



DERIVATIVE AND QUANTITATIVE RESEARCH

Thinking about Theta: An Analysis of Time Decay, Early Unwind and Closeout Feedback Effects

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SUMMARY

- Near-the-money options experience a rapid decline in time value over the weeks leading up to the expiry date. A possible strategy to alleviate the impact of the time decay effect is to unwind the hedge prior to expiry.
- If an investor has a view that the underlying will not decline significantly over the period remaining then an early unwind should be considered.
- In this work, historical data is examined in order to determine whether or not a South African equity index exhibits any abnormal behaviour prior to expiry that would affect one's view on the likelihood of a hedge expiring in-the-money.
- Such expiry-related behaviour has been investigated in international markets in order to test for the existence of feedback effects where trading in the underlying by agents hedging derivative positions has an effect on underlying price movements.
- Two types of expiry-related effect in the local market were examined, namely abnormal returns for periods immediately prior to expiry, and price clustering where the underlying exhibits a higher probability of closing near a strike price on an expiry date.
- It is shown that the Swix40 index has historically delivered a higher average return over the week prior to expiry compared to non-expiry 1 week periods. However, further analysis is required in order to determine if a genuine closeout-related effect exists or if the result is a manifestation of other events that coincidentally occur around the closeout period.
- An analysis based on data for Top40 and Swix40 options is presented which reveals that while a higher likelihood of closing near a strike is associated with expiry dates and 1 week prior for a small measure of closeness (0.1% of spot), the proportions are fairly low, making it difficult to extract any predictive benefit that would assist in the early unwind decision.
- As a strategy to reduce exposure to the time decay of a hedge in a systematic manner, a hedged equity portfolio featuring a put option rolled well before the expiry date is considered. Historical performance is presented of representative examples in which options of various terms (6, 9 and 12) are rolled every 3 months and compared a strategy in which a 3 month option is held for the full term.
- It is shown that historically, an investor who wished to roll a hedge every 3 months achieved an appreciable increase in realised return for a moderate increase in risk by rolling a longer term option every 3 months compared to holding a 3 month option for the full term.

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1. INTRODUCTION

An investor holding an option is generally interested in how the value of the option responds to changes in one or more factors. A collection of sensitivity measures, “the Greeks”, is familiar to most users of options and consists of quantities that give an indication of option price sensitivity to factors such as changes in the underlying, volatility and the risk free rate. For many investors, the sensitivity of the option's value to changes in the underlying (i.e. delta) is of primary interest. For example, a fund manager using put options as insurance will be interested in the extent to which losses on an equity portfolio will be offset by gains on the value of the put option.

Although not as dramatic, the sensitivity of an option's value to the passage of time (i.e. theta) is also an important consideration. This is particularly true for near-the-money options which experience a significant decline in value as the expiry date approaches. Fund managers should therefore be aware of the time decay effect and the extent to which it affects the values of their option positions.

Generally, holders of option positions are familiar with the concept of the time value of an option as the premium in excess of the intrinsic value:

$$\text{Time Value} = \text{Option Premium} - \text{Intrinsic Value}$$

An important point to note is that the time value of an option is not constant across all underlying levels at a particular date. Instead, it varies based on the uncertainty around whether or not the option will expire in-the-money. One can obtain a good sense of the time sensitivity of an option across a range underlying levels by considering option value curves at various times to expiry. An example can be seen in figure 1 which shows the value of a put option struck at 100% as a function of the underlying for a selection of times to expiry ranging from 24 weeks to the expiry date itself. Figure 2 shows the corresponding time value as the difference between each pre-expiry curve and the expiry profile.

FIGURE 1: OPTION VALUE CURVES FOR SELECTED TIMES TO EXPIRY (WEEKS)

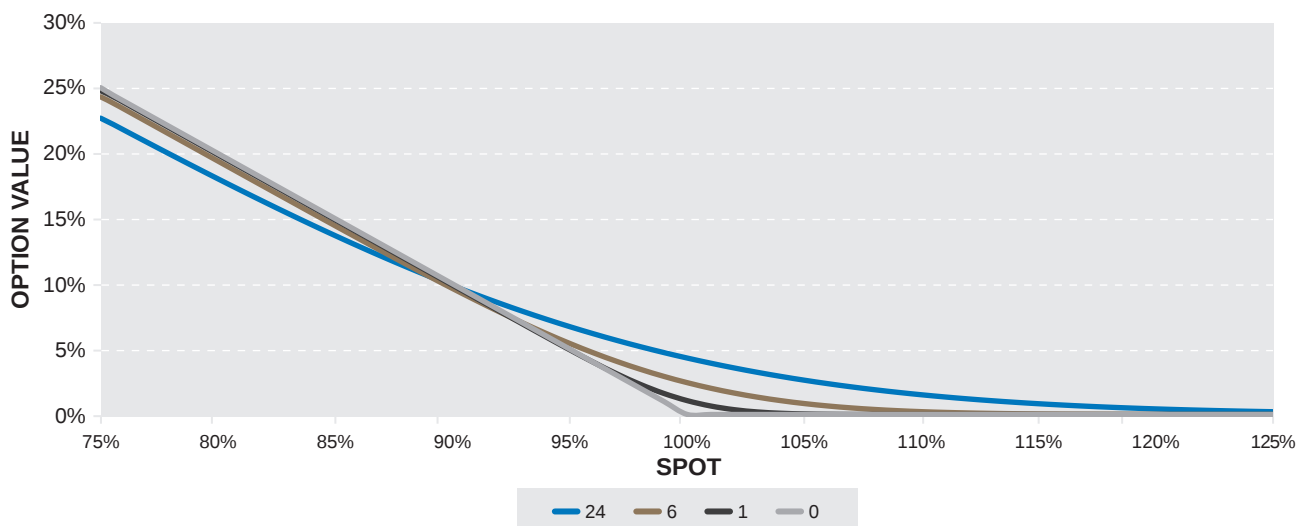
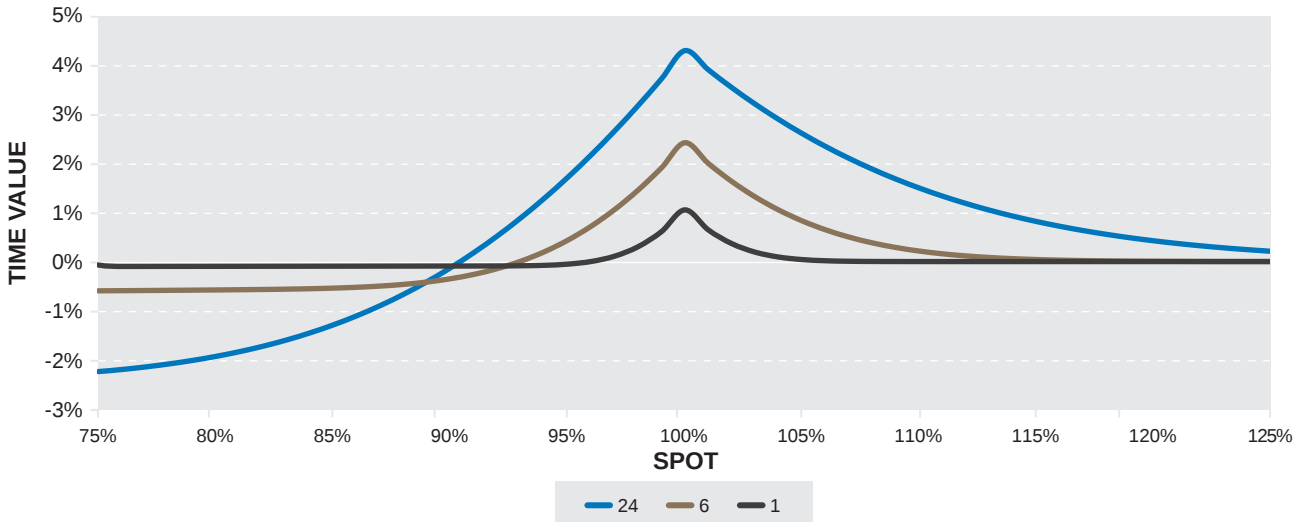


FIGURE 2: SPOT LEVEL DEPENDENCE OF TIME VALUE FOR SELECTED TIMES TO EXPIRY (WEEKS)

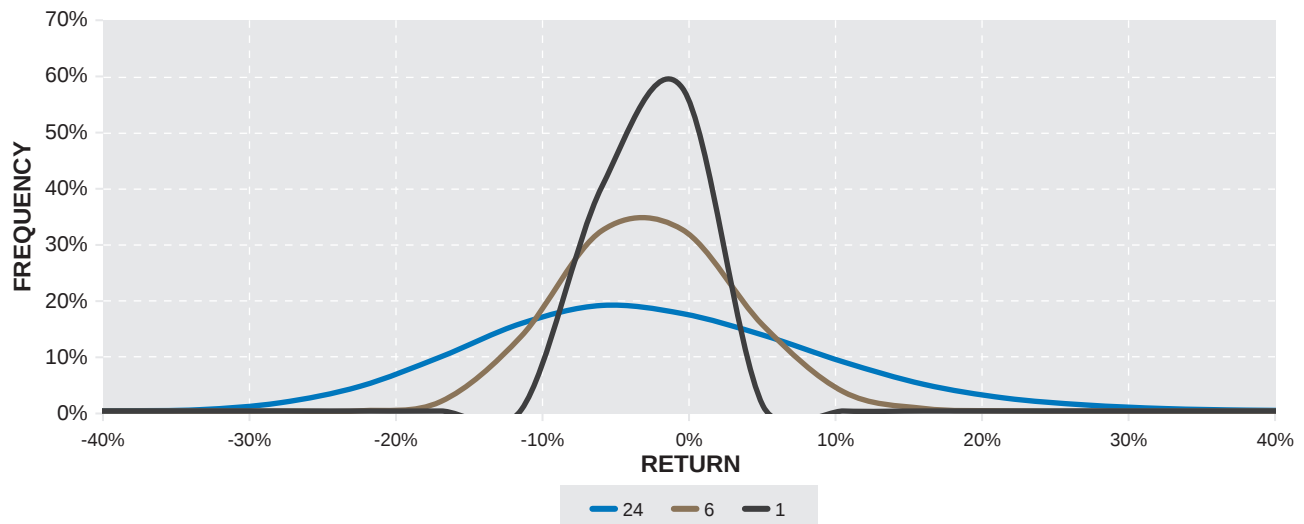


Some immediate observations on the variation of time value are the following:

- For all maturities, time value is greatest when the spot value is equal to the strike
- The range of underlying values leading to time value significantly in excess of zero decreases with decreasing time to expiry
- Deep in-the-money put options have negative time value

All of these features can be related to the uncertainty associated with an in-the-money option expiration. For deep in- and out-of-the money options, there is little uncertainty around the final payoff of the option and the current value of the option is approximately the present value of the anticipated payoff. For underlying levels far in excess of the strike, the anticipated value is zero and there is thus negligible value in excess of the intrinsic value of zero. For underlying levels well below the strike, there is a high probability that the expiry value of the option will be $Strike - Spot$, and the current value will therefore be approximately equal to the present value of this quantity. Since $PV(Strike) < Spot$, the current value of the option is less than its intrinsic value and one observes negative time value for deep in-the-money put options.

In contrast to options for which the underlying is far away from the strike, at-the-money options have a high degree of uncertainty around the possible value of the option at expiry. One finds that such options have the greatest time value. The reduction in the range of non-zero time values with decreasing time to expiry is as a result of the narrowing distribution of future values for the underlying. This is illustrated graphically in figure 3 which displays future simple (and hence log-normal) return distributions for terms from 24 weeks to 1 week prior to expiry. As the time horizon decreases, the range of probable returns for the underlying decreases. By way of a specific example, consider an underlying value at 115% of the strike price. In this case, the 24 week return distribution is such that there is still a chance that the underlying will be below the strike price at expiry and consequently, the option has appreciable time value (see figure 2). However, at 6 weeks to expiry, the future return distribution has narrowed to the extent that the likelihood of a decline from 115% to below 100% of the strike is very small. As a result, the time value for a put option with an underlying value equal to 115% of the strike price at 6 weeks prior to expiry is close to zero.

FIGURE 3: SIMPLE RETURN DISTRIBUTIONS FOR VARIOUS TIME HORIZONS (WEEKS)

The discussion above showed that when time value is positive, it decreases with decreasing time to expiry. An additional important factor to consider is the *rate* of time decay. Figure 4 shows the value of an at-the-money put option as a function of time to expiry (weeks), while the corresponding daily change in option value can be seen in figure 5. In this example, the daily decrease in time value increases in magnitude gradually between 24 and 5 weeks to expiry, ranging from 0.07% to 0.19%. Closer to expiry, the rate of time decay increases markedly with a 1% reduction over the final weeks.

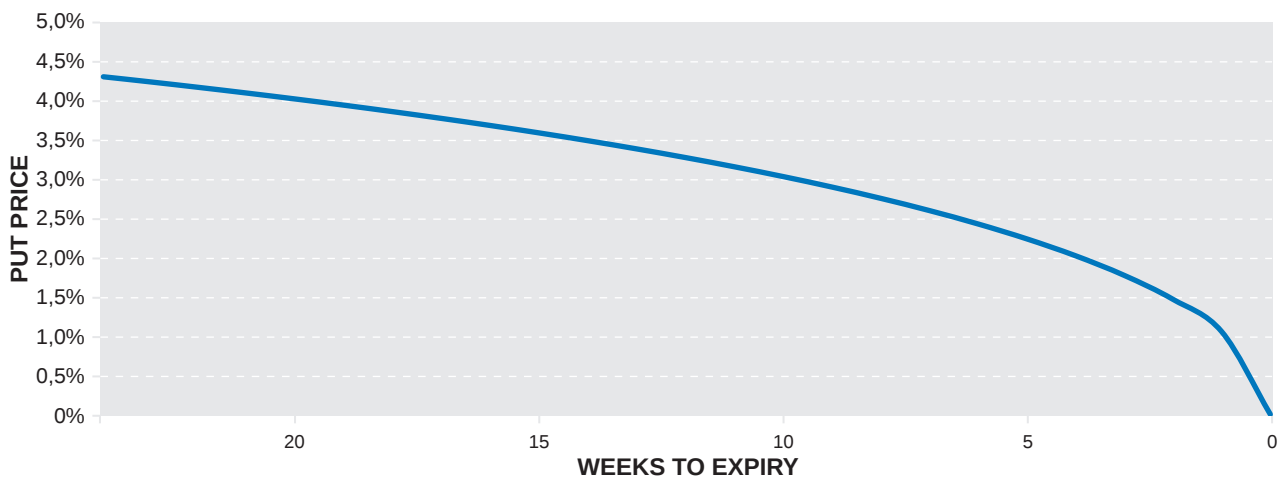
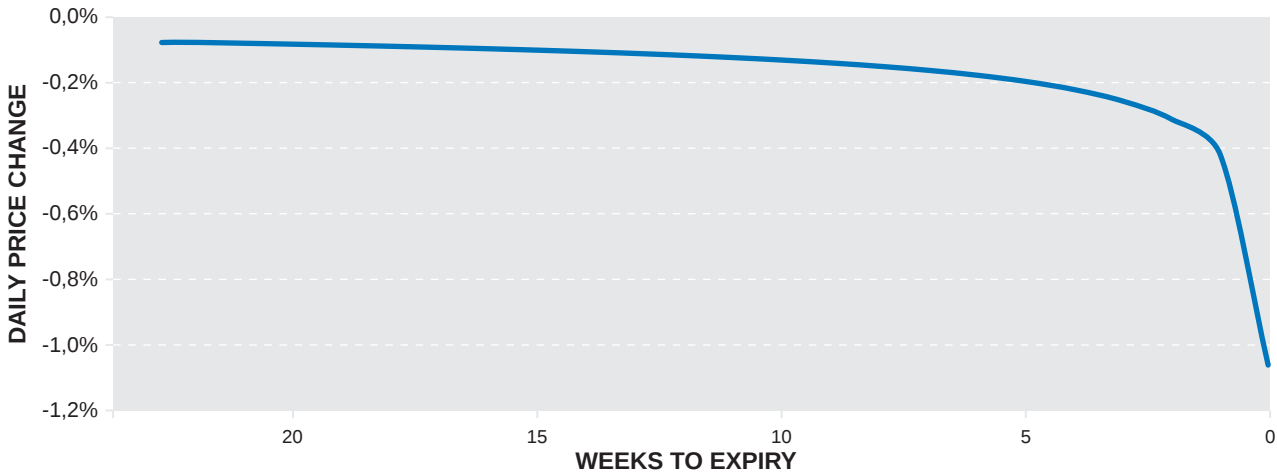
FIGURE 4: AT-THE-MONEY PUT OPTION VALUE

FIGURE 5: DAILY CHANGE IN OPTION VALUE



Based on the preceding discussion, it is clear that an investor holding a put option should be aware of the time value inherent in the position as the expiry date approaches and consider whether or not it is worthwhile holding the option until the expiry date. The choice is not, however, a straightforward one.

If an option has appreciable time value, it is because the final expiry value the option is uncertain and that there is a chance that the option could increase further in value. On the other hand, the investor may have reason to believe that the value of the underlying will not decline significantly over the period remaining until expiry, leading to erosion of the existing time value. In this case, the investor would consider unwinding the hedge before expiry in order to avoid the reduction in hedge value owing to the passage of time.

In this work we consider the early unwind of equity index put options as a strategy to alleviate the impact of time decay on the hedge value. In particular, we examine historical data in an attempt to answer the following questions:

- Does the underlying index exhibit any abnormal behaviour prior to and at expiry that would affect one's view on the likelihood of a hedge expiring in the money?
- Has there been any benefit historically from following a rolling hedge strategy in which the hedge is rolled prior to the expiry date of the option?

The remainder of the report is structured in line with the aforementioned aims. Section 2 examines the historical price movements of a representative South African equity index, the Swix40, in terms of realised returns over the weeks leading up to expiry. Furthermore, the existence of evidence of a tendency of an equity index to be at or near a strike price of a put option with significant open interest at expiry (an effect known as *price clustering*) is considered. Section 3 discusses the historical performance of selected hedged equity strategies in which the hedge is rolled prior to the expiration date, while section 4 concludes.

2. ABNORMAL INDEX RETURNS AT EXPIRY

2.1 Introduction

A fair amount of research has been devoted to the question of whether or not the introduction of derivatives on a particular underlying leads to feedback effects where trade in derivatives has an effect on the properties of the underlying itself. If such feedback effects exist, they might impart some degree of predictability to the future changes in the underlying and therefore provide insight into the merits of unwinding a hedge prior to the expiry date.

Different types of feedback effects have been examined, although not all are relevant to the early unwind decision. One strand of research has focussed on index derivatives and their effect on market volatility on expiration dates. As described by Bollen et al. (1999), cash settlement of index derivatives means that arbitrageurs holding positions on the expiry date need to liquidate their stock holdings at the same prices used to calculate the expiry values of futures and options. This leads to two main types of possible effects:

- Abnormal trading activity owing to simultaneous liquidation of positions by several arbitrageurs
- Abnormal stock market volatility if the unwindings are generally in the same direction

Bollen et al. (1999) survey investigations of the US, Japanese and Australian markets and present additional results for the Hong Kong futures exchange. It is found that while increased trading volume is observed on expiration dates, no evidence of increased stock market volatility is observed.

