1 Viscosity

where	Building 49, Rm 303
when	Dec-9, 9:00-11:30 am

1.1 Objective

- Understand the concept: fluid and viscosity.
- Understand the viscosity in our daily life. Recall the content of terminal velocity.
- Explain the kinetic status when an object moving in a fluid.
- Measure the physical quantities and calculate the viscosity.

1.2 Theory and background

In the class, "fundamental of physics", we discussed the "terminal velocity" which is the final velocity of a falling object with the present of the air friction. A cat falls from a high building but the air friction drag it and balance the gravity and yields a relatively small terminal velocity for the cat to land safely (with a high possibility). Such friction exists widely in fluid which includes gas, liquid, plasmas and etc. For example, a person who swims in water feels the friction as the resistance force. On the other hand, let us consider the thickness of the fluid. Obviously, milk shake is thicker than coffee. And we can image that swimming in a pond of milk shake is way more difficult than swim in a pond of coffee. Such property which indicate the resistance of a fluid can be described by the physical quantity: viscosity. The typical examples of the fluids with different viscosity are shown in Figure 1.

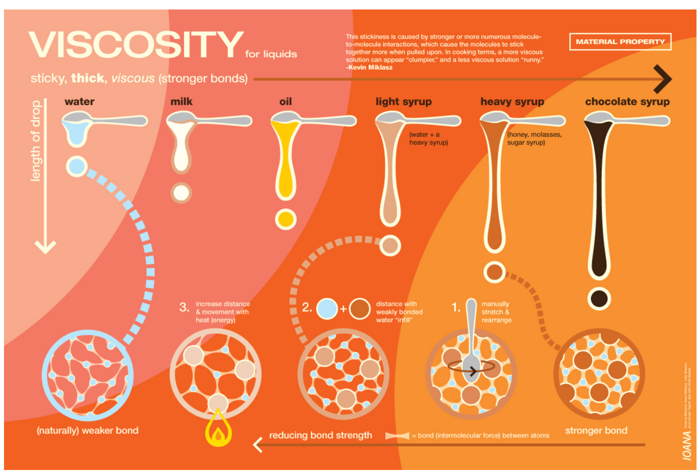

Figure 1: The viscosity of different liquids. (https://ediblesciencefaire.wordpress.com/viscosity-poster/)

It is also shown in the figure that the viscosity is originated from the inter molecular forces. Generally, a fluid with larger molecules and stronger molecule-molecule interaction has larger viscosity.

The resistance caused by the viscosity of the fluid is called viscous force, which can be calculated as

$$F = \eta\, A\, \frac{dv}{dy}\,. \tag{1}$$

The force is proportional to the viscosity, η, the contact area, A and the velocity gradient, dv/dy.

The physical detail of the fluid is very difficult. There is a special branch of physics: Fluid Dynamics for the detailed study and prediction of the behavior of fluid. The analysis of the fluid is very common in the industry. For example, the auto design as shown in Figure 2. When we see a car moving on the road, we cannot see the air flow. But in fact, as a fluid, the air flows in a complicated behavior as indicated by the stream lines in the figure. By the proper consideration of the flow, cars runs smoothly with fewer resistance which yields a lower cost of the gasoline.

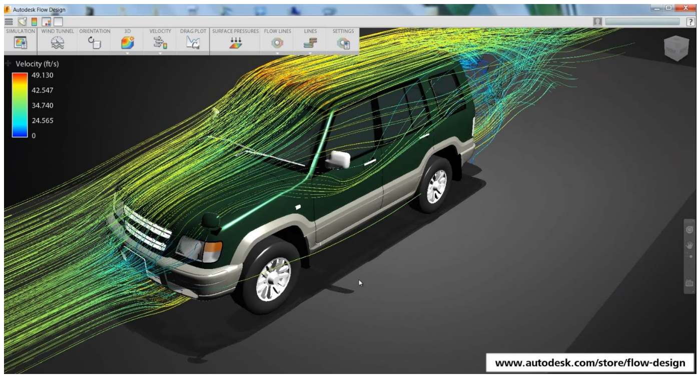

Figure 2: The demonstration of the flow of the car design. The software is autodesk. (https://www.youtube.com/watch?v=2RBOtd-Z8O8)

The faster the object motion in the fluid is, the more complicated the dynamics of the fluid is. Of course, the dynamics of air is even more important in the aerospace industry. As shown in Figure 3, there is a large vortex on the win tip of a moving aircraft called turbulence. Such flow is sometimes very harmful for its instability. When you travel in an airplane, you may hear the captain warns people stay safe with the safety belts on when he observes a strong turbulence.

