

Design and Analysis of Algorithms

Math review (1)

The goal of this document is to review some of the math needed for this course. It also contains links to web sites that provide added information on some of the topic. It contains also additional formulas and procedures.

Proofs and Mathematical Induction

<http://www.purplemath.com/modules/inductn.htm>

<http://zimmer.csufresno.edu/~larryc/proofs/proofs.html>

<http://www.teachers.ash.org.au/mikemath/mathsc/structureproof/>

Logarithms

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^y = y \log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$x^{\log_b y} = y^{\log_b x}$$

$$\lg x = \log_2 x$$

$$\ln x = \log_e x$$

$$\log^k n = (\log n)^k$$

$$\lg \lg n = \lg(\lg n)$$

<http://www.purplemath.com/modules/logs.htm>

Other

$$m \bmod n = m - n \left\lfloor \frac{m}{n} \right\rfloor$$

Simple Series

1. **Arithmetic series:** The first n values are: $a, a + d, a + 2d, \dots, a + (n - 1)d$ where a is an initial value and d is a fixed increment. The sum of the first n values is:

$$\sum_{i=0}^{n-1} a + id = \frac{(2a + (n-1)d)n}{2}.$$

Let $a_1 = a$, $a_n = a + (n-1)d$ the sum is also $\frac{(a_1 + a_n)n}{2}$.

2. **Geometric series:** The first n values are: $a, ar, ar^2, \dots, ar^{n-1}$ where a is an initial value and $r \neq 1$ is a fixed multiplier. The sum of the first n values is:

$$\sum_{i=0}^{n-1} ar^i = \frac{a(1-r^n)}{1-r}.$$

When $-1 < r < 1$ and $r \neq 1$ the sum of the infinite geometric

progression converges to: $\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}.$

3. **Harmonic series:** The first n values are: $1, 1/2, 1/3, \dots, 1/n$. The sum of the first n values is: $H_n = 1 + 1/2 + 1/3 + \dots + 1/n$ satisfies $\ln(n+1) < H_n \leq 1 + \ln n$.

[Arithmetic and geometric progressions](#)

L'Hopital's Rule:

If $f(x)$ and $g(x)$ are both differentiable with derivatives $f'(x)$ and $g'(x)$, respectively, and if $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} g(x) = \infty$ then

$$\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = \lim_{x \rightarrow \infty} \frac{g'(x)}{f'(x)}$$

whenever the limit on the right exists

Review of limits

$$\lim_{x \rightarrow c} f(x) = l \text{ iff } \begin{cases} \text{for each } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that} \\ \text{if } 0 < |x - c| < \delta, \text{ then } |f(x) - l| < \varepsilon. \end{cases}$$

If $\lim_{x \rightarrow c} f(x) = l$ and $\lim_{x \rightarrow c} g(x) = m$ then:

a) $\lim_{x \rightarrow c} [f(x) + g(x)] = l + m$

b) $\lim_{x \rightarrow c} af(x) = al$ for any real a

c) $\lim_{x \rightarrow c} [f(x)g(x)] = lm$

d) If $m \neq 0$ then $\lim_{x \rightarrow c} \frac{1}{g(x)} = \frac{1}{m}$ and $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{l}{m}$

Review of derivatives

$$p(x) = x^n \text{ then } p'(x) = nx^{n-1} \text{ for } n \neq 0$$

$$(fg)'(x) = f(x)g'(x) + g(x)f'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}(f(g)) = f'(g(x))g'(x)$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)}f'(x)$$

$$\frac{d}{dx}(p^x) = p^x \ln p$$

$$\frac{d}{dx}[p^{g(x)}] = p^{g(x)}g'(x) \ln p$$

$$\frac{d}{dx}(\log_p x) = \frac{1}{x \ln p}$$

$$\frac{d}{dx}(\log_p g(x)) = \frac{g'(x)}{g(x) \ln p}$$