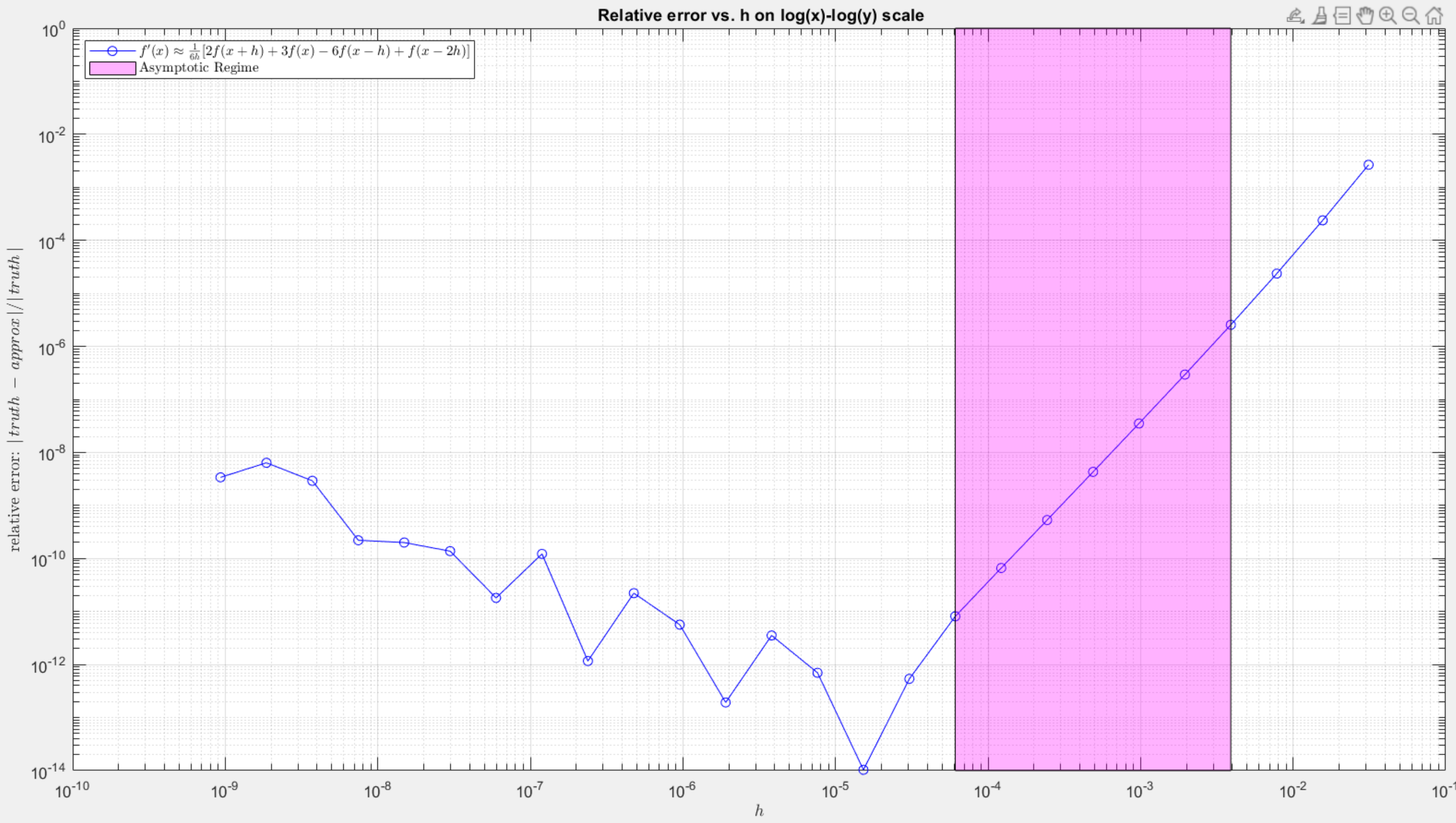
Part 2:

Using a numerical study similar to Problem one, the rate of convergence for this approximation is 3.04385

