

ECSE 446/546: Realistic/Advanced Image Synthesis

Assignment 4: Path Tracing

Due: Thursday, December 12th, 2024 at 11:59pm EST on myCourses

Final weight: 30%

Contents

1 Assignment Policies
1.1 Assignment Submission
1.2 Late policy
1.3 Collaboration & Plagiarism
2 Explicit Path Tracing
3 Explicit Path Tracing
4 Russian Roulette
5 Caustics
6 You're Done!

1 Assignment Policies

As with previous assignments, this one builds on top your previous code bases.

1.1 Assignment Submission

Gather **all the python source files within the `taichi_tracer/` folder** (i.e., everything except the `scene_data_dir/` folder) and compress them into a single zip file. Name your zip file according to your student ID, as:

`YourStudentID.zip`

For example, if your ID is `234567890`, your submission filename should be `234567890.zip`.

DO NOT ADD ANYTHING BEFORE OR AFTER THE MCGILL ID.

Info Every time you submit a new file on myCourses, your previous submission will be overwritten. We will only grade the **final submitted file**, so feel free to submit often as you progress through the assignment.

In accordance with article 15 of the Charter of Students' Rights, students may submit any written or programming components in either French or English.

1.2 Late policy

All the assignments are to be completed *individually*. You are expected to respect the **late day policy** and **collaboration/plagiarism** policies, discussed below.

Late Day Allotment and Late Policy

Every student will be allowed a total of **six (6)** late days during the entire semester, without penalty. Specifically, failure to submit a (valid) assignment on time will result in a late day (rounded up to the nearest day) being deducted from the student's late day allotment. Once the late day allotment is exhausted, any further late submissions will obtain a score of **0%**. Exceptional circumstances will be treated as per [McGill's Policies on Student Rights and Responsibilities](#).

If you require an accommodation, please advise [McGill Student Accessibility and Achievement](#) (514-398-6009) as early in the semester as possible. In the event of circumstances beyond our control, the evaluation scheme as detailed on the course website and on assignment handouts may require modification.

1.3 Collaboration & Plagiarism

Plagiarism is an academic offense of misrepresenting authorship. This can result in penalties up to expulsion. It is also possible to plagiarize *your own work*, e.g., by submitting work from another course without proper attribution. **When in doubt, attribute!**

You are expected to submit your own work. Assignments are individual tasks. This does not need to preclude forming an environment where you can be comfortable discussing **ideas** with your classmates. **When in doubt, some good rules to follow include:**

- fully understand every step of every solution you submit,
- only submit solution code that was *written* (not copy/pasted/modified, not ChatGPT-ed, etc.) by you, and
- never refer to another student's code — if at all possible, we recommend that you avoid *looking* at another classmate's code.

McGill values academic integrity and students should take the time to fully understand the meaning and consequences of cheating, plagiarism and other academic offenses (as defined in the Code of Student Conduct and Disciplinary Procedures — see [these two links](#)).

Info Computational plagiarism detection tools are employed as part of the evaluation procedure in ECSE 446/546. Students may only be notified of potential infractions at the end of the semester.

Additional policies governing academic issues which affect students can be found in the Handbook on Student Rights and Responsibilities.

2 Implicit Path Tracing

You will implement a 1-sample/path MC estimator of the (recursive) global illumination equation:

$$L(\mathbf{x}, \omega_o) = L_c(\mathbf{x}, \omega_o) + \int_{S^2} L(\text{ray}(\mathbf{x}, \omega_i), \omega_i) f_r(\mathbf{x}, \omega_o, \omega_i) \max(\mathbf{n} \cdot \omega_i, 0) d\omega_i,$$

as

$$L(\mathbf{x}, \omega_o) \approx L_c(\mathbf{x}, \omega_o) + \frac{L(\text{ray}(\mathbf{x}, \omega_j), \omega_j) f_r(\mathbf{x}, \omega_o, \omega_j) \max(\mathbf{n} \cdot \omega_j, 0)}{p(\omega_j)},$$

