

## Use Overfitting To Evaluate Different Models

The process of Machine Learning and using [Overfitting to evaluate Linear Regression Model and Non-linear Regression](#) .

- Please compare the following two Regression Models to see which one has more serious overfitting issue.
  - [Linear Regression Model 1](#)
  - [Non-Linear Regression Model 2](#)
- Suppose we collect a set of sample data and [distribute](#) the sample data by
  - Training phase: 50%
  - Validation phase: 25%
  - Test phase: 25%

Training Phase				Validation Phase				Test Phase	
Real Data Set 1 50% of the collected data	<a href="#">Model 1: Linear Regression</a>	<a href="#">Model 2: Non-Linear Regression</a>		Real Data Set 2 25% of the collected data	<a href="#">Model 1: Linear Regression</a>	<a href="#">Model 2: Non-Linear Regression</a>		Real Data Set 3 25% of the collected data	The better model ( <a href="#">Model 1</a> or <a href="#">Model 2</a> ) selected from the <b>Validation Phase</b> based on the analysis of <a href="#">overfitting</a> will be used to calculate $\hat{y}$
<ul style="list-style-type: none"> <li>▪ After calculating <b>a1, b1, a2, b2</b> in <b>Training Phase</b>, the values are not changed with the new <b>Real Data Sets</b> in <b>Validation Phase</b> and <b>Test Phase</b>.</li> <li>▪ Only <math>\hat{y}</math> values are changed with the new <b>Real Data Sets</b>.</li> </ul>									
x	y	$\hat{y}=a1 + b1 * x$	$\hat{y}=a2 + b2 * x^2$	x	y	$\hat{y}=a1 + b1 * x$	$\hat{y}=a2 + b2 * x^2$	x	$\hat{y}=a1 + b1 * x$ or $\hat{y}=a2 + b2 * x^2$
1	1.8	1.37	1.75	1.5	1.7	1.28	1.41	1.4	1.88
2	2.4	2.23	2.15	2.9	2.7	2.54	2.31	2.5	2.46
3.3	2.3	3.35	3.08	3.7	2.5	3.26	3.08	3.6	3.36
4.3	3.8	4.22	4.10	4.7	2.8	3.61	3.53	4.5	4.34
5.3	5.3	5.08	5.39	5.1	5.5	4.51	4.87	5.4	5.53
1.4	1.5	1.71	1.88	X	X	X	X	X	X
2.5	2.2	2.66	2.45	X	X	X	X	X	X
2.8	3.8	2.92	2.67	X	X	X	X	X	X
4.1	4.0	4.04	3.88	X	X	X	X	X	X
5.1	5.4	4.91	5.12	X	X	X	X	X	X

Note:

- Real Data Set 1 can be used to determine the formulas for [Model 1: Linear Regression](#) and [Model 1: Linear Regression](#). That is, to determine the values of  $a_1$ ,  $b_1$ ,  $a_2$ , and  $b_2$  in the following formulas:
  - $\hat{y} = a_1 + b_1 * x$
  - $\hat{y} = a_2 + b_2 * x^2$
- After the formulas are determined, you can use the formulas to calculate the  $\hat{y}$  values in the following phases:
  - Training Phase
  - Validation Phase
  - Test Phase
- Note: The values of " $x$ " in " $\hat{y} = a_1 + b_1 * x$ " and " $\hat{y} = a_2 + b_2 * x^2$ " are the same as the " $x$ " list on the "[Real Data Set](#)".
- Optional: You may want to implement the following 3 programs:
  - Program 1: To implement [Linear Regression Model 1](#)  
Note:
    - This program is to use RealData Set 1 to determine  $a_1$  and  $b_1$  based on [Model 1](#).
    - The program can be used to fill part of the blank spaces in above table.
  - Program 2: [Non-Linear Regression Model 2](#)  
Note:
    - This program is to use RealData Set 1 to determine  $a_2$  and  $b_2$  based on [Model 2](#).
    - The program can be used to fill part of the blank spaces in above table.
  - Program 3: Calculate [MSE](#)

