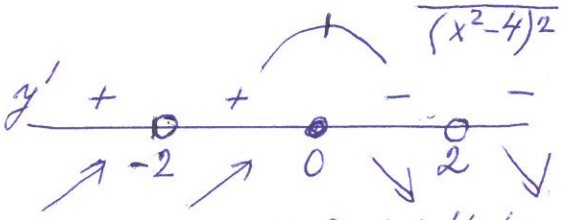
Př1: Vyšetřete průběh funkce $f: y = \frac{x^2}{x^2-4}$

1) $D(f) = \mathbb{R} - \{-2, 2\}$

2) $f(-x) = \frac{(-x)^2}{(-x)^2-4} = \frac{x^2}{x^2-4} = f(x) \Rightarrow$ fce f je sudá

3) $y' = \frac{2x\cdot(x^2-4) - x^2\cdot 2x}{(x^2-4)^2} = \frac{2x^3-8x-2x^3}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$

4) $\frac{-8x}{(x^2-4)^2} = 0 \Rightarrow x = 0$

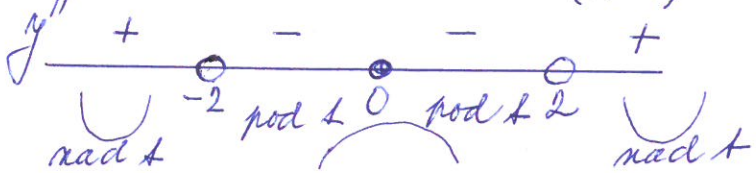


$x = 0$ lokální maximum

5) $y'' = \frac{-8\cdot(x^2-4)^2 + 8x\cdot 2\cdot(x^2-4)\cdot 2x}{(x^2-4)^4} = \frac{(x^2-4)\cdot[-8\cdot(x^2-4) + 8x\cdot 2\cdot 2x]}{(x^2-4)^{4\cdot 3}} =$

$= \frac{-8x^2+32+32x^2}{(x^2-4)^3} = \frac{24x^2+32}{(x^2-4)^3}$

$\frac{24x^2+32}{(x^2-4)^3} = 0 \Rightarrow x \in \emptyset \Rightarrow$ není inflexní bod



6) a) asymptota bez směrnice:

$x = -2 \quad x = 2$ (v bodech nespojitosti)

b) asymptota se směrnicí:

$y = ax + b$

$a = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{x^2}{x^2-4}}{x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\pm\infty} \frac{x}{x^2-4} = \lim_{x\to\pm\infty} \frac{1}{2x} = 0 =$

$b = \lim_{x\to\pm\infty} [f(x) - ax] = \lim_{x\to\pm\infty} \left(\frac{x^2}{x^2-4} - 0\cdot x\right) = \lim_{x\to\pm\infty} \frac{x^2}{x^2-4} =$

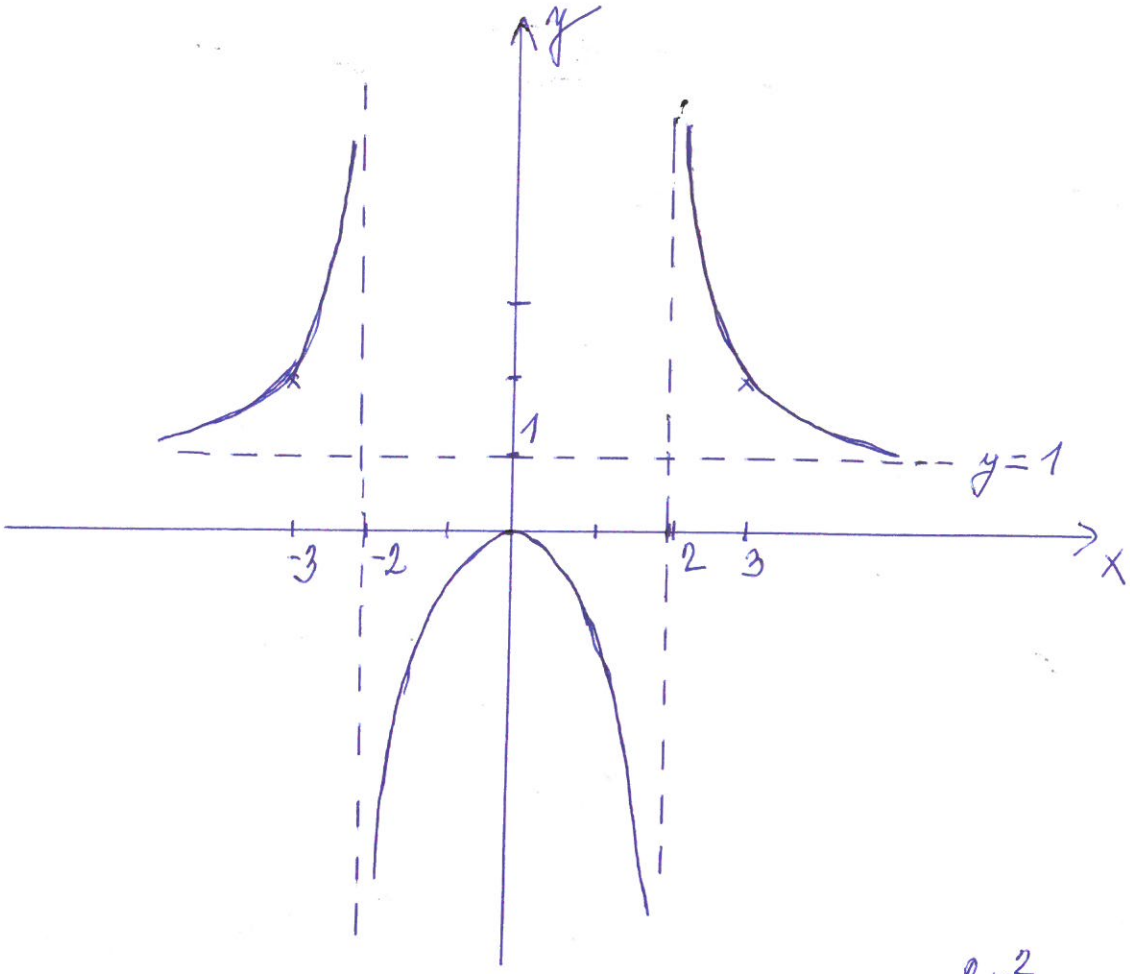
$= \lim_{x\to\pm\infty} \frac{2x}{2x} = \lim_{x\to\pm\infty} 1 = 1 =$

$y = 1$

7)

x	0	-3	-1	3
y	0	$\frac{9}{5}$	$-\frac{1}{3}$	$\frac{9}{5}$

8) graf



<u>Př.2</u>: Vyšetřete průběh funkce $f: y = \frac{2x^2}{1-x^2}$

-příklad na procvičení