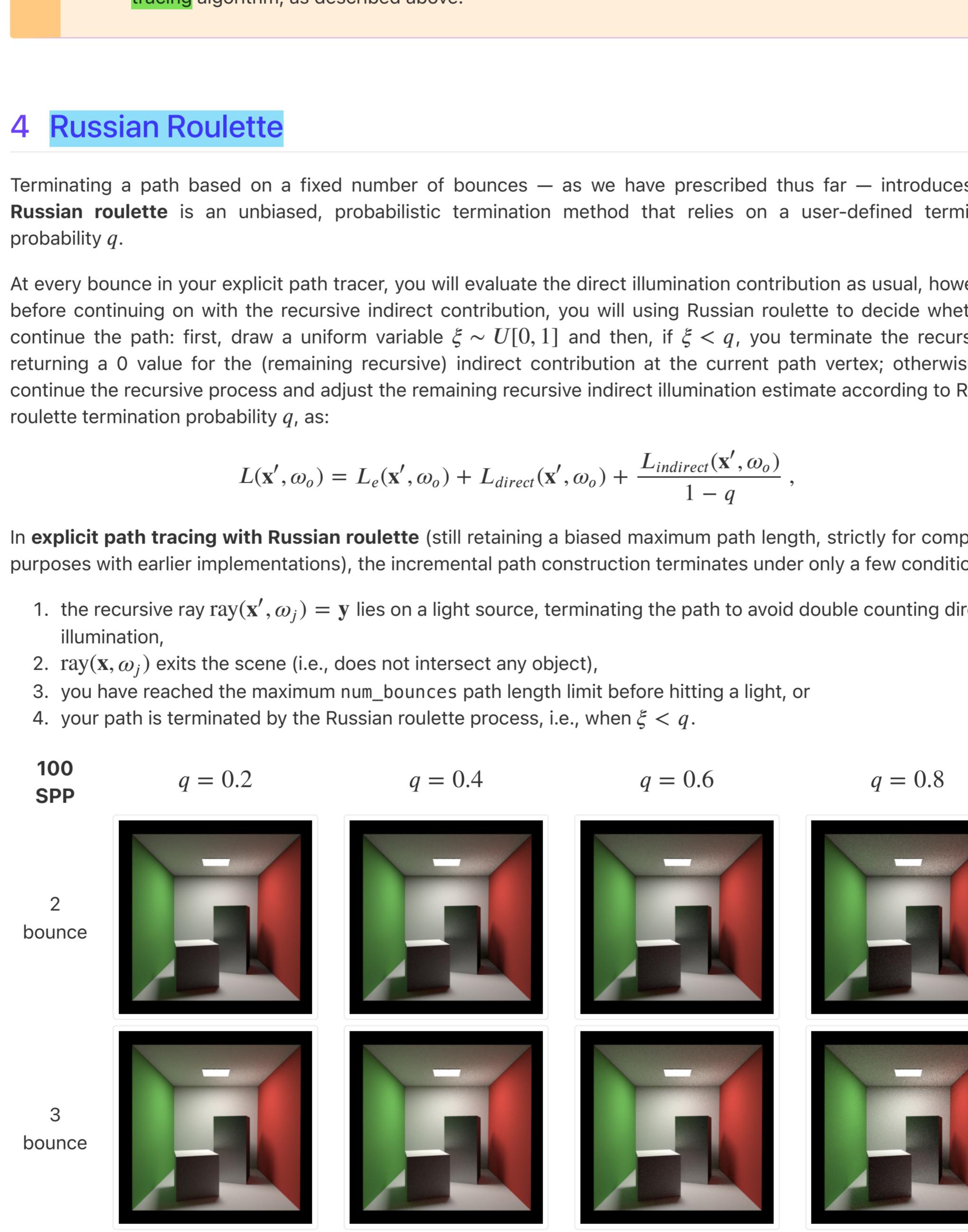
where the ω_j will be drawn using **BRDF importance sampling**, i.e., $\omega_j \sim p(\omega) \propto f_r(\mathbf{x}, \omega_o, \omega)$.

