1 Young's modulus

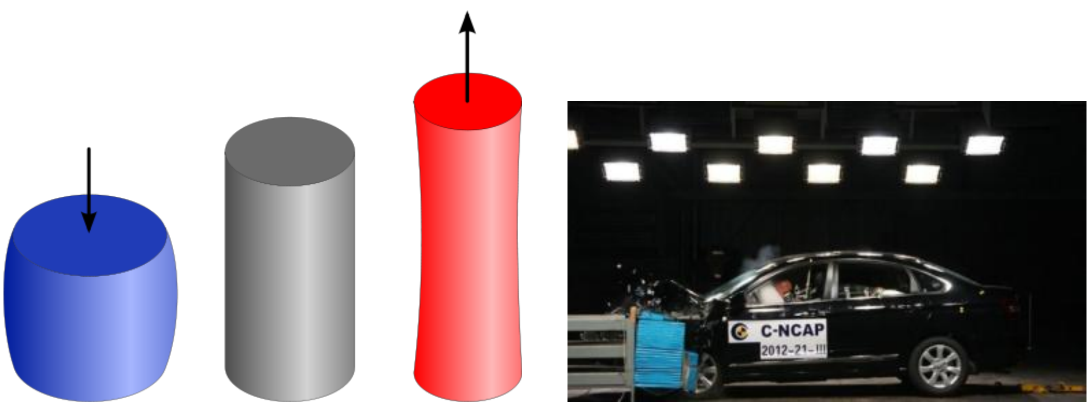
where	Building 49, Rm 301
when	Sep-30, 9:00-11:30 am

1.1 Objective

- Measure and calculate Young's modulus of the steel string.
- Understand the rules of the data collection and error estimation.
- Learn to use the micrometer.
- Represent your data as tables and graphs.
- Explain the physical content of Young's modulus.

1.2 Theory and background

The deformations can be simple or complicated as shown in Figure 1. But there are ways to classify the deformations. The reversible deformations are elastic deformations. Once the forces are no longer applied, the object returns to its original shape. On the other hand, the inelastic deformations refer to the ones that the objects cannot return to its original shape after the forces are removed. Inelastic deformations are also called "plastic" deformation.



(www.viscoelasticity.info)

Figure 1: Examples of deformations.

There is a certain limit for the elastic deformation. In general, an elastic deformation can be turned into an inelastic deformation when the stress exceeds some limit. A break (fracture) occurs after the material has reached the end of the inelastic deformation. The elastic deformation, inelastic deformation and fracture are summarized in Figure 2. The application of a deforming force is called "STRESS". A stress may cause the change in length, volume or shape. The deformation is often described by "STRAIN". A strain is measure of deformation representing the displacement relative to a reference length.

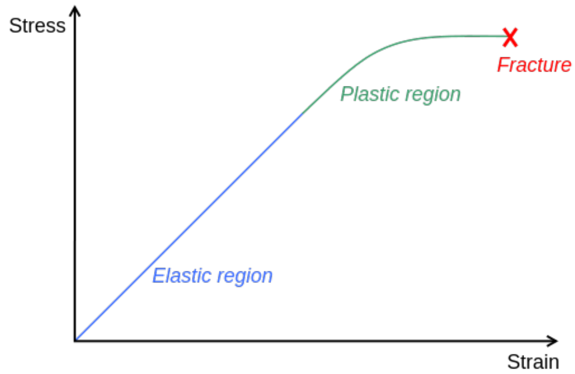


Figure 2: Elastic deformation, inelastic deformation and fracture (taken from Wikipedia)

In this experiment, we will study the relation of the stress and strain. Recall the Hooke's law we learned in the "Fundamentals of Physics" class:

$$k = \frac{F}{x - x_0} \tag{1}$$

In Equation 1, F is the force applied to the spring and $x - x_0$ is the change of the spring in length. As the elastic deformation of the spring, the ratio of the force and the length change is a constant. Such a constant indicates the property of a spring.

Young's modulus is a wider concept, it applies to most elastic materials than the Hooke's law. Young's modulus is named after the 19th-century British scientist Thomas Young. The definition of Young's modulus is

$$E = \frac{F/A}{(l - l_0)/l_0}. \tag{2}$$

The Young's modulus, E, is express as the ratio of the stress and the strain. The stress, F/A, is the force per unit area. The strain, $(l - l_0)/l$, is the change of length per original length. The strain can also be understood as the change of the length in percentile. Young's modulus represents the mechanical property of the material.

