Taxing Carbon, Not Competition: Optimizing Market and Environment in a World of Imperfections

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Abstract

This paper analyzes carbon taxation under market power. We develop a dynamic general equilibrium model with an oligopolistic carbon-intensive intermediate sector to study the effects of carbon taxation on emissions and aggregate welfare. In the United States, carbon-intensive industries tend to have higher measures of market power. In the absence of industrial policy, the optimal carbon tax levied on oligopolistic sectors must be differentiated based on the industry market power levels. We derive a formula for the optimal carbon tax that depends on the firm's price elasticity of demand. Using data on the size and markups of the U.S. coal mining industry, the calibrated model shows that the optimal carbon tax on unregulated oligopolistic coal producers is, on average, 27 percent lower than the conventional Pigouvian tax. Moreover, the difference between the optimal and Pigouvian taxes increases as within-industry competition decreases.

JEL Codes: E62, H21, H23, L13, L52, Q48, Q54, Q58.

Keywords: Carbon taxes, Market power, Climate change, Externalities, Optimal taxation, Industrial policy, Dynamic integrated assessment modeling.

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1 Introduction

For some time now, the theory of optimal environmental policy has been concerned with the implications of market power in polluting industries; however, the literature on optimal climate change policy has not yet addressed this issue. When a production externality, such as pollution, is the only economic distortion, the Pigouvian tax, equal to marginal external damages or the difference between marginal social and private costs, is the first-best policy. However, when polluting firms have market power, the Pigouvian tax is no longer the first-best policy. Indeed, imposing the Pigouvian tax on an unregulated, imperfectly competitive industry may lead to welfare loss by reducing the output by more than the socially optimal amount. In this case, the optimal policy should include two policy instruments: a subsidy on output to increase production to competitive levels and a tax on pollution to reduce production to the socially optimal level. However, competition and environmental authorities are often separate, and thus, policies are uncoordinated. The environmental policy should account for market power since the Pigouvian tax is not the first-best policy without competition regulation. How much the policy must be adjusted depends on market structure, characterized by the elasticity of demand and industry size.

This simple theoretical observation, studied for local pollutants' regulation in the past, e.g., Buchanan (1969), Barnett (1980), Levin (1985), Shaffer (1995), Simpson (1995), Katsoulacos and Xepapadeas (1996), Lee (1999), and Requate (2006), has been overlooked in the literature on climate change policy. Unlike local pollutants, greenhouse gases, measured in carbon dioxide (CO_2) equivalent units, in the atmosphere do not cause immediate damage; instead, the accumulation of global greenhouse gas emissions drives climate change, which increases the frequency and influences the magnitude of extreme weather events, such as hurricanes, heat waves, and droughts. Thus, regulating carbon emissions has dynamic implications, and the optimal policy should account for the intertemporal nature of climate change. The literature on optimal carbon tax has focused on the intertemporal nature of climate change and the uncertainty surrounding the damages caused by carbon emissions, e.g., Nordhaus (1994), Manne and Richels (2005), Anthoff and Tol (2013), and Golosov et al. (2014). However, it has not yet addressed the intratemporal tradeoff between lack of competition and climate damage. This paper aims to address this gap by analyzing the implications of market power in the context of climate change policy.

Our research theoretically derives the optimal policy for an imperfectly competitive and unregulated carbon-intensive industry in the context of climate change. It further quantitatively evaluates the implications of market power on the optimal policy. Using a seminal dynamic general equilibrium climate change model, we find that the optimal carbon tax assuming no entry is smaller when competition decreases. The underlying mechanism is that since the equilibrium output is lower with market power, the optimal tax does not have to be as high as it would be in a competitive industry to achieve the same reduction in emissions. This result depends on the assumption that the industry structure is exogenous, does not change over time, and is not affected by policy. We also find that the optimal policy is more sensitive to market power in the short rather than the long run. This result is because the accumulation of emissions increases damages over time, and the welfare loss due to damages supersedes the welfare loss from underproduction due to market power.

The literature on optimal climate change policy has focused on the assumption that carbon-intensive industries are perfectly competitive when designing optimal policy. Market power is a valid concern in the context of climate change policy due to the concentrated structure of carbon-intensive industries. To see this, we examine the data on the market structure of carbon-intensive industries in the U.S. The carbon intensity of an industry is the ratio of aggregate industry greenhouse gas emissions, measured in metric tons of CO_2 equivalent (mtCO2e), and the industry value added, measured in millions of dollars. The concentration ratio of N largest firms in an industry is the ratio of the total revenues of the N largest firms in the industry to the industry total. As the concentration ratio increases, the largest firms have more market power. We calculate carbon intensity by aggregating detailed facility-level direct greenhouse gas emissions data from the U.S. Environmental Protection Agency's (2024) Greenhouse Gas Reporting Program (GHGRP) and industry total value-added data from the U.S. Bureau of Economic Analysis's (2024b) Input-Output Accounts' Use Tables. We obtain the concentration ratio of the largest 50 firms from the U.S. Census Bureau's (2023) 2017 Economic Census, the latest available data year for the Economic Census's industry concentration measures.[1]

The data suggests that imperfect competition is a reasonable assumption for the market structure of carbon-intensive industries. In Figure 1, we plot these two measures against each other to show the significant positive relationship between the concentration ratio and the carbon intensity of U.S. industries in 2017. Each scatter point represents an industry labeled by its 2017 NAICS code. The vertical axis shows the carbon intensity of the industry, plotted using a log scale to accommodate the wide range of carbon intensity across industries. The horizontal axis shows the concentration ratio of the largest 50 firms in each industry, measuring market power. Furthermore, each scatter point is scaled (i.e., weighted) by that industry's share of total value added, which measures the industry's size relative to the aggregate economy.

[1]The Economic Census is conducted every five years. The 2022 Economic Census market concentration data release date is June 2025.

The red straight line is the weighted least squares regression line, which shows a significant positive correlation coefficient between the concentration ratio and the carbon intensity of U.S. industries. The scatter plot shows that industries with higher concentration ratios tend to have greater carbon intensities. These industries include cement manufacturing (32731), gas manufacturing (32411, 32512), coal mining (2121), waste management and remediation services (562), paper mills (32212), electric power generation, transmission, and distribution (2211), pipeline transportation (486), and fats and oils refining and blending (311225). The concentration ratios of these highly carbon-intensive industries are significant, ranging from 60 to 100 percent.

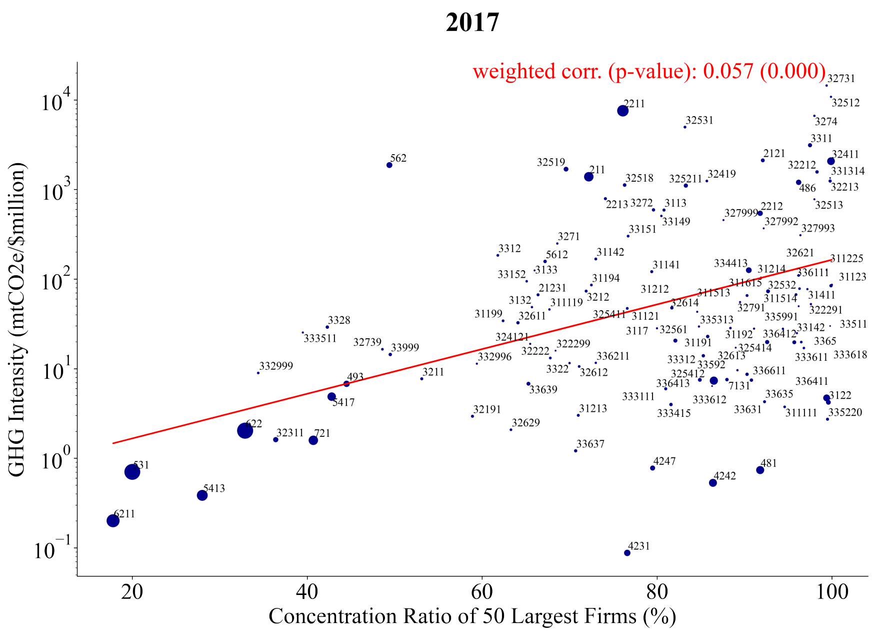

Figure 1: Concentration ratio of 50 largest firms and carbon intensity of U.S. NAICS industries in 2017. Sources: Concentration ratio from U.S. Census Bureau (2023) and carbon intensity from the author's calculations using data from U.S. Environmental Protection Agency (2024) and U.S. Bureau of Economic Analysis (2024b).

Based on this evidence, we develop a theoretical model to derive the optimal policy for an oligopolistic and unregulated carbon-intensive industry in the context of climate change. Our study is based on the dynamic general equilibrium climate change model from Golosov et al. (2014). We extend their model to allow for imperfect competition and quantitatively evaluate the implications of market power on the optimal policy.

We first extend Golosov et al.'s (2014) model to include market power in the carbon-intensive industry. We assume that the carbon-intensive energy sector's market structure is exogenous, and we assume that a finite number of imperfectly substitutable energy sources, such as different types of coal, are used to produce an energy composite using a constant elasticity of substitution (CES) production function.

Under this new assumption, we derive an optimal carbon tax formula for the imperfectly competitive and unregulated coal industry. We normalize energy outputs to units of carbon content and show that the optimal carbon tax is the sum of the differences between the marginal social cost and the marginal private cost of carbon emissions and between the marginal revenue and the marginal cost of energy production evaluated at the optimal allocation. The latter difference is a simple function of the energy price and the elasticity of energy demand. Moreover, by the law of demand, the elasticity of energy demand is negative; thus, our formula implies that the optimal carbon tax is less than the Pigouvian tax, equal to the marginal external damage at the optimal allocation.

We quantitatively evaluate the adjustment of the optimal tax formulation by calibrating parameters governing the extension of the benchmark model to match the U.S. coal industry's characteristics. We calibrate the industry size of the coal industry to match the U.S. coal industry's HHI and the elasticity of energy demand to match the U.S. coal industry's average markups. Furthermore, we adjust the benchmark calibration of the output share of energy in the final goods production to match the U.S. output share of coal in the final goods production. Our quantitative model replicates the benchmark results from Golosov et al. (2014) and shows that the optimal carbon tax with blockaded entry is smaller when competition decreases. Moreover, we find that the optimal policy is more sensitive to market power in the short rather than the long run.