Another strand of research examines the price behaviour of *individual stocks* around expiration dates. One study of particular relevance to the decision of whether or not to unwind a hedge prior to the expiry date is that of Cinar and Vu (1987). In that work, the impact of option expiration on six underlying stocks was considered. It was found that in the case of four of the stocks, the return over the week prior to expiry was not significantly different to non-expiry weeks. One stock displayed a significantly positive expiry week return, while the remaining stock's expiry week return was significantly negative. The expiry week volatilities for all stocks were indistinguishable from non-expiry weeks.

Another interesting and relevant study on the expiration date behaviour of stocks is that of Ni et al. (2004) which presents evidence of stock price clustering on expiration dates. It was found that a higher proportion of stocks close within a certain range of an associated option strike on an expiry date compared to a non-expiry date. The study was based on data for all optionable stocks in the US between January 1996 and August 2002, and considered the percentage of stocks that closed within \$0.25 of a strike price for dates ranging from 10 trade days before expiry to 10 trade days after expiry. The analysis showed that more than 19% of optionable stocks close within the specified range of a strike price on expiration dates. For dates one week prior and one week after expiry, the proportion is less than 18%.

In order to explain the stock price clustering effect, Ni et al. (2004) refer to a model proposed by Avellaneda and Lipkin (2003) which describes how delta hedging activity of option market participants with net purchased option holdings can lead to underlying price clustering at strike prices. The basis of the model is the time sensitivity of option delta close to expiry. Specifically, Avellaneda and Lipkin (2003) derive the following result for both call and put options:

- $Spot > Strike \Rightarrow \frac{\partial \Delta}{\partial t} > 0$
- $Spot < Strike \Rightarrow \frac{\partial \Delta}{\partial t} < 0$

In the case of a market participant who has a *long* position in an option and who wishes to carry out a delta hedging strategy, the required delta neutral portfolio is

$$Option - \Delta S$$

In other words, the agent holds $-\Delta$ shares. If the spot is above the strike, the delta of the option increases as time passes. The increase in Δ implies a decrease in $-\Delta$ and the holder of the option sells stock, driving the spot price toward the strike price. Similarly, the *negative* time derivative for delta when the spot is below the strike as time passes the agent is required to buy stock, pushing the spot price up towards the strike price.

An additional potential consequence of delta hedging that is well known to market participants is the *impact on the volatility of the underlying*. Whereas the analysis of clustering around strike prices focussed on the way that delta changes with time, the impact on volatility is as a result of the way that delta responds to changes in the underlying, as measured by the option's gamma. A delta hedging agent is said to hold a long gamma portfolio if the delta of the option and replicating portfolio position increases as the underlying increases. An example of such a case is when the delta hedger is long a put option. In order to maintain delta neutrality, the agent would need to sell the underlying when the underlying increases. In contrast, the delta of a short gamma portfolio *decreases* as the underlying increases, meaning

that the underlying would need to be purchased in order to maintain delta neutrality. Intuitively, it is expected that long gamma positions will lead to a decrease in the volatility of the underlying, while short gamma positions will lead to an increase in volatility.

Theoretical models such as that of Frey and Stremme (1997) have been derived to quantify the effect of delta hedging on the volatility of the underlying and provide explicit formulae that show the relationship between gamma and underlying volatility. Generally, volatility of the underlying at a particular instant of time is expressed as

$$\sigma \times \text{scaling function}$$

In the above formula σ is constant and the scaling function has a dependence on the gamma of the delta hedger's net option position.

Empirical evidence for the impact of delta hedging on underlying volatility has been presented by Pearson et al. (2006). Based on data from the Chicago Board Options Exchange (CBOE) between 1990 and 2001, it was found that a negative relationship exists between long gamma positions and underlying volatility.

In this section, we aim to determine whether or not any index option feedback effects exist in the South African market that might lead to some degree of predictability in the behaviour of the underlying leading up to the expiry date, and assist in the early unwind decision. In particular, we consider two main questions:

- Does an equity index display abnormal returns over the weeks immediately prior to expiry?
- Is there any evidence of clustering at an expiry date?

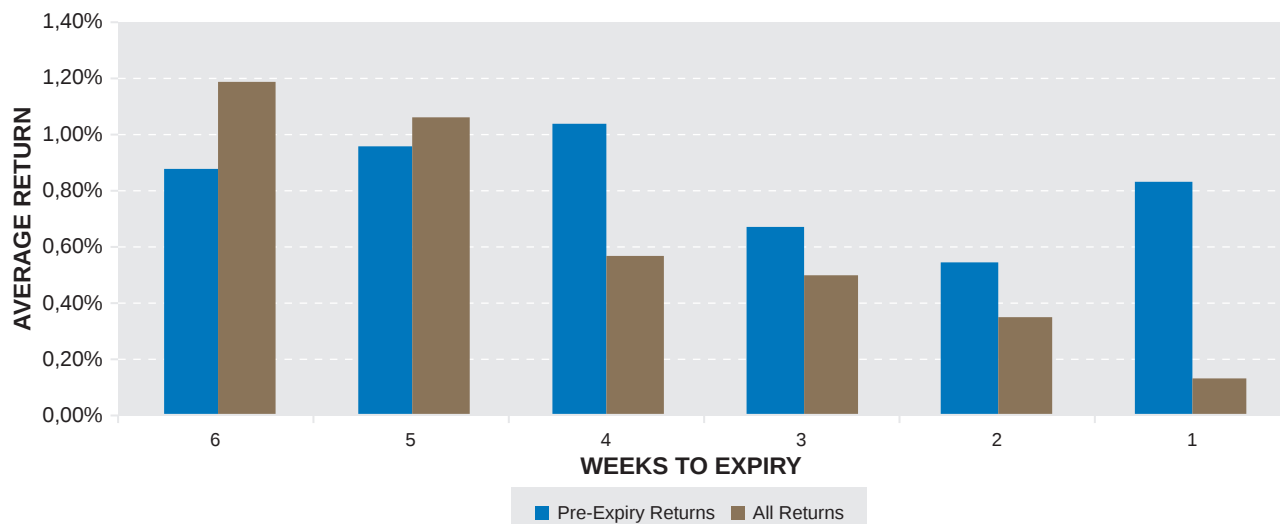
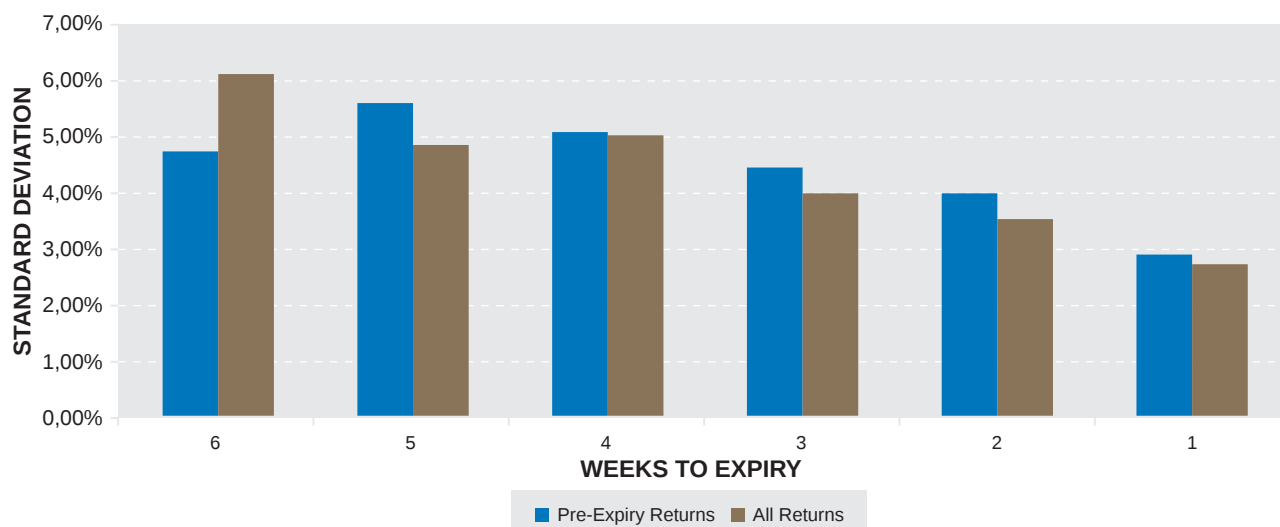
The types of effect that can be examined are limited by available data. For example, while it would have been interesting to consider the impact of net gamma on underlying volatility, the lack of detail on the holdings of delta hedgers made this impossible.

In the course of carrying out the analysis, the data set that was constructed made it straightforward to also consider the variation in the ratio of put to call options, and we include these results for interest in section 2.4.

2.2 Index Returns Prior to Expiry

In this section we examine returns on the Swix40 index leading up to closeout. The closeout dates that we consider are all quarterly dates from 15 December 2005 to 15 December 2016, giving 45 expiry observations. For each of these dates, returns of the Swix40 index over periods ranging from 6 weeks to 1 week prior to expiry were calculated. Also estimated were returns for the same terms across the entire dataset, excluding weeks immediately prior to expiry. A comparison of the properties of these returns with those observed leading up to closeout should give some insight into whether or not the index exhibits abnormal returns as a result of an impending expiry date.

The average returns and standard deviations for the terms considered are shown in figures 6 and 7 respectively. For an underlying with a positive drift rate, one would expect both the average return and volatility to decrease as the term decreases. This is indeed the pattern observed for returns across the entire dataset. The average return displays a decrease from 1.31% for a term of 6 weeks to 0.23% for 1 week, while the standard deviation decreases from 5.55% to 2.81%. In the case of returns immediately prior to expiry, volatility follows the same pattern as those of general returns with values that are not too dissimilar. However, the pattern of average returns is noticeably different. In particular, while the average return decreases as expected for terms between 4 and 2 weeks to expiry, *the 1 week pre-expiry average return is higher than those corresponding to both the 3 and 2 week terms*. Furthermore, it is also *higher than the average general 1 week return*.