Figure 3: Turbulence in the tip vortex from an airplane wing. The red smoke is used to show the air flow. (wikipedia)

Depends on the texture of the fluid, we classify the fluid into Newtonian and non-Newtonian fluids. What we will study in this lab, is one of the simplest Newtonian fluid which has a viscosity that can be described by Equation 1. The flow can also be classified into laminar and turbulent flow. From Figure 2, we can see the flow in the front part is the laminar flow while the flow behind the car is the turbulent flow. Again, we will study the laminar flow in this lab.

1.3 Experiment Design

In this lab we will measure the viscosity of castor oil. But it is difficult for us to follow the equation directly: by measuring F, A and dv/dy and calculating the viscosity since the viscosity only shows in the flowing fluid. Therefore, we use a small metal ball and drop it into the oil for the measurement. When the ball moves in the castor oil, it will perform a laminar motion in a Newtonian fluid. As a result, the viscosity equation can be simplified as

$$F = 3\,\pi\,\eta\,v\,d \tag{2}$$

η is the viscosity. v is the speed of the ball. d is the diameter of the ball. In addition, we don't measure the force directly. In stead we wait until the ball moves in a uniform motion. Then by applying the Newton's First Law, we immediately know that the net force exerted on the ball must be 0. Thus, the gravity is balanced by the viscous force (resistance drag) and the buoyancy.

$$mg = 3\pi\eta v d + \rho_{liq} g V \tag{3}$$

m, the mass of the metal ball; ρ_{liq}, the density of the fluid; g, the acceleration of gravity; V , the volume of the ball. When we assume the ball is a perfect sphere, then the volume must be $\frac{1}{6}\pi d^3$. It is also possible to find the density of the ball, ρ_{ball}. The speed can be calculated as l/t, the falling distance divided by the falling time (as the uniform motion). Then we can solve for the viscosity as

$$\eta = \frac{(\rho_{liq} - \rho_{ball}) t g d^2}{18 l}\,. \tag{4}$$

Equation 4 can be used to calculate the viscosity of a liquid. But in reality, the falling distance of the ball is usually smaller than expected. This is due to the container: when the liquid is pushed away by the ball, the container also exerts a viscous force to the liquid. Therefore, the falling distance, l should be corrected as $l(1 + 2.4d/D)$. D is the diameter of the container. In our experiment, the diameter of the ball, d is much smaller than the diameter of the container, D. So the correction is very small. By substituting the correction into the Equation, we have

$$\eta = \frac{(\rho_{liq} - \rho_{ball}) t g d^2}{18 l (1 + 2.4\frac{d}{D})}\,. \tag{5}$$

Based on the equation 5, we should measure the density of the liquid, falling time, the diameter of the ball, the diameter of the container and the falling distance.

Density of the liquid
We use the hydrometer to measure the density of the castor oil. The density of the ball is $7.670 \pm 0.014\, g \cdot cm^{-3}$.

Falling time
There are 2 marks on the container, we will use a stop watch to measure the falling time for the ball between the 2 marks.

The diameter of the ball
The ball is very small. So we use the "traveling microscope" to measure the diameter.

The diameter of the container
We use the vernier caliper to measure the diameter of the container.

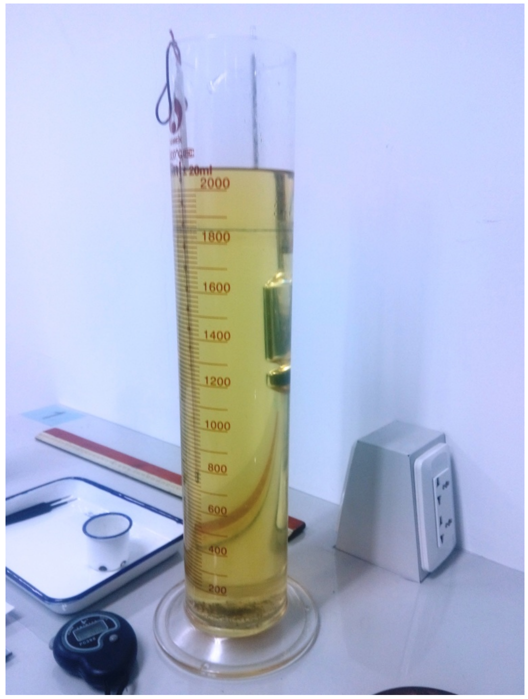

Figure 4: Experiment setup: castor oil and the container.

1.4 Experiment

1. Check if you have at least 6 balls on the small container on the table.
2. Measure the diameter of the container 6 times using the vernier caliper. Record your measurement.
3. Read the density of the liquid from the hydrometer.
4. Measure the distance between the 2 marks once. Record your measurement.
5. Take a ball to the traveling microscope and record the meter reading of the left and right side of the ball. The subtraction is the diameter of the ball.
6. Use the tweezers to move the ball on top of the oil surface. (The closer the better but **NO TOUCH!!!**)
7. Release the ball and the ball start to fall in the oil. Start the timer when the ball passes the first marker.
8. Stop the timer when the ball passes the second marker. Record the falling time.
9. Repeat from step 5. Record the 2 readings on traveling microscope and the falling times. Stop until you have 6 sets of measurement.
10. Finish the calculations and error analysis.