In the implicit path tracing scenario (with biased maximum path length), the incremental path construction **terminates** under only a few conditions:

1. $\text{ray}(\mathbf{x}, \omega_j) = \mathbf{y}$ lies on a **light source**, yielding a (non-recursive, deterministic) value for $L(\text{ray}(\mathbf{x}, \omega_j), \omega_j) = L_s(\mathbf{y}, -\omega_j)$,
2. $\text{ray}(\mathbf{x}, \omega_j)$ exits the scene (i.e., **does not intersect** any object), or
3. you have reached the **maximum num_bounces** path length limit before hitting a light.

The following images show the incremental path construction for implicit path tracing:

1 SPP 10 SPP 10^2 SPP



Info A common design pattern in implicit path tracing implementations is to **incrementally update the path throughput**, starting with an initial value of **(1, 1, 1)** and updating according to **throughput += $(f_r(\mathbf{x}, \omega_{j-1}, \omega_j) \max(\mathbf{n} \cdot \omega_j, 0)) / p(\omega_j)$** ;; then, upon termination, the final 1-sample/path estimate is set as either $L = \text{throughput} \cdot L_s(\mathbf{y}, -\omega_j)$ or zero (e.g., when the recursive ray does not hit an emitter).

- Deliverable 1 [30 points]**
- Complete the implementation of your `A4Renderer's render()` function to support the **implicit path tracing** algorithm, as described above.

3 Explicit Path Tracing

You will next implement an explicit path tracer, separating out the direct (non-recursive) and indirect (recursive) lighting contributions as:

$$L(\mathbf{x}, \omega_o) = L_c(\mathbf{x}, \omega_o) + L_{\text{direct}}(\mathbf{x}, \omega_o) + \int_{S^2 \cap \mathcal{M}_i} L(\text{ray}(\mathbf{x}, \omega_i), \omega_i) f_r(\mathbf{x}, \omega_o, \omega_i) \max(\mathbf{n} \cdot \omega_i, 0) d\omega_i,$$

where the (non-recursive) direct illumination contribution $L_{\text{direct}}(\mathbf{x}, \omega_o)$ will be computed using **light importance sampling**, and the (recursive) indirect term $L_{\text{direct}}(\mathbf{x}, \omega_o)$ is computed using **BRDF importance sampling**. Here, unlike the implicit case above, **direct path vertices should be rejected** if they contribute (locally, i.e., at the current path vertex) to **direct illumination**, adding a certain complexity to the algorithmic logic; this is denoted mathematically. In part, where the exclusion of points $\mathbf{y} \in \mathcal{M}_i$ from the integration domain of the recursive indirect contribution, i.e., where $L(\text{ray}(\mathbf{x}, \omega_i), \omega_i) = L_s(\mathbf{y}, -\omega_i) \neq 0$.

Validation against the implicit path tracing results can be used to confirm that the explicit path tracer converges to the same result (albeit, usually, with far fewer accumulated samples per pixel). As discussed in class, recall that — unlike the implicit case — each 1-sample estimate that your Scene.render() explicit path tracer generates may potentially correspond to a contribution from *more than one* energy carrying light path.

In the explicit path tracing scenario (with biased maximum path length), the incremental path construction **terminates** under only a few conditions:

1. $\text{ray}(\mathbf{x}, \omega_j) = \mathbf{y}$ lies on a **light source**, terminating the path to avoid double counting direct illumination,
2. $\text{ray}(\mathbf{x}, \omega_j)$ exits the scene (i.e., **does not intersect** any object), or
3. you have reached the **maximum num_bounces** path length limit before hitting a light.

The following images show the incremental path construction for explicit path tracing:

1 SPP 10 SPP 10^2 SPP

Info Deliverable 2 [10 points]

- Complete the implementation of your `A4Renderer's render()` function to support the **explicit path tracing** algorithm, as described above.