1.3 Experiment Design

In this experiment, we measure the Young's modulus for a steel string. The string, when being stretched by some force, extends in length. In equation 2, we need to measure the force, area, change in length and the original length of the string. Multiple measurements are required for a convincing result. The calculation of the error is also needed. The experiment setup is shown in Figure 3.

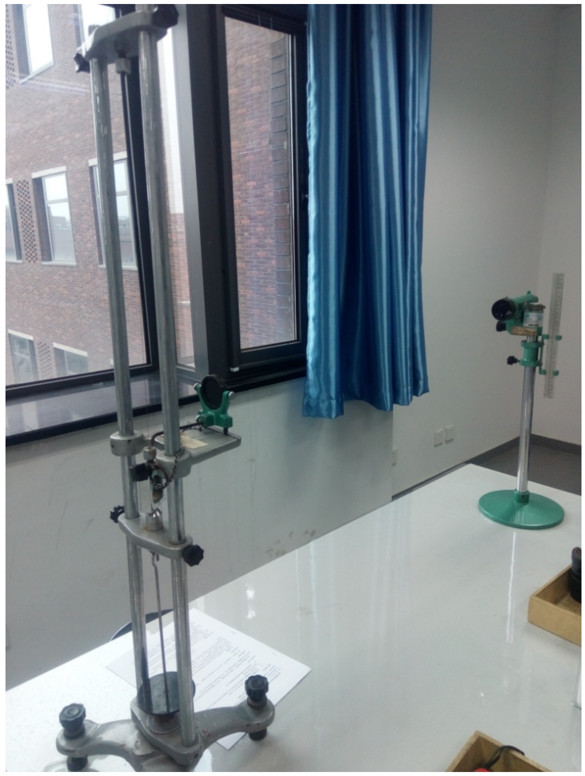


Figure 3: Experiment setup overview

Force
For our measurement, the steel string is installed in the shelf for convinence. One end of the string is attached to the top of the shelf, while the other end is attached to a cylinder as shown in the left panel of Figure 4. There is also a weight carrier attached to the end of the string where one can place disc weight to apply force to the string (right panel of Figure 4). By using the disc weight, the force can be controlled, and we read the magnitude of the weight easily.

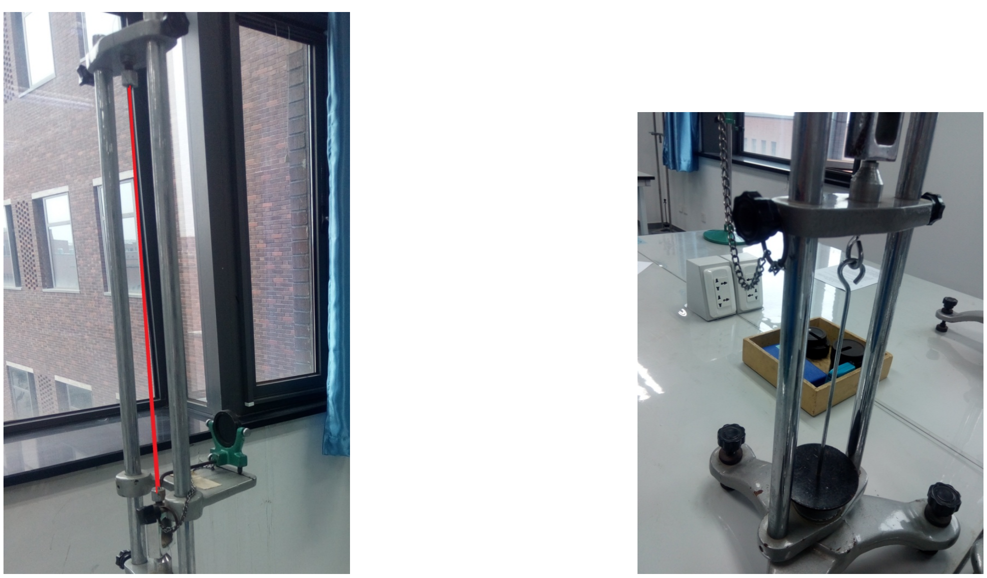


Figure 4: Details of string and weight.

Area, the diameter of the string
We use micrometer to measure the diameter of the string. If the diameter is d, then the area can be calculated as $A = \frac{\pi d^2}{4}$. For the usage of the micrometer, please refer to the appendix .

The change in length
The change in length is measured by the optical level. An optical level usually contains 2 parts, a telescope (with base) and a meter. But in our experiments, there is an extra mirror for the measure of the string. The major parts of the optical level are shown in Figure 5. It is a special optical level designed for this experiment.