Finally, we test the sensitivity of our results to changes in model parameters and assumptions. First, we check the sensitivity of our results to the changes in parameters governing the market power extension of the benchmark model. As expected, the optimal carbon tax gets smaller when competition decreases. When we relax the assumption of an oligopolistic coal industry to a monopolistic competitive one, we find that our results do not change significantly. In contrast, introducing returns to scale in coal production changes optimal policy significantly.

Immediate extensions we would like to explore include the introduction of clean renewables in the energy composite and the endogenous determination of the energy sector's market structure. Introducing clean renewables is crucial for the optimal policy's implications for the energy transition. Moreover, the endogenous determination of the energy sector's market structure is an essential extension of our model. Existing literature has shown that environmental policy can affect market structure, and market structure can affect environmental policy. The optimal policy's adjustment direction is unclear in an endogenous market structure.

Other extensions could include introducing dynamic changes in the energy markups and inflation using a non-CES energy demand system, such as the Kimball (1995) demand system. Our analysis relies on the time consistency of the optimal policy, which is not a realistic assumption. Thus, considering time-inconsistent policy would be an essential extension of our model. Finally, we could relax the linearity assumption between energy output and carbon emission units and derive a more general formula for the optimal carbon tax. Under this relaxation, achieving the first-best allocation may not be feasible using a single policy instrument, and the optimal policy may only achieve the second-best.

1.1 Related Literature

This paper contributes to two strands of literature. First is the extensive literature on optimal policy to control the problem of climate change while maintaining economic growth. Many macroeconomic studies of optimal carbon taxes focus on the climate externality as the only distortion in the modeled economy, e.g., Nordhaus (1994), Manne and Richels (2005), Anthoff and Tol (2013), and Golosov et al. (2014). In such a setting, the optimal carbon tax is equal to the marginal damage of carbon emissions, which is Pigouvian, named after the seminal work of Pigou (1920). Few optimal carbon tax studies consider other pre-existing distortions, e.g., Acemoglu et al. (2012) consider a static monopoly distortion in the market for machines used in carbon-intensive intermediate production, and Barrage (2019) considers other taxes on labor and capital. However, these analyses do not consider monopoly distortions in carbon-intensive industries. If implemented, carbon taxes will interact with the market structure of carbon-intensive industries. On the one hand, carbon taxes reduce the production of carbon-intensive goods. On the other hand, carbon taxes could exacerbate monopoly distortions in carbon-intensive industries by increasing costs and reducing competition. This paper contributes to this literature by considering the interaction between carbon taxes and monopoly distortions in carbon-intensive industries by incorporating imperfect competition into a dynamic integrated assessment model.

Second, several studies have considered the interaction between environmental policy and market structure. Buchanan (1969) was the first to consider the implications of monopoly distortions in the context of environmental policy. He argued that the optimal tax on a polluting monopolist should be lower than the Pigouvian tax because the monopolist's output is lower than the competitive level. He graphically illustrated the welfare loss associated with levying a Pigouvian tax on a polluting monopolist. Barnett (1980) formalized Buchanan's insight in a general equilibrium model. He describes that the ideal policy solution for a polluting monopolist would incorporate two policy actions: One device increases the monopolist's production, and another controls the external diseconomic effects of the monopolist's production. Instead, Barnett (1980) assumes that policymakers cannot directly correct the product market distortion, and the pollution tax is the only tool available to achieve the socially optimal allocation. He concludes that the optimal pollution tax on an unregulated monopolist is smaller than marginal external damages, and the difference between the two increases as the price elasticity of demand for the polluter's product decreases, i.e., the demand is more inelastic. Baumol and Oates (1988) summarizes that the optimal tax on an unregulated monopolist depends on the marginal external damages, the demand elasticity, and the abatement cost function of the monopolist's production.

Beyond the polar cases of perfect competition and monopoly, the analysis of optimal environmental policy under an oligopoly market structure has been carried out extensively. Levin (1985) examines various forms of taxation to control pollution from a Cournot oligopoly of homogenous goods. This paper uses the insight that the optimal tax on polluting, imperfectly competitive, and unregulated firms is less than the marginal external damages and asks its implications for regulating stock pollutants driving climate change.

The study of optimal tax on polluting oligopolists further developed from a static setting to a dynamic setting. Shaffer (1995), Katsoulacos and Xepapadeas (1996), and Lee (1999) extend the analysis to a Cournot oligopoly of homogenous goods to incorporate the possibility of endogenous market structure, i.e., the possibility of entry and exit of firms. Imposing a tax on polluting firms may affect the number of firms in the industry, affecting the optimal tax rate. Under the endogenous market structure setting, the optimal tax on polluting oligopolists may be lower, equal, or higher than the Pigouvian tax because now the regulator has to consider three effects: the beneficial effect of reducing pollution, the negative effect of reducing already distorted output due to imperfect competition, and a third positive effect of bringing the number of firms closer to the second-best optimum. If the regulator omits the third effect, the optimal tax is less than the marginal external damages. However, if the third effect is taken into account, the result depends on the curvature of market demand. The intuition behind this internalization result is that the equilibrium number of firms may be above (or below) the socially optimal level, resulting in an additional distortion. The optimal tax could reduce this distortion by raising (or lowering) its rate, which is why it may be optimal to have a tax greater (or less) than marginal external damages. Finally, Requate (2006) provides a comprehensive review of the literature on environmental policy under imperfect competition and summarizes optimal policies for oligopoly models, such as Bertrand competition with homogenous goods, price competition with differentiated goods, and monopolistic competition. This paper is in the process of further incorporating endogenous market structure and studying the dynamic interaction between policy and market structure in a dynamic general equilibrium model.

1.2 Structure of the Paper

The rest of the paper is organized as follows: In Section 2, we review the benchmark dynamic general equilibrium climate change model from Golosov et al. (2014) and introduce extensions to include market power in the carbon-intensive energy industry. Once we introduce the model and its assumptions, we show the derivation of Golosov et al.'s (2014) simple optimal social cost of carbon formula. Then, we look at the decentralized equilibrium outcomes under our new assumptions of market power and derive the optimal carbon tax formula for the oligopolistic and unregulated coal industry. Section 3 outlines our quantitative analysis and computation strategy. In Section 4, we calibrate the model to match the U.S. coal industry's characteristics and quantitatively evaluate the implications of market power on the optimal policy. Section 5 tests the sensitivity of our results to changes in model parameters and assumptions. Finally, Section 6 concludes and discusses future extensions.

2 Model

We begin by describing the general setting in Golosov et al. (2014). We then introduce their key assumptions to derive their simple optimal social cost of carbon result and our extension assumptions that simplify our adjusted optimal tax formulation. We state the planning problem and derive the optimal social cost of carbon. Then, we describe the decentralized equilibrium and introduce an adjustment to the degree of competition in the carbon-intensive intermediate sector. Finally, we show that the optimal tax formula that brings decentralized equilibrium allocations with the optimal ones is no longer dependent on the social cost of carbon alone but also the degree of competition in the carbon-intensive intermediate sector.

2.1 General Specification

Consider a version of the multi-sector neoclassical growth model outlined in Golosov et al. (2014). Time is discrete and infinite. There is a representative household with a utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where U is the strictly increasing and concave one-period utility function, C_t is consumption at period t, $\beta \in (0, 1)$ is the discount factor, and $\mathbb{E}_0$ is the mathematical expectation conditioned on the consumer's time 0 information.

The production process consists of a final-goods sector with output Y_t and an intermediate energy-goods sector that produces energy E_t for use in the final-goods sector.

The feasibility constraint in the final-goods sector, denoted as sector 0, is:

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t, \tag{1}$$

where K_t is capital stock and $\delta \in (0,1)$ is the depreciation rate. The following aggregate production function describes output in the final goods sector:

$$Y_t = F_{0,t}(K_{0,t}, N_{0,t}, E_t, S_t), \tag{2}$$

where $K_{0,t}$ and $N_{0,t}$ are capital and labor used in sector 0 at time t, E_t is the energy input used in final-goods production at time t, and S_t is the climate variable which affects output. The production function $F_{0,t}$ depends on $K_{0,t}$, $N_{0,t}$, E_t, and S_t. We view the climate as sufficiently well represented by one variable, S_t, which is the amount of carbon in the atmosphere. In studies from natural sciences, for example, Schneider (1989), National Research Council (2010), and IPCC (2021), the current atmospheric carbon concentration is considered to be approximating the current climate well. We assume that S_t affects final goods production only. We measure S_t as atmospheric carbon concentration above pre-industrial times in billions of tons of carbon (GtC) units.

The intermediate energy sector produces energy composite input E_t for use in the final goods sector. We assume that coal is the only type of energy produced, and there are J imperfectly substitutable coal sources. Each energy producer $j = 1, \ldots, J$ has its own technology $F_{j,t}$ to produce energy $E_{j,t}$ at time t by using labor at time t, $N_{j,t}$, as the only input:

$$E_{j,t} = F_{j,t}(N_{j,t}) \geq 0. \tag{3}$$

In each period, production factors are mobile across sectors. The final goods sector is perfectly competitive, whereas studying different energy market structures is this paper's objective.

We assume that each energy source emits fossil carbon to the atmosphere. We normalize that one unit of $E_{j,t}$ produces one unit of carbon content. Thus, the total carbon emissions from the energy sector at time t, E_t^f, is the sum of the emissions from each energy source j at time t, $E_{j,t}$, and:

$$E_t^f = \sum_{j=1}^{J} E_{j,t}. \tag{4}$$

The climate variable S_t is affected by the total carbon emissions from the energy sector,

E_t^f, and the carbon cycle. To describe the evolution of the climate, let $\tilde{S}_t$ be a function that maps a history of human-made (anthropogenic) emissions into the current atmospheric concentration level, S_t. History is defined to start at the time of industrialization, denoted as T periods before period 0:

$$S_t = \tilde{S}_t \left(E_{-T}^f, E_{-T+1}^f, ..., E_t^f \right). \tag{5}$$

2.2 Additional Assumptions

In this section, we summarize the three key assumptions outlined in Golosov et al. (2014) that simplify the planning problem analyzed in the next section. We also introduce a fourth assumption that simplifies the carbon tax formulation in the presence of imperfect competition in the energy sector. Golosov et al. (2014) discuss the plausibility of these assumptions. First, households have a logarithmic utility function:

Assumption 1. $U(C) = \log(C)$.