>>



Relative error vs. h on log(x)-log(y) scale
Foward-Difference
Backward-Difference
Central-Difference
Asymptotic Regime Foward-Difference
Asymptotic Regime Backward-Difference
Asymptotic Regime Central-Difference
relative error: |truth − approx|/|truth|
h
$10^{0}$
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{-12}$
$10^{-10}$
$10^{-9}$
$10^{-8}$
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$



Relative error vs. h on log(x)-log(y) scale
$f'(x) \approx \frac{1}{6h}[2f(x+h)+3f(x)-6f(x-h)+f(x-2h)]$
Asymptotic Regime
relative error: $|truth - approx|/|truth|$
$h$
$10^{0}$
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{-12}$
$10^{-14}$
$10^{-10}$
$10^{-9}$
$10^{-8}$
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$

## Table of Contents



# House keeping

```
clear all; close all; clc;

%{
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%}
```

# Part 1

```
% define function
f_p1 =@(x) sin(4.8 * pi * x);
%define functions first derivative found by hand
f1_truth =@(x) 4.8 * pi * cos(4.8 * pi * x);
%initilize values for h and pont to be evaluated
h = 2.^-(5:30);
x_p1 = 0.2;

%compute relative error vs h for each three methods

%intialize storage and vars

truth = f1_truth(x_p1); %computed truth
len_h = length(h);
rel_err = zeros(len_h, 3);

for i = 1 : len_h

    %compute approximations
    fwd_approx = fwd_diff(f_p1 , x_p1, h(i));
    bck_approx = back_diff(f_p1 , x_p1, h(i));
    cnt_approx = cent_diff(f_p1 , x_p1, h(i));

    %compute relative error for the three methods
    rel_err(i,1) = abs(fwd_approx - truth) / abs( truth);
    rel_err(i,2) = abs(bck_approx - truth) / abs( truth);
    rel_err(i,3) = abs(cnt_approx - truth) / abs( truth);

end
```

```
% code for plot will be shown in plotting section

%find beginning of asymptotic range

% initialize arrays
log_h = log(h);
log_errf = log(rel_err(:,1)); %fwd diff
log_errb = log(rel_err(:,2)); %back diff
log_errc = log(rel_err(:,3)); %central diff

%find change points
%used
%https://www.mathworks.com/help/signal/ref/findchangepts.html#bu3nws1-
ipt
%hopefully it works
ptsf = findchangepts(log_errf, 'MinThreshold', len_h);
ptsb = findchangepts(log_errb, 'MinThreshold', len_h);
ptsc = findchangepts(log_errc, 'MinThreshold', len_h);

%define regime for three methods
rngef = [ptsf(1) , ptsf(end)];
rngeb = [ptsb(1) , ptsb(end)];
rngec = [ptsc(1) , ptsc(end)];
%added to plots function

%find convergence rates
conv_f = ( log_errf(rngef(2)) - log_errf(rngef(1)) ) /
 ( log_h(rngef(2)) - log_h(rngef(1)) );
conv_b = ( log_errb(rngeb(2)) - log_errb(rngeb(1)) ) /
 ( log_h(rngeb(2)) - log_h(rngeb(1)) );
conv_c = ( log_errc(rngec(2)) - log_errc(rngec(1)) ) /
 ( log_h(rngec(2)) - log_h(rngec(1)) );
```

# Part 2

```
%initialize method function
f1_p2 =@(x , h , f)  ( 1 /( 6*h ) )  * ( 2 * f(x+h) + 3 * f(x) - 6 *
 f(x-h) + f(x - 2*h) );
%intialize storage and vars
rel_errp2 = zeros(len_h,1);
for i = 1 : len_h
    %compute approximation
    p2_approx = f1_p2( x_p1 , h(i) , f_p1 );

    %compute relative error
    rel_errp2(i) = abs( p2_approx - truth) / abs( truth );

end

% code for plot will be shown in plotting section

%define asymptotic regime for finite difference approximation
```

```
%initialize
log_errp2 = log(rel_errp2);
%find change points
%used
%https://www.mathworks.com/help/signal/ref/findchangepts.html#bu3nws1-
ipt
%hopefully it works
ptsp2 = findchangepts(log_errp2, 'MinThreshold', len_h);
%define aymptotic regime for p2 plot
%looking at result
rngep2 = [ptsp2(1) , ptsp2(end-1)];

%plot in plotting section

%determine convergence rate
conv_p2 = ( log_errp2(rngep2(2)) - log_errp2(rngep2(1)) ) /
 ( log_h(rngep2(2)) - log_h(rngep2(1)) );
```