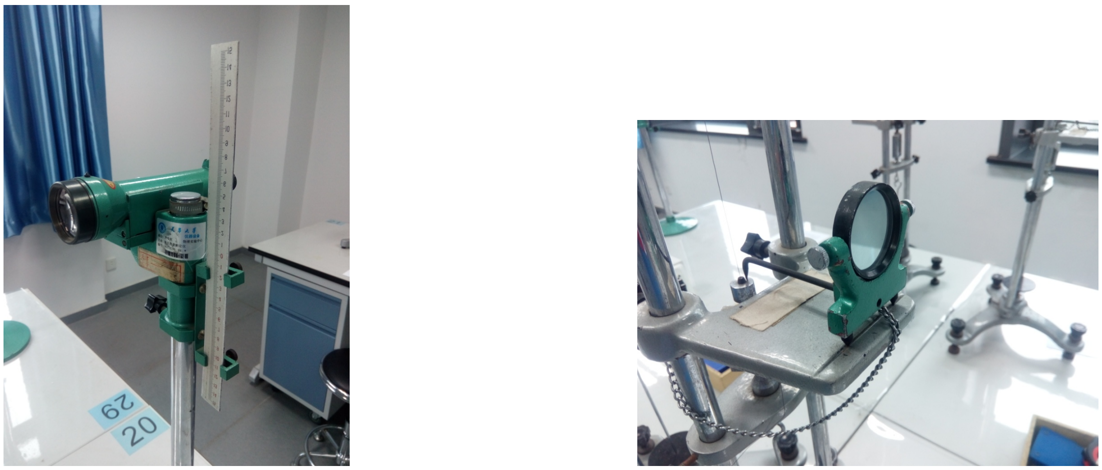


Figure 5: Optical level: the telescope, the meter and the mirror.

In the setup, we see the meter is placed next to the telescope, while the mirror is connected to the end of the string through the cylinder. When the mirror faces the telescope, we can read the meter through the telescope. Thus, a small change in the direction of the mirror leads to the image change of the telescope. The separation of the telescope and the mirror is very large compare to the length change of the string. Therefore, the string deformation is magnified by the optical level for an accurate reading.

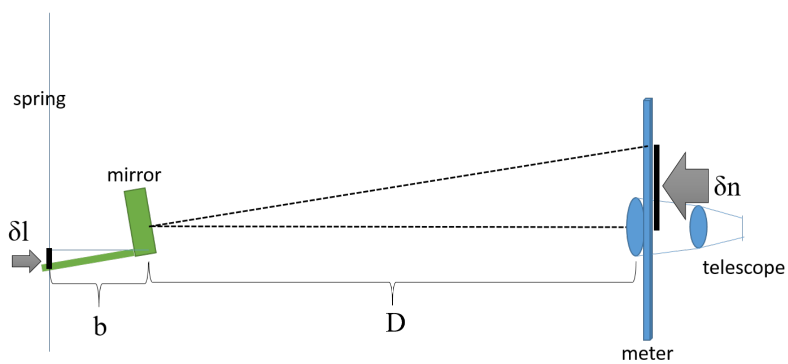


Figure 6: Demonstration for the optical level.

The optical level is demonstrated in Figure 6. D is the distance between the telescope and the mirror. b is the arm which is attached to the string. δl is the change in length of the string. δn is the change in reading shown on the meter. When the string changes, the optical level magnifies the change for us to read a more significant outcome from the meter. Typically, the distance between the telescope and the mirror is 1~2 meter. The arm of the mirror is about 7 cm. Then magnification gain for the change of the string is about 30~60. The change of the string can be calculated as $\delta l = \delta n \frac{b}{2D}$.

Summary
In this experiment, Young's modulus can be calculated as

$$E = \frac{8Mgl_0D}{\pi d^2 b \delta \bar{n}} \tag{3}$$

M is the mass of the disc weight, l_0 is the original length of the string, D is the distance between the telescope and the mirror, d is the diameter of the string, b is the length of the arm of the mirror, $\delta \bar{n}$ is the averaged change of reading for the string strech.

1.4 Experiment

1. Check if the string is attached firmly. There should be no friction between the weight carrier, the cylinder and the shelf. Check if the mirror is correctly placed: 2 front legs in the groove; 1 rear leg on the cylinder. Secure the mirror by hanging the security chain onto the shelf.

2. Check to see if you can read the meter from the telescope. You should see the black tics and numbers NOT red ones. If you cannot, ask your teacher.

3. Read the meter for the original length from the telescope. Add weights to the weight carrier one by one and read the meter each time. You should have 8 records (1 original reading and 7 readings for 7 weights).

4. Reduce the weight one by one and read the meter each time. You should have 8 records (1 original reading and 7 readings for 7 weights). Starting from 7 weights and measure it again.