FIGURE 6: AVERAGE RETURNS FOR SELECTED TERMS**FIGURE 7: STANDARD DEVIATION OF RETURNS FOR SELECTED TERMS**

The finding of an average 1 week pre-expiry return in excess of the general average 1 week return would seem to suggest that the Swix40 index experiences unusually high returns as a consequence of some effect related to futures and options expiry. However, it is possible that general factors that influence domestic equities positively could have occurred coincidentally at or near closeout dates. Figure 8 shows the 1 week pre-expiry return for all the closeout dates under consideration. An initial visual inspection confirms that the returns are generally positive with the obvious exceptions of dates in 2008, with the most recent positive return observed over the week prior to the 17 March 2016 expiry. An examination of the daily returns over that week reveals that the positive performance of the Swix40 could be linked to global events and not anything specifically related to closeout. In particular, as shown in figure 9, the Swix40 index yielded a return in excess of 1% on three of the days of the March 2016 closeout week. The following events were noted for these dates:

- 14 March 2016: Global markets were generally positive as a result of expectations of further stimulus measures by central banks.
- 16 March 2016: Expectation of no rate hike from the US Federal Reserve lifted markets further.
- 17 March 2016: Confirmation of the Fed decision to keep interest rates unchanged led to an emerging market rally.

It is therefore difficult to make a definitive statement on whether or not a closeout effect exists for equity index returns. Initial results do seem to indicate an abnormal average return in the week immediately prior to closeout, but further analysis is required to determine if this is simply the manifestation of other events that coincidentally occur around the closeout period.

FIGURE 8: SWIX40 1-WEEK PRE-EXPIRY RETURNS

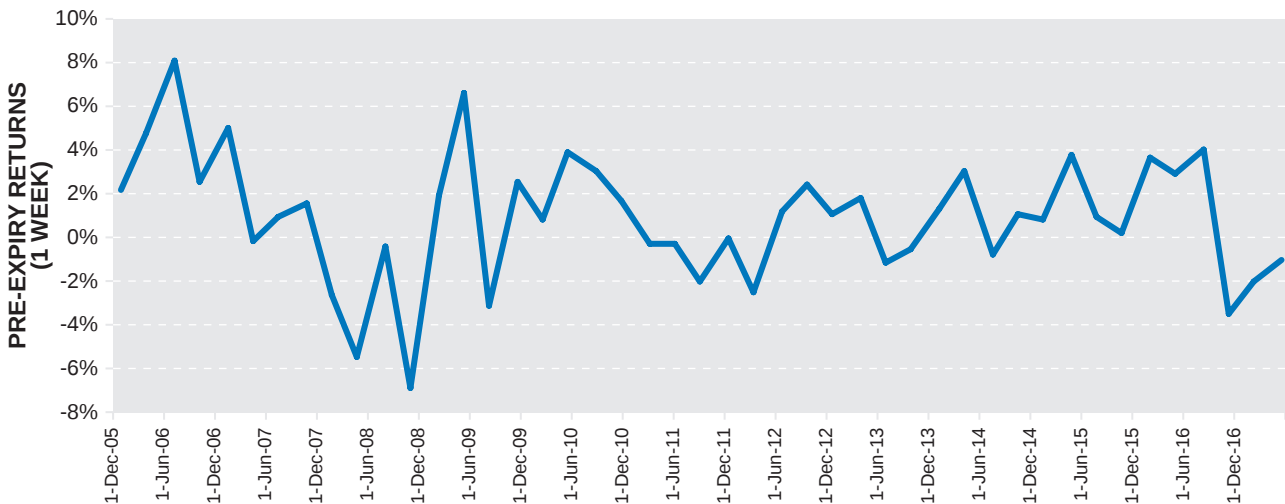
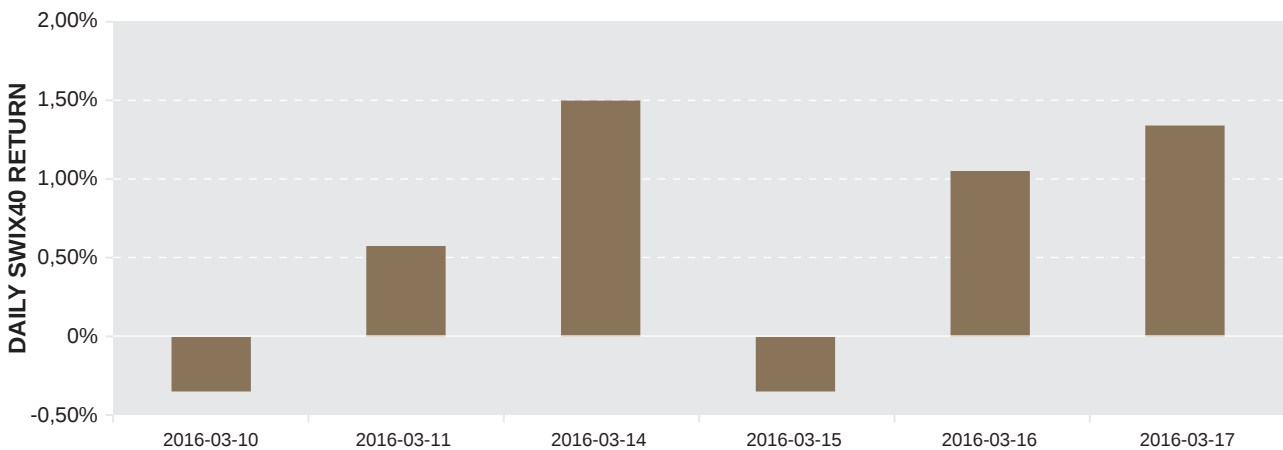


FIGURE 9: SWIX40 DAILY RETURNS FOR MARCH 2016 CLOSEOUT WEEK



2.3 Price Clustering on Expiry Dates

In section 2.1 it was shown how rebalancing of hedge portfolios in response to the change in delta with the passage of time may lead to the level of the underlying being driven towards the strike price. In this section we examine historical data to determine if such an effect exists in the South African market. Specifically, the main aim of the analysis is to determine if there is a higher probability of the Top40 futures spot level closing near an option strike on a closeout date compared to a non-closeout date.

The available data consisted of weekly values for parameters defining listed futures and options contracts between November 2008 and December 2016, retrieved from the JSE. Parameters included the expiry date, open interest, strike price (for options) and futures spot level. Since agents carrying out delta hedging strategies for positions in option contracts on the Swix40 index often use Top40 futures contracts, Swix40 options were considered in addition to Top40 options, with Swix40 option strikes and open interest numbers converted to equivalent Top40 values.

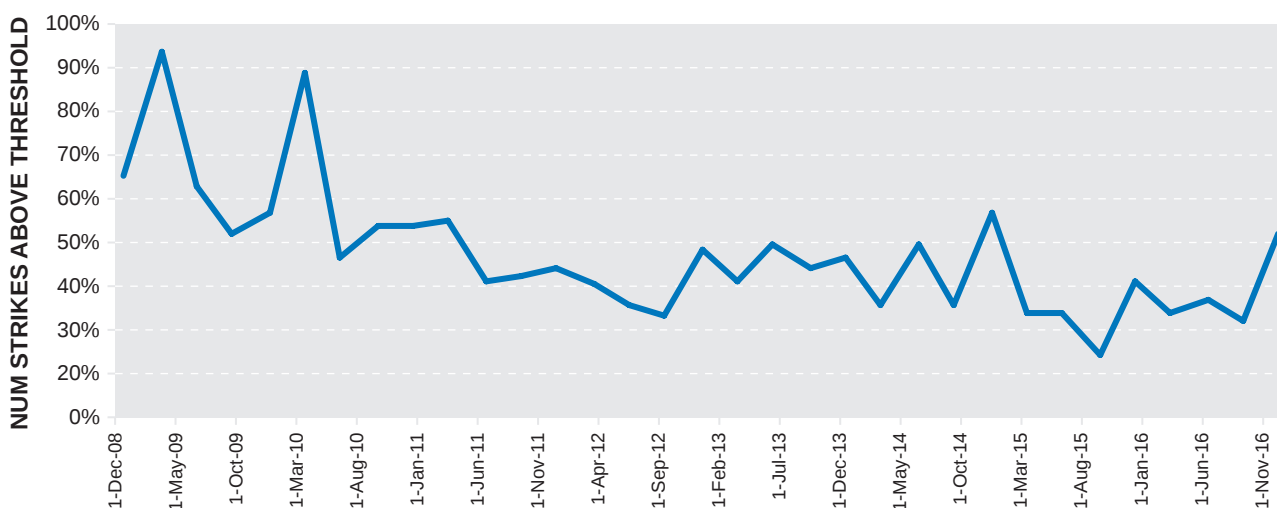
The analysis was carried out as follows:

- On each closeout date the set of near Top40 and Swix40 option contracts with open interest above a specified threshold was determined.
- For each of these contracts the distance between the strike price and the prevailing futures spot level was determined, and the minimum recorded.
- The number of dates for which the minimum distance was less than the specified threshold was calculated and expressed as a proportion of the total number of closeout dates.

