Written Exercises

1.  
Maximize: aT B a

Subject to: aT W a = 1

Apply Lagrange Multiplier:  
l(λ) = aT B a – λ (aT W a − 1)

∂l/∂a = (B + BT) a – λ (W + WT) a = 0

= (B + BT) a = λ (W + WT) a

= (W + WT)-1 (B + BT) a = λ a  
because matrices B and W are symmetric, we reduce this to  
W-1 B a = λ a

This becomes a standard eigenvalue problem T(a) = λ a where we apply transformation of (W-1 B) over a.

2.

(a)

δk(x) = xT Σ −1µk – ½ µTk Σ −1µk + log πk

When δ2(x) > δ1(x), LDA rule classifies to class 2 if  
xT Σˆ−1 (ˆµ2 − µˆ1) > ½ (ˆµ2 + ˆµ1)T Σˆ−1 (ˆµ2 − µˆ1) + log(N1/N) − log(N2/N)  
= ½ ˆµ2T Σˆ−1 ˆµ2 − ½ ˆµ1T Σˆ−1 µˆ1 + log(N1/N) − log(N2/N)

(b)

(c)  
ΣˆBβ = (ˆµ2 − µˆ1)(ˆµ2 − µˆ1)T β

Since (ˆµ2 − µˆ1)T β is a scalar and Σˆβ is a linear combination of B in the direction of (ˆµ2 − µˆ1), βˆ ∝ Σˆ−1(ˆµ2 − µˆ1).

(d)

We can replace N with N1N2/N (y2-y1):

β = N1N2/N(N-2) [(y2-y1) - (ˆµ2 − µˆ1)T β] Σˆ−1(ˆµ2 − µˆ1) ∝ Σˆ−1(ˆµ2 − µˆ1)

(e)

βˆo = 1/N 1T (Y − Xβ)

= -1/N(N1 µˆ1T+ N2 µˆ2T) β

f(x) = βo + βT x

🡪 f(x) = 1/N(Nx T - N1 µˆ1T- N2 µˆ2T) β

= 1/N(Nx T - N1 µˆ1T- N2 µˆ2T) λ Σˆ−1(µˆ2 - µˆ1)

When f(x) > 0,

NxT λΣˆ −1 (ˆµ2 − µˆ1) > (N1µˆ T 1 + N2µˆ T 2 ) λΣˆ −1 (ˆµ2 − µˆ1)

xT Σˆ −1 (ˆµ2 − µˆ1) > 1/N (N1µˆ T1 + N2µˆ T2) Σˆ −1 (ˆµ2 − µˆ1)

3.

(a)

Reduce M to row reduced form:  
R2 = R2 – 3R1  
[[1, 0, 3], [0, 7, -7], [2, -2, 8], [0, -1, 1], [5, 8, 7]]

R3 = R3 – 2R1

[[1, 0, 3], [0, 7, -7], [0, -2, 2], [0, -1, 1], [5, 8, 7]]

R5 = R5 – 5R1  
R2 = R2 / 7

R3 = R3 + 2R2

R4 = R4 + R2

R5 = R5 – 8R2

[[1, 0, 3], [0, 1, -1], [0, 0, 0], [0, 0, 0], [0, 0, 0]]

MMT  = [[1, 0, 3], [0, 1, -1], [0, 0, 0], [0, 0, 0], [0, 0, 0]] [[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [3, -1, 0, 0, 0]] = [[10, -3, 0, 0, 0], [-3, 2, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]

MMT = [[10, 9, 26, 3, 26],

[ 9, 62, 8, -5, 85],

[ 26, 8, 72, 10, 50],

[ 3, -5, 10, 2, -1],

[ 29, 85, 50, -1,138]]

MTM = [[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [3, -1, 0, 0, 0]] [[1, 0, 3], [0, 1, -1], [0, 0, 0], [0, 0, 0], [0, 0, 0]] = [[1, 0, 3], [0, 1, -1], [3, -1, 10]]

MTM = [[ 39, 57, 60],

[ 57, 118, 53],

[ 60, 53, 127]]

(b)

Eigenvalues for MMT:

10x1 – 3x2 = 0  
+3/2(-3x1 + 2x2 = 0)

🡪 x1 = 2/11, x2 = 22/33

Eigenvalues for MTM

(c)

Eigenvectors for MMT

Eigenvectors for MTM