Př. 3: Vyšetřeta průběh funkce $f: y = \dfrac{x}{1-x^2}$

1) $D(f) = R - \{-1; 1\}$ body nespojitosti: $x_1 = -1,\ x_2 = 1$

2) $f(-x) = \dfrac{-x}{1-(-x)^2} = -\dfrac{x}{1-x^2} = -f(x) \Rightarrow$ fce f je _lichá_

3) 4)

$$y' = \frac{(1-x^2) - x\cdot(-2x)}{(1-x^2)^2} = \frac{1-x^2+2x^2}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2}$$

$$\frac{x^2+1}{(1-x^2)^2} = 0 \Rightarrow x \in \emptyset \Rightarrow \underline{\text{není extrém}}$$

$$y' \quad + \quad\quad + \quad\quad +$$
$$\nearrow \quad -1 \quad \nearrow \quad 1 \quad \nearrow$$

5)
$$y'' = \frac{2x\cdot(1-x^2)^2 - (x^2+1)\cdot 2\cdot(1-x^2)\cdot(-2x)}{(1-x^2)^4} = \frac{2x\cdot(1-x^2)\cdot(1-x^2+2x^2+2)}{(1-x^2)^{4\ 3}} =$$

$$= \frac{2x\cdot(x^2+3)}{(1-x^2)^3} \qquad\qquad \frac{2x\cdot(x^2+3)}{(1-x^2)^3} = 0 \Rightarrow \underline{x=0}$$

$$y'' \quad + \quad\quad - \quad\quad + \quad\quad -$$
$$\text{nad } t \quad -1 \quad \text{pod } t \quad 0 \quad \text{nad } t \quad 1 \quad \text{pod } t$$
$$\smile \qquad \frown \qquad \smile \qquad \frown$$
$$x=0 \quad \underline{\text{inflexní bod}}$$

6) a) asymptoty bez směrnice:
$$\underline{x=-1}, \quad \underline{x=1} \qquad (\text{v bodech nespojitosti})$$

b) asymptoty se směrnicí: $y = ax + b$

$$a = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{x}{1-x^2}}{x} = \lim_{x\to\pm\infty} \frac{1}{1-x^2} = \underline{\underline{0}}$$

$$b = \lim_{x\to\pm\infty} [f(x) - ax] = \lim_{x\to\pm\infty} \frac{x}{1-x^2} = \lim_{x\to\pm\infty} \frac{1}{-2x} = \underline{\underline{0}}$$
$$\frac{\infty}{\infty}$$

$$\underline{y=0}$$

7)

x	0	$\frac{1}{2}$	2	3	-2
y	0	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{3}{8}$	$\frac{2}{3}$

8) graf