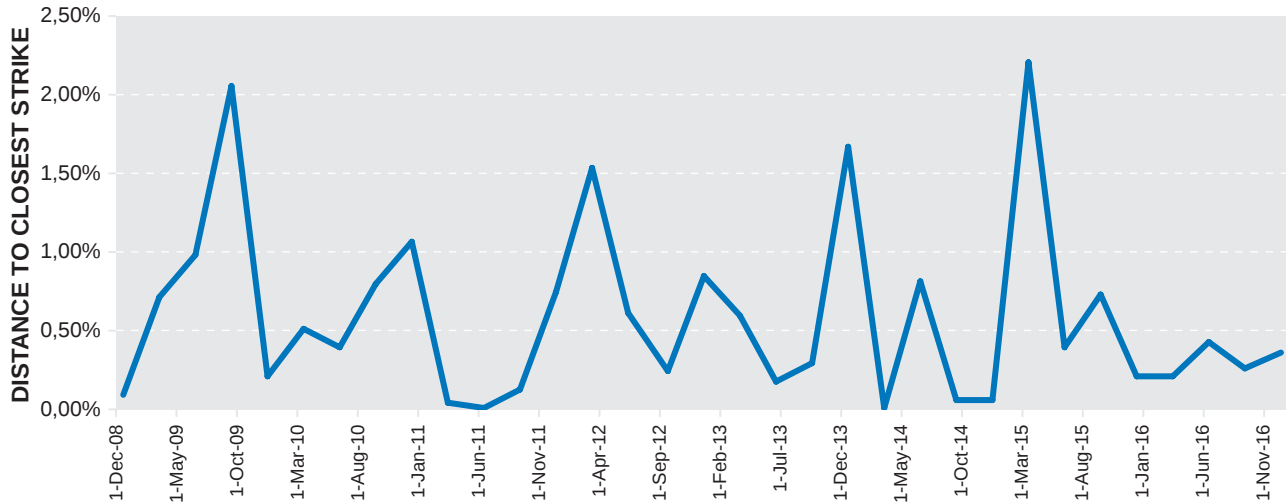
The analysis was also carried out for selected weeks prior to expiry in order to determine whether or not option expiry is associated with a higher probability of the underlying ending close to a strike price.

The open interest threshold was set to 2000 Top40 contracts. Figure 10 shows the number of eligible strikes for each closeout date. There were 93 different strikes in March 2009, but for the most part, the number has ranged between 30 and 50. The number of strikes is important to consider since there will be a higher chance of the spot closing near a strike if the density of strikes is high.

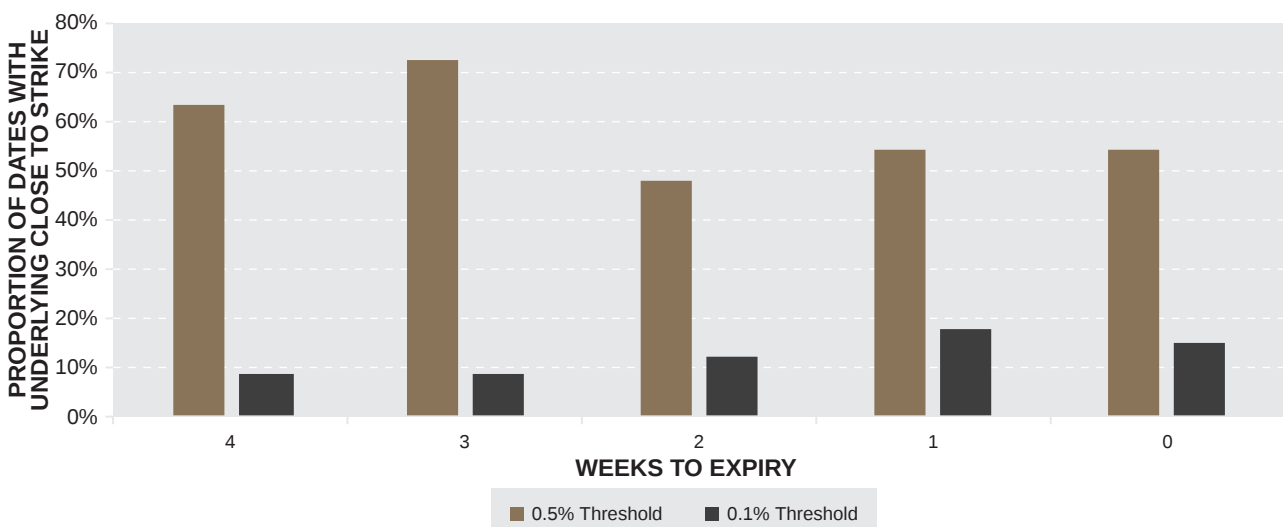
FIGURE 10: STRIKE DENSITY (NUMBER OF DIFFERENT NEAR CONTRACTS WITH OPEN INTEREST ABOVE THE THRESHOLD)



The distance between the futures spot level and the closest strike for each closeout date is shown in figure 11. The values are expressed as a percentage of the futures spot level and are seen to range between 0.01% and 2.22%.

FIGURE 11: DISTANCES TO CLOSEST STRIKE AT EXPIRY

We consider now the proportion of pre-expiry and expiry dates for which the Top40 futures spot price was close to an option strike. Figure 12 summarises results for two distance threshold values, namely 0.5% and 0.1%. In the larger threshold case, the proportions range between 48.5% and 72.7%. Interestingly, the proportions are highest for terms of 4 and 3 weeks, indicating no abnormal clustering at the strike date. Furthermore, it is possible that longer terms are associated with more available strikes since contracts are rolled or unwound as the expiry date approaches. As a result, the likelihood of the underlying being within range of a strike on a date well before expiry is higher owing to a higher strike density. Reducing the threshold to 0.1% results in a reduction of the proportions of all terms. In this case, the expiry date and the date 1 week prior to expiry produce the highest number of clustering events, hinting at some degree of closeout-linked price clustering. However, the given that the proportions are fairly low and not much in excess of longer term dates, it would be difficult to extract any predictive benefit that would assist in the early unwind decision.

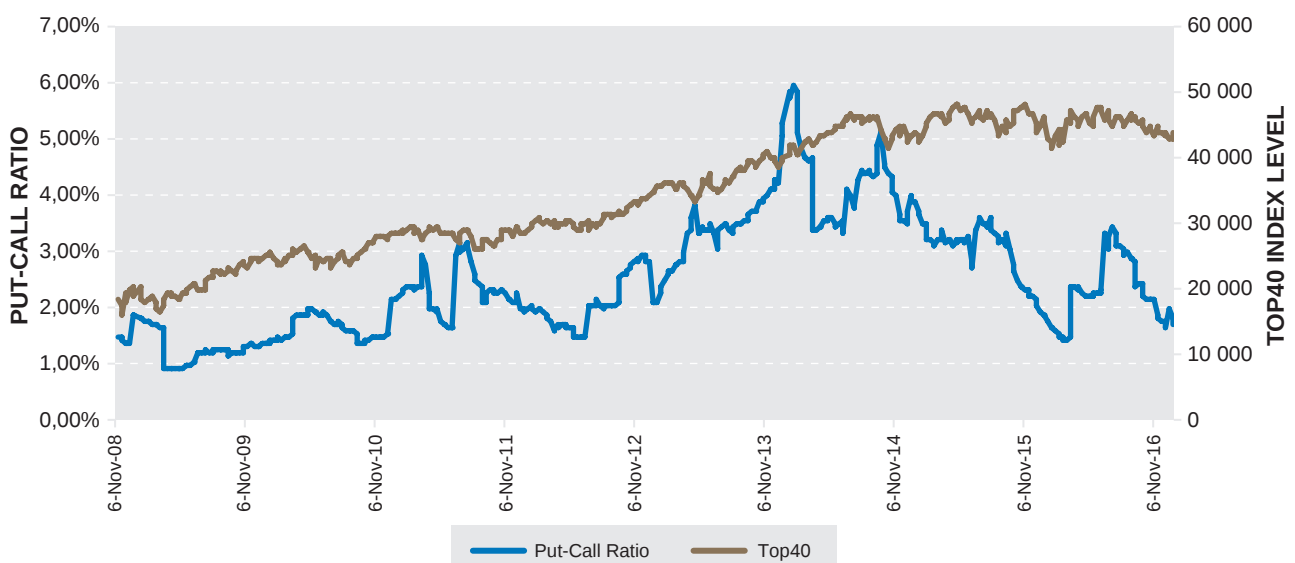
FIGURE 12: PROPORTION OF DATES WITH UNDERLYING CLOSE TO A STRIKE

2.4 Ratio of Put to Call Options

We include for interest the historical series of the ratio of put to call option nominal value for the South African market based on the same data used for the analyses presented in sections 2.2 and 2.3. In certain markets, put and call options are often used to gain exposure to the underlying in order to take advantage of a bullish or bearish view. In particular, put options are used to obtain a short exposure to the underlying, while call options offer a geared long exposure. The ratio of put to call options could thus be interpreted as a measure of the bullish or bearish sentiment of the market.

In South Africa the situation is different, with call options often used in structures to offset the cost of a long put option position. Thus the interpretation of the ratio of put to call options as an indicator of bullish or bearish sentiment is less valid. However, it is still interesting to consider how these values have changed over time in the local market, and are presented in figure 13. Also shown for reference is the level of the Top40 index. The ratio has ranged between 0.88 and 5.97 with an average of 2.5. One sees that the high point was reached in January 2014 during a sustained bull run. It is possible that with the equity market trending upwards, demand for capped strategies was reduced with a preference for uncapped structures such as outright put options and put spreads. Since November 2014, with the market moving sideways, the put-call ratio has declined steadily to be in the region of 2 in December 2016, hinting at increased use of capped strategies.

FIGURE 13: PUT-CALL RATIO (COMBINED TOP40 AND SWIX40)



3. ROLLING HEDGES WITH EARLY UNWIND

3.1 Introduction

In section 2 the behaviour of an equity index in the period leading up to closeout was examined in order to determine if any patterns exist which would suggest that the unwind of a hedge with appreciable time value would be beneficial. A key point to note is that the period over which feedback effects were considered was close to the expiry date of the hedge. In this section we discuss the historical performance of a rolling hedge strategy in which the hedge is unwound well before the expiry date. The aim of such a strategy is to maintain a hedged equity position at all times, but reduce exposure to the time decay of the hedge in a systematic manner.