Second, they assume that production damages are multiplicative:

Assumption 2. *The production technology can be represented as:*

$$F_{0,t}(K_{0,t}, N_{0,t}, E_t, S_t) = [1 - D(S_t)]\tilde{F}_{0,t}(K_{0,t}, N_{0,t}, E_t), \tag{6}$$

where $1 - D(S_t) = \exp[-\gamma_t(S_t - \bar{S})]$ *with* $\bar{S}$ *being the pre-industrial atmospheric* CO_2 *concentration, and* γ_t *is the marginal damage measured as a share of GDP per marginal unit of carbon in the atmosphere assumed to be constant, and before damages production function* $\tilde{F}_{0,t}$ *is strictly increasing and strictly concave in all its inputs* $K_{0,t}, N_{0,t}$*, and* E_t *and it exhibits constant returns to scale.*

Third, they assume a simplified carbon cycle:

Assumption 3. *The function* $\tilde{S}_t$ *is linear with the following depreciation structure:*

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s) E_{t-s}^f,$$

where $d_s \in [0, 1]$ *for all* s *and* T *denotes the number of periods since industrialization.*

The fraction $1 - d_s$ represents the amount of carbon left in the atmosphere s periods in the future. A three-parameter family determines the depreciation structure, where (i) ψ_L is the share of carbon emitted into the atmosphere that stays in it forever; (ii) $1 - \psi_0$ is the share of the remaining emissions exiting the atmosphere immediately, and (iii) ψ is the geometric decay rate of the remaining share of emissions. This parameterization corresponds to a linear system with the depreciation rate at horizon s given by:

$$1 - d_s = \psi_L + (1 - \psi_L)\psi_0(1 - \psi)^s. \tag{7}$$

Golosov et al. (2014) show that this depreciation structure is consistent with the existence of two "virtual carbon stocks" S_1 (the part that remains in the atmosphere forever) and S_2 (the part that depreciates at rate ψ), with $S_{1,t} = S_{1,t-1} + \psi_L E_t^f$ and $S_{2,t} = \psi S_{2,t-1} + \psi_0(1 - \psi_L)E_t^f$, and $S_t = S_{1,t} + S_{2,t}$.

Golosov et al. (2014) use these three assumptions to obtain a closed-form solution to the optimal SCC to output ratio. This expression simplifies the computation of the model.

Next, we introduce two additional assumptions as an extension to the Golosov et al. (2014) model and yield constant energy markups. Constant markups simplify the second-best carbon tax formulation due to imperfect competition in the energy sector:

Assumption 4. *The energy sector is imperfectly competitive. The energy composite production combines imperfectly substitutable energy types using a constant elasticity of substitution (CES) technology with an elasticity of substitution parameter, η, greater than one. So, the monopoly problem of each intermediate energy producer is well-defined.*

Assumption 5. *The energy market structure is a Cournot oligopoly and a finite number of firms E_j for $j = 1, \ldots, J$ produces energy inputs to a CES energy composite production function:*

$$E_t = \left[\sum_{j=1}^{J} \kappa_j E_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{8}$$

where κ_j is the share parameter of firm j and $\sum_{j=1}^{J} \kappa_j = 1$.

Following Atkeson and Burstein (2008), we assume that the intermediate good is small compared to the entire economy, and each firm does not consider the influence of its production decision on the final goods production (or the general price level). Nevertheless, it is sufficiently large within each sector so that it is aware of its influence on E_t (and the intermediate energy price p_t).

2.3 The Planning Problem

The social planner's problem is to maximize the representative household's discounted lifetime expected utility subject to technology, feasibility, and carbon cycle constraints of the economy. The social planner's problem is:

$$\max_{\left\{C_t, K_{t+1}, N_{0,t}, E_t, \{N_{j,t}, E_{j,t}\}_{j=1}^{J}, E_t^f, S_t, Y_t\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

subject to (1), (2), (3), (4), (5), (8), and $N_t = \sum_{j=0}^{J} N_{j,t}$, where N_t is the inelastic labor supply at period t. Importantly, the social planner accounts for the climate externality in the planning problem.

2.4 The Social Cost of Carbon

Following Hassler, Krusell, and Smith (2016), we define the social cost of carbon (SCC) as the marginal externality damage of carbon emissions, keeping constant behavior in the given allocation. In this context, the social cost of carbon at time t for dirty sector j, in consumption units at this point is given by:

$$\mathrm{SCC}_{j,t} = \sum_{s=0}^{\infty} \beta^s \frac{U'(C_{t+s})}{U'(C_t)} \frac{\partial Y_{t+s}}{\partial S_{t+s}} \frac{\partial S_{t+s}}{\partial E_{j,t}^f}.$$

Since each unit of E_j produces one unit of carbon by construction, the SCC is independent of j for all $j = 1, \dots, J$. Therefore, we can drop the subscript j and denote the SCC as SCC_t. The SCC captures the externality of carbon emission. It depends on structural parameters in complicated ways. When Assumptions 1, 2, and 3 are satisfied, the expression for the SCC simplifies dramatically:

$$\mathrm{SCC}_t = \sum_{s=0}^{\infty} \beta^s C_t \frac{Y_{t+s}}{C_{t+s}} \gamma_{t+s}(1 - d_s),$$

which expresses the costs of the externality only in terms of exogenous parameters and the endogenous saving rate, along with initial output.

If one further assumes that the saving rate is constant (which does not tend to vary so much over time), then the SCC expression becomes:

$$\mathrm{SCC}_t = Y_t \left[\sum_{s=0}^{\infty} \beta^s \gamma_{t+s}(1 - d_s) \right],$$

which expresses the SCC as a proportion of GDP. This result is Proposition 1 of Golosov et al. (2014).

A final expression for their SCC is obtained by assuming that the expected time path for the damage parameter is constant, i.e., $\gamma_{t+j} = \bar{\gamma}_t$ for all j and d_j defined as in equation (7):

$$\frac{\text{SCC}_t}{Y_t} = \bar{\gamma}_t \left(\frac{\psi_L}{1-\beta} + \frac{(1-\psi_L)\psi_0}{1-(1-\psi)\beta} \right). \tag{9}$$

The optimal SCC (OSCC), denoted by Λ_t, is the marginal external damage of a unit of carbon emissions evaluated at the optimal allocation and is the optimal carbon tax implied by Pigou's principle. Finally let $\hat{\Lambda}_t$ denote the optimal SCC to output ratio, or $\hat{\Lambda}_t \equiv \Lambda_t / Y_t$.

2.5 Decentralized Equilibrium

2.5.1 Consumers

The representative individual solves the problem:

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$\text{subject to}$$

$$\sum_{t=0}^{\infty} q_t (C_t + K_{t+1}) = \sum_{t=0}^{\infty} q_t ((1 + r_t - \delta) K_t + w_t N_t + T_t) + \Pi,$$

where r_t is the net rental rate of capital at time t, w_t is the wage rate at time t, N_t is the inelastic labor supply at time t, T_t is the government transfer at time t, Π is the present value of all production sectors' profits from the future, and q_t are the Arrow-Debreu prices (i.e., probability-adjusted state-contingent prices of the consumption good) at time t.

2.5.2 Producers

All output and input markets are assumed to be competitive. The representative firm in the final good sector solves the following:

$$\Pi_0 \equiv \max_{\left\{Y_t, K_t, N_{0,t}, E_t, \{E_{j,t}\}_{j=1}^{J}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} q_t \left[Y_t - r_t K_t - w_t N_{0,t} - p_t E_t \right],$$

subject to nonnegativity constraints and technology constraints (6) and (8), where the price p_t is the price of the energy composite at time t, and Π_0 is the present value of all maximized profits of the final-goods producer from the future. The cost-minimization of final goods producer 0 implies that:

$$\frac{p_{j,t}}{p_t} = \kappa_j E_{j,t}^{-\frac{1}{\eta}} E_t^{\frac{1}{\eta}}, \tag{10}$$

where $p_{j,t}$ is the price of energy source j at time t and the energy composite price p_t is the CES aggregator of the energy sources' prices at time t:

$$p_t = \left[\sum_{j=1}^{J} \kappa_j^{\eta} p_{j,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

The first-order condition of the final goods producer 0 and the energy composite price index implies that $p_t E_t = \sum_{j=1}^{J} p_{j,t} E_{j,t}$.

The problem of the energy firm j is to maximize the discounted value of its profits given the per-unit tax $\tau_{j,t}$ on the carbon content it produces:

$$\Pi_j \equiv \max_{\{E_{j,t}, N_{j,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} q_t \left[(p_{j,t} - \tau_{j,t}) E_{j,t} - w_t N_{j,t} \right],$$

subject to nonnegativity, energy technology (3), energy composite (8) constraints, and the inverse energy j demand (10). Here, Π_j is the present value of all maximized profits of the energy j producer from the future. Total profits are $\Pi = \Pi_0 + \sum_{j=1}^{J} \Pi_j$.

In the absence of taxes, the first-order condition of the energy firm j and inverse energy j demand function (10) implies:

$$p_{j,t} = \frac{\sigma(s_{j,t})}{\sigma(s_{j,t}) - 1} \frac{w_t}{F'_{j,t}(N_{j,t})},$$

where $F'_{j,t}$ is the marginal product (i.e., the derivative) of the energy j production function at time t, $w_t / F'_{j,t}(N_{j,t})$ is the marginal cost of energy j production, and $\sigma(s_{j,t})$ is the price elasticity of energy j demand, expressed as a function of j's sales share $s_{j,t} = p_{j,t} E_{j,t} / p_t E_t = p_{j,t} E_{j,t} / \sum_{j=1}^{J} p_{j,t} E_{j,t}$ and elasticities following Atkeson and Burstein (2008). Specifically, $\sigma(s_{j,t})$ has the following expression:

$$\sigma(s_{j,t}) = \left[\frac{1}{\eta}(1 - s_{j,t}) + \frac{E_t / p_t}{|\partial E_t / \partial p_t|} s_{j,t} \right]^{-1}, \tag{11}$$

where $\frac{E_t/p_t}{|\partial E_t/\partial p_t|}$ is the absolute value of inverse price elasticity of energy composite demand. In other words, $\frac{\sigma(s_{j,t})}{\sigma(s_{j,t})-1}$ is the markup of the energy firm j. The Appendix provides a detailed derivation of the expression for $\sigma(s_{j,t})$ given by equation (11).

