Q4. a)

Given the Jacobian at the $k+1$-th iteration as:

$$J(x_{k+1}) = J(x_k) + u_k v_k^T$$

Where u_k and v_k^T are vectors representing the rank-one update to the Jacobian, the Sherman-Morrison formula can be used to compute the inverse of the updated Jacobian. The inverse is given by:

$$J(x_{k+1})^{-1} = J(x_k)^{-1} - \frac{J(x_k)^{-1} u_k v_k^T J(x_k)^{-1}}{1 + v_k^T J(x_k)^{-1} u_k}$$

b)

Benefits:

1. *Efficiency:* Updating the Jacobian inverse using Sherman-Morrison can be computationally more efficient than recalculating the inverse from scratch.
2. *Memory Usage:* Updating can save memory since the full Jacobian doesn't have to be stored at each step.

Drawbacks:

1. *Accuracy:* A rank-one update might not capture significant changes in the system, leading to a loss in accuracy.
2. *Convergence Issues:* The formula requires that $1 + v^T A^{-1} u$ is not zero. If it approaches zero, numerical instabilities may arise.