As representative examples we present scenarios in which an investor wishes to roll the hedge position every three months, but avoid the time decay that would be associated with holding a three month option for the full term. In particular, we compare strategies in which at-the-money options of various terms (3, 6, 9 and 12) are rolled every 3 months. As an example, consider the 9 month strategy. In this case, the hedge would be unwound 6 months prior to its expiry date and replaced with another 9 month hedge.

As with any strategy there are trade-offs to consider and these are discussed in section 3.2 for the case of a rolling put option. In particular, the performance of the early unwind strategies relative to the full term strategy as a function of the underlying return for the quarter are summarised. The aim is to obtain an understanding of the kinds of scenarios for the quarter in which the early unwind strategy would deliver a superior value at closeout. Section 3.3 presents the historical performance of hedging an equity portfolio with put options of various terms rolled every 3 months. There are clearly many other combinations that could be considered, such as different roll terms as well as put spreads with varying degrees of downside risk. The historical performance of these variations generally follows the same patterns described in this report and are not discussed. However, interested readers are welcome to contact Peregrine Securities for any required detail on alternative strategies.

Note that the terms selected are such that the roll date always coincides with closeout. It is also possible though to consider the historical performance of strategies in which the hedge is rolled a specified number of weeks prior to closeout. This would allow one to study the performance of strategies in which hedges are rolled closer to the expiry date, and with a term similar to the full term options. However, a complication when comparing the historical return performance of such strategies with that of the full term hedge is that they are rolled on different dates and are thus struck at different underlying levels. We present the historical performance for the 3 month roll of a 4.5 month hedge separately in section 3.4.

3.2 Relationship between Underlying Return and Strategy Profit

While rolling a put option prior to its expiry date reduces the effect of time decay on the position, it also leads to a different value profile relative to a shorter term option held until expiry. In other words, the relationship between the underlying and the value of the hedge is different, as discussed in section 1 (see figure 1). In particular, for underlying values close to the strike price the delta of the put option can be of a significantly lower magnitude, leading to a hedge that provides less offset to declines in the underlying. An additional important factor to consider is the initial cost of the hedge, with longer term options being generally more expensive. We therefore consider similar graphical representations of a put option's value to those shown in section 1, but incorporate the initial cost of the option and consider the *profit* of the strategies at a roll date as a function of the return on the underlying over the quarter.

Figure 14 shows the profit across underlying returns at a roll date based on an arbitrarily selected set of option pricing inputs. In particular, the initial at-the-money volatility was assumed to be 20% at the inception and roll dates. The relative profit of the strategies is displayed more clearly in figure 15 which shows the difference in profit between each early unwind strategy and the full term hedge. It is evident that there is a range of underlying returns around the initial strike for which the early unwind of a longer dated put options delivers a superior profit relative to that which is held until expiry. For more extreme underlying returns, the full term strategy is preferable. Furthermore, increasing the term of the put option increases the relative profit for underlying values close to the strike as the exposure to time decay is reduced. However, for underlying returns less than -5%, the shorter terms deliver higher profits.

FIGURE 14: PUT OPTION PROFIT AS A FUNCTION OF UNDERLYING RETURNS

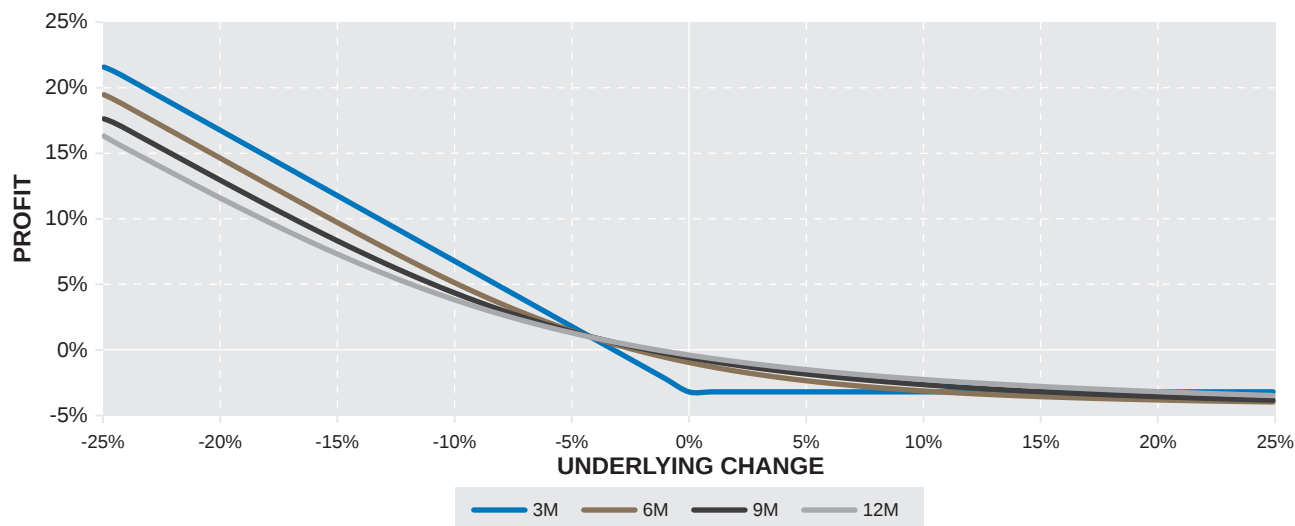
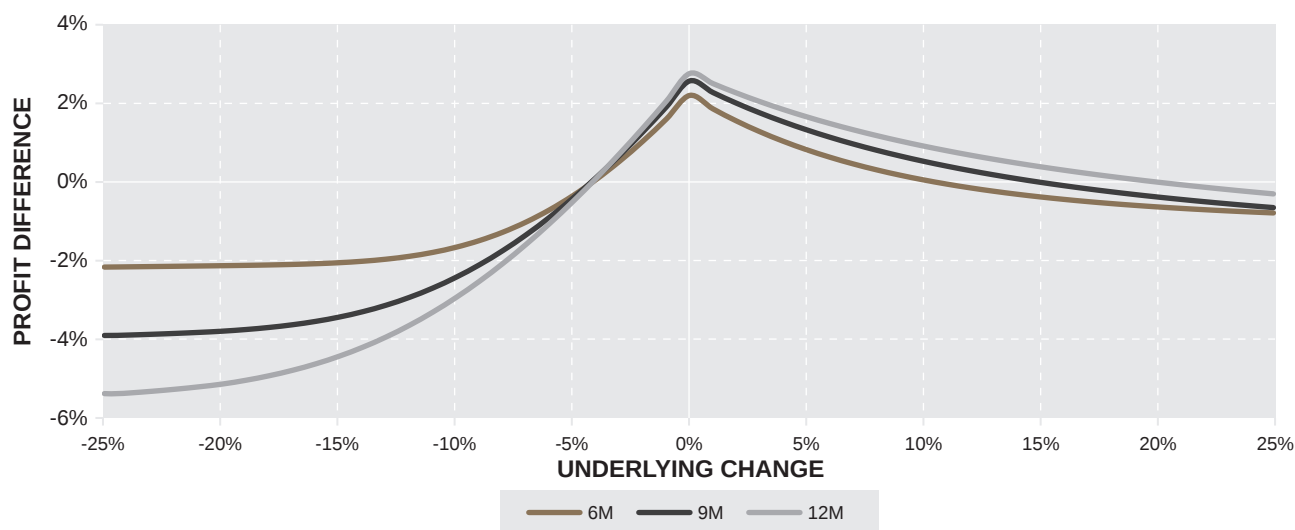
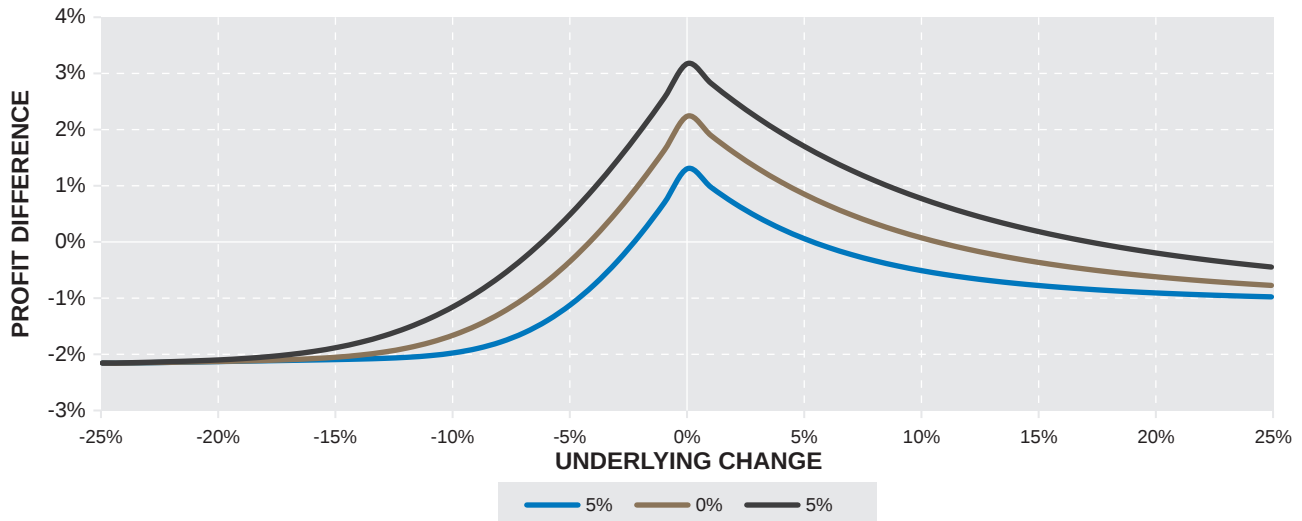


FIGURE 15: PUT OPTION RETURNS RELATIVE TO FULL TERM HEDGE



In practice it is likely that the prevailing volatility at the roll date will be different to that at the inception date. This will clearly have an impact on the profitability on hedges rolled prior to expiry, as their value is still dependent on the volatility of the underlying. Since put options increase in value with an increase in volatility, the relative profit of an early unwind roll is positively related to volatility. This is demonstrated graphically for the case of a quarterly roll of a 6 month put option in figure 16. For a 5% parallel shift in volatility, profit of the strategy relative to the full term hedge increases or decreases. Note that for underlying returns less than 15%, the sensitivity to changes in volatility is much less as there is less uncertainty around the future expiry value of the option.

