MAT167

Yuchen Liu

August 2023

Matrices can be partitioned into several submatrices known as "blocks." Using a blocked approach, matrix-matrix multiplication can be performed with a divide and conquer approach. Supposed we have two matrices, a) Show that C can be writen as

$$C = \begin{pmatrix} M_1 + M_4 + M_5 - M_7 & M_3 + M_5 \\ M_2 + M_4 & M_1 - M_2 + M_3 + M_6 \end{pmatrix}$$

Given:

$$M1 = (A_{11} + A_{22})(B_{11} + B_{22})$$
$$M2 = (A_{21} + A_{22})B_{11}$$
$$M3 = A_{11}(B_{12} - B_{22})$$
$$M4 = A_{22}(B_{21} - B_{11})$$
$$M5 = (A_{11} + A_{12})B_{22}$$
$$M6 = (A_{21} - A_{11})(B_{11} + B_{12})$$
$$M7 = (A_{12} - A_{22})(B_{21} + B_{22})$$

for the first column and first row
$M_1 + M_4 + M_5 - M_7 =$
$A_{11}B_{22}$ + $\cancel{A_{22}B_{11}}$ + $\cancel{A_{22}B_{22}}$ - $\cancel{A_{22}B_{21}}$ - $\cancel{A_{22}B_{11}}$ + $\cancel{A_{11}B_{22}}$ + $\cancel{A_{12}B_{22}}$ + $A_{12}B_{21}$
+ $\cancel{B_{22}A_{12}}$ - $\cancel{A_{22}B_{21}}$ - $\cancel{A_{22}B_{22}}$
$= A_{11}B_{11} + A_{12}B_{21}$
for the first column and second row
$M3 + M5 =$
$A_{11}B_{12}$ - $\cancel{A_{11}B_{22} + A_{11}B_{22}}$ $+A_{12}B_{22}$
$= A_{11}B_{12} + A_{12}B_{22}$
for the second column and first row
$M2 + M4 =$
$A_{21}B_{11}$+$\cancel{A_{22}B11}$+$A_{22}B_{21}$-$\cancel{A_{22}B_{11}}$
$= A_{21}B_{11} + A_{22}B_{21}$
for the second column and second row
$M1 - M2 + M3 + M6 =$
$\cancel{A_{11}B_{22}}$ + $\cancel{A_{22}B_{11}}$ + $A_{22}B_{22}$ - $\cancel{A_{22}B_{21}}$-$\cancel{A_{21}B_{11}}$-$\cancel{A_{22}B11}$+$\cancel{A_{11}B_{12}}$ - $\cancel{A_{11}B_{22}}$+$\cancel{A_{21}B_{11}}$+$A_{21}B_{12}$-
$\cancel{A_{11}B_{11}}$-$\cancel{A_{11}B_{12}}$
$=A_{21}B_{12}+A_{22}B_{22}$

So we can have :

$$C = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12}+A_{22}B_{22} \end{pmatrix}$$