

## Model

$$y(s) = \mu(s) + w(s) + \varepsilon(s)$$

- $\mu(s) = x(s)^T \beta$
- $w(s) \sim GP(0, \sigma^2 \rho(\phi; \|s_1 - s_2\|))$
- $\varepsilon(s) \stackrel{iid}{\sim} N(0, \tau^2)$

## Hierarchical Modeling

First stage:

$$y | \beta, w, \tau^2 \sim \prod_{i=1}^n N(y(s_i) | x(s_i)^T \beta + w(s_i), \tau^2)$$

Second stage:

$$w | \sigma^2, \phi \sim N(0, \sigma^2 R(\phi))$$

Third stage:

$$\text{Priors on } \Omega = (\beta, \tau^2, \sigma^2, \phi)$$

Marginalized likelihood:

$$y | \Omega \sim N(x\beta, \sigma^2 R(\phi) + \tau^2)$$

矩阵表示:

$$y = X\beta + \epsilon, \quad \epsilon \sim N(0, \Sigma), \quad \Sigma = \sigma^2 R(\phi) + \tau^2$$