2.5.3 Market Clearing

The final goods market clears:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t.$$

The labor market clears:

$$N_t = \sum_{j=0}^{J} N_{j,t}.$$

Finally, the energy market clears by Assumption 5.

2.5.4 Government

The government runs a balanced budget:

$$\sum_{t=0}^{\infty} q_t T_t = \sum_{t=0}^{\infty} q_t \sum_{j=1}^{J} \tau_{j,t} E_{j,t}.$$

2.5.5 Market Structure

Golosov et al. (2014) assume that all production sectors are perfectly competitive. When the climate externality is the only distortion, the optimal policy is to tax carbon emissions at the marginal externality damage at the optimal output level, i.e., the OSCC, Λ_t, known as the Pigouvian tax.

However, as argued in the introduction, imperfect competition is a more realistic market structure assumption for carbon-intensive production sectors. Under an imperfectly competitive carbon-intensive intermediate production sector, there will be two different distortions: a climate externality and a market distortion.

2.6 The Optimal Carbon Tax

Unless environmental and competition regulators operate harmoniously, a regulator is restricted with a constrained policy tool to improve aggregate welfare. This carbon-intensive and imperfectly competitive intermediate sector subjects society to two costs. The first is the external costs associated with climate change, and the second is the costs resulting from the output restriction. The simple Pigouvian tax, while reducing the climate externality, also increases the welfare loss resulting from reduced production. Thus, the net social welfare effect is uncertain. The optimal carbon tax will combine the policies that address each distortion individually: the Pigouvian tax and the output subsidy. We can formally show this by finding the optimal carbon tax in the presence of imperfect competition in the energy j sector.

Accounting for the pricing power changes the carbon tax formulation when competition policy is constrained. Let $\tau^*_{j,t}$ denote the optimal carbon tax on energy source j. The optimal carbon tax, $\tau^*_{j,t}$ is sector-specific and depends on the marginal externality damages from input j, the energy j price, and the price elasticity of energy j demand:

$$\tau^*_{j,t} = \Lambda_t - \frac{p_{j,t}}{\sigma(s_{j,t})}, \tag{12}$$

where $\sigma(s_{j,t})$ is the price elasticity of energy j demand defined in equation (11). Thus, the optimal carbon tax is always smaller than the Pigouvian tax and can even be negative, i.e., a subsidy, in the presence of market power.

Proposition 1. *Suppose that $\tau_{j,t}$ follows equation (12) and that the tax proceeds are rebated lump-sum to the representative consumer. Then, the oligopoly equilibrium allocation coincides with the solution to the social planner's problem.*

Proof.

To derive the optimal tax formula, we first solve the social planner's problem, stated in detail in the Appendix, and derive its first-order condition with respect to the labor used in the energy j sector. The first-order condition of the social planner's problem with respect to the labor used in the energy j sector is:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{j,t}} + \sum_{s=0}^{\infty}\beta^j \frac{U'(C_{t+s})}{U'(C_t)}\frac{\partial Y_{t+s}}{\partial S_{t+s}}\frac{\partial S_{t+s}}{\partial E^f_t}\frac{\partial E^f_t}{\partial E_{j,t}}\right] = -\frac{\partial Y_t}{\partial N_{0,t}}\frac{\partial N_{0,t}}{\partial N_{j,t}}.$$

Under Assumptions 1, 2, and 3, the first-order condition simplifies to:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{j,t}} + \sum_{s=0}^{\infty}\beta^j \frac{C_t}{C_{t+s}} Y_{t+s}\gamma_{t+s}(1-d_s)\right] = \frac{\partial Y_t}{\partial N_{0,t}}.$$

This expression is further simplified when we assume the savings rate is constant over time:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{j,t}} + \Lambda_t\right] = \frac{\partial Y_t}{\partial N_{0,t}}. \tag{13}$$

Next, we derive the corresponding first-order condition of the energy firm j's problem with respect to its labor input choice under Assumption 4. This condition is:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[\frac{\partial p_{j,t}}{\partial E_{j,t}}E_{j,t}+p_{j,t}-\tau_{j,t}\right]=w_t.$$

Since the final goods production sector is perfectly competitive, factor price for labor, w_t equals the marginal product of labor in final goods production, $\frac{\partial Y_t}{\partial N_{0,t}}$, in equilibrium. Further imposing Assumption 5, we obtain:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[\frac{\partial p_{j,t}}{\partial E_{j,t}}E_{j,t}+p_{j,t}-\tau_{j,t}\right]=\frac{\partial Y_t}{\partial N_{0,t}}.$$

As shown in the Appendix, the first-order condition of the energy firm j's problem with respect to its labor input choice is:

$$\frac{\partial E_{j,t}}{\partial N_{j,t}}\left[p_{j,t}\frac{\sigma(s_{j,t})-1}{\sigma(s_{j,t})}-\tau_{j,t}\right]=\frac{\partial Y_t}{\partial N_{0,t}}. \tag{14}$$

The first-order condition of the social planner's problem with respect to the labor used in the energy j sector is the same as the first-order condition of the energy firm j's problem with respect to its labor input choice when $\tau_{j,t}$ is equal to the optimal tax formula given by equation (12):

$$\tau^*_{j,t}=\Lambda_t-\frac{p_{j,t}}{\sigma(s_{j,t})}.$$

□

Since emissions and energy output are in the same units (GtC), the optimal tax on oligopolistic coal producers is a linear combination of the optimal tax on competitive coal producers and the optimal output subsidy on oligopolistic clean energy producers. We know the optimal tax on competitive coal producers is the Pigouvian tax, Λ_t, as shown in Golosov et al. (2014).

The optimal output subsidy on oligopolistic producers in the absence of carbon externality is equal to $\frac{p_{j,t}}{\sigma(s_{j,t})}$. We know that this is the optimal output subsidy by considering the first-order condition with respect to energy producer j's labor input choice. Absent the carbon externality, the SCC term Λ_t is absent from the planner's first-order condition, equation (13). Thus, the optimal tax on oligopolistic producer j that would equate its first-order condition (14) with the planner's first-order condition (13) is minus $\frac{p_{j,t}}{\sigma(s_{j,t})}$, i.e. a subsidy.

3 Quantitative Analysis

In this section, we describe a complete characterization of the general multi-sector model used for the quantitative analysis in Golosov et al. (2014) described above. Assume that Assumptions 1, 2, 3, 4, and 5 hold. Moreover, we present quantitative model results for a baseline calibration. We explain the calibration process of the new components of our extension of the Golosov et al.'s (2014) model to match the United States coal industry's market structure in 2010.

Assume the energy sector produces coal, a resource in infinite supply that yields carbon emissions. Further, suppose that there are J coal firms that produce imperfectly substitutable types of coal − which is a reasonable assumption given coal's high transportation costs. Each type j producer produces coal using the following constant returns to scale technology:

$$E_{j,t} = \chi_{j,t} N_{j,t}, \tag{15}$$

where $\chi_{j,t}$ is the exogenous labor productivity of the energy j producer that grows at a constant rate g_{χ_j} and $N_{j,t}$ is the labor input used in j's production. The energy composite E_t that enters the final goods production is a CES aggregate of the different types of coal as outlined in Assumption 5 equation (8).

The final-goods production function is of Cobb-Douglas specification:

$$Y_t = \exp\{-\gamma_t(S_t - \bar{S})\} A_{0,t} K_t^{\alpha} N_{0,t}^{1-\alpha-\nu} E_t^{\nu}$$

where $A_{0,t}$ is the exogenous total factor productivity in the final-goods sector that grows at a constant rate g_0. The Cobb-Douglas production function is a special case of the CES production function with price elasticity of demand equal to unity. Labor employed by all producers has to sum up to the exogenous labor supply level N_t. Finally, we assume that there is full capital depreciation, given that each period is a decade.

The Cobb-Douglas production function and complete capital depreciation assumptions deliver a constant saving rate under optimality conditions, which was the assumption behind the OSCC expression derived in equation (9). We now characterize the solutions to the planning problem and three types of market equilibria.

3.1 The Planning Problem

Given the constant saving rate result, there is a closed-form solution to the planner's problem defined in Appendix A.2, pinned down by the first-order condition with respect to labor used in energy j production, $N_{j,t}$. The optimal path of energy j use satisfies:

$$E_{j,t} = \varepsilon_{j,t} E_t^{-(\eta-1)}, \tag{16}$$

where the $\varepsilon_{j,t}$ for the planner is denoted by $\varepsilon^*_{j,t}$ and is given by:

$$\varepsilon^*_{j,t} \equiv \left(\frac{\nu \kappa_j \chi_{j,t} N_{0,t}}{1 - \alpha - \nu + \chi_{j,t} \hat{\Lambda}_t N_{0,t}} \right)^{\eta}. \tag{17}$$

The $\varepsilon^*_{j,t}$ can be expressed in terms of exogenous parameters and $N_{0,t}$. We obtain the solution by guessing the path of $N_{0,t}$ and solving for the paths of $\varepsilon^*_{j,t}$, E_t, $E_{j,t}$, and $N_{j,t}$. Using the solution for the path of $N_{j,t}$, we update the path of $N_{0,t}$ guess and repeat until convergence. The details of the solution algorithm are available in Appendix A.2.1.

3.2 Decentralized Equilibrium

In a decentralized equilibrium, the energy producer j chooses the path of labor input $N_{j,t}$ to maximize its profits, given the path of taxes $\{\tau_{j,t}\}_{t=0}^{\infty}$, which are set by the government. Notably, the energy producer does not internalize its influence over the final goods production possibilities through the effect of its accumulated emissions on climate change.

First, we replicate the competitive equilibrium from Golosov et al. (2014) and then consider the case of equilibrium with an oligopolistic energy sector.

3.2.1 The Competitive Equilibrium

In a competitive equilibrium, each producer is a price taker. Specifically, energy producers do not internalize their market power over energy prices. Appendix A.3.1 shows the complete competitive equilibrium definition.

