Math review (2)

1. $x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$
2. For any integer n: $\lceil n/2 \rceil + \lfloor n/2 \rfloor = n$

   When n is even, $n=2k$ and $\lfloor n/2 \rfloor = \lceil n/2 \rceil = k$
   When n is odd $n=2k+1$. $n/2=k+1/2$. So $\lfloor n/2 \rfloor = k$ and $\lceil n/2 \rceil = k+1$.
   In both cases the sum of $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$

3. $\lceil \lceil n/a \rceil / b \rceil = \lceil n/ab \rceil$ (symmetrical for floor)

   Let $n=abk$. In this case $\lceil n/a \rceil = n/a = bk$, and $\lceil bk/b \rceil = k$. $\lceil n/ab \rceil = n/ab) = k$.

   Let $n=abk + r$ where $0 < r < ab$. In this case $\lceil n/a \rceil = \lceil bk + r/a \rceil = bk+1$, and $\lceil (bk+1)/b \rceil = k+1$. $\lceil n/ab \rceil = \lceil k + r/ab \rceil = k+1$.

4. $\lceil a/b \rceil \le (a+(b-1))/b$.
   Let $a=kb$. $a+(b-1)=kb+(b-1)$. $(a+(b-1))/b=k+(b-1)/b$. $\lceil a/b \rceil = k < k+(b-1)/b$
   Let $a=kb+r$ where $0<r<b$. $a+(b-1)=kb+(b-1)+r$. $(a+(b-1))/b=(k+1)+(r-1)/b$. Ceiling $(a/b)=k+1 \le k+1+(r-1)/b$

5. Let $a = kn + r$. Then $r = a \bmod n = a - kn = a - \text{floor}(a/n)n$

6. k-permutations – ordered subsequence of k distinct elements of set S. For example when S={a, b, c} the 2-permutations are ab, ac, ba, bc, ca, cb.
   The number of k-permutation of a set of size n are: $n(n-1)(n-2)\dots(n-k+1) = \frac{n!}{(n-k)!}$

7. k-combinations –k subset of S. Example S={a, b, c}, the 2-combinations are ab, ac, bc.
   The number of k-combinations of a set of size n are: $\frac{n!}{(n-k)!k!}$

8. Binomial coefficients. We use the notation $\binom{n}{k}$ to denote the number of k combinations.
9. It is easy to see that $\binom{n}{k} = \binom{n}{n-k}$
10. It is easy to see that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
11. The binomial expansion: $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$. A special case occurs when $x = a = 1$.

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

12. Some inequalities: For $1 \le k \le n$ we have $\binom{n}{k} \ge \left(\frac{n}{k}\right)^k$, $\binom{n}{k} \le \frac{n^k}{k!}$