

Design and Analysis of Algorithms
CS575, Spring 2023

Theory Assignment 2.1

Due on 3/2/2023 (Thursday)

1. (16 points) Use the iteration method or recursion tree method to solve the following recurrence equation.

a) (8 points)

$$T(n) = \begin{cases} T(n-1) + n & \text{if } (n > 1) \\ 1 & \text{if } (n = 1) \end{cases}$$

b) (8 points) You can assume $n^{1/2^k} = 2$ for some integers n and k .

$$T(n) = \begin{cases} 0 & \text{if } n = 2 \\ T(\sqrt{n}) + 1 & \text{if } n > 2 \end{cases}$$

2. (6 points) Use Master method to solve $T(n) = 4T(n/2) + n^2$ and $T(1)=1$.
3. (10 points) Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's Algorithm. His algorithm will use divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems, then the recurrence for the running time $T(n)$ becomes $T(n) = aT(n/4) + \theta(n^2)$. What is the largest integer value of a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?