# Display

```
%part 1
fprintf('\n------------------------------------------------------------------
\n')
fprintf('Part 1:')
fprintf('\n------------------------------------------------------------------
\n\n\n')
fprintf('-------------------------------------------------------------------------
\n')
fprintf('            Method             |    Estimated Convergence Rate
    |    Theory Convergence Rate  ');
fprintf('|
\n--------------------------------------------------------------------------------
\n')
fprintf(' Forward Difference     |                %0.5f
          |                       1    ', conv_f)
fprintf('                   |
\n--------------------------------------------------------------------------------
\n')
fprintf(' Backward Difference  |                  %0.5f
        |                       1    ', conv_b)
fprintf('                   |
\n--------------------------------------------------------------------------------
\n')
fprintf(' Central Difference      |                %0.5f
           |                       2    ', conv_c)
fprintf('                   |
\n--------------------------------------------------------------------------------
\n')

%part 2
fprintf('\n
\n------------------------------------------------------------------
\n')
```

```
fprintf('Part 2:')
fprintf('\n------------------------------------------------------------------
\n\n\n')

fprintf('Using a numerical study similar to Problem one, the rate of
 convergence for this approximation is %0.5f', conv_p2)
```

```
------------------------------------------------------------------
Part 1:
------------------------------------------------------------------


----------------------------------------------------------------------------------
           Method                |    Estimated Convergence Rate    |
 Theory Convergence Rate   |
----------------------------------------------------------------------------------
 Forward Difference     |                 0.99875
   |                       1                      |
----------------------------------------------------------------------------------
 Backward Difference  |                 1.00245
 |                       1                      |
----------------------------------------------------------------------------------
 Central Difference      |                 2.00005
    |                       2                      |
----------------------------------------------------------------------------------


------------------------------------------------------------------
Part 2:
------------------------------------------------------------------


Using a numerical study similar to Problem one, the rate of
 convergence for this approximation is 3.04385
```

