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Expected Time Value Decay of Options: Implications for Put-Rolling Strategies

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Abstract

Assuming the underlying asset price remains constant, previous studies show that the time value of an option decays gradually at a rate that accelerates over time and peaks at the expiration date. Thus, a significant portion of time value is lost in the four weeks leading up to expiration. This paper shows the time value of currently at- or near-the-money options should be expected to decay at a rate that decreases over time. The time values of options that are currently deep-in- or deep-out-of-the-money are expected to initially rise and then resume the normal decay pattern.

Keywords: options, time value decay, portfolio insurance

JEL Classifications: G11, G12, G19, D46

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1. Introduction

The premium of an option can be decomposed into two components: intrinsic value and time value. The intrinsic value is the dollar amount that can be received if the option is exercised now. The time value, also known as the time premium, is the option premium minus the intrinsic value.

The time value of an option is important for practitioners. For out-of-the-money options, the entire premium is the time value. Nelson (1997) indicates that time value is the profit margin that large trading firms hope to earn from writing options. He argues that option writers often write out-of-the-money options and receive the time value, hoping that the options expire worthless. Nelson (1997) states, "a majority of options do expire worthless, and the sellers pocket the premium unharmed" (p. 50).

Determining the value of an option is the topic of countless journal articles. Introductory finance textbooks often devote one or more chapters to consider option trading and pricing. In addition, a reader can find a multitude of books devoted exclusively to this topic. Yet, our understanding of the time value of options remains incomplete. In particular, it is frequently suggested that as time passes the time value of an option follows a decay pattern similar to the one shown in Figure 1. This means the time value drops gradually at a rate that accelerates over time and peaks at the expiration date.

This characterization of time value decay is frequently expressed in books, web sites and journal articles. Labuszewski and Sawa (1988) state, "Many traders are also aware that options experience 'accelerated' time value decay—they fall more and more sharply as the time until expiration becomes very short" (p. 54). Baird (1993) writes "option prices tend to decline as expiration approaches, and indeed, decline more rapidly the closer the expiration is" (p. 43). He adds, "the time value decay is slowest with long trading life remaining in the option" (p. 43).

The Montréal Exchange (2007) equity options manual states, "an option is a wasting asset in the sense that part of its value, the time value, decreases with the passage of time" (p. 11). This description is accompanied by a chart similar to Figure 1 showing the time value decaying with the passage of time at an increasing rate. The manual adds, "all other factors being constant (which is rarely the case, but a useful assumption) the time value portion [...] declines at an accelerating rate such that perhaps over one-half of the premium is 'lost' in the last one-third of the option's life" (p. 11). Similarly, the discussion of index option concepts on the Chicago Board Options Exchange (2007) web site displays a graph similar to Figure 1 to explain the time value decay in Leaps.

Many studies suggest trading strategies based on the shape proposed in Figure 1. Figlewski, Chidambaran and Kaplan (1993) analyze the performance of protective put strategies for portfolio insurance. They state, "so far we have looked only at one-month options. Since option time value decays to zero at expiration, and the most rapid decay occurs in the last month, perhaps these strategies are more costly than similar ones using longer-dated puts" (p. 54). Figlewski, Chidambaran and Kaplan

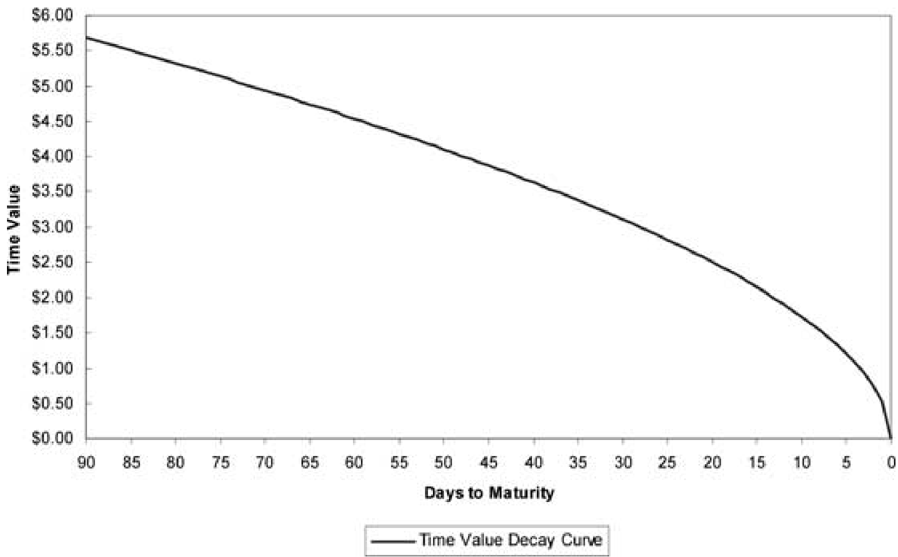

Figure 1

Perceived time value curve of a call option

The time value is obtained as the call premium computed using the Black and Scholes (1973) option-pricing model minus the intrinsic value. Time to expiration ranges from 90 days to 0 days. The other parameters are the volatility of the asset return, $\sigma = 25\%$, the risk-free rate, $r = 6\%$, the exercise price, $K = \$100$, and the price of the underlying asset, $S = \$100$.

(1993) suggest that an alternative would be to buy two-month puts for the purpose of holding them for only one month, after which the puts can be sold to avoid the biggest loss in time value.

Chidambaran and Figlewski (1995) write, "on the other hand, an option experiences the fastest time decay as it gets close to maturity, which can make buying many short-term puts costly in terms of time decay" (p. 47). They add, "an interesting variant on the protective put strategy is to buy two-month puts, but roll over at the end of the first month, when the puts still have a month to run. This avoids losses associated with holding the option during its period of maximum time decay" (p. 47). Similarly, the Montréal Exchange (2007) reference manual states that "positions with less than six weeks to maturity ought to be examined closely for resale value. That is, it is often best to resell call options with less than six weeks to maturity and roll forward (i.e., buy) to options with a more distant expiration month" (p. 8).

The pattern of time value decay shown in Figure 1 and expressed by previous studies is accurate under the assumption that the option remains at-the-money over its entire life. This assumption is unrealistic as the price of the underlying asset changes stochastically. Even if an option is purchased at-the-money the probability of occupying another at-the-money position at any future point in time is very small. Thus, the probability that the option remains at-the-money over its whole life is almost negligible. The characterization of time value decay portrayed in Figure 1, therefore, is misleading and a more realistic and practical characterization is needed. In addition, for a complete understanding of time value decay, options that are purchased either out-of-the-money or in-the-money must be considered.

Given the stochastic nature of the underlying asset, we propose the expected time value curve is a better indicator of how the passage of time affects option premiums. Each point on the curve represents the traditional expected time value of the option measured at a unit of time that precedes the expiration date. It suggests the time value of an option purchased at- or near-the-money should be expected to decline at a decreasing rate. Investors should expect large time value losses shortly after purchasing at- or near-the-money options. In contrast, the common perception is that time value decays at a rate that increases over time, implying that most of the time value decays shortly before expiration. Further, our findings suggest that if an option is deep-out-of-, or deep-in-, the-money, other decay patterns can be expected.

The expected time value curve is a good approximation of the time value of a portfolio of options that share similar characteristics, such as remaining time to expiration, volatility and the growth rate of the underlying asset. In addition, the expected time value curve is experienced, on average, if we frequently purchase the same option under similar conditions. Therefore, the results of this study are important for option portfolio managers and for practitioners that use options on a regular basis for hedging or speculation. For example, option writers should be interested to know that a large portion of the time value they obtain from selling options is earned shortly after selling at-the-money options rather than during the last three weeks before expiration.

Furthermore, utilizing the expected time value curve enables us to derive important conclusions for put-rolling portfolio insurance strategies. This paper shows that the expected time value costs of holding a single long-term put to protect a portfolio over a planning horizon are equivalent to the expected time value costs of purchasing a series of short-term puts that span the same horizon. Previous studies suggest that the single-put strategy is cheaper. Our result has significant practical implications.

Bookstaber and Langsam (1988), Choie and Novomestky (1989), Brooks (1989), Benet and Luft (1995), Tian (1996), and Ghosh and Arize (2003) agree that put options can be used to protect a portfolio against a drop in the market. However, Tian (1996) lists several reasons that prohibit the extensive use of puts for portfolio insurance. In particular, he suggests it may not be possible to use index options to insure a portfolio over a long period of time as listed options have a short maturity, usually less than three months. In addition, Tian (1996) points out that supply limits, poor liquidity and the regulations that limit the size of positions constrain the wider use of index and other puts for insurance activities. More recently, Bharadwaj and Wiggins (2003) report that the supply of put options in the S&P 500 Long-term Equity Anticipation Securities (LEAPs) is significantly lower than demand. Fung, Mok and Wong (2004) find that supply limits and poor liquidity are serious problems in the Hang Seng Index Options Market.