FIGURE 16: SENSITIVITY OF 3 MONTH ROLL/ 6 MONTH OPTION STRATEGY TO VOLATILITY



3.3 Historical Performance of Early Unwind Strategies

In section 3.2 it was shown that by following a rolling put option strategy with an early unwind of a long-dated option, one is introducing the potential for higher return if the underlying is within a certain range of the strike, along with higher risk in the form of higher exposure to negative returns on the downside. Furthermore, the strategy is also exposed to the change in volatility between the inception and roll dates. In this section we consider how these properties would have affected the cumulative return and statistical properties of a hedged equity strategy over the period between September 2005 and December 2016.

Figure 17 compares graphically the cumulative returns of the strategies under consideration. It is immediately apparent that strategies aimed at reducing exposure to option time decay have outperformed the full term strategy considerably. Furthermore, increasing the term of the rolled put option has led to an increase in realised cumulative return. However, it is important to consider the associated risk. Consequently, in table 1 we display annualised average return and standard deviation values. It is clear that the increase in average returns is not for free, and there is a concomitant increase in standard deviation. Fund managers would therefore need to decide to what extent they are willing to expose their equity portfolios to more downside in order to have the potential of enhanced returns relative to a full term hedge.

FIGURE 17: HISTORICAL CUMULATIVE RETURN COMPARISON OF ROLLING PUT OPTION STRATEGIES

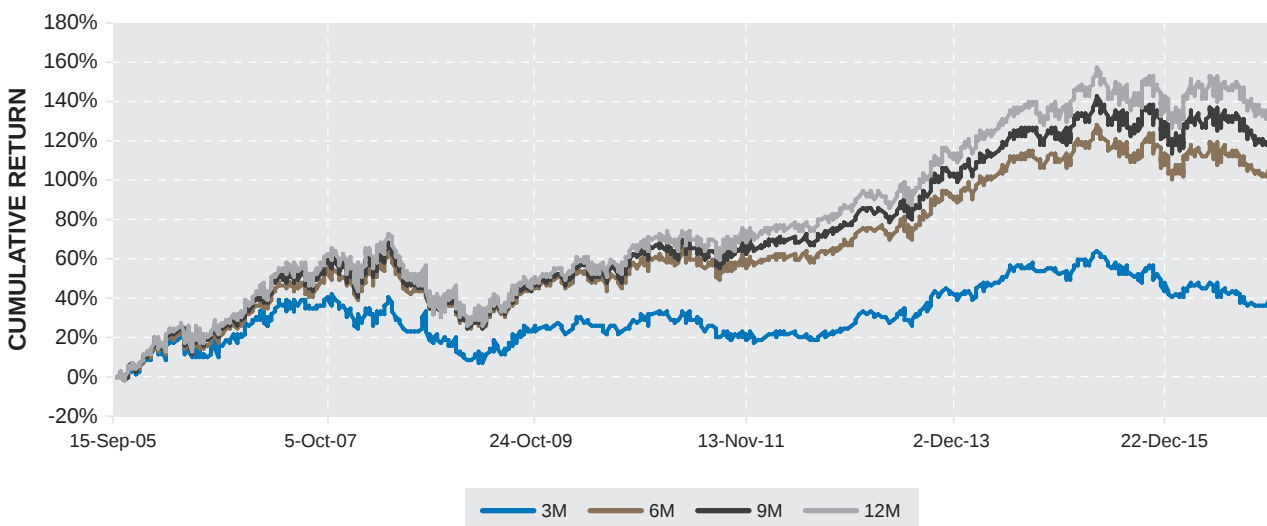


TABLE 1: RETURN STATISTICS OF HEDGED EQUITY STRATEGIES

Option Term	Annual Mean	Annual Vol
3M	2.94%	12.86%
6M	6.61%	14.28%
9M	7.33%	15.36%
12M	7.96%	16.35%

The fact that the early unwind strategies have outperformed the full term strategy suggests that for on each roll date the profit associated with the unwind of a long dated option has been on average higher. This is confirmed for the case of the 6 month strategy by Figure 18 which shows the profit of the early unwind strategy in excess of that of the full term strategy for each closeout date. The average of this series is 0.89% with 33 out of 45 closeout dates yielding a positive relative profit.

FIGURE 18: EXCESS PROFIT OF EARLY UNWIND OF 6M OPTION RELATIVE TO FULL TERM (3M) HEDGE

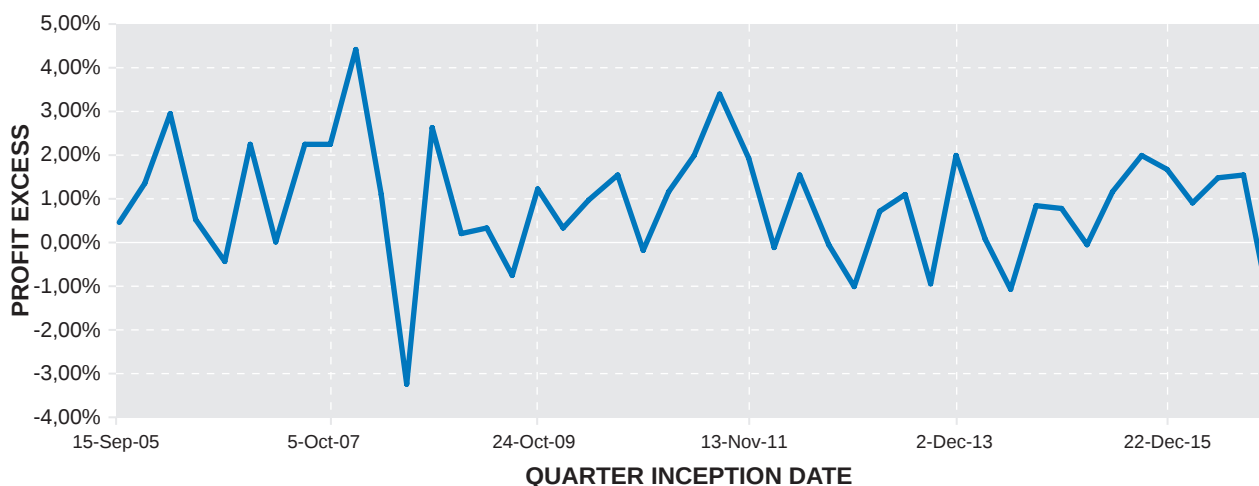


FIGURE 19: CUMULATIVE RETURNS FOR QUARTER STARTING ON 20 DECEMBER 2007

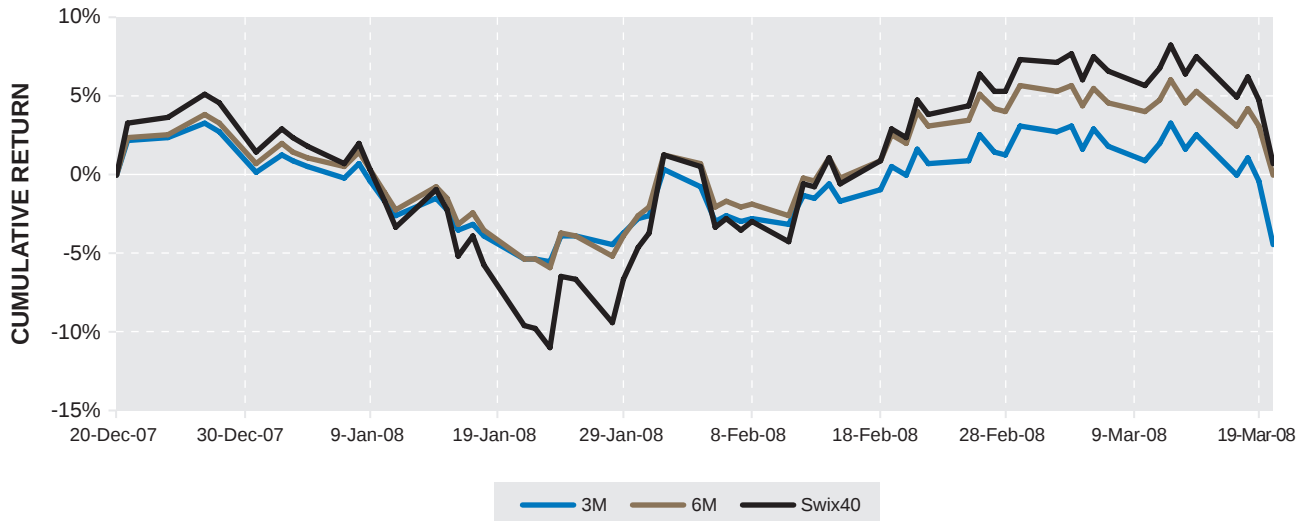
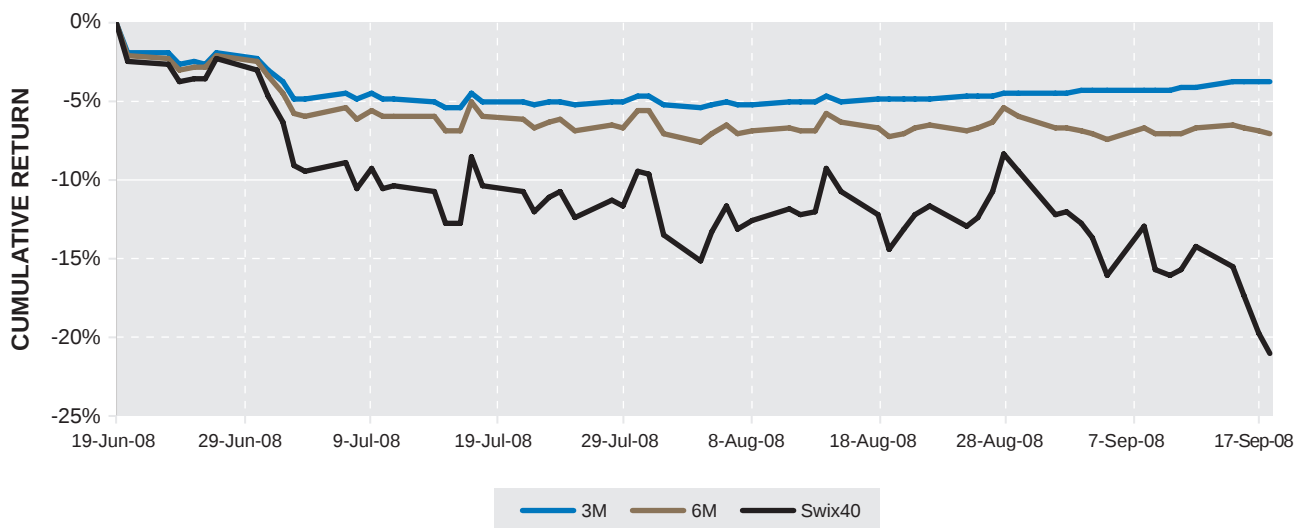