4 Russian Roulette

Terminating a path based on a fixed number of bounces — as we have prescribed thus far — introduces bias. **Russian roulette** is an unbiased, probabilistic termination method that relies on a user-defined termination probability q :

At every bounce in your explicit path tracer, you will evaluate the direct illumination contribution as usual, however — before continuing on with the recursive indirect contribution, you will use Russian roulette to decide whether to continue the path: first, draw a uniform variable $\xi \sim U[0, 1]$ and then, if $\xi < q$, you terminate the recursion — returning a 0 value for the (remaining recursive) indirect contribution at the current path vertex; otherwise, you continue the recursive process and adjust the remaining recursive indirect illumination estimate according to Russian roulette termination probability q , as:

$$L(\mathbf{x}, \omega_o) = L_c(\mathbf{x}, \omega_o) + L_{\text{direct}}(\mathbf{x}, \omega_o) + \frac{L(\text{ray}(\mathbf{x}, \omega_i), \omega_i) f_r(\mathbf{x}, \omega_o, \omega_i) \max(\mathbf{n} \cdot \omega_i, 0)}{1 - q} d\omega_i,$$

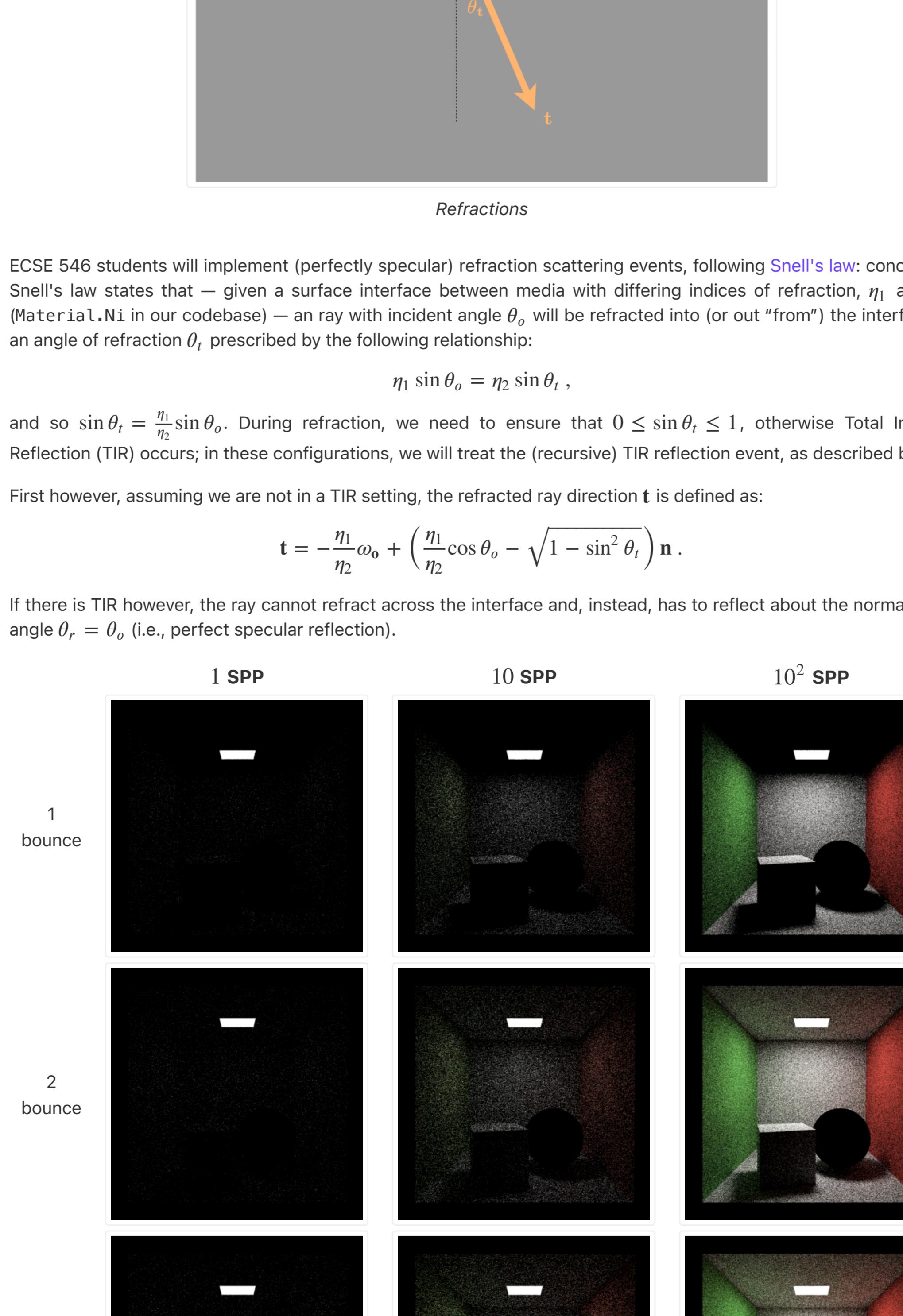
Validation against the implicit path tracing results can be used to confirm that the explicit path tracer converges to the same result (albeit, usually, with far fewer accumulated samples per pixel). As discussed in class, recall that — unlike the implicit case — each 1-sample estimate that your Scene.render() explicit path tracer generates may potentially correspond to a contribution from *more than one* energy carrying light path.

