HW1

2. [20 points] Prove the following using the original definitions of O, Ω, θ, o, *and* ω.

(a) $3n^3 + 50n^2 + 4n - 9 \in O(n^3)$

(b) $1000n^3 \in \Omega(n^2)$

(c) $10n^3 + 7n^2 \in \omega(n^2)$

(d) $78n^3 \in o(n^4)$

(e) $n^2 + 3n - 10 \in \Theta(n^2)$

(b) $1000n^3 \in \Omega(n^2)$

From the definition, there must exist $c>0$ and integer $N>0$, such that

$$1000n^3 >= cn^2 \text{ for all } n>=N$$

$$1000n >= c$$

Choose $c=1$, $N=1$, the definition is satisfied.

(c) $10n^3 + 7n^2 \in \omega(n^2)$

From the definition, for every $c>0$, there exists integer $N>0$, such that

$$10n^3 + 7n^2 >= cn^2 \text{ for all } n>=N$$

$$10n + 7 >= c$$

$$n >= (c-7)/10$$

Choose $N = \max(1, (c-7)/10)$, the definition is satisfied.

(a) $3n^3 + 50n^2 + 4n - 9 \in O(n^3)$

From the definition, there must exist c>0 and N>0, such that

$$3n^3 + 50n^2 + 4n - 9 <= cn^3 \text{ for all } n>=N$$

$$3n^3 + 50n^2 + 4n - 9 <= 3n^3 + 50n^3 + 4n^3 - 9$$
$$<= 57n^3$$

Choose c=57, N=1, the definition is satisfied.

(d) $78n^3 \in o(n^4)$

From the definition, for every c>0, there exists N>0, such that

$$78n^3 <= cn^4 \text{ for all } n>=N$$

$$78 <= cn$$

$$n >= 78/c$$

Choose N = 78/c, the definition is satisfied.

5. [20 points] Let $f(n)$ and $g(n)$ be asymptotically positive functions. For each of the following conjectures, either prove it is true or provide a counter example to show it is not true.

a. $\big(f(n) + g(n)\big) \in \Theta\big(\max\big(f(n), g(n)\big)\big)$.

b. $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$.

c. $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$.

d. $f(n) \in O\big(g(n)\big)$ implies $\lg\big(f(n)\big) \in O\big(\lg\big(g(n)\big)\big)$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for sufficiently large n.

a. $\left(f(n) + g(n)\right) \in \Theta\left(\max\left(f(n), g(n)\right)\right).$

For big O,

Wrong:

1. "Assume f(n) = n, g(n) = n^2 …"
2. "Assume f(n) <= g(n), then max(f(n), g(n)) = f(n) …"

f(x) = x + sinx, g(x) = x - sinx

f(n) <= max(f(n), g(n))
g(n) <= max(f(n), g(n))

f(n) + g(n) <= 2*max(f(n), g(n)), satisfied.

For big Omega,

f(n) + g(n) >= max(f(n), g(n)), satisfied.

Based on the definition of theta, it is true.

b. $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$.

$f(n) = 2n$, $g(n) = n$, $f \in O(g)$

$2^{f(n)} = 2^{2n} = 4^n$, $2^{g(n)} = 2^n$, $2^f \notin O(2^g)$

Wrong:

1. "$f(n) = n$, $g(n) = n^2$"

d. $f(n) \in O(g(n))$ implies $\lg(f(n)) \in O(\lg(g(n)))$, where lg(g(n))≥1 and $f(n)$ ≥1 for sufficiently large n.

If f(n) ∈ O(g(n)), then there exist positive constants c and N, s.t.

f(n) <= cg(n) for all n >= N

lgf(n) <= lgc + lgg(n)
<= lgc*lgg(n) + lgg(n)
<= (lgc + 1)*lgg(n)

c2 = lgc+1, lgf(n)<= c2*lgg(n).
So lgf(n) ∈ O(lgg(n)).
True.

7 (b) What is the time complexity T(n). You may assume that n is divisible by 4.

```
for ( i = 2; i <= n; i++) {          n-1
    for ( j = 0; j <= n) {           5
        cout << i << j;
        j = j + ⌊n/4⌋;
    }
}
```

T(n) = 5x(n-1) ∈ O(n)