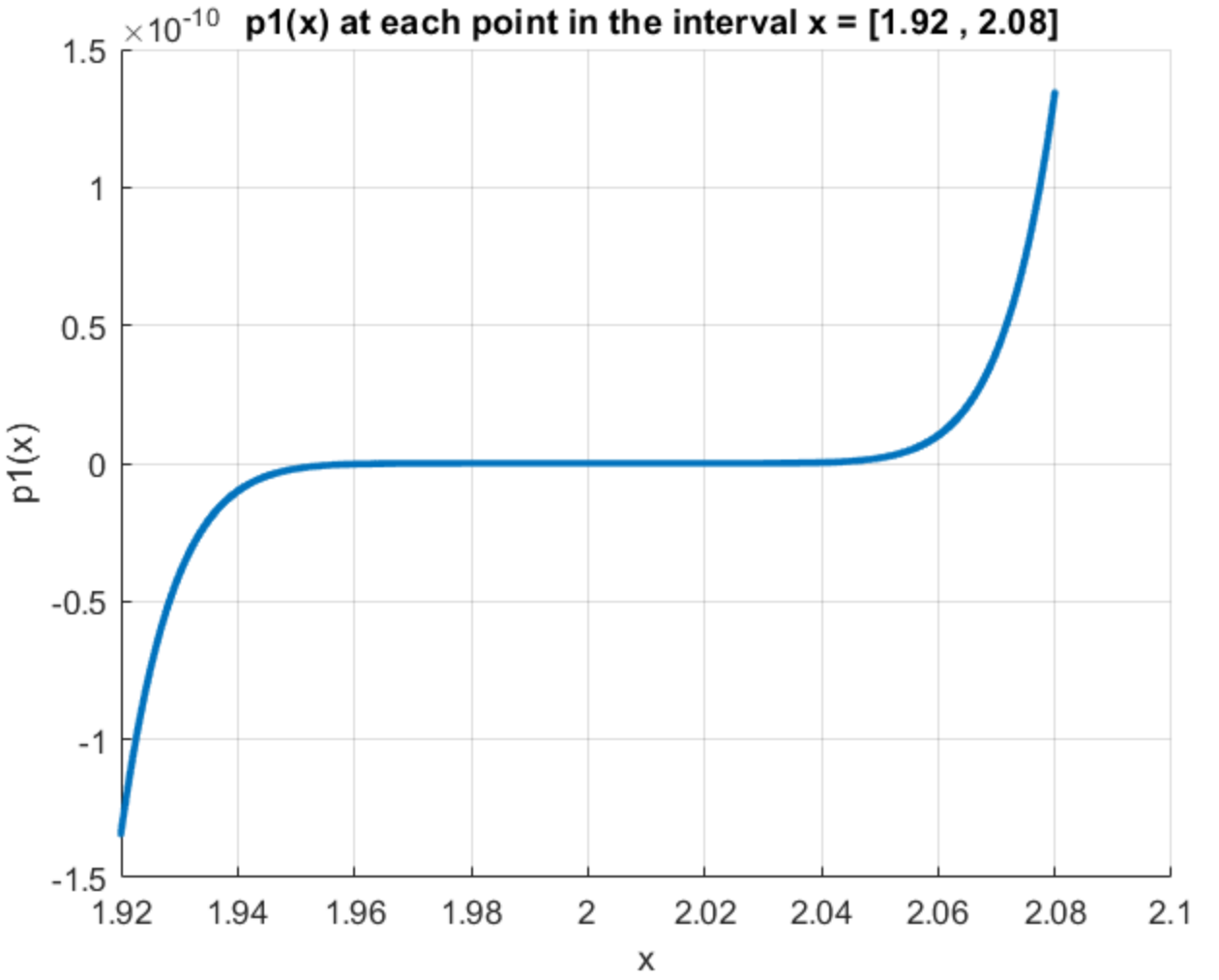
Q1

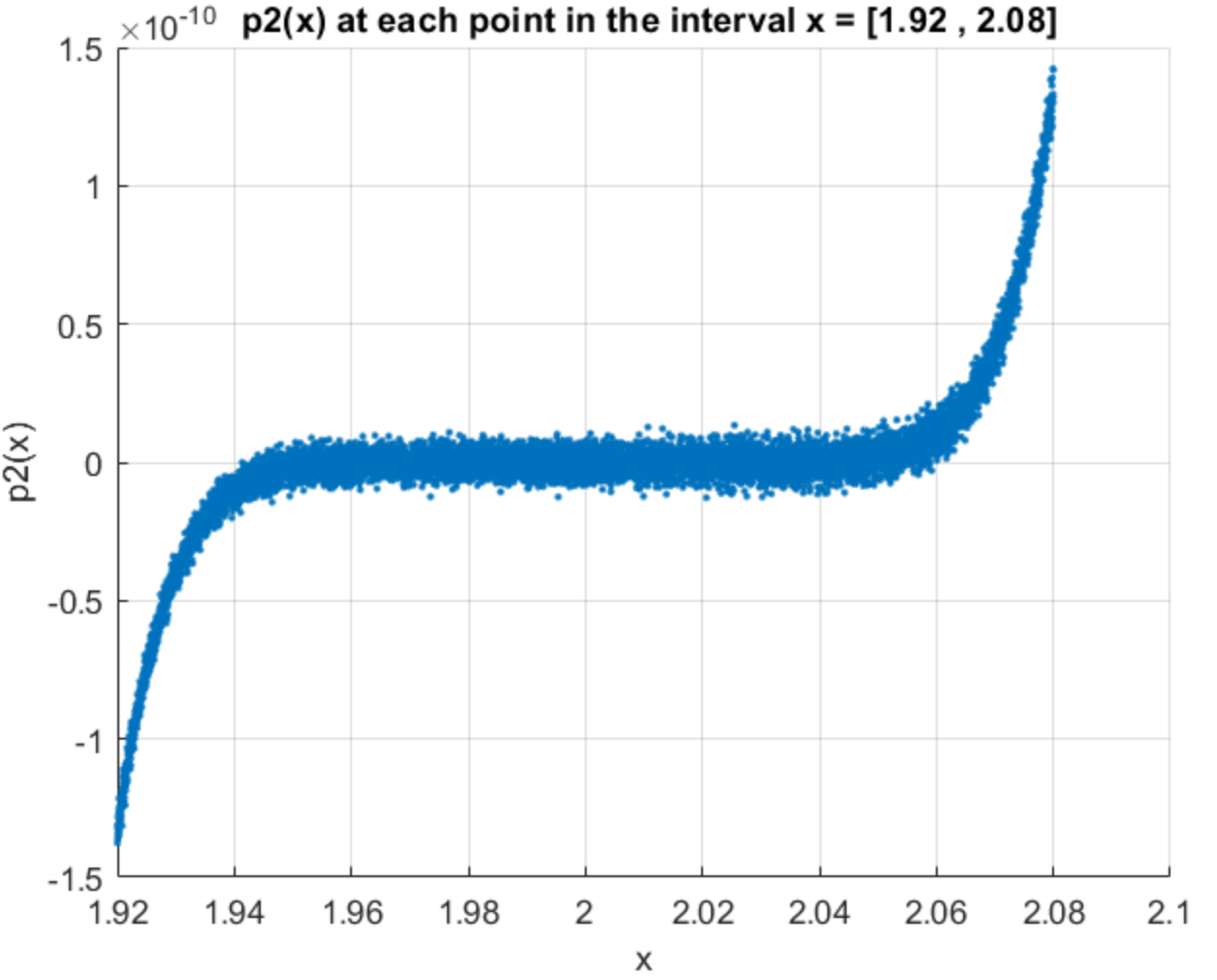
After reading the blog post my trust in computers was not affected. I could not recall a time that I ran into overflow of underflow problems using matlab in my aerospace classes maybe some minor round off errors from ugly small coefficients but nothing crazy. It wasn't until I noticed the real world example on the wikipedia round off error page that my trust was affected. The section described how a round off error in a missile defence system caused a failed interception and cost lives. The section definitely made me more aware of how these errors are worth accounting for.

2) a) Convinced myself using wolfram

b) implemented in code

c) Horner's method uses two operations, addition and multiplication in every iteration causing errors due to the use of floating point numbers. Using an array of 8000 equally spaced points on the interval [1.92, 2.05] introduces roundoff error.


p1(x) at each point in the interval x = [1.92 , 2.08]
×10-10
1.5
1
0.5
0
-0.5
-1
-1.5
p1(x)
1.92
1.94
1.96
1.98
2
2.02
2.04
2.06