The problems indicated by Tian (1996) were particularly severe prior to the introduction of new index options in the 1990s. As a result, many studies, for example Bookstaber and Langsam (1988), Choie and Novomestky (1989), Figlewski, Chidambaran and Kaplan (1993) and Chidambaran and Figlewski (1995), propose put-rolling strategies to insure a portfolio over a long period of time. Instead of using a single long-term put for insurance, a put-rolling strategy involves trading a series of short-term puts. These studies, however, suggest that put-rolling strategies are more costly than strategies that use a single long-term put. Arnott (1998) estimates the costs of rolling monthly options are 12.68% of the portfolio's value (1% per month compounded monthly) while the costs of a one-year insurance program are approximately 5%.

Bookstaber and Langsam (1988) argue that under stable market conditions the conventional strategy of rolling options leads to higher expected costs. They observe an option with six months to expiration costs less than twice the cost of a three-month option. They conclude the higher expected costs are due to the value of the reset opportunities provided by the put-rolling strategy. This conclusion implies, in the absence of reset opportunities, the expected cost of the two strategies should be the same. This paper proves this conclusion by showing that the expected time value costs of a single long-term put are equivalent to the expected time value costs of a series of short-term puts. In this paper's analysis, the investor does not use the reset opportunities.

Figlewski, Chidambaran and Kaplan (1993) and Chidambaran and Figlewski (1995) propose rolling puts before expiration can reduce the loss of time value. Their propositions are based on the assumption that time value decays as suggested in Figure 1. Under this assumption, the major portion of time value is lost during the last month leading to expiration. However, the results of Figlewski, Chidambaran and Kaplan (1993) do not demonstrate a significant improvement when the options are rolled before expiration. This paper shows that improvement should not be expected, given that a large portion of time value decays shortly after buying the options, rather than shortly before expiration.

Our results are derived by Monte Carlo simulation. Boyle (1977) suggests the use of simulation to price options. In particular, simulation is widely accepted as a proper method to examine portfolio insurance strategies. Previous studies that use simulation include Zhu and Kavee (1988), Brooks (1989), Figlewski, Chidambaran and Kaplan (1993), Chidambaran and Figlewski (1995), and Tian (1995).

2. The valuation model and time value

We use the Merton (1976) jump-diffusion model to price options for three reasons. First, previous studies, for example, Chernov, Gallant, Chysels and Tauchen (2003) and Andersen and Andreasen (2000), suggest that the Merton (1976) model performs better than the Black and Scholes (1973) option-pricing model (hereafter the B–S model). Second, the Merton (1976) model provides a closed form solution to European option prices. The Monte Carlo procedure we use requires calculating option prices a very large number of times. For example, every random time value path (for example, one of the ten shown in Figure 4) requires calculating the put price 90 times, one time every day leading to expiration. Therefore, it is computationally prohibitive to use option pricing models that must be solved by numerical methods such as those suggested by Andersen and Andreasen (2000) and Levendorskii (2004). Third, the qualitative results do not seem to be sensitive to the choice of the option-pricing model. The results based on the B–S model (available from the authors) are identical to the results that we report in detail.

The Merton (1976) model generates asset returns over time as a mixture of "normal" and "abnormal" vibrations. A standard geometric Brownian motion that has a constant variance per unit of time and a continuous sample path, generates the "normal" vibrations. The "abnormal" vibrations come from a process that produces changes only when important news arrives. Merton (1976) presents the mixture of vibrations using the stochastic differential equation:

$$dS(t)/S(t) = (\alpha - \lambda y)\,dt + \sigma\,dZ + dq(t), \tag{1}$$

where $S(t)$ is the asset price at time t, α is the instantaneous expected return on the underlying asset, σ is the instantaneous standard deviation of the asset return conditional on no occurrence of jumps (hereafter normal return volatility), dZ is a standard Gauss–Weiner process, $q(t)$ is an independent Poisson process, λ is the mean number of jump arrivals per unit of time and y is the expected value of the random variable $(Y - 1)$ that represents the percentage jump in the underlying asset price if the Poisson event occurs (if the asset price before a jump is $S(t)$ then the price after the jump is $S(t)Y$). Hereafter, we omit (t) from S and q except where the time index is necessary for clarity.

Equation (1) is generalized to allow for continuous dividend payments. The resulting equation is:

$$dS/S = (\alpha - \xi - \lambda y)\,dt + \sigma\,dZ + dq, \tag{2}$$

where ξ is the dividend yield. This formulation is proposed by previous studies. Merton (1973) suggests that the B–S model can be extended to options on assets that pay dividends continuously over time and dividends are proportional to the asset price. Hull (2000) proposes that the Merton (1973) results are still true when the dividends are paid at discrete points of time. In this case, the continuous dividend rate is set equal to the average annualized dividend yield during the life of the option. Andersen and Andreasen (2000) use the same formulation to price index options where the underlying asset is affected by frequent dividend payments that may be approximated by a continuous process. Thus, the results may not apply to options on assets that pay dividends at discrete points in time.

Previous studies, for example, Pindyck (1988) and Chung (1993), use ξ to model situations where the growth rate of the underlying asset is higher or lower than the riskless rate. For example, when ξ is positive it can represent the dividend rate or a situation where the underlying asset is growing at a rate lower than the riskless rate. In contrast, a negative ξ represents a situation where the underlying asset is growing at a rate higher than the risk-free rate.

Let $C(\tau, S)$ denote the premium of a European call with τ years to expiration (in later sections we use T to denote time to expiration in days) and current underlying asset price S. Assuming Y has a log-normal distribution, Merton (1976) shows that Equation (2) leads to the option-pricing formula:

$$C(\tau, S) = \sum_{n=0}^{\infty} \frac{e^{-\lambda(1+y)\tau}[\lambda(1+y)\tau]^n}{n!} B_n(\tau, S), \tag{3}$$

where

$$B_n(\tau, S) = Se^{-\xi\tau} N(d_1) - Ke^{-r_n\tau} N(d_2) \tag{4}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_n - \xi + \frac{v_n^2}{2}\right)\tau}{v_n\sqrt{\tau}} \tag{5}$$

$$d_2 = d_1 - v_n\sqrt{\tau}, \tag{6}$$

where

$N(\cdot) \equiv$ is the cumulative probability density function of a standard normal distribution
$e =$ 2.71828 is the base of the natural log function
$K \equiv$ is the exercise price
$r \equiv$ is risk-free rate and is equal to α the instantaneous expected return on the underlying asset
$r_n = r - \lambda y + n\gamma/\tau$
$v_n^2 = [\sigma^2 + n\delta^2/\tau]$
$n \equiv$ is the number of jumps during the life of the option and it is Poisson distributed with parameter $\lambda(1 + y)\tau$
$\gamma =$ $\log(1 + y)$, and
$\delta \equiv$ is standard deviation of the logarithm of Y, hereafter jump size volatility.

$B_n(\tau, S)$ is the value of a standard B–S European option on an asset that pays a continuous dividend at the rate ξ where the "formal" instantaneous rate of interest is r_n and the "formal" standard deviation per unit of time is v_n. $B_n(\tau, S)$ is the value of the option conditional on knowing that exactly n Poisson jumps will occur during the life of the option. $C(\tau, S)$ is therefore the weighted average of these prices where the weight of the nth price, $B_n(\tau, S)$, is the probability a Poisson random variable with characteristic parameter $\lambda(1 + y)\tau$ takes the value n.

Hall (2000) argues that the call-put parity relation holds even when the asset prices follow a jump diffusion process, because the relation follows from a simple arbitrage argument that requires no assumptions about the underlying asset price's probability distribution. Therefore, the price of a put that has the same underlying asset, exercise price and time to expiration can be calculated as:

$$P(\tau, S) = C(\tau, S) + Ke^{-r\tau} - Se^{-\xi\tau}. \tag{7}$$

For the remainder of this paper, the discussion is presented in terms of puts. This is done for three reasons. First, it keeps the paper at a reasonable length and avoids complexity. Second, no generality is lost as the analysis and the results related to calls are exactly parallel to the analysis and results related to puts. Differences are pointed out as appropriate. Third, presenting the analysis in terms of puts prepares the reader for later sections where the put-rolling strategies are considered.

Given Equations (3)–(7), the time value of a put option can be determined. Consider a put on day 0 when the option has $\tau \times 365$ days to maturity. After $t \times 365$ days, $t < \tau$, the time value of the put option with $\tau - t$ years remaining to maturity and underlying asset price S is:

$$TV_t(\tau - t, S) = \left\{ \begin{array}{ll} P(\tau - t, S) - (K - S) & \text{If } S \leq K \\ P(\tau - t, S) & \text{If } S > K \end{array} \right\}. \tag{8}$$

3. Time value surface

This section demonstrates that over the life of an option, the possible time values that may be realized by a given option form a three-dimensional surface.