The equilibrium path of energy use again follows equation (16), but now $\varepsilon_{j,t}$ does not internalize $\hat{\Lambda}_t$ unless the tax $\tau_{j,t}$ is set equal to it. The $\varepsilon_{j,t}$ for the competitive equilibrium is denoted by $\varepsilon^C_{j,t}$ and is given by:

$$\varepsilon^C_{j,t} \equiv \left(\frac{\nu \chi_{j,t} N_{0,t}}{1 - \alpha - \nu + \chi_{j,t} \tau_{j,t} N_{0,t}} \right)^{\eta}.$$

The $\varepsilon^C_{j,t}$ is a function of exogenous parameters and $N_{0,t}$. We obtained the solution in the same way as we did for the planning problem.

3.2.2 The Oligopoly Equilibrium

On the other hand, in an oligopoly equilibrium, each energy producer j internalizes its influence over the energy composite and price but not the final output and the general price level. The definition of the complete oligopoly equilibrium is available in Appendix A.3.2.

The $\varepsilon_{j,t}$ in the oligopoly equilibrium is denoted by $\varepsilon_{j,t}^O$ and is given by:

$$\varepsilon_{j,t}^O \equiv \left(\frac{\nu \kappa_j \chi_{j,t} N_{0,t}}{1 - \alpha - \nu + \chi_{j,t} \tau_{j,t} N_{0,t}} \frac{\sigma(s_{j,t}) - 1}{\sigma(s_{j,t})} \right)^\eta,$$

which can be expressed in terms of exogenous parameters, $N_{0,t}$, and $s_{j,t}$.

If the energy producers are symmetric, i.e., they have the same labor productivity $\chi_{j,t}$ and the share parameters κ_j, then they also have the same sales share $s_{j,t}$. Together with the Cobb-Douglas final goods production function assumption, this implies that $\sigma(s_{j,t})$ expression in equation (11) is constant across j and σ equals:

$$\sigma = \left[\frac{1}{\eta} \frac{J-1}{J} + \frac{1}{J} \right]^{-1}.$$

The symmetry assumption implies that $\varepsilon_{j,t}^O$ is constant across j, and the solution method is the same as in Appendix A.2.1.

3.3 Calibration

The baseline model closely follows calibration of Golosov et al. (2014) for model comparability except for the energy share parameter in final goods production. The start period is 2010. Table 1 presents all calibrated parameter values. The energy share parameter, ν, is adjusted to account for the fact that the extension model only includes coal in the energy sector, rather than also including oil and renewables as energy sources as Golosov et al. (2014) do.

Table 1: Calibration summary

Parameter	Value
β (decadal)	0.985^{10}
N (normalized)	1
γ	2.3794e-5
α	0.3
ν	0.04
K_0	0.1289
δ (decadal)	1
$A_{0,0}$ (normalized)	1
χ_0	7,693
g_{A_0} (annual)	1
g_χ (annual)	1.02
ψ_0	0.393
ψ_L	0.2
ψ	0.0228
$\bar{S}$ (GtC)	581
$S_{1,-1}$ (GtC)	684
$S_{2,-1}$ (GtC)	118
η	11.16
J	13

3.3.1 Energy Sector: Output Share

The energy share parameter, ν, is calibrated to match the United States Gross Domestic Product (GDP) share of coal production. We obtain the nominal coal expenditure data from the U.S. Energy Information Administration (2023b) and nominal annual nonseasonally adjusted GDP data from the U.S. Bureau of Economic Analysis (2024a). We calibrate ν to match the United States coal expenditure share of GDP in 2010, which is 0.00334.

3.3.2 Energy Sector: Size

The industry size parameter, J, is calibrated to match the Herfindahl-Hirschmann Index (HHI) of the United States coal mining industry. HHI is calculated by squaring the market share of each competing firm in the industry and then summing the resulting numbers The result can range from close to zero to 10,000. Increases in HHI indicate a decrease in competition and an increase in market power.

If all firms have an equal share, the reciprocal of the index shows the number of firms in the industry. When firms have unequal shares, the reciprocal of the index indicates the "equivalent" number of firms in the industry.

The only data available for the HHI of the coal mining industry is from U.S. Census Bureau (2023). The HHI of the coal mining industry, with the NAICS code 2121, is 750.6/10,000. We calibrate J to equal 13 to match the reciprocal of the HHI.

3.3.3 Energy Sector: Markups

Finally, we choose the elasticity of substitution parameter between coal types, η, to match the average markup, μ, of coal miners in the United States in the base year 2010.

To calculate the average markup, μ, we use the production approach method proposed by De Loecker, Eeckhout, and Unger (2020). Following De Loecker, Eeckhout, and Unger (2020), we obtain annual data from Compustat (2023) on the revenue and cost of goods sold for publicly traded firms for the period from 1956 to 2020. Our calculations suggest that the average sales-weighted markup of coal miners, firms with NAICS code 2121, in the United States in 2010 is 1.19, which implies a 19 percent profit margin for coal producers.

We calibrate η to match the average markup of coal miners in the United States in 2010. The average markup level, 1.19, and the industry size, 13, implies an elasticity of substitution parameter, η, of 11.16. We provide details of the markup calculation in Appendix B following equation (24).

4 Quantitative Results

We quantitatively evaluate the model by simulating the optimal and equilibrium paths of allocations and prices using the baseline calibration described. In this section, we present the model's simulation results of mean global surface temperature change, the amount of coal used, the ratio of optimal to equilibrium level of outputs, and the optimal policy paths from 2010 to 2200. Our results show that the net distortion of the oligopoly is lower than that of the competitive equilibrium, and the difference between the optimal carbon tax on on the oligopolistic carbon-intensive industry and the optimal subsidy on the oligopolistic industry decreases over time.

Figure 2 shows the paths of optimal and decentralized equilibrium temperature increase relative to the pre-industrial mean global surface temperature level. The optimal temperature increase reaches 2 degrees Celsius, the oligopoly equilibrium temperature increase is almost 3.2 degrees Celsius, and the competitive equilibrium temperature increase is 3.5

degrees Celsius by 2100. The equilibrium temperatures increase exponentially, while the optimal temperature increase is almost linear.

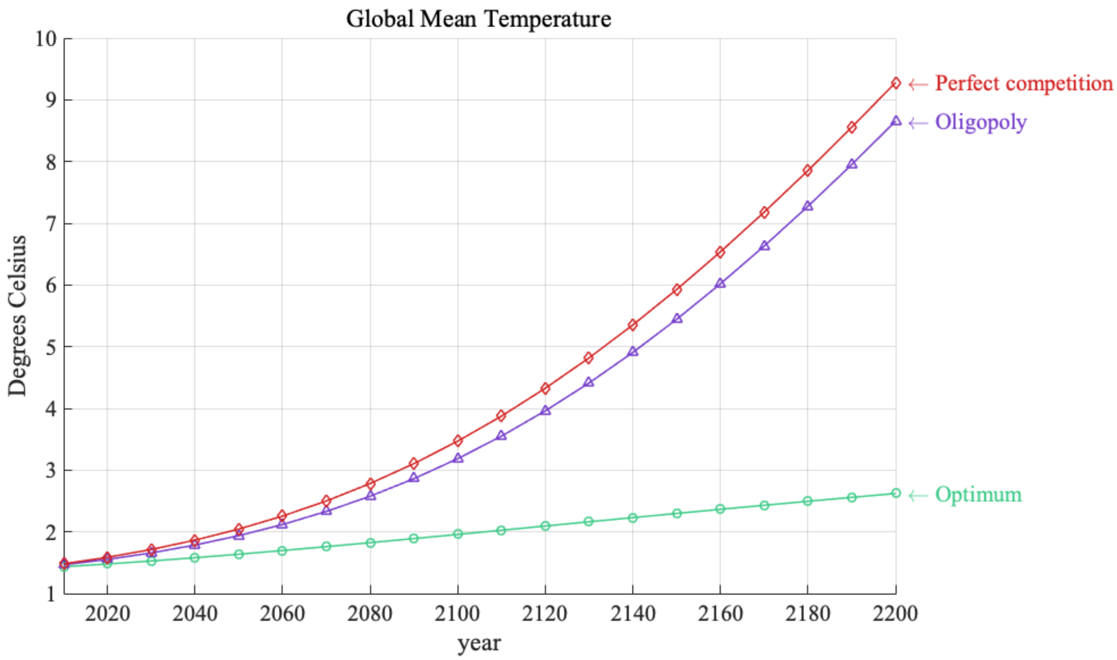

Figure 2: Temperature change

The oligopoly equilibrium temperature increase is lower than the competitive equilibrium temperature increase because the oligopoly equilibrium level of energy use is lower than the competitive equilibrium level, as shown in Figure 3.

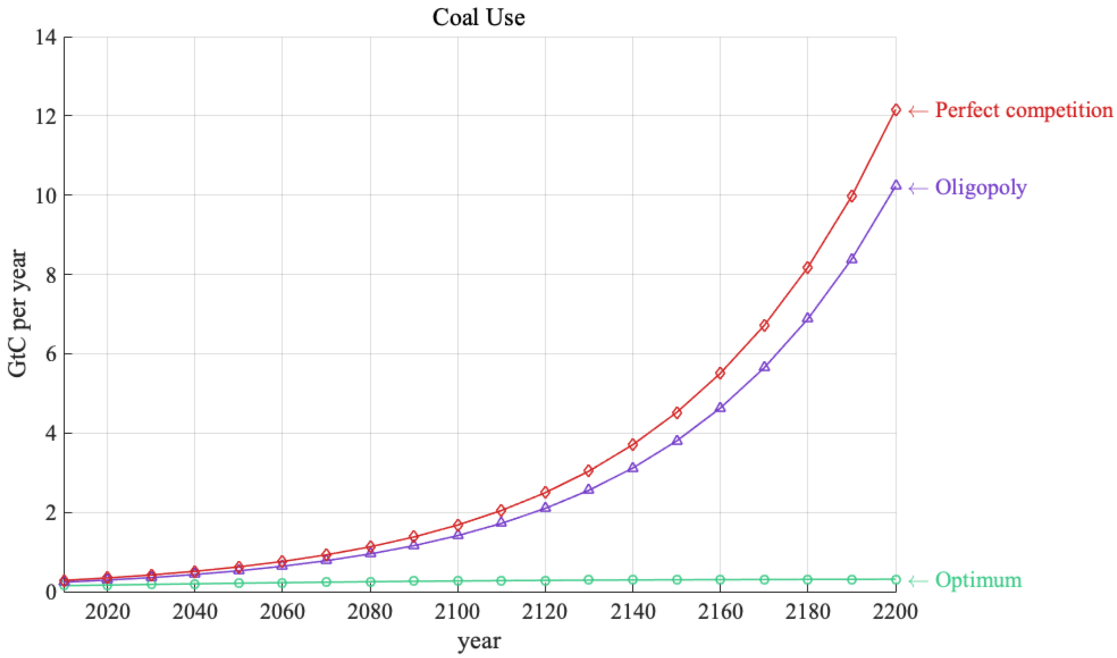

Figure 3: Energy use

Figure 4 shows the paths of relative output, defined as the ratio of the optimal-to-equilibrium levels of outputs. Although the oligopoly level of energy production is always less than the competitive level, the final goods production level is higher under an oligopoly than under competition due to the climate change externality of energy production.