FIGURE 20: CUMULATIVE RETURNS FOR QUARTER STARTING ON 20 JUNE 2008



3.4 Early Unwind on a Non-Closeout Date

As mentioned previously, it is possible to consider a strategy in which the hedge is rolled at a date that is different to the roll of the full term hedge. An example is rolling into a near contract 6 weeks prior to expiry so that in effect one is rolling a 4.5 month option every 3 months. The historical cumulative return of such a strategy based on an at-the money Swix40 put option is shown in figure 21, with annualised average return and standard deviation given in table 1.

FIGURE 21: CUMULATIVE RETURN OF HEDGED EQUITY WITH 3 MONTH UNWIND OF 4.5 MONTH PUT OPTION

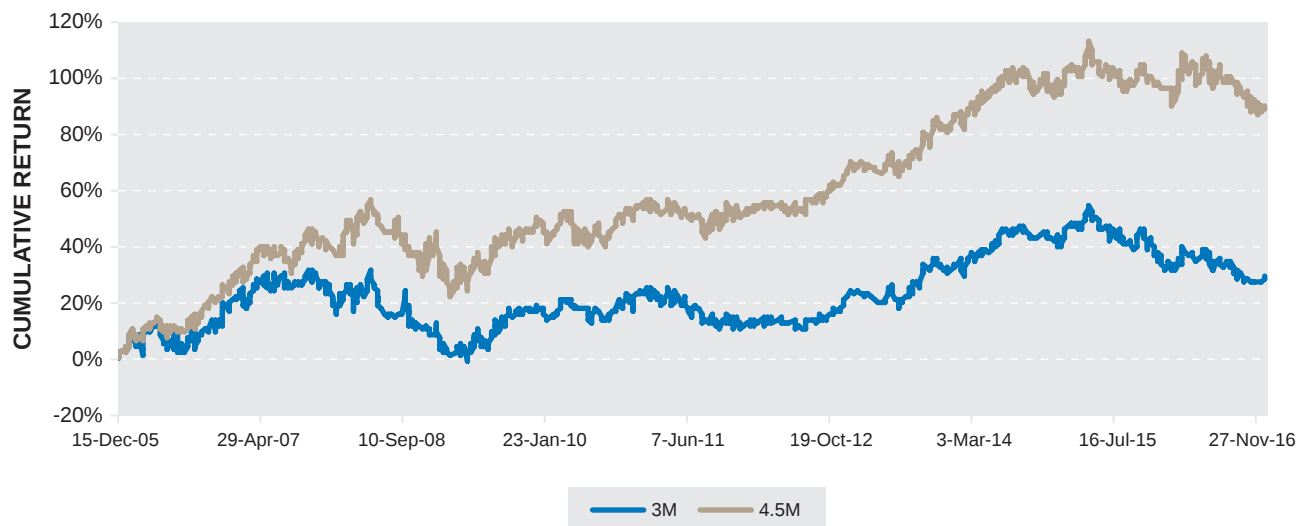
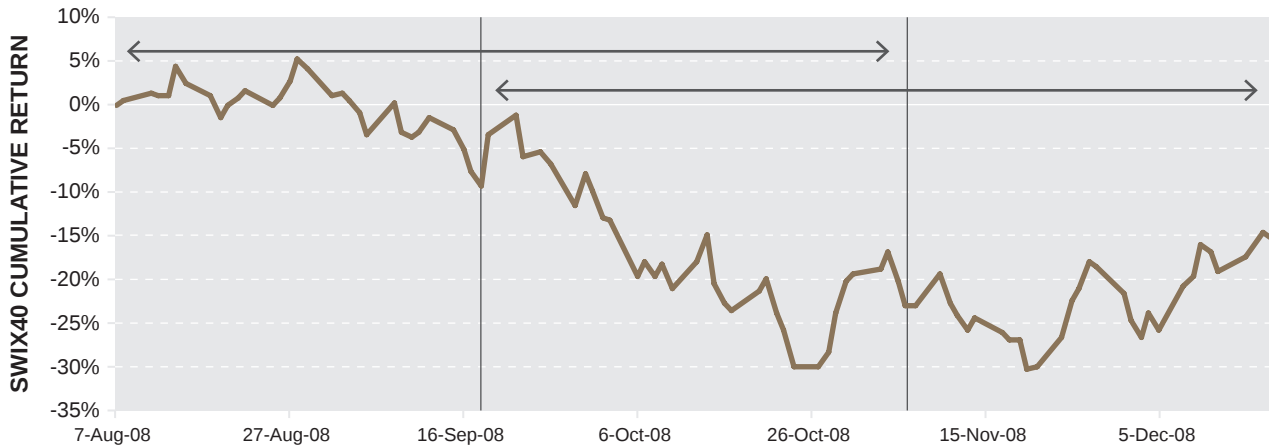


TABLE 2: 3 MONTH AND 4.5 MONTH STRATEGY RETURN STATISTICS

Option Term	Annual Mean	Annual Vol
3M	2.94%	12.86%
4.5M	6.02%	13.23%

As before, rolling a longer term option has historically delivered a significantly superior realised return for a moderate increase in risk relative to a 3 month hedge held for the full term. However, it is difficult to make a direct comparison to the performance of a rolled 3 month option as the roll dates are different, leading to potentially different strike levels and final payoff or unwind values that are dependent on the underlying return over different periods. To illustrate this point, we consider the quarter beginning on 18 September 2008. A strategy rolled 6 weeks prior to this date on 7 August 2008 and unwound 6 weeks prior to the following closeout date yielded a hedge profit of 14.9%, while the 3 month strategy with inception date 18 September yielded a hedge profit of 1.59%. A large part of this difference is due to the return on the underlying over the two periods under consideration. Between 7 August 2008 and 5 November 2008, the return on the Swix40 index was -20.1%, while between 18 September 2008 and 18 December 2008 it was -6.5%. The cumulative performance of the Swix40 between 7 August 2008 and 18 December 2008 is shown in Figure 22 with the hedge sub-periods indicated.

FIGURE 22: SWIX40 CUMULATIVE RETURN



4. CONCLUSION

Near-the-money options experience a rapid decline in time value over the weeks leading up to the expiry date. A possible strategy to alleviate the impact of the time decay effect is to unwind the hedge prior to expiry. However, it is not guaranteed that such a strategy will deliver a superior return as there is the possibility the option expiring sufficiently far in-the-money to result to a higher payoff. On the other hand, if the investor has a view that the underlying will not decline significantly over the period remaining then an early unwind should be considered.

In this work, historical data was examined in order to determine whether or not a South African equity index exhibits any abnormal behaviour prior to expiry that would affect one's view on the likelihood of a hedge expiring in-the-money. Such expiry-related behaviour has been investigated in international markets in order to test for the existence of feedback effects where trading in the underlying by agents hedging derivative positions has an effect on underlying price movements. Two types of expiry-related effect in the local market were examined, namely abnormal returns for periods immediately prior to expiry, and price clustering where the underlying exhibits a higher probability of closing near a strike price on an expiry date.

It was found that the Swix40 index has historically delivered a higher average return over the week prior to expiry compared to non-expiry 1 week periods. However, further analysis is required in order to determine if a genuine closeout-related effect exists or if the result is a manifestation of other events that coincidentally occur around the closeout period.

In terms of price clustering, international research has shown that rebalancing of hedge portfolios of net bought option positions in response to the change in delta with the passage of time may drive the underlying towards the strike price. An analysis based on data for Top40 and Swix40 options revealed that while a higher likelihood of closing near a strike is associated with expiry dates and 1 week prior for a small measure of closeness (0.1% of spot), the proportions are fairly low, making it difficult to extract any predictive benefit that would assist in the early unwind decision.

As a strategy to reduce exposure to the time decay of a hedge in a systematic manner, a hedged equity portfolio featuring a put option rolled well before the expiry date was considered. Historical performance was presented of representative examples in which options of various terms (6, 9 and 12) were rolled every 3 months and compared to a strategy in which a 3 month option is held for the full term. It was found that historically, an investor who wished to roll a hedge every 3 months achieved an appreciable increase in realised return for a moderate increase in risk by rolling a longer term option compared to holding a 3 month option for the full term.

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