# Plotting

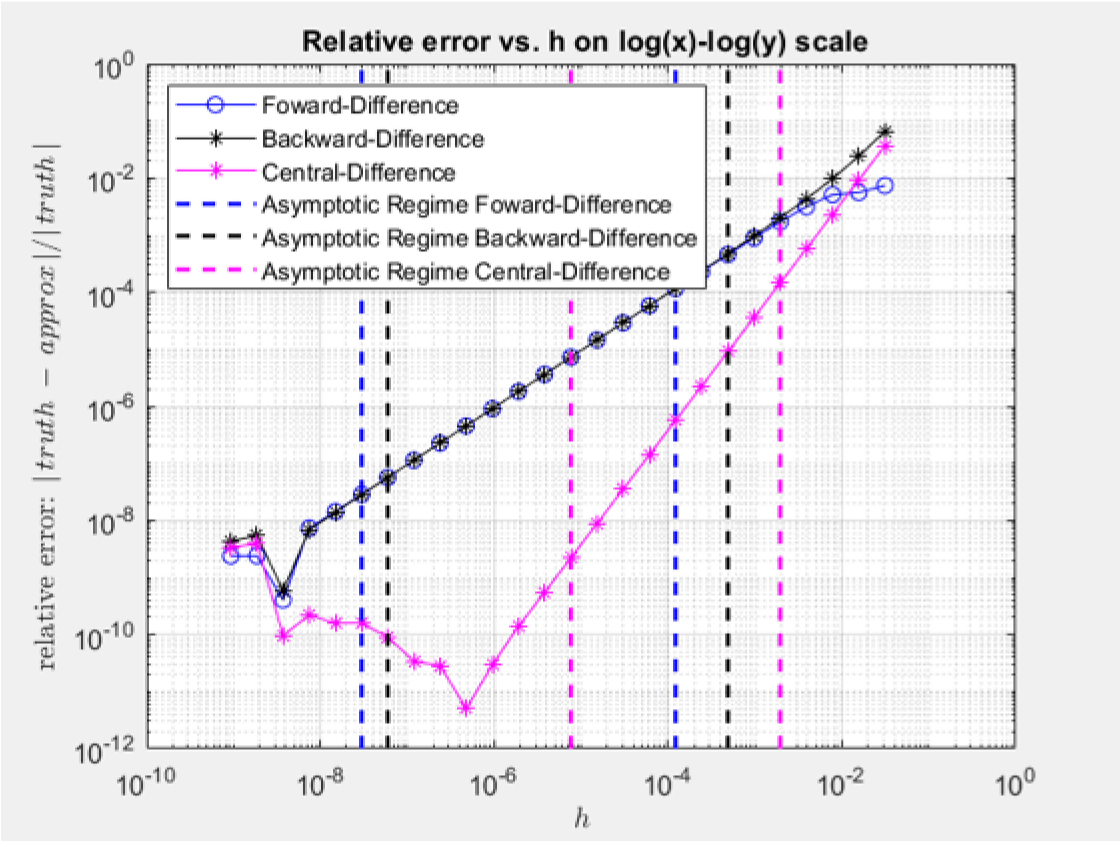
part 1 plot

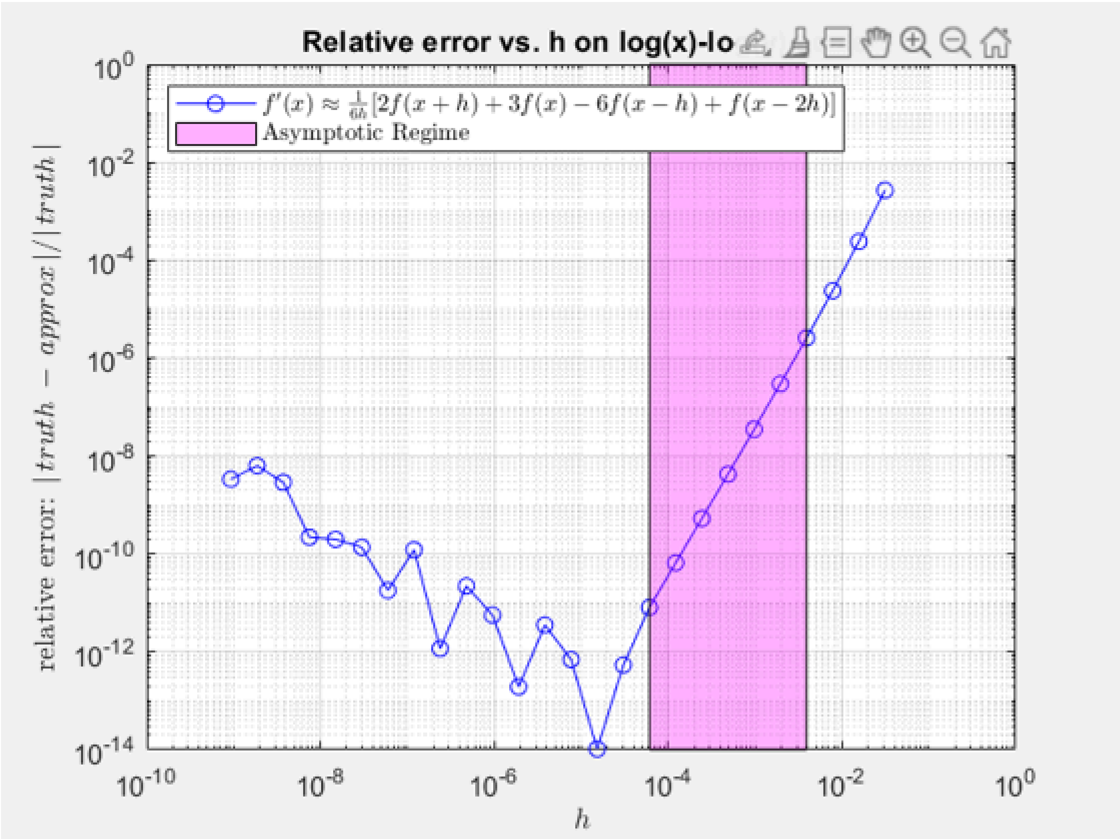
```
%error vs h on a log-log scale for all approximations
figure(1)
loglog(h, rel_err(: , 1),'bo-')
hold on
loglog(h, rel_err(: , 2), 'k*-')
loglog(h, rel_err(: , 3) , 'm*-')
%add vertical lines for asymptotic range
%for foward diff
loglog([h(rngef(1)) h(rngef(1))], [10^-12 1], '--b', 'Linewidth',1.5)
loglog([h(rngef(2)) h(rngef(2))], [10^-12 1],'--b', 'Linewidth',1.5)
%backward diff
loglog([h(rngeb(1)) h(rngeb(1))], [10^-12 1], '--k', 'Linewidth',1.5)
loglog([h(rngeb(2)) h(rngeb(2))], [10^-12 1],'--k', 'Linewidth',1.5)
%central difference
```

```
loglog([h(rngec(1)) h(rngec(1))], [10^-12 1], '--m', 'Linewidth',1.5)
loglog([h(rngec(2)) h(rngec(2))], [10^-12 1],'--m', 'Linewidth',1.5)
grid on;
legend('Foward-Difference' , 'Backward-Difference', 'Central-
Difference' ,'Asymptotic Regime Foward-Difference', '', 'Asymptotic
 Regime Backward-Difference','','Asymptotic Regime Central-
Difference','', 'Location', 'northwest' )
title(' Relative error vs. h on log(x)-log(y) scale')
%from given code
ylabel('relative error: $|\,truth\,-\,approx\,|/|\,truth\,|
$', 'interpreter', 'latex');
xlabel('$h$', 'interpreter', 'latex');
hold off;



%part 2
figure(2)
loglog(h, rel_errp2,'bo-')
hold on;
%plot asymptotic region
patch([h(rngep2(1)) h(rngep2(2)) h(rngep2(2)) h(rngep2(1)) ],[1 1
 10^-14 10^-14],'magenta','FaceAlpha',.3);
grid on;
title(' Relative error vs. h on log(x)-log(y) scale')
%from given code
ylabel('relative error: $|\,truth\,-\,approx\,|/|\,truth\,|
$', 'interpreter', 'latex');
xlabel('$h$', 'interpreter', 'latex');
legend('$f''(x) \approx \frac{1}{6h} [ 2f(x+h)
 + 3f(x) - 6f(x-h) + f(x-2h) ] $','Asymptotic
 Regime', 'Interpreter','latex','location','northwest')
hold off;
```