3.1. Time value as a function of the time to expiration

Hull (2000, pp. 319–321) shows using the B–S option-pricing formula that the time value of an option decreases with the passage of time if the underlying asset price is constant. The result is derived by finding the option's theta, Θ, the rate of change of the option's value with respect to the passage of time when all else remains constant. Θ is also the rate of change of the option's time value with respect to the passage of time. This result follows from Equation (8). When the price of the underlying asset, S, is constant, it is an easy exercise to show that:

$$-\frac{\partial TV_t}{\partial \tau} = \Theta = -\frac{\partial P}{\partial \tau} = -\frac{Sn(d_1)\sigma e^{\xi\tau}}{2\sqrt{\tau}} + rKe^{-r\tau}N(-d_2) + \xi SN(-d_1)e^{\xi\tau}, \tag{9}$$

where $n(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$.

Hull (2000) argues that Θ is usually negative with a possible exception of the theta of a deep-in-the-money European put on a nondividend paying asset. Alexander and Stutzer (1996) provide additional scenarios where Θ can be negative. Appendix A proves that Θ is usually negative for the case when the asset price dynamics are determined by the jump diffusion process described in Equation (2).

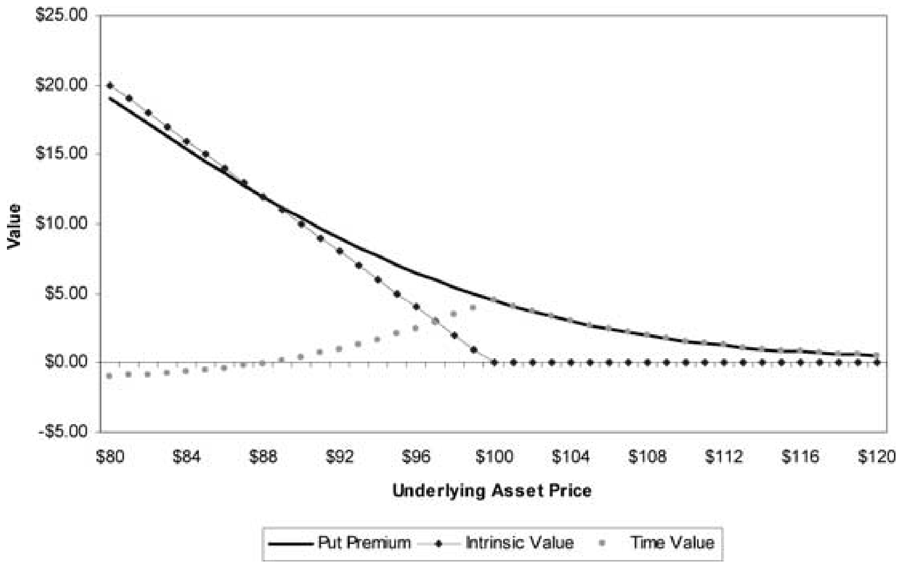

Figure 2

European put premium, intrinsic value, and time value as functions of the asset price

The put premium is computed using the Merton (1976) jump diffusion model with an underlying asset that pays continuous dividend. Asset price ranges from \$80 to \$120. The other parameters are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, the time to expiration $T = 90$ days, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$.

3.2. Time value as a function of the underlying asset price

It is widely known that for a given exercise price and time to maturity, a put option premium is a decreasing function of the underlying asset price. Textbooks often draw the premium line and the intrinsic value line on the same graph. Figure 2 shows such a graph. Visual inspection suggests that the time value, the distance between the put premium and the intrinsic value lines, is largest when the underlying asset price is equal to the exercise price, and decreases as the price moves away from the exercise price, either higher or lower. The time value starts below zero when the asset price is zero and increases until it reaches its maximum level when the option is at-the-money.[1] Once the price of the underlying asset exceeds the exercise price, the time value begins to decrease as the underlying asset price increases.[2]

[1] Hull (2000: pages 171–178) explains the reasons for the negative time value of European puts.

[2] Using Equation (8), the derivative of time value with respect to S is $N(d_1)$ in the interval $(0, K)$ and $(N(d_1) - 1)$ in the interval (K, ∞). Therefore, the derivative is always positive in $(0, K)$ and always negative in (K, ∞). The time value rises monotonically until the asset price reaches K and drops steadily as the asset price rises beyond K.

The time value curve of a call as a function of the underlying asset price is very similar in shape. The only exception is that the time value of a call option is always positive.

The Merton (1976) formula is derived for European options. Therefore, the results of this study are proved only for these options. We suggest that the qualitative results could apply to American options also. We show for American put options on nondividend paying assets, the shape of the time value line (computed using the binomial tree method) is the same as that for European put options (the related graph is omitted for brevity). The time value line peaks when the option is at-the-money and slopes downwards as the option moves out-of-the-money. The unique shape of the line partially explains the findings of this study and suggests the results apply to American options. We leave the proof of this proposition for all American options to future research.

3.3. Time value surface

The time value line in Figure 2 represents the set of possible time values from which only one value is realized when the option has 90 days remaining to expiration. The realized value depends on the realized asset price. As the option's time to expiration decreases to 89 days, the line of possible time values is similar in shape to the time value line in Figure 2. However, the time value at a particular asset price drops over time. As the option's value changes with time to expiration, the set of all possible time values forms a surface. Figure 3 shows the surface of all the possible time values for a European put.

Figure 3 shows that for any given time to expiration, there is a line analogous to the time value line shown in Figure 2 and each has a maximum level. The maximum level is obtained if the underlying asset price is equal to the exercise price. For a particular time to expiration, the realized time value can be any point on the corresponding time value line. For the entire period of time over which the option is outstanding, the upper bound for the time value is the line that joins the maximum values in the individual time value lines. This line can be labeled the maximum time value frontier. It is indeed the line in Figure 1 that is commonly used to characterize time value decay. However, an option realizes this time value pattern only if the option stays at-the-money over the entire period leading to expiration. Such a scenario is highly unlikely. In the majority of the situations, the realized time value line traces a path other than the maximum time value frontier and the majority of the points on the path lie below the frontier. The realized line cannot be determined a priori as it depends on the realization of the underlying asset price.

4. Expected time value curve

The previous section showed how for a given time to maturity the maximum time value is obtained when the option is at-the-money. If the option is out-of-the-

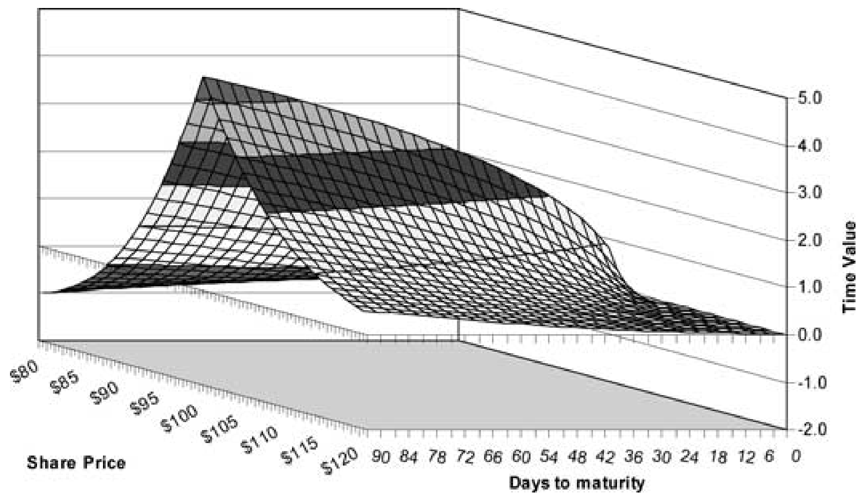

Figure 3

The time value surface for European puts as a function of the asset price and time to maturity

The surface is calculated for a 90-day European put on an underlying asset worth \$100 on day 0 and follows a jump diffusion motion. The parameters are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. Put premiums are computed with the Merton (1976) formula modified to account for a continuous dividend yield and time value is computed as put premium minus intrinsic value.

money, the time value is less than the maximum. This section proves the following proposition:

Proposition: *The expected time value decreases as time to expiration approaches and at any time preceding the expiration date the expected time value is less than the time value calculated for the option at-the-money.*

Proof: Consider an at-the-money put option with T days ($T = \tau \times 365$) remaining to expiration. For simplicity, the unit is chosen to be one day. Today, the time value is:

$$TV_0 = TV(S_0 = K, T). \tag{10}$$

Let us change the time by one unit. Given the time change and the stochastic nature of the price, S_0 also changes to $S_{1,-}$, $S_{1,0}$, or $S_{1,+}$ depending on whether the change in S is negative, zero, or positive. If the change is zero, a possible event, the option continues to be at-the-money, and $S_{1,0}$ is equal to K. In this case, the time value on day 1 is:

$$TV_{1,0} = TV(S_{1,0} = K, T - 1). \tag{11}$$

While the price of the underlying asset did not change, $TV_{1,0}$ is less than TV_0 due to the passage of time.