Ans:

Training Phase - Liner Regression Model:

```
import numpy as np
import matplotlib.pyplot as plt

x = np.array([1, 2, 3.3, 4.3, 5.3, 1.4, 2.5, 2.8, 4.1, 5.1])
y = np.array([1.8, 2.4, 2.3, 3.8, 5.3, 1.5, 2.2, 3.8, 4.0, 5.4])

n = x.size

ss_xy = np.sum(x*y)
ss_xx = np.sum(x*x)
s_x = np.sum(x)
s_y = np.sum(y)
```

```
b = (n*ss_xy-s_x*s_y)/(n*ss_xx-s_x*s_x)
a = (s_y - b*s_x)/n
```

```
print ("Sum of x*y", ss_xy)
print ("Sum of x*x", ss_xx)
print ("Sum of x", s_x)
print ("Sum of y", s_y)
print ("Slope_b", b)
print ("Intercept_a", a)
```

```
def plot_regression_line(x, y, a, b):
    # plotting the actual points as scatter plot
    plt.scatter(x, y, color = "m", marker = "o", s = 30)

    # predicted response vector
    y_pred = a + b*x

    # plotting the regression line
    plt.plot(x, y_pred, color = "g")

    # putting labels
    plt.xlabel('x')
    plt.ylabel('y')

    # function to show plot
    plt.show()
```

```
def main():
    for i in range(0, n):
        y_bar = a + b * x[i]
        print("Y_Value: ", y_bar)

    # plotting regression line
    plot_regression_line(x, y, a, b)
```

```
if __name__ == "__main__":
    main()
```

```
D:\Files\UNewFiles\NPU-File\MSCS\2021Summer\CS550\  
Sum of x*y 120.79999999999998  
Sum of x*x 121.34  
Sum of x 31.800000000000004  
Sum of y 32.5  
Slope_b 0.8631776810447154  
Intercept_a 0.5050949742778048  
Y_Value: 1.3682726553225202  
Y_Value: 2.2314503363672356  
Y_Value: 3.3535813217253656  
Y_Value: 4.216759002770081  
Y_Value: 5.079936683814796  
Y_Value: 1.7135437277404062  
Y_Value: 2.663039176889593  
Y_Value: 2.9219924812030076  
Y_Value: 4.044123466561137  
Y_Value: 4.907301147605853  
  
Process finished with exit code 0  
|
```

Non-Linear Model:

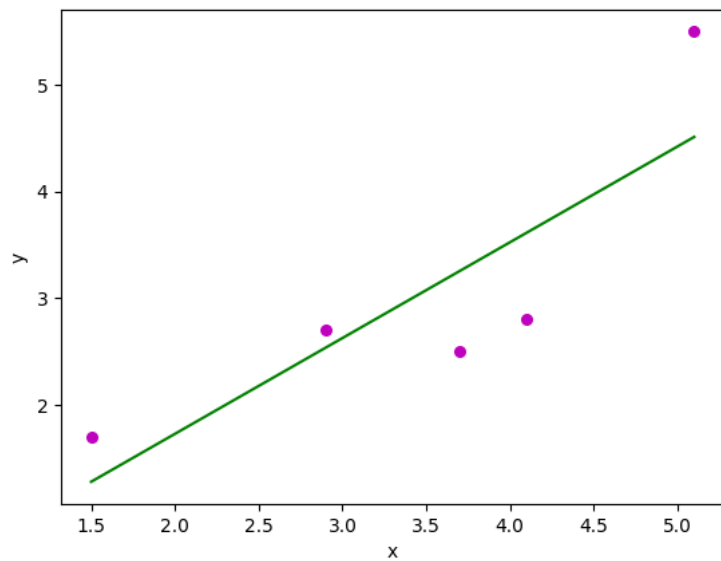
```
D:\Files\UNewFiles\NPU-File\MSCS\2021Summer\CS550\Assig
Sum of x_bar 121.34
Sum of y 32.5
Sum of x_bar * y 509.76199999999994
Sum of x_bar * x_bar 2329.9862
Slope_b 0.13456241139124608
Intercept_a 1.61721970017862
Y_Value: 1.751782111569866
Y_Value: 2.1554693457436045
Y_Value: 3.08260436022929
Y_Value: 4.105278686802761
Y_Value: 5.397077836158722
Y_Value: 1.8809620265054623
Y_Value: 2.458234771373908
Y_Value: 2.672189005485989
Y_Value: 3.879213835665466
Y_Value: 5.11718802046493

Process finished with exit code 0
|
```