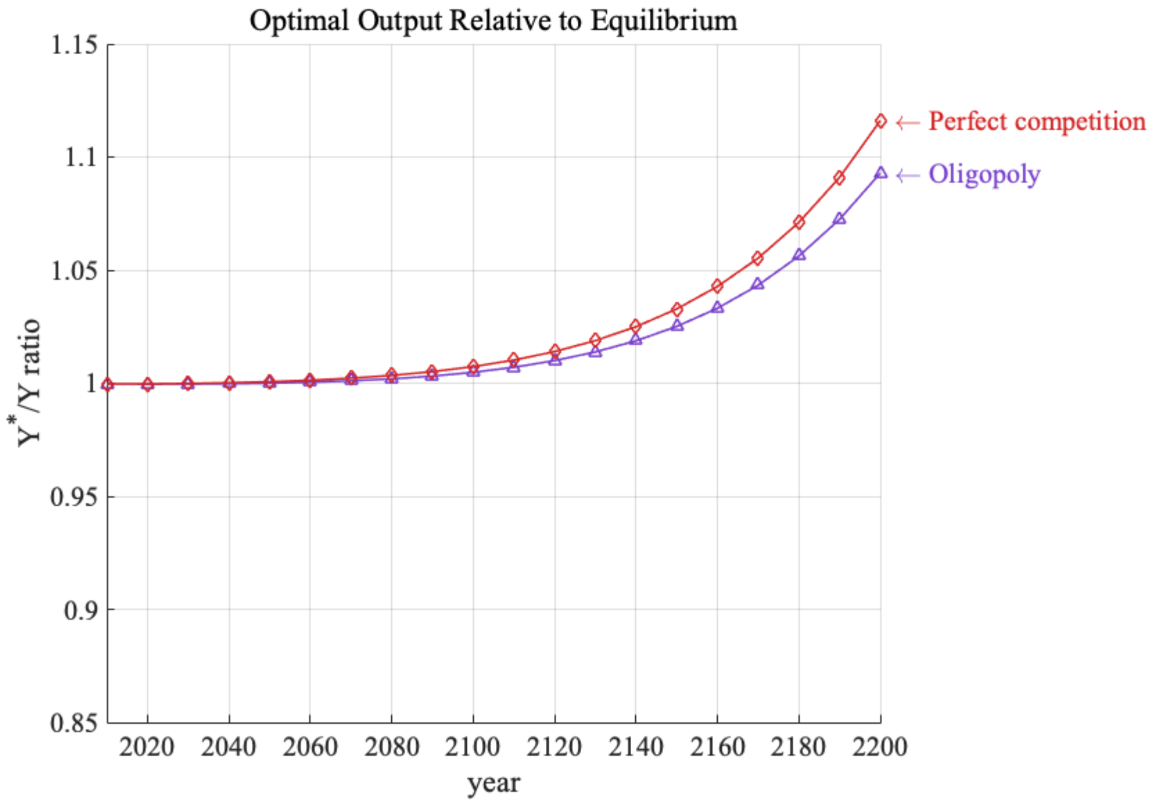

Figure 4: Relative optimal output paths

Figure 5 shows the paths of the optimal tax on the competitive carbon-intensive energy sector, the optimal tax on the oligopolistic carbon-intensive industry, and the optimal subsidy on the oligopolistic industry. The Pigouvian carbon tax is proportional to output. The energy production subsidy is proportional to energy price and decreases over time. The price decreases because the equilibrium path of energy production increases at a decreasing rate over time while the output path increases at a lower decreasing rate until 2060 and then decreases at a decreasing rate.

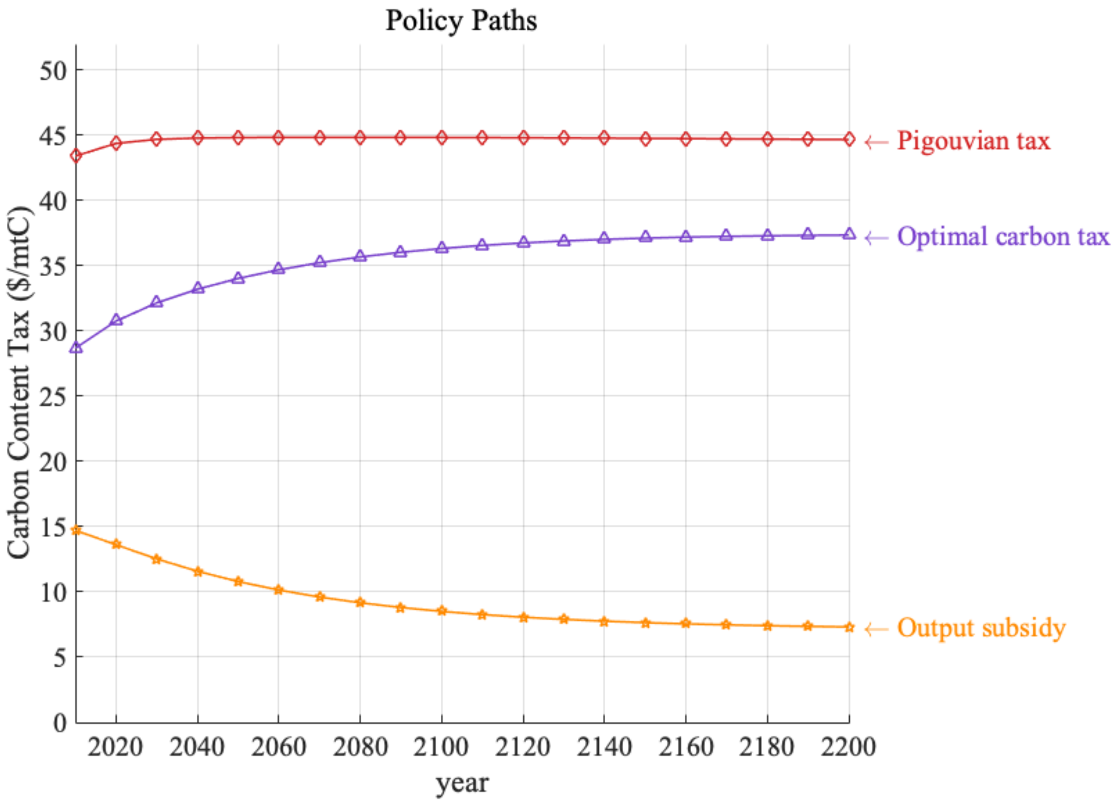

Figure 5: Energy sector regulation paths

The optimal carbon tax is the difference between the optimal tax on a competitive carbon-intensive producer and the optimal subsidy on an oligopolistic producer, and increases over time.

5 Sensitivity and Extensions

The next step in our analysis is to conduct a sensitivity analysis to examine the robustness of our results to alternative parameter values and assumptions. Our first set of sensitivity analyses considers the comparative statics of model simulations to various markup and industry size values. We consider these two parameter targets because they describe the degree of competition in the intermediate carbon-intensive sector. Our paper focuses on studying the optimal carbon output tax formulation as the degree of competition changes among carbon emitters. In further sections, we consider extensions to our baseline model assumptions. Each extension relaxes one of the assumptions in the baseline model to examine the robustness of our results to alternative assumptions.

5.1 Comparative Statics: Coal Industry Markups and Size

To consider the sensitivity of markup calculations to the choice of methodology, we consider the comparative statics of model simulations to various markup and industry size values.

First, holding industry size constant, we consider the sensitivity of model simulations to the choice of average markup value. Figure 6 shows the simulated path of optimal tax on the oligopolistic coal industry comparative statics with respect to the average coal industry markup μ. We observe that the optimal tax on the coal industry is decreasing in the average markup value. This observation is consistent with the intuition that higher markups imply less competition and, thus, lower output levels. Thus, the optimal output subsidy is higher. In contrast, the optimal Pigouvian tax is constant since the markup is constant at unity under perfect competition.

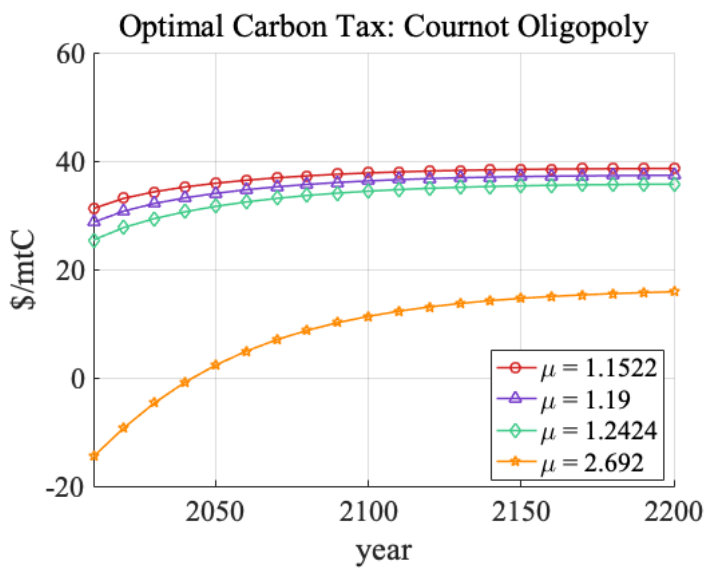

Figure 6: The simulated path of optimal tax on the oligopolistic coal industry comparative statics with respect to the average coal industry markup μ.

We confirm this intuition by comparing the simulated path of optimal tax on a competitive coal industry and optimal output subsidy on clean oligopolistic industry comparative statics with respect to the average coal industry markup μ in Figures 7 and 8. The optimal tax on the competitive coal industry is constant, while the optimal output subsidy on the clean oligopolistic industry is increasing in the average markup value.