5. Measure the diameter of the string using the micrometer. Measure the diameter at different places and collect 6 readings.

6. Measure the distance between the telescope and the mirror (from the meter to the groove).

7. Remove all weights and measure the original length of the string (from the cylinder to the fixed point).

8. Carefully remove the mirror and put it onto a blank paper. Press the mirror against the paper until you have 3 dots for the 3 legs of the mirror. Measure the distance between the middle of the 2 front legs and the rear leg.

1.5 Data

Here is a sample table for you to organize your data collection. To collect your reading on the optical level.

Measurement	Weight (kg)	Optical level Reading 1 (cm)	Optical level Reading 2 (cm)
0(orig.)			
1			
2			
3			
4			
5			
6			
7			

Table 1: Data collection for the optical level readings.

To collect your reading on the micrometer:

Unit (mm). zero reading for the micro meter:					
1	2	3	4	5	6

Table 2: Data collection for the diameter measurement.

Other data:

String original length (cm)	
Distance between the telescope and the mirror (cm)	
Length for the arm of the mirror (cm)	

Table 3: Other measurements.

1.6 Calculations and Results

Careful! The units must be consistent! It is recommended to convert everything to meter. But the simplest way is to convert micrometer readings to cm and keep other units.

Optical level reading For the readings of the optical level, we use the following method to calculate. First, take the average of the two readings. Second, calculate $\delta\bar{n_i}$ as $\delta\bar{n_1} = (\delta\bar{n_4} - \delta\bar{n_0})/4$, $\delta\bar{n_2} = (\delta\bar{n_5} - \delta\bar{n_1})/4$, $\delta\bar{n_3} = (\delta\bar{n_6} - \delta\bar{n_2})/4$ and $\delta\bar{n_4} = (\delta\bar{n_7} - \delta\bar{n_3})/4$.

Measurement	Opt. level 1 (cm)	Opt. level 2 (cm)	Average
0(orig.)			
1			
2			
3			
4			
5			
6			
7			

Change in length (cm)
Total Average, $\delta\bar{n}$

Table 4: Calculation for the length change of the string.

Calculate the standard deviation:

$$S_{\delta n} = \sqrt{\frac{(\delta\bar{n_1} - \delta\bar{n})^2 + (\delta\bar{n_2} - \delta\bar{n})^2 + (\delta\bar{n_3} - \delta\bar{n})^2 + (\delta\bar{n_4} - \delta\bar{n})^2}{4 \times (4-1)}}$$

Look up $t_{0.95}$ in the table and calculate $u_A(\delta n) = t_{0.95} S_{\delta n}$. $u_B(\delta n) = \sqrt{0.4/300}$ cm. Now calculate $u(\delta n)$ as:

$$u(\delta n) = \sqrt{u_A^2(\delta n) + u_B^2(\delta n)}$$

Then write your result as:

$$\delta n = \delta\bar{n} \pm u(\delta n)$$

String diameter We perform a similar calculation as the one above.

Actual diameter (zero reading + measurement) (mm)						Total Average
1	2	3	4	5	6	

Table 5: Calculation for the diameter measurement.

Calculate the standard deviation and then the $u_A(d)$.

$$S_d = \sqrt{\frac{(d_1 - \bar{d})^2 + (d_2 - \bar{d})^2 + \cdots + (d_6 - \bar{d})^2}{6 \times (6-1)}}$$

$$u_A(d) = t_{0.95} S_d$$

$u_B(d) = 2/3$ mm. Calculate $u(d) = \sqrt{u_A^2(d) + u_B^2(d)}$. Then write your results as:

$$d = \bar{d} \pm u(d)$$

Distance between the telescope and the mirror $u(D) = \sqrt{5}/15$ cm. No other calculation needed:

$$D = D \pm u(D)$$

length of the arm of the mirror $u(b) = 0.2/3$ cm. No other calculation needed:

$$b = b \pm u(b)$$

original length of the string $u(l_0) = \sqrt{2}/15$ cm. No other calculation needed:

$$l_0 = l_0 \pm u(l_0)$$

final result

Use Equation 3 to calculate the Young's modulus, E for the steel string. The uncertainty is calculated via the following equation:

$$u(E) = E\sqrt{\left(\frac{u(l_0)}{l_0}\right)^2 + \left(\frac{u(D)}{D}\right)^2 + \left(\frac{u(b)}{b}\right)^2 + 4\left(\frac{u(\bar{d})}{\bar{d}}\right)^2 + \left(\frac{u(\bar{\delta n})}{\bar{\delta n}}\right)^2}$$

Eventually,

$$E = E \pm u(E)$$