

On the Non-Existence of Universal **P**/**NP** Classifiers

Yanlin Li

Abstract

This note clarifies a structural limitation on complexity classification. Even when a rule and parameter system is fully fixed, there is in general no universal procedure that takes an arbitrary problem description and decides whether it belongs to **P** or **NP**. We further observe that such classification may become possible once the admissible rule system and problem description space are sufficiently restricted.

Setting

Let Θ denote a fixed parameter system specifying admissible encodings, rules, and interpretation constraints under which problems induce languages $L(\Theta) \subseteq \Sigma^*$. We write $\mathbf{P}(\Theta)$ and $\mathbf{NP}(\Theta)$ for feasibility predicates evaluated relative to this fixed system.

A *problem description* q is assumed to induce, together with Θ , a language $L(q, \Theta)$.

No Universal Classifier in General

Note 1. *There does not exist, in general, a total computable function*

$$F(q, \Theta) \in \{\mathbf{P}, \mathbf{NP}\}$$

that correctly decides, for all problem descriptions q and parameter systems Θ , whether $L(q, \Theta) \in \mathbf{P}$ or $L(q, \Theta) \in \mathbf{NP}$.

Proof sketch. Membership in **P** or **NP** is a nontrivial semantic property of the induced language $L(q, \Theta)$, defined via existential quantification over algorithms (e.g., the existence of a polynomial-time decider or verifier). Under any sufficiently expressive description formalism, problem instances (q, Θ) can uniformly encode arbitrary Turing-machine behavior. A universal classifier deciding **P**/**NP** membership would therefore decide undecidable properties as a special case. Hence no such total computable function exists in full generality. \square

Restricted Rule Systems

Proposition 1. *If, in addition to fixing Θ , the admissible problem descriptions are restricted to a decidable fragment \mathcal{Q}_Θ whose feasibility classification is syntactically characterizable, then a total classifier*

$$F_\Theta : \mathcal{Q}_\Theta \rightarrow \{\mathbf{P}, \mathbf{NP}\}$$

may exist.

Justification. This situation arises when Θ collapses the space of admissible problems into a family with effective classification rules. Typical cases include:

- Finite admissible problem catalogs, where membership is decidable by lookup.
- Syntactically restricted fragments with known polynomial-time bounds.
- Rule systems admitting tractability/intractability dichotomies under fixed encodings.

In such settings, feasibility classification becomes computable by construction. □

Remark 1. *Fixing Θ renders feasibility comparisons well-typed, but does not, by itself, guarantee the existence of a universal mechanical classifier. Classification becomes possible only after further restricting the rule system and admissible description space.*