If S changes by a positive amount, for example S changes to $K + \varepsilon$, where ε is a positive number, the time value on day 1 is:

$$TV_{1,+} = TV(S_{1,+} = K + \varepsilon,\ T - 1). \tag{12}$$

The previous section proves that $TV_{1,+}$ is less than $TV_{1,0}$ for all $\varepsilon > 0$. In turn, $TV_{1,0}$ is less than TV_0 due to the passage of time.

If S changes by a negative amount, for example S changes to $K - \eta$ where η is a positive number less than K (as S cannot be negative), the time value on day 1 is:

$$TV_{1,-} = TV(S_{1,-} = K - \eta,\ T - 1). \tag{13}$$

The previous section proves that $TV_{1,-}$ is less than $TV_{1,0}$ for all η belonging to the interval $(0, K)$. In turn, $TV_{1,0}$ is less than TV_0 due to the passage of time.

When all the possibilities for the underlying asset price are considered, the time value for the option on day 1 varies with the underlying asset price as shown by the time value curve in Figure 2.

By definition, the expected time value on day 1 is:

$$E(TV_1) = \int_0^K TV(S, T - 1) f(S)\, dS + \int_K^\infty TV(S, T - 1) f(S)\, dS. \tag{14}$$

It is shown above that $TV_1(S, T - 1) < TV_{1,0}$ for all $S < K$ and $TV_1(S, T - 1) < TV_{1,0}$ for all $S > K$. Therefore:

$$E(TV_1) < TV_{1,0} \int_0^\infty f(S)\, dS = TV_{1,0}. \tag{15}$$

The expected time value on day 1 is less than the maximum time value $TV_{1,0}$ when the option is at-the-money.

On day 2, the maximum time value $TV_{2,0}$ is obtained if the underlying asset price is equal to the exercise price. Again, $TV_{2,0}$ is less than $TV_{1,0}$ and less than TV_0. Using the same arguments used for day 1, the expected time value on day 2, $E(TV_2)$, is less than $TV_{2,0}$ and less than $E(TV_1)$.

In general, on day t, the maximum time value $TV_{t,0}$ is obtained if the underlying asset price is equal to the exercise price. $TV_{t,0}$ is less than $TV_{t-1,0}$ and less than $TV_{t-2,0}$. Using the same arguments as for day 1, the expected time value on day t, $E(TV_t)$, is less than $TV_{t,0}$ and less than $E(TV_{t-1})$. Therefore, the following relation holds:

$$E(TV_{T-t}) < E(TV_{T-t+1}) < E(TV_{T-t+2}) < \cdots < E(TV_{T-1}), \quad \text{and} \tag{16}$$

$$E(TV_{T-t}) < TV_{T-t,0} \quad \text{for all } t < T. \tag{17}$$

5. Simulation model and results

This section uses Monte Carlo simulation to determine the expected time value curve as a function of the days remaining to expiration.

5.1. Simulation model

The general simulation procedure consists of several steps. First, the random asset price after the passage of one time unit is generated. For simplicity, the time unit is considered to be one day. Second, Equations (3)–(7) are used to calculate the corresponding random premium of the option. Third, the corresponding intrinsic value is subtracted to find the time value. Fourth, the first three steps are repeated at the end of every time unit leading to expiration to create the desired sample of observations. Fifth, the expected put premium and the expected time value for a given time to maturity are determined by taking the average over the sample observations for that time to maturity. Finally, the results are graphed to show the expected time value curve. Unless otherwise stated, each graph is produced from 1,500 trials.

The random asset prices are generated as follows. On day t consider a put option that has many days remaining to expiration and underlying asset price of $S(t)$. Assume that the asset price evolves according to the jump diffusion model presented by Equation (2). Under this assumption, Merton (1976) suggests that the price one day later, on day $t + 1$, can be obtained as follows:

$$S_{t+1} = S_t \exp\left[\left(r - \xi - \lambda y - \frac{\sigma^2}{2}\right)\frac{1}{365} + \left(\sigma\sqrt{\frac{1}{365}}\right) z_1\right] \exp\left[\frac{n\gamma}{365} + \left(\delta\sqrt{\frac{n}{365}}\right) z_2\right], \tag{18}$$

where exp is the exponential function, z_1 is a random number from a standardized normal distribution that determines the stochastic vibrations related to the normal stochastic process, and z_2 is a random number from a standardized normal distribution that determines the stochastic vibrations related to the jump component of the stochastic process.

5.2. Base-case parameter values

Throughout the remainder of this paper, the base-case parameter values are the normal return volatility, $\sigma = 25\%$, the risk-free rate, $r = 6\%$, the exercise price, $K = \$100$, the probability of a jump, $\lambda = 10\%$, the expected jump size, $y = -10\%$, the volatility of the jump size, $\delta = 25\%$, the dividend yield, $\xi = 1.25\%$, the initial asset price, $S_0 = \$100$ and time to expiration, $T = 90$ days. When different parameter values are used, they are explicitly identified.

The choice of -10% and 25%, respectively for y and δ deserves comment. These two values are chosen to be consistent with the findings of Andersen and Andreasen (2000). The authors suggest that the jump diffusion model of Merton (1976) provides

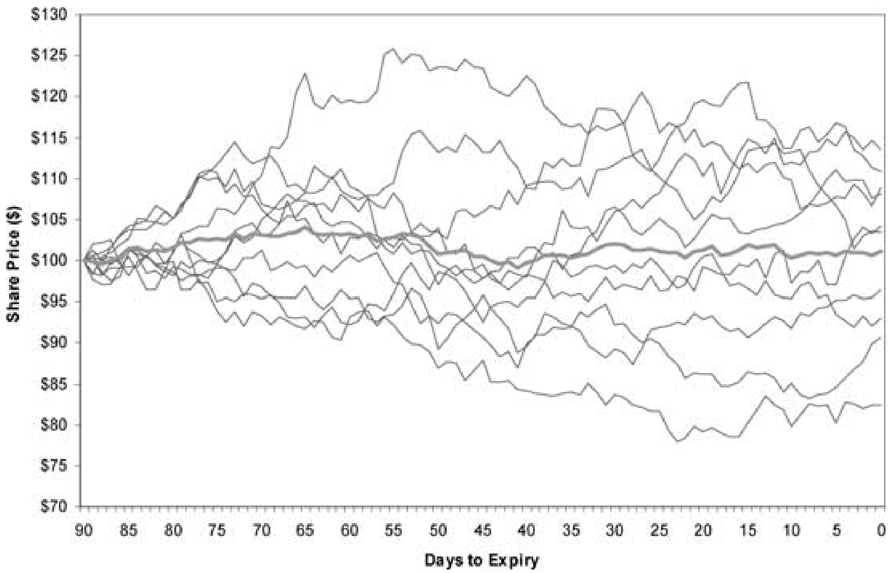

Figure 4

Random paths of the underlying asset price

The price paths are for an asset that follows a jump diffusion motion. The parameters are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. Each path starts with an initial asset price of \$100 and the price is followed over a 90-day period. The line in the middle represents the average price of the ten sample random walks.

a good fit for option prices observed in the market when the jumps are expected to generate negative changes in the underlying assets. They estimate the expected jump (y) to be approximately -50% and the volatility of the jump (δ) to be 45%. Andersen and Andreasen (2000) argue that during their sample period, around April 1999, investors may have priced options with the anticipation that securities would experience negative price jumps. This conclusion is reasonable given the high prices of technology shares at the time and the eventual collapse of the prices shortly after. Therefore, the base-case values of y and δ are chosen to be -10% and 25%, respectively to represent normal investor expectations. Section 6 examines the sensitivity of the results to changes in y, δ and other parameters.