In the explicit path tracing scenario (with biased maximum path length), the incremental path construction **terminates** under only a few conditions:

1. $\text{ray}(\mathbf{x}, \omega_j) = \mathbf{y}$ lies on a **light source**, terminating the path to avoid double counting direct illumination,
2. $\text{ray}(\mathbf{x}, \omega_j)$ exits the scene (i.e., **does not intersect** any object), or
3. you have reached the **maximum num_bounces** path length limit before hitting a light.

The following images show the incremental path construction for explicit path tracing using Russian roulette:

1 SPP 10 SPP 10^2 SPP



Info Deliverable 3 [5 points]

- Modify your implementation of `A4Renderer's render()` to implement an explicit path tracer with support **Russian roulette**, as described above.

5 Caustics

At every bounce in your explicit path tracer, you will evaluate the direct illumination contribution as usual, however — before continuing on with the recursive indirect contribution, you will use Russian roulette to decide whether to continue the path: first, draw a uniform variable $\xi \sim U[0, 1]$ and then, if $\xi < q$, you terminate the recursion — returning a 0 value for the (remaining recursive) indirect contribution at the current path vertex; otherwise, you continue the recursive process and adjust the remaining recursive indirect illumination estimate according to Russian roulette termination probability q , as:

$$L(\mathbf{x}, \omega_o) = L_c(\mathbf{x}, \omega_o) + L_{\text{direct}}(\mathbf{x}, \omega_o) + \frac{L(\text{ray}(\mathbf{x}, \omega_i), \omega_i) f_r(\mathbf{x}, \omega_o, \omega_i) \max(\mathbf{n} \cdot \omega_i, 0)}{1 - q} d\omega_i,$$

If there is TIR however, the ray cannot refract across the interface and, instead, has to reflect about the normal at an angle $\theta_r = \theta_o$ (i.e., perfect specular reflection).

$$\mathbf{t} = \frac{\eta_1}{\eta_2} \omega_o + \left(\frac{\eta_1}{\eta_2} \cos \theta_o - \sqrt{1 - \sin^2 \theta_o} \right) \mathbf{n}.$$

First however, assuming we are not in a TIR setting, the refracted ray direction \mathbf{t} is defined as:

$$\mathbf{t} = \frac{\eta_1}{\eta_2} \omega_o + \left(\frac{\eta_1}{\eta_2} \cos \theta_o - \sqrt{1 - \sin^2 \theta_o} \right) \mathbf{n}.$$

and so $\sin \theta_r = \frac{\eta_1}{\eta_2} \sin \theta_o$. During refraction, we need to ensure that $0 \leq \sin \theta_r \leq 1$, otherwise Total Internal Reflection (TIR) occurs; in these configurations, we will treat the (recursive) TIR reflection event, as described below.

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