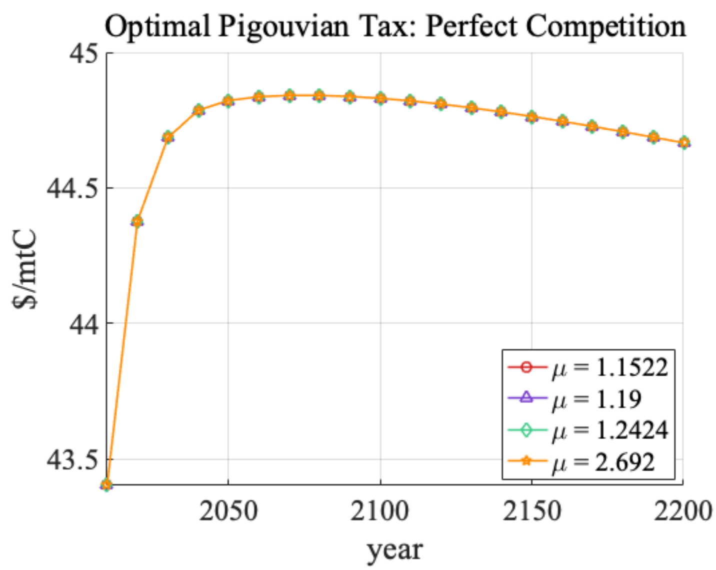

Figure 7: The simulated path of optimal tax on a competitive coal industry comparative statics with respect to the average coal industry markup μ.

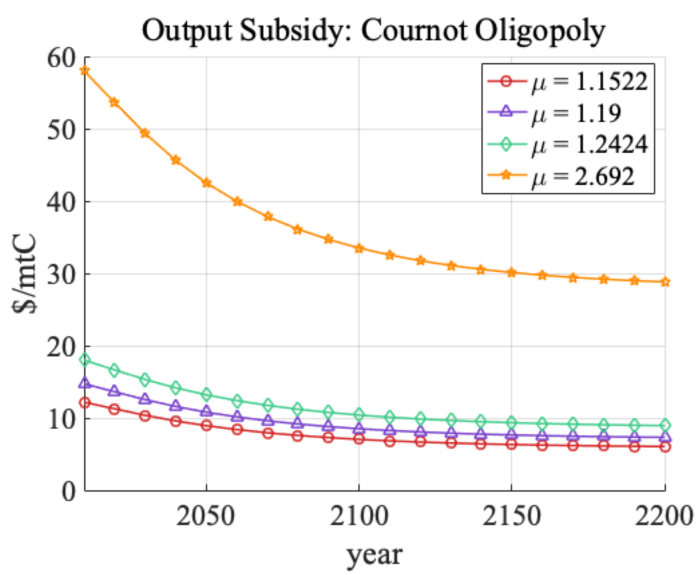

Figure 8: The simulated path of optimal output subsidy on an oligopolistic clean industry comparative statics with respect to the average coal industry markup μ.

Second, holding the average markup value constant, we consider the sensitivity of model simulations to the choice of industry size. Figure 9 shows the simulated path of optimal tax on the oligopolistic coal industry comparative statics with respect to the industry size J. We observe that the optimal tax on the coal industry decreases with the industry's size. Intuitively, the direction of this effect is ambiguous, as a larger industry size implies a higher output level, which would imply a lower optimal output subsidy. However, dividing the optimal Pigouvian tax across more firms implies a lower tax rate for each firm. The direction of the effect of increasing industry size depends on the relative magnitude of these two effects. Figure 9 suggests that the former effect dominates the latter, as the optimal tax on the coal industry is decreasing in the industry size J.

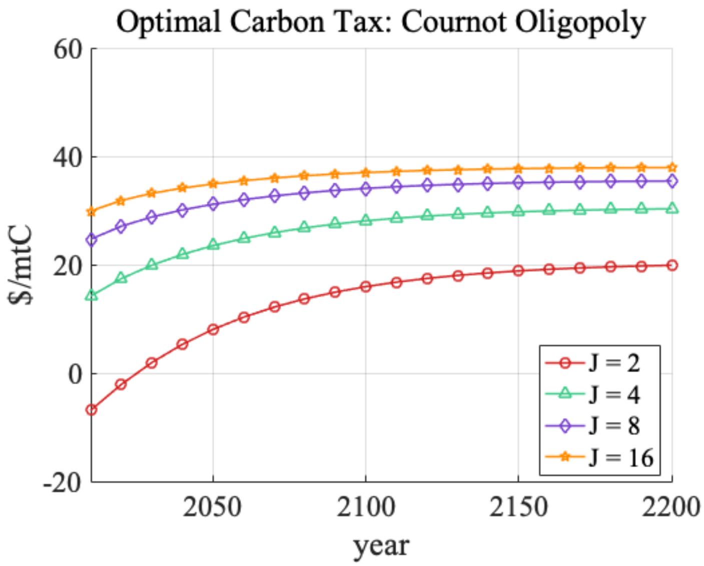

Figure 9: The simulated path of optimal tax on the oligopolistic coal industry comparative statics with respect to the industry size J.

We examine these two opposite effects by comparing the simulated path of optimal tax on a competitive coal industry and optimal output subsidy on clean oligopolistic industry comparative statics with respect to the industry size J in Figures 10 and 11. We indeed observe that the optimal tax on the competitive coal industry is decreasing in the industry size J, while the optimal output subsidy on the clean oligopolistic industry is increasing in the industry size J. Moreover, the increase in the optimal output subsidy has a larger magnitude than the decrease in the optimal tax, suggesting that the latter effect dominates the former.

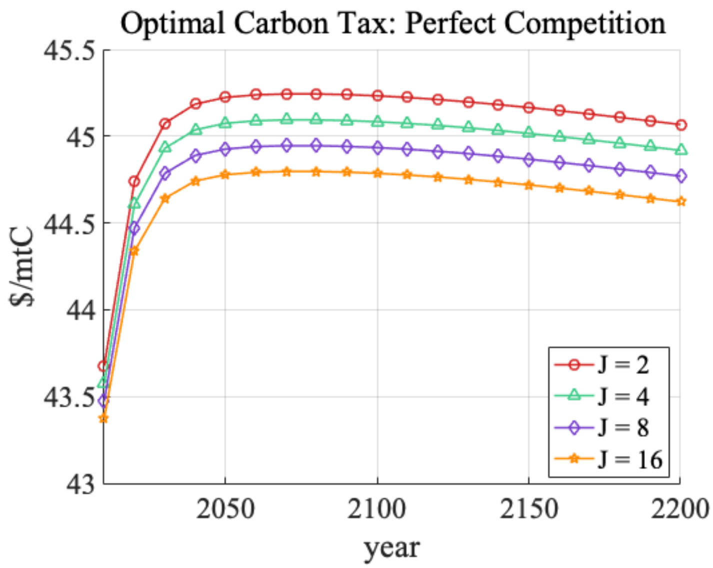

Figure 10: The simulated path of optimal tax on a competitive coal industry comparative statics with respect to the industry size J.

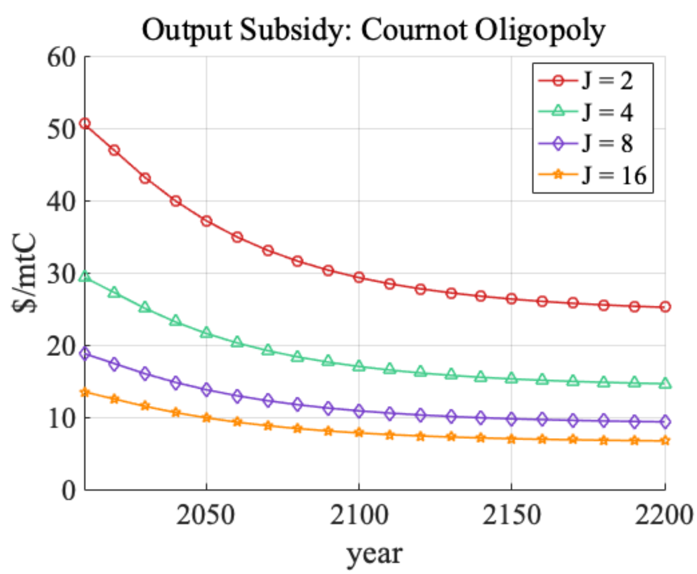

Figure 11: The simulated path of optimal output subsidy on an oligopolistic clean industry comparative statics with respect to the industry size J.

5.2 Monopolistic Competition in the Energy Sector

In this section, we relax Assumption 5, where there are a finite number of differentiated energy sources, and instead assume a continuum of energy sources indexed by $j \in [0, 1]$:

Assumption 5′. *As competition increases in the energy sector ($J \to \infty$), a continuum* *of monopolistically competitive firms E_j for $j \in [0,1]$ produces the energy inputs to a CES production function:*

$$E_t = \left[\int_0^1 E_{j,t}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}.$$

The first-order condition for the planner's choice of labor input $N_{j,t}$ is given by:

$$\chi_{j,t}\left[\frac{\nu}{E_t^{\frac{\eta-1}{\eta}} E_{j,t}^{\frac{1}{\eta}}} - \frac{\mathrm{SCC}_t}{Y_t}\right] = \frac{1-\alpha-\nu}{N_{0,t}}.$$

This condition pins down the optimal labor input $N_{j,t}$ for each firm j in the energy sector as in equation (16), where $\varepsilon_{j,t}$ of the planner $\varepsilon^*_{j,t}$ is now given by:

$$\varepsilon^*_{j,t} \equiv \left(\frac{\nu\chi_{j,t}N_{0,t}}{1-\alpha-\nu+\chi_{j,t}\hat{\Lambda}_t N_{0,t}}\right)^{\eta}.$$

Similarly, the competitive equilibrium solution for the path of labor input $N_{j,t}$ for each firm j in the energy sector is given by equation (16), where $\varepsilon_{j,t}$ of the competitive equilibrium $\varepsilon^C_{j,t}$ is now given by:

$$\varepsilon^C_{j,t} \equiv \left(\frac{\nu\chi_{j,t}N_{0,t}}{1-\alpha-\nu+\chi_{j,t}\tau_{j,t}N_{0,t}}\right)^{\eta}.$$

The critical assumption in monopolistic competition is that each firm is small compared to the aggregate so that it ignores the effect of its energy production $E_{j,t}$ on the aggregate energy composite E_t. Therefore, each firm takes E_t as given. The first-order condition for the monopolistically competitive energy producer j's choice of labor input $N_{j,t}$ is given by:

$$\chi_{j,t}\left[\frac{\nu}{E_t^{\frac{\eta-1}{\eta}} E_{j,t}^{\frac{1}{\eta}}}\frac{\eta-1}{\eta} - \frac{\mathrm{SCC}_t}{Y_t}\right] = \frac{1-\alpha-\nu}{N_{0,t}}.$$

Then the $\varepsilon_{j,t}$ of the monopolistically competitive equilibrium $\varepsilon^M_{j,t}$ is given by:

$$\varepsilon^M_{j,t} \equiv \left(\frac{\nu\chi_{j,t}N_{0,t}}{1-\alpha-\nu+\chi_{j,t}\tau_{j,t}N_{0,t}}\frac{\eta-1}{\eta}\right)^{\eta}.$$

Keeping the calibration targets constant, we compare the simulated path of optimal tax on the monopolistically competitive coal industry with the oligopolistic coal industry. To keep the markup calibration target the same, we need to adjust the elasticity of substitution parameter η in the monopolistic competition case. To match the markup to the calibration target 1.19, we set η to equal 6.26.