5.3. Expected time value curve for puts purchased at-the-money

Figure 4 depicts ten random asset price paths. Each price path is generated from a simulation run where the price starts at \$100 and follows a jump diffusion process over a 90-day period. As expected, the asset price does not stay close to the initial

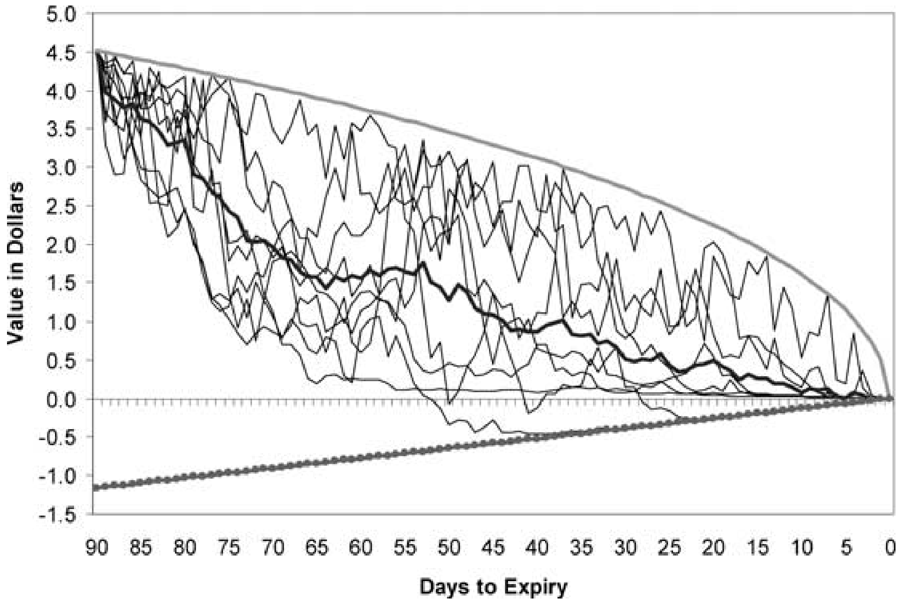

Figure 5

Random time value paths of ten European puts purchased initially at-the-money

Each random time value path is based on a 90-day European put option on a stock price that follows a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. On day 0, the put is purchased with an exercise price of \$100 when the security price was \$100. The time value on a particular day is computed by subtracting the intrinsic value from the put premium. The Merton (1976) option-pricing formula is used to calculate the premium.

\$100. Instead, it moves either higher or lower and sometimes crosses the initial \$100 price.

For each simulation, the time value path for a 90-day put purchased at-the-money (the exercise price is \$100) is computed. Figure 5 shows the resulting ten random time value paths, the expected time value curve related to the ten random paths and the maximum time value frontier. The expected time value curve (shown by the solid line) of the ten puts that were purchased at-the-money is always lower than the maximum time value frontier and slopes downwards at a decreasing rate.

Figure 6 shows one random time value path during the last 40 days leading to expiration. The path is generated for a 90-day European put option purchased at-the-money when the stock was priced at \$100. For clarity, the figure shows only the path during the last 40 days. After purchase, the asset price follows the jump diffusion motion. As a result of the stochastic changes leading to 40 days prior to expiration, the stock price is \$97.30 providing a time value for the put equal to \$1.82. Twenty-one

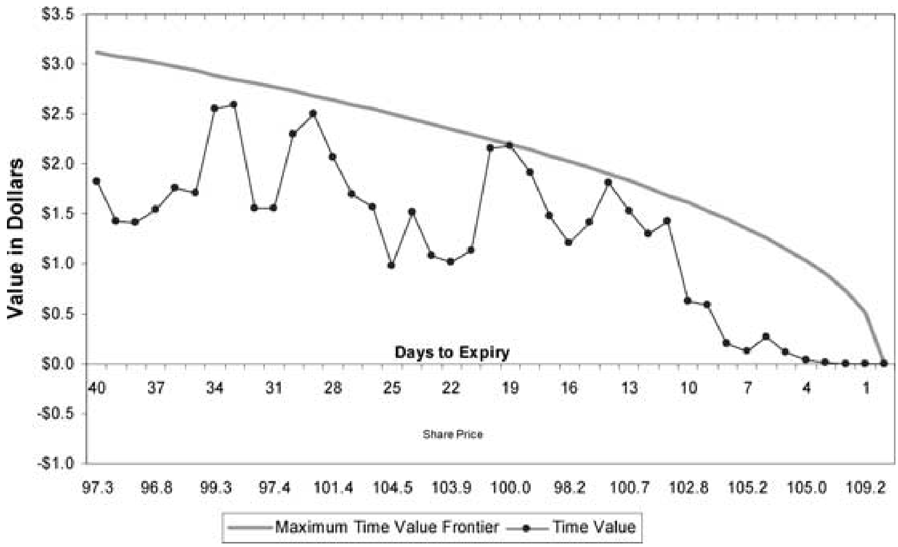

Figure 6

A random time value path for a European put during the last 40 days leading to the expiration date

The path is generated for a 90-day European put option purchased at-the-money when the stock was priced at \$100. Although the path was generated for the entire 90 days, the figure shows the path only during the last 40 days. After purchase the asset price followed a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. Thus, at 40 days prior to expiration, the asset price is approximately \$97.30 and the corresponding time value is \$1.07. The other simulated stock prices are shown below the horizontal axis of the chart. The time value on a particular day is computed by subtracting the intrinsic value from the put premium. The Merton (1976) option-pricing formula is used to calculate the premium.

days later, the price of the asset is back at-the-money (\$100.03) where the time value of the put is \$2.18. For the remaining days, the put is either out-of- or in-the-money but in either case, the time value is less than the maximum time value for the given time to expiration.

5.4. Varying the exercise price

The expected time value curve shown in Figure 5 is generated from ten trials for a put option purchased on day 0 at-the-money. The same simulation process can be used to generate the expected time value curve of a single option given any set of simulation parameters. Figure 7 shows the simulated time value curves of puts with exercise prices that range from \$80 to \$120 while the underlying asset price on day 0 is \$100. Each expected time value curve is generated from 1,500 trials.

For a European put purchased at- or near-the-money, the expected time value curve decreases at a decreasing rate as expiration approaches. For these options, a

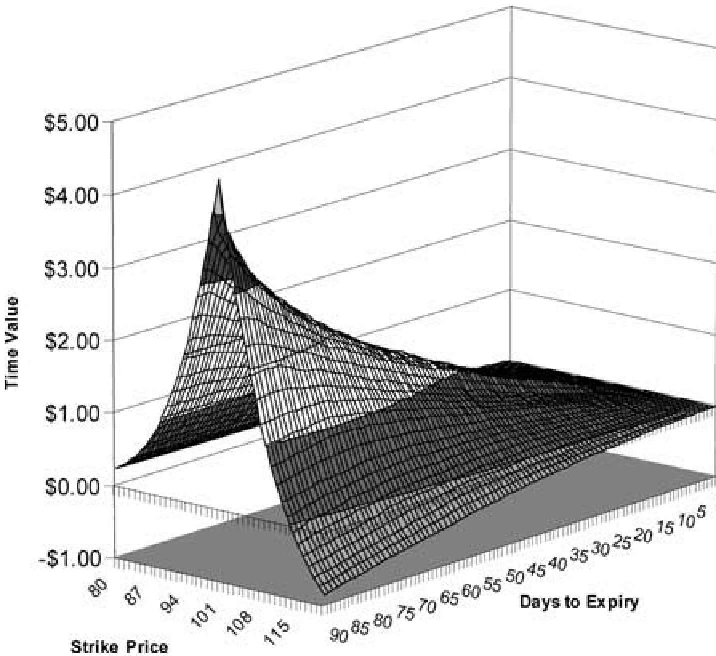

Figure 7

The expected time value surface of a European put as a function of the initial exercise price at which it is purchased on day 0

The puts are purchased with an initial 90-day time to expiration and exercise prices that range from \$80 to \$120. Each simulation starts with an initial asset price of \$100 after which the price varies stochastically following a jump diffusion motion. The parameters are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The sample time values at a given time to expiration are calculated as the put premium obtained from the Merton (1976) formula minus the intrinsic value. The expected time value for a given time to expiration is the average of the sample time values.

large portion of the time value is expected to be lost in the initial days of holding the option. This result is obtained since for the majority of the options the underlying asset price is either higher or lower than the exercise price.

Furthermore, Figure 7 shows that as puts are purchased further out-of-the-money the time value is expected to rise for a short period after purchase and then it starts declining. Once the decline starts, the decay is generally similar to that observed for puts purchased at-the-money, but the overall time value decay for these options is slower. If the option is purchased deeper out-of or in-the-money, the time value is expected to rise for a longer period of time before it starts dropping at a slower rate. In extreme cases where put options are purchased significantly deep-in-the-money, the initial time value is negative. In this case, the time value is expected to rise continuously until it reaches zero at expiration.