Validation Phase - Liner Regression Model:

```
D:\Files\UNewFiles\NPU-File\MSCS\2021Summe
Sum of x*y 59.16
Sum of x*x 67.169999999999999
Sum of x 17.3
Sum of y 15.2
Slope_b 0.8982494529540502
Intercept_a -0.06794310722101393
Y_Value: 1.2794310722100615
Y_Value: 2.5369803063457317
Y_Value: 3.255579868708972
Y_Value: 3.6148796498905917
Y_Value: 4.513129102844642

Process finished with exit code 0
```



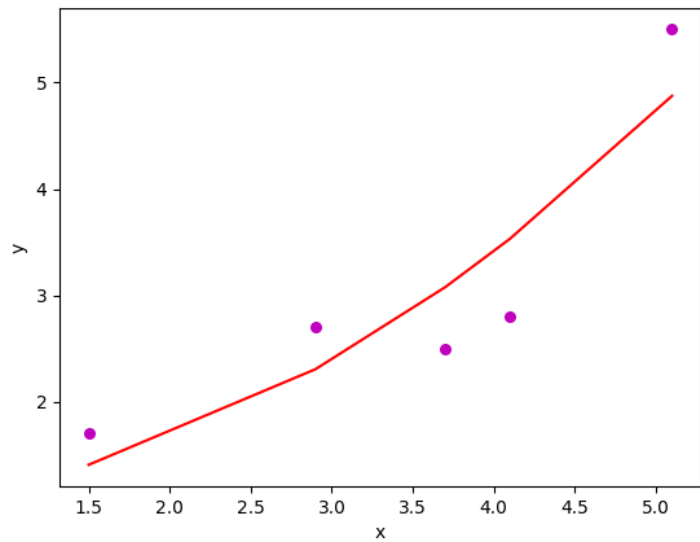
Non-Liner Model:

```

D:\Files\UNewFiles\NPU-File\MSCS\2021Summer\CS
Sum of x_bar 67.16999999999999
Sum of y 15.2
Sum of x_bar * y 250.87999999999997
Sum of x_bar * x_bar 1222.3028999999997
Slope_b 0.14591184777999153
Intercept_a 1.079820236923594
Y_Value: 1.408121894428575
Y_Value: 2.306938876753323
Y_Value: 3.0773534330316785
Y_Value: 3.532598398105251
Y_Value: 4.874987397681173

Process finished with exit code 0

```



MSE – Python Program

```

y = [11,20,19,17,10]
y_bar = [12,18,19.5,18,9]

```

```
summation = 0 #variable to store the summation of differences
n = len(y) #finding total number of items in list
for i in range (0,n): #looping through each element of the list
    difference = y[i] - y_bar[i] #finding the difference between observed and predicted value
    squared_difference = difference**2 #taking square of the difference
    summation = summation + squared_difference #taking a sum of all the differences
MSE = summation/n #dividing summation by total values to obtain average
print ("The Mean Square Error is: ", MSE)
```

Model\_1 – Training Phase = 0.28129

Model\_2 – Training Phase = 0.2348

Model\_1 – Validation Phase = 0.48316

Model\_2 – Validation Phase = 0.3

Compare:

Model\_1 =  $0.48316 / 0.28129 = 1.717$

Model\_2 =  $0.3 / 0.2348 = 1.277$

So:

Model\_2 is better model