The model simulations are summarized in Appendix C. The results suggest that the simulated paths do not change as we switch from oligopoly to monopolistic competition assumption in the energy sector as long as the markup calibration target is the same.

5.3 Heterogeneous Markups

In this section, we relax the assumption that energy producers are symmetric and assume that energy producers have different labor productivities $\chi_{j,t}$. The firms in the energy sector are non-symmetric, and their markups are endogenous and depend on their sales share.

Following Atkeson and Burstein (2008), we suppose each firm in the energy sector has a constant returns-to-scale production function with labor as the only input. The production functions for firm j in the energy sector are given by $E_j = Az_jN_j$, where z_j differs across firms and A denotes aggregate productivity that affects all firms based in the energy sector. That is, the firm-specific labor productivity is given by $\chi_{j,t} = Az_j$. Each firm j in the energy sector draws its idiosyncratic productivity z_j from a log-normal distribution with mean zero and standard deviation σ_z, that is, $\log z \sim \mathcal{N}(0, \sigma_z^2)$. This idiosyncratic component of productivity is fixed over time. The marginal cost of production for a firm with productivity z_j is W/Az_j. The first-order condition implies that:

$$p_j = \frac{\sigma(s_j)}{\sigma(s_j) - 1}\frac{W}{Az_j},$$

where p_j is the price charged by firm j in the energy sector, W is the wage rate, and $\sigma(s_j)$ is price elasticity of energy j demand, with the following formula:

$$\sigma(s_j) = \left[\frac{1}{\eta}(1 - s_j) + s_j\right]^{-1}.$$

We can write the sales shares as functions of prices:

$$s_j = \frac{p_j^{1-\eta}}{\sum_{s=1}^{J} p_s^{1-\eta}}.$$

The solution algorithm is similar to the exogenous constant markup case, with one difference. Now, the sales share of each firm in the energy sector is endogenous and depends on the prices charged by the firms. Our solution algorithm before was to guess and update the labor share path of the final goods sector. Now, we need additional information regarding each energy firm's sales share. We obtain this information by additionally guessing the path of each energy firm's labor share and updating the guess using a convex combination of the initial guess and updated value until all labor share paths converge. The initial guess for the labor share path of the final goods sector is constant at 0.99, and the initial guess for the labor share of each energy firm is $0.01/J$.

5.3.1 Calibration

The majority of the extended model's calibration follows Table 1. The critical parameters in this analysis are the dispersion of the idiosyncratic productivity component, σ_z, and the share parameter, κ_j, of the CES production function that aggregates imperfectly substitutable J energy inputs into an energy composite E.

First, we choose κ_j to match the ratio of energy j prices in equilibrium and $\sum_{j=1}^{J} \kappa_j = 1$. This implies that:

$$\kappa_j = \frac{p_j E_j^{1/\eta}}{\sum_{s=1}^{J} p_s E_s^{1/\eta}}.$$

Ideally, we would calibrate κ_j to match data on energy prices and consumption. Data on regional coal spot prices are available through S&P Capital IQ database, which is propriety. We are in the process of obtaining access to this data.

For now, for our quantitative exercise, we set $\kappa_j = z_j / \sum_{s=1}^{J} z_s$ for all j. Thus, the highest productivity firm also has the highest κ_j, and κ_j decreases as firm productivity decreases.

Initially, we set $\sigma_z = 0.385$ following on Atkeson and Burstein (2008). However, this value produces a higher temperature increase than the benchmark model with symmetric energy firms. The temperature change decreases as σ_z decreases. We choose $\sigma_z = 0.01$ to match the benchmark model's temperature change path.

5.3.2 Quantitative Results

We quantitatively evaluate the model by simulating the optimal and equilibrium paths of allocations and prices. This section presents the model's simulation results for the optimal policy paths from 2010 to 2200.

Figure 12 shows the simulated optimal policy paths for the lowest, median, and highest productivity energy producers in the oligopoly equilibrium and the Pigouvian tax rate for comparison.

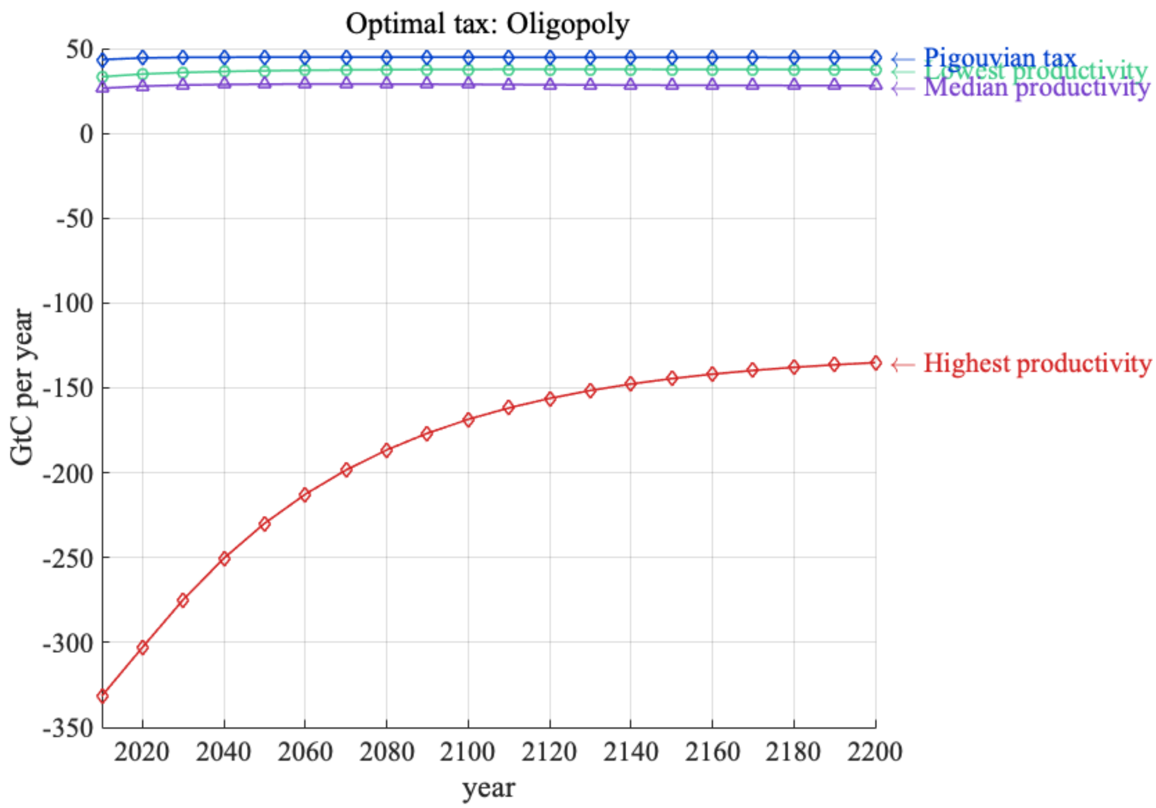

Figure 12: Energy sector regulation paths

We observe that the optimal policy is to promote the production of the highest productivity energy producer and to discourage production more as productivity decreases. So the optimal policy to subsidize the highest productivity producer while taxing less productive producers at close to the Pigouvian rate will discourage competition in the energy sector.

5.4 Renewables in the Energy Mix

In this section, we introduce an additional energy source, renewable energy, into the energy sector. We assume that the energy sector is composed of two energy sources: coal and renewable energy. The energy sector's production function is now given by:

$$E_t = \left[\alpha_c \left(\sum_{j=1}^{J} \kappa_j E_{j,t}^{\frac{\eta-1}{\eta}} \right) + \alpha_r \left(\sum_{m=1}^{M} \kappa_m E_{m,t}^{\frac{\eta-1}{\eta}} \right) \right]^{\frac{\eta}{\eta-1}},$$

where α_c and α_r are the shares of coal and renewable energy in the energy mix, respectively, $\alpha_c + \alpha_r = 1$, $E_{m,t}$ is the energy input from the renewable energy producer $m \in \{1, \ldots, M\}$, and κ_m is the renewable energy share of producer m.

Importantly, the renewable energy does not produce carbon emissions, thus, it doesn't add up to the atmospheric carbon stock. In the planner's solution, the first-order condition for the renewable energy producer m's choice of labor input $N_{m,t}$, analogous to equation (13), is given by:

$$\frac{\partial E_{m,t}}{\partial N_{m,t}} \frac{\partial Y_t}{\partial E_t} \frac{\partial E_t}{\partial E_{m,t}} = \frac{\partial Y_t}{\partial N_{0,t}}, \tag{18}$$

but now there is no external distortion from the carbon emissions. In the decentralized equilibrium, the oligopolistic renewable energy producer's first-order condition with respect to labor input $N_{m,t}$ is given by:

$$\frac{\partial E_{m,t}}{\partial N_{m,t}} \left[\frac{\partial p_{m,t}}{\partial E_{m,t}} E_{m,t} + p_{m,t} - \tau_{m,t} \right] = \frac{\partial Y_t}{\partial N_{0,t}}.$$

Now the only difference between the planner's and the energy m producer's first-order conditions is $\partial p_{m,t}/\partial E_