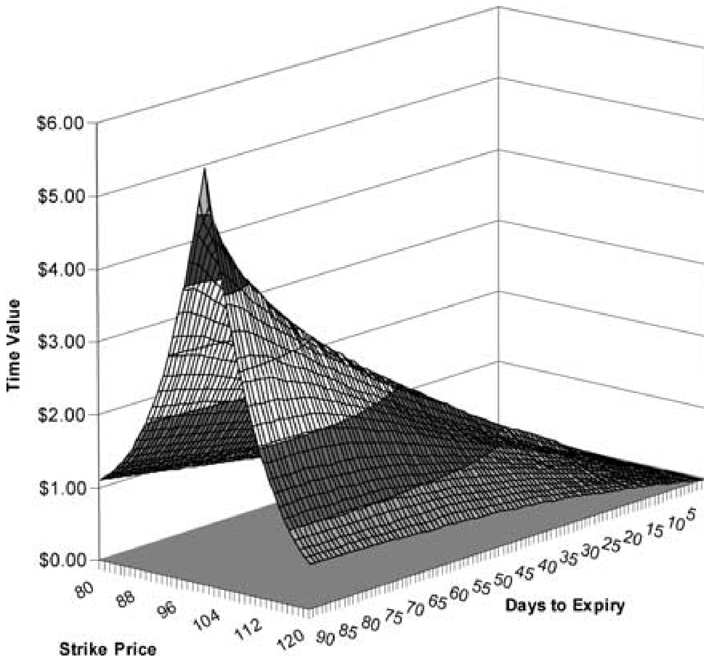

Figure 8

The expected time value surface of a European call as a function of the initial exercise price at which it is purchased on day 0

The calls are purchased with an initial 90-day time to expiration and exercise prices that range from \$80 to \$120. Each simulation starts with an initial asset price of \$100 after which the price varies stochastically following a jump diffusion motion. The parameters are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The sample time values at a given time to expiration are calculated as the call premium obtained from the Merton (1976) formula minus the intrinsic value. The expected time value for a given time to expiration is the average of the sample time values.

Figure 8 shows that the expected time value curves of call options are similar to those of put options. The only exception is that the time values of call options are never negative.

6. Sensitivity analysis

We analyze the sensitivity of the expected time value curve against changes in the expected jump size, y, the volatility of jump size, δ, the normal return volatility, σ, the riskless rate, r and the dividend yield, ξ. For this analysis, we generated data for five figures corresponding to the five parameters of interest. Each figure displays the expected time value curve for puts purchased at-the-money under three scenarios of the parameter. Figure 9 is a sample. The figures for δ, σ, r, and ξ are available from the authors.

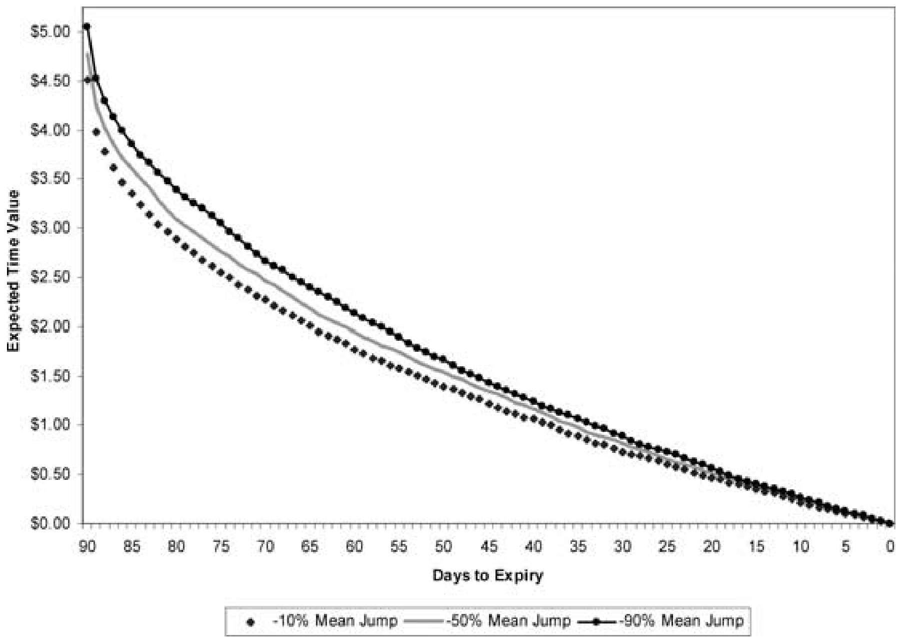

Figure 9

The impact of the jump size on the expected time value curve of a 90-day European put purchased at-the-money

Puts are purchased at-the-money (exercise price = \$100) with initial 90-day time to expiration. The expected time value curves are obtained from simulation sample sets generated by varying the expected jump size, y, over the values -10%, -50%, and -90%. The purchase prices are respectively \$4.5103, \$4.7694, and \$5.0518. For a given y, the simulation starts with an initial asset price of \$100 after which the price varies following a jump diffusion motion. The other parameter values are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The sample time values at a given time to expiration are calculated as the put premium obtained from the Merton (1976) formula minus intrinsic value. The expected time value on day t is the average of the sample time values on day t.

As Figure 9 demonstrates, the values of the parameters have no impact on the overall shape of the expected time value curve for puts or calls. The time value is expected to decay over time at a decreasing rate regardless of the magnitudes of the various parameters.

In contrast, we find that the changes in the parameters affect the time value decay rates as shown in Table 1. When an increase in the magnitude of a parameter increases (decreases) the initial time value of an option, the time value decay rate for this option increases (decreases). This change occurs as the initial higher (lower) time value eventually decays to zero at maturity, while the time over which the decay occurs is constant. For example, Figure 9 suggests the higher the expected magnitude of the negative jump, the higher the initial value of the put and the higher the decay rate.

Table 1

The impact of various parameters on the time value decay rates of puts and calls

Each entry indicates the change in the time value decay rate of a put or call option in response to an increase in the given parameter. The opposite change occurs in response to a decrease in the parameter. The change in the time value decay rate of a call in response to an increasing negative jump cannot be determined a priori. A larger expected negative jump (y) increases the probability that call options would expire out-of-the-money. Thus, call prices should be lower. However, when the expected jump size is negative the "formal" instantaneous rate of interest, r_n ($r_n = r - \lambda y + n\gamma/T$), is larger than the risk-free rate, and the more negative the jump, the larger is the "formal" rate of interest. Therefore, the larger the expected negative jump, the larger the value of call options.

	Increase in the parameter				
	y	δ	σ	r	ξ
Change in time value decay rate of puts	Higher	Higher	Higher	Lower	Higher
Change in time value decay rate of calls	Lower or higher	Higher	Higher	Higher	Lower

7. Expected time value decay in put-rolling strategies

This section shows the time value loss expected from put-rolling insurance strategies is equivalent to the expected time value loss from an insurance strategy that uses a single long-term protective put. The loss in time value does not depend on the frequency of rollovers or on the time remaining to expiration of the puts used in the rollover strategy. In addition, rolling puts before expiration does not affect the cumulative time value loss.

Consider an investor who wishes to insure the value of an asset over a given time horizon. For simplicity, assume the current value of the asset is \$100 and the time horizon is 90 days. The investor can buy a single European put on the asset with 90 days to expiration and an exercise price equal to \$100. Alternatively, the investor can follow a put-rolling strategy, which requires trading a series of puts to insure the asset. While there are many variations of put-rolling strategies, this study focuses on two. In one strategy, the investor rolls three 30-day puts on the asset with exercise prices of \$100 each. The first put is purchased on day 0, the second is purchased after the expiration of the first put on day 30 and the third put is purchased after the expiration of the second put on day 60. The expiration of the third put coincides with the end of the insurance horizon.

In the other strategy, the investor rolls three 90-day puts on the asset with exercise prices of \$100 each. The first put is purchased on day 0, the second is purchased after the sale of the first put on day 30 and the third put is purchased after the sale of the second put on day 60. The third 90-day put is sold on day 90, the end of the insurance horizon. Previous studies, for example Figlewski, Chidambaran and Kaplan (1993) and Chidambaran and Figlewski (1995), as well as the Montréal Exchange (2007) reference manual, propose that an investor can reduce the loss resulting from the decay in time value by rolling the puts before expiration.

The choice of a \$100 exercise price for all put options requires comment. Figlewski, Chidambaran and Kaplan (1993) and Chidambaran and Figlewski (1995) propose three strategies for choosing the exercise price: the fixed strike strategy, the fixed percentage strategy, and the ratchet strategy. The first involves purchasing all the put options at the same exercise price. This strategy determines the price at the beginning of the insurance strategy. The second sets the exercise price at a fixed percentage of the underlying asset's price at the time of rollover. The third strategy sets the initial exercise price at a given percentage of the asset price on day 0. If the asset price rises during the rollover period, the exercise price of the next put is set at a fixed percentage of the new asset price. If the asset price falls during the rollover period, the exercise price of the next put is chosen equal to the exercise price of the expiring put. The expected time value decay is considered under the three strategies. The results are equivalent. For brevity, only the results related to the fixed strike strategy are presented.

The simulation procedure presented in the previous section is followed to determine the expected time value curves and the expected put premium lines of the three alternative strategies. Each of the curves is generated from 1,500 trials. For the purpose of the simulations, the underlying asset's price changes stochastically over time according to the jump diffusion process defined in Equation (2). The parameter values are the same base-case values listed earlier in Section 5.2. Furthermore, assume transaction costs are zero and the put-rolling strategy will not be modified to exploit opportunities at the roll-over points.