2.08
2.1
x


p2(x) at each point in the interval x = [1.92 , 2.08]
×10-10
1.5
1
0.5
0
-0.5
-1
-1.5
p2(x)
1.92
1.94
1.96
1.98
2
2.02
2.04
2.06
2.08
2.1
x

3) $f_1(x) = \frac{1-\cos(x)}{\sin^2(x)}$ $\quad f_2(x) = \frac{1}{1+\cos(x)}$

$$\sin^2(x) = 1 - \cos^2(x)$$
$$\sin^2(x) = (1+\cos(x))(1-\cos(x))$$

$$f_1(x) = \frac{1-\cos(x)}{(1+\cos(x))(1-\cos(x))}$$

$$f_1(x) = \frac{1}{1+\cos(x)} = f_2(x) = \frac{1}{1+\cos(x)}$$

Explenation: $f_1(x)$ experiences loss of significant digits due to subtraction of nearly equal numbers. as x approaches zero, $\cos(x)$ approaches 1. around $K = 5$ these errors become significant and I assume this becomes so great that round off error occurs.

x_k = 1.0000000 0.1000000 0.0100000 0.0010000 0.0001000 0.0000100 0.0000010 0.0000001 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000

f1 = 0.6492232 0.5012521 0.5000125 0.5000001 0.5000000 0.5000000 0.5000445 0.4996004 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000

f2 = 0.6492232 0.5012521 0.5000125 0.5000001 0.5000000 0.5000000 0.5000000 0.5000000 0.5000000 0.5000000 0.5000000 0.5000000 0.5000000