1.5 Data

You can use the following sample table to organize your result and calculate. The diameter of the container:

Measurement	Diameter, D (cm)	Averaged(D) (cm)
1		
2		
3		
4		
5		
6		

Table 1: Parameters of the container inner diameter.

Calculate the standard deviation for D:

$$S_D = \sqrt{\frac{(D_1 - Ave.(D))^2 + (D_2 - Ave.(D))^2 + \cdots + (D_6 - Ave.(D))^2}{6 \times (6-1)}} \tag{6}$$

and the A type uncertainty:

$$u_A(D) = t_{0.95} S_D \tag{7}$$

The B type uncertainty, $u_B = 0.003$cm. Then we can calculate the uncertainty as

$$u(D) = \sqrt{u_A^2(D) + u_B^2(D)} \tag{8}$$

liquid density, ρ_{liq} ($g \cdot cm^{-1}$)	room temp. (before exp.)	room temp. (after exp.)	falling distance, l (cm)

Table 2: Single measured quantities.

For the liquid density $u(\rho_{liq})$ is 0.028 $g \cdot cm^{-3}$ and the uncertainty of l is 0.1 cm.

#	left (mm)	right (mm)	diameter, d (mm)	Averaged(d) (mm)	falling time, t (s)	Averaged(t) (s)
1						
2						
3						
4						
5						
6						

Table 3: Falling ball: diameter and the falling time.

Calculate the standard deviation for d:

$$S_d = \sqrt{\frac{(d_1 - Ave.(d))^2 + (d_2 - Ave.(d))^2 + \cdots + (d_6 - Ave.(d))^2}{6 \times (6-1)}} \tag{9}$$

and the A type uncertainty:

$$u_A(d) = t_{0.95} S_d \tag{10}$$

The B type uncertainty, $u_B = 0.007$mm. Then we can calculate the uncertainty as

$$u(d) = \sqrt{u_A^2(d) + u_B^2(d)} \tag{11}$$

Calculate the standard deviation for t:

$$S_t = \sqrt{\frac{(t_1 - Ave.(t))^2 + (t_2 - Ave.(t))^2 + \cdots + (t_6 - Ave.(t))^2}{6 \times (6-1)}} \tag{12}$$

and the A type uncertainty:

$$u_A(t) = t_{0.95} S_t \tag{13}$$

The B type uncertainty, $u_B = 0.07$s. Then we can calculate the uncertainty as

$$u(t) = \sqrt{u_A^2(t) + u_B^2(t)} \tag{14}$$

Before you move on to calculate the viscosity, please convert all the lengths to cm. The apply Equation 5 for η.

Finally, the uncertainty of η is:

$$u(\eta) = \eta \sqrt{\left(\frac{u(\rho_{liq})}{\rho_{liq} - \rho_{ball}}\right)^2 + \left(\frac{u(\rho_{ball})}{\rho_{liq} - \rho_{ball}}\right)^2 + 4\left(\frac{u(d)}{Ave(d)}\right)^2 + \left(\frac{u(l)}{l}\right)^2 + \left(\frac{u(t)}{Ave(t)}\right)^2 + \left(\frac{u(D)}{Ave(D)}\right)^2} \tag{15}$$

1.6 Results and Summary

The room temperature before and after the experiment is ____________ $^\circ C$ and ____________ $^\circ C$.
The averaged temperature is ____________ $^\circ C$.
Using the hydrometer: The density of the oil is ____________ $\pm$ ____________ $g \cdot cm^{-3}$.
Using the meter: The falling distance is ____________ $\pm$ ____________ cm.
Using vernier caliper: The inner diameter of the container is ____________ $\pm$ ____________ cm.
Using the traveling microscope: The diameter of the ball is ____________ $\pm$ ____________ cm.
Using the stop watch: The falling time is ____________ $\pm$ ____________ s.
We know from other resources that:
The density of the ball is $7.670 \pm 0.014\, g \cdot cm^{-3}$.
The the acceleration of gravity is $980.1\, cm \cdot s^{-2}$.
The calculated viscosity of the castor oil is ____________ $\pm$ ____________ $g \cdot cm^{-1} \cdot s^{-1}$.
Answer the following questions:

- What could be the problem of using larger balls to do this experiment. List the possible issues and explain the reason.

- The same method can be used to measure other liquid. Suppose we want to measure the viscosity of water which has a much smaller viscosity. Which equipment/part/object should be upgraded? Why?

- Explain the speed, forces and the acceleration changes during its falling since it is released by the tweezers.