7.1. Buying a single 90-day put versus rolling 30-day puts at expiration

Figure 10 presents the expected time value curves and the expected put premium lines for the two strategies. A point on the expected time value curve represents the average of the simulated put time values for the corresponding time to maturity. Similarly, a point on the expected put premium line represents the average premium of the simulated puts for the corresponding time to maturity. The figure shows the time value of the 90-day put is expected to decay over the 90-days from \$4.51 to zero. The expected time value curve is downward sloping with the decay rate decreasing over time. In contrast, the expected put premium line is expected to rise over time at the risk-free rate (this fact may not be visible in the graphs as the difference between the average premium on day 0 and day 90 is approximately \$0.0677 (\$4.51 $\times$ 0.06/4) plus or minus simulation and round-off errors).

For the 30-day put strategy, Figure 10 indicates the time value of the first put is expected to decay in 30 days from \$2.71 to zero. The second put is purchased with an exercise price of \$100 on the asset that started at a price of \$100 on day 0 and followed a jump diffusion motion during the previous 30-day period. We expected the time value of the second put to decay from \$1.04 to zero. Similarly, the third put is purchased with an exercise price of \$100 on an asset that started at a price of \$100 on day 0 and followed a jump diffusion motion over the previous 60-day period. The time value of the third put is expected to decay from \$0.78 to zero.

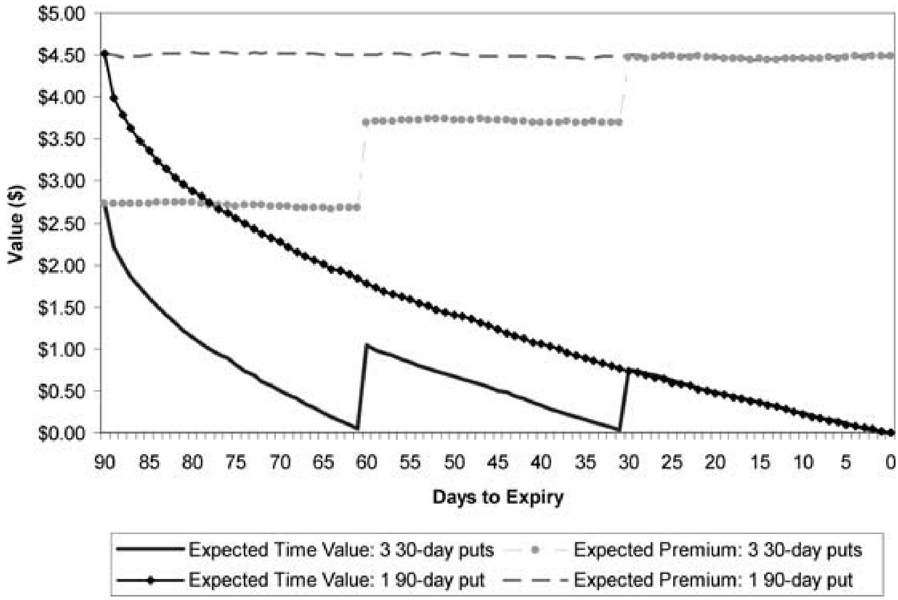

Figure 10

Comparing the expected time value curves: a single 90-day put versus a series of three 30-day European puts purchased and rolled every 30 days

The 90-day put is purchased at-the-money. The initial underlying asset price is \$100 after which it changes stochastically following a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, the time to expiration $T = 90$ days, $\lambda = 10\%$, $y = -0.1\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The purchase price of the put is \$4.51. For the three 30-day put strategy, the exercise price for each of the three puts is \$100 and the time to expiration of each put is 30 days. The first put is purchased at-the-money while the other two are purchased either in- or out-of-the-money depending on how the asset price evolved over time. The initial asset price is \$100 after which it follows a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, $\lambda = 10\%$, $y = -0.1\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The purchase price of the first put is \$2.71. A point on the expected put premium line represents the average of the simulated put premiums for the corresponding time to maturity.

The initial time values of the second and third puts in the put-rolling strategy are expected to be below the initial time value of the first put as the simulated price of the underlying asset deviates from the exercise price at the roll-over points. The time value is lower since the sample puts are no longer at-the-money. Yet, the present value of the cumulative time value decay of the put-rolling strategy is \$4.5172. We obtain this value by adding the time value loss of the second put discounted at the risk-free rate (6%) over 30 days (\$1.0349), the time value loss of the third put discounted at the risk-free rate over 60 days (\$0.7723), and the time value loss of the first put. The value is \$0.0069 more than the present value of the time value loss of the single 90-day put (\$4.5103). The difference may be attributed to rounding and simulation errors.

At the roll-over points the expected premium line rises by the amount of the time value associated with the new put. This rise occurs since at each roll-over point the investor exercises the expiring put to receive the intrinsic value. The proceeds plus an additional amount are used to buy the new put. As the exercise of the expiring put and the purchase of the new put should take place simultaneously and their exercise prices are equal, their intrinsic values should be equal. Thus, the additional amount needed to buy the new put is equal to the time value of the new put.

Figure 10 shows the expected time value decay from holding one put over a long period of time is equal to the expected loss in time value from holding a series of short-term puts that span the same long period. Furthermore, we expect the overall investment needed to implement one strategy to be exactly equal to the investment needed to implement the other strategy.

7.2. Buying a single 90-day put versus rolling 90-day puts prior to expiration

Figure 11 shows the expected time value curves and the expected put premium lines of the two strategies. Over the first 30 days of the insurance period, the expected time value and the expected premium of the first put in the put-rolling strategy coincide with those of the single 90-day put strategy. This is obvious as over the initial 30 days the two strategies are using the same 90-day put on the same underlying asset. On day 30 of the insurance plan, the investor sells the first put in the rolling strategy and buys the second. As the second put starts with 90 days remaining to maturity, its purchase price is higher than the price obtained from selling the first put. Therefore, the expected time value and premium lines of the rolling strategy jump to levels higher than the counter parts for the single 90-day put strategy. Similarly, on day 60 the expected time value and premium lines of the rolling strategy move up by the additional premium required to buy the third put as replacement for the second. On day 90, the third put in the rolling strategy is expected to have positive time value as it still has 60 days to expiration. Figure 11 shows that the expected time value that is gained on day 90 from selling the third put for more than its intrinsic value is completely offset by the additional premiums that are paid to replace the first and second put. Therefore, the expected cumulative time value decay from rolling puts before expiration is equal to the expected time value decay of employing a single 90-day put strategy.

7.3. Buying out-of-the-money or in-the-money puts

Figure 12 shows the expected time value decay when the put-rolling strategy is implemented with out-of-the-money puts the exercise prices of which are 10% below the initial (day 0) market price of the underlying asset. As the underlying asset price on day 0 is \$100, each put has an exercise price of \$90. The simulation results

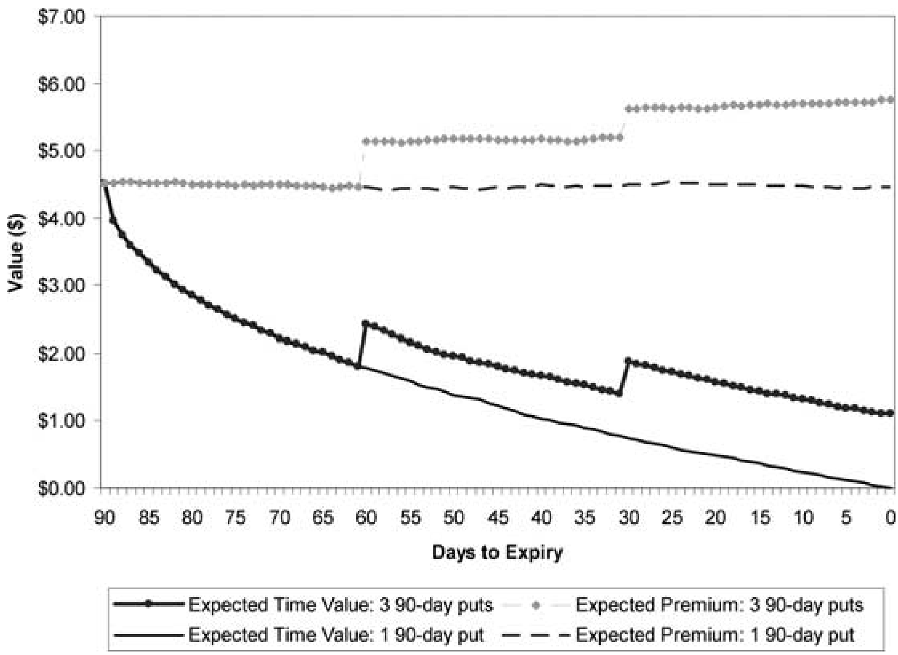

Figure 11

Expected time value curve—single 90-day put versus a series of three 90-day puts each held for 30 days

Each put in the put-rolling strategy is initially purchased with 90 days remaining to expiration and held for 30 days. The put is sold when it has 60 days remaining to expiration. The initial underlying asset price is \$100 after which it follows a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, the time to expiration $T = 90$ days, $\lambda = 10\%$, $y = -0.1\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The purchase price of the first 90-day put is \$4.51. A point on the expected put premium line represents the average of the simulated put premiums for the corresponding time to maturity.

indicate that the expected time value loss from rolling three 30-day puts is equal to the expected time value loss from insuring with a single 90-day put.

We obtain a similar conclusion when we implement the put-rolling strategy with in-the-money puts. We tested a strategy of buying puts the exercise prices of which are 5% above the initial (day 0) market price of the underlying asset. As the underlying asset price on day 0 is \$100, each put has an exercise price of \$105. The simulation results produced graphs almost identical to those shown in Figure 12. For brevity, these graphs are omitted but they are available from the authors. The results confirm that the expected time value loss from rolling three 30-day puts is equal to the expected time value loss from insuring with a single 90-day put.

In conclusion, ignoring transaction and financing costs and the benefits of reset opportunities, the time value loss expected from a fixed exercise put-rolling insurance

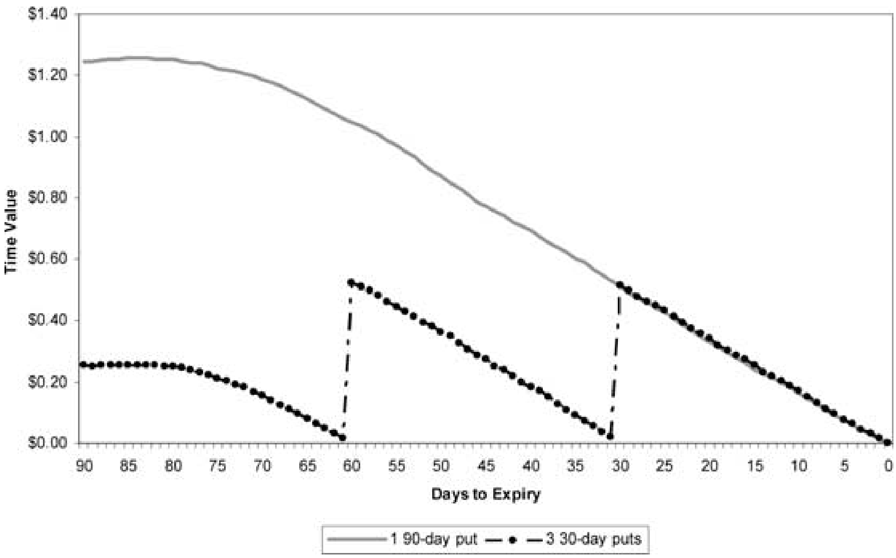

Figure 12

Expected time value curve—single 90-day put versus a series of three 30-day puts when the initial puts are purchased 10% out-of-the-money

This figure compares the expected time value decay of a 90-day European put purchased 10% out-of-the-money with that of rolling three 30-day European puts. The first of the three 30-day puts is purchased 10% out-of-the-money and the other two are purchased following a fixed exercise strategy. The initial underlying asset price is $100 after which it follows a jump diffusion motion the parameters of which are the volatility $\sigma = 25\%$, the risk-free rate $r = 6\%$, the exercise price $K = \$100$, the time to expiration $T =$ 90 days, $\lambda = 10\%$, $y = -0.1\%$, $\delta = 25\%$, and $\xi = 1.25\%$. The purchase price of the 90-day put is $1.2432 while the purchase price of the first 30-day put is $0.2545.

strategy is equivalent to the expected time value loss from an insurance strategy that uses a single long-term protective put. The loss in time value does not depend on the frequency of rollovers or on the time remaining to expiration of the puts used in the rollover strategy. In addition, trading puts before expiration does not affect the cumulative time value loss.

Simulations are conducted to examine whether the choice of parameters affects these conclusions. As shown in Section 6, a change in a parameter changes the magnitude of the initial premium paid to buy a put. However, the main conclusion remains unchanged regardless of the choice of the exercise price or the levels of the various parameters. For brevity, the graphs that show these conclusions are not included.

8. Summary and conclusions

The current common perception is that the time value of an option decays at a rate that accelerates with time. The time value decay is often shown as a downward sloping curve with the slope getting larger as time to expiration approaches. At expiration, the curve becomes almost vertical. In practice, this pattern occurs if the option is purchased at-the-money and remains at-the-money until expiration. However, with a stochastic underlying asset, the likelihood of achieving such a time decay curve is negligible.

This study uses Monte Carlo simulation to determine the expected time value curve of European put and call options. The analysis shows the expected time value curve depends on whether the option is purchased out-of, at-, or in-the-money. We expect the time value of an option purchased at-the-money to decrease at a decreasing rate as expiration approaches. Thus, a large portion of the time value is expected to be lost in the initial days of holding the option. For puts purchased far out-of-the-money, we expect the time value to rise for a short period after purchase and then start to decline.

Sensitivity analysis indicates the shape of the expected time value curve does not change with the levels of the normal return volatility, the riskless rate, the dividend yield (or the drift of the underlying asset), the expected jump size, or the volatility of the jump size. However, changes in these parameters affect the overall expected rate of time value decay for puts and calls.

Given the pattern of expected time value decay, we consider the implications for put-rolling strategies. The results suggest that the time value loss expected from an insurance strategy that rolls a sequence of short-term puts with the same exercise price is the same as the expected time value loss of an insurance strategy that uses a single long-term protective put. The expected loss in time value does not depend on the frequency of rollovers or on the time remaining to expiration of the options used in a rollover strategy. Trading the puts before expiration does not affect the expected amount of cumulative time value decay. The analysis indicates the levels of the various parameters and the choice of the exercise price do not affect these conclusions.

Finally, the results are obtained for European options and under the assumptions that dividends are paid continuously over time and the normal return volatility is constant over the insurance horizon. We provide evidence suggesting that the qualitative results may apply to American options. In addition, we speculate these results hold for options on assets that pay dividends at discrete points of time. Yet, we leave the endeavor of proving these propositions to future research. Furthermore, future studies should try to confirm the results using stochastic volatility option pricing models.

Appendix A

Proposition: *Under the Merton (1976) option-pricing formula, Theta (Θ) is usually negative with a possible exception of the theta of a deep-in-the-money put.*

Proof: Merton (1976) shows that the value of a call option $C(\tau, S)$ is a weighted average of the B–S prices $B_n(\tau, S)$ where the weight of the nth price is the probability that a Poisson random variable with characteristic parameter $\lambda(1 + y)\tau$ takes the value n. Hull (2000) argues that Θ of a B–S option is usually negative except perhaps for the Θ of a deep-in-the-money put. Thus, $\Theta(B_n) = -\partial B_n/\partial\tau$ is usually negative for $n = 0, 1, 2, \ldots$. Let W_n denote the weight of price $B_n(\tau, S)$. As the weights are positive and add up to 1, $\Theta(C)$, the theta of the call option, is usually negative; it is:

$$\Theta(C) = -\frac{\partial C}{\partial\tau} = \sum_{n=0}^{\infty} W_n \left(-\frac{\partial B_n}{\partial\tau}\right). \tag{A.1}$$

Define $A_n(\tau, S)$ to be the B–S price of a put that has the same exercise price, time to expiration, and underlying asset as the call the price of which is $B_n(\tau, S)$. Then, the two must satisfy call-put parity:

$$A_n(\tau, S) = B_n(\tau, S) + Ee^{-r\tau} - Se^{\xi\tau}. \tag{A.2}$$

This equation leads to:

$$\sum_{n=0}^{\infty} W_n A_n(\tau, S) = \sum_{n=0}^{\infty} W_n B_n(\tau, S) + \sum_{n=0}^{\infty} W_n(Ee^{-r\tau}) - \sum_{n=0}^{\infty} W_n(Se^{\xi\tau}). \tag{A.3}$$

As the weights W_n are positive and sum up to 1, it follows that:

$$\sum_{n=0}^{\infty} W_n A_n(\tau, S) = \sum_{n=0}^{\infty} W_n B_n(\tau, S) + Ee^{-r\tau} - Se^{\xi\tau} = C(\tau, S) + Ee^{-r\tau} - Se^{\xi t} \text{ and} \tag{A.4}$$

$$\sum_{n=0}^{\infty} W_n A_n(\tau, S) = P(\tau, S). \tag{A.5}$$

Therefore,

$$\Theta(P) = -\frac{\partial P}{\partial\tau} = \sum_{n=0}^{\infty} W_n \left(-\frac{\partial A_n}{\partial\tau}\right) = \sum_{n=0}^{\infty} W_n \Theta(A_n). \tag{A.6}$$

The weights W_n are positive and sum up to 1. In addition, for $n = 0, 1, \ldots, \infty$, $\Theta(A_n) = -\partial A_n/\partial\tau$ is usually negative except perhaps when the put is deep-in-the-money. Therefore, $\Theta(P)$ should be negative.

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