Relative error vs. h on log(x)-log(y) scale
Foward-Difference
Backward-Difference
Central-Difference
Asymptotic Regime Foward-Difference
Asymptotic Regime Backward-Difference
Asymptotic Regime Central-Difference
relative error: |truth − approx|/|truth|
h


Relative error vs. h on log(x)-lo
$f'(x) \approx \frac{1}{6h}[2f(x+h)+3f(x)-6f(x-h)+f(x-2h)]$
Asymptotic Regime
relative error: |truth − approx|/|truth|
h

# Functions

```
function [x1] = fwd_diff(f, x, h)
%{
    Purpose: matlab implementation of one-sided foward differerence to
 approximate
    first dereivative of f (f')

    Inputs:
        f: function to derive
        x: evaluation point
        h: step size
    Output:
        x1: approximation of derivative at x
%}

    x1 = ( f(x+h) - f(x) ) / h;

end

function [x1] = back_diff(f, x, h)
%{
    Purpose: matlab implementation of one-sided backward differerence
 to approximate
    first dereivative of f (f')

    Inputs:
        f: function to derive
        x: evaluation point
        h: step size
    Output:
        x1: approximation of derivative at x
%}

    x1 = ( f(x) - f(x-h) )/h;

end

function [x1] = cent_diff(f, x, h)
%{
    Purpose: matlab implementation of central differerence to
 approximate
    first dereivative of f (f')

    Inputs:
        f: function to derive
        x: evaluation point
        h: step size
    Output:
        x1: approximation of derivative at x
%}

    x1 = ( f(x+h) - f(x-h) )/( 2 * h);
```

```
end
```

*Published with MATLAB® R2021a*