Proof of Proposition 7

Meng Yang

1 Proof

For the first inequality, if the opposite holds

$$\Pi_{CTR}(\bar{p}, \bar{r}, \Delta\bar{p}(\bar{p}, \bar{r}, \bar{w})) > \Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*)) \quad (1)$$

Under the CTR scheme, fix the first-stage variable to (p^*, r^*) and solve the sub "max-min" problem. Denote the obtained worst case as $\bar{w}^{'}$ and the corresponding second-stage variable as $\Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'})$.

1) If $\bar{w}^{'} = w^*$, then according to Proposition 4, we have

$$\Pi_{CTR}(p^*, r^*, \Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'})) \leq \Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*))$$

As a result, together with (11), we get

$$\Pi_{CTR}(\bar{p}, \bar{r}, \Delta\bar{p}(\bar{p}, \bar{r}, \bar{w})) > \Pi_{CTR}(p^*, r^*, \Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'}))$$

which is contradict to the assumption that $\bar{p}, \bar{r}$ is the optimal solution of robust model under CTR scheme.

2) If $\bar{w}^{'} \neq w^*$, then If

$$\Pi_{CTR}(p^*, r^*, \Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'})) \leq \Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*))$$

Similar contradiction as 1) can be found. Otherwise, if

$$\Pi_{CTR}(p^*, r^*, \Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'})) > \Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*))$$

suppose $\Delta\bar{p}^{*'}$ is the optimal solution under scenario $\bar{w}^{'}$, then because of Proposition 4, we have

$$\Pi_{SMK}(p^*, r^*, \Delta\bar{p}^{*'}(p^*, r^*, \bar{w}^{'})) \geq \Pi_{CTR}(p^*, r^*, \Delta\bar{p}^{'}(p^*, r^*, \bar{w}^{'}))$$

As a result,

$$\Pi_{SMK}(p^*, r^*, \Delta\bar{p}^{*'}(p^*, r^*, \bar{w}^{'})) > \Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*))$$

which is contradict to the assumption that w^* is the worst case for the robust model under sharing scheme. Similarly, we can prove that

$$\Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*)) \leq \Pi_{IND}(\tilde{p}, \tilde{r}, \Delta\tilde{p}(\tilde{p}, \tilde{r}, \tilde{w}))$$

and $\Pi_{SMK}(p^*, r^*, \Delta p^*(p^*, r^*, w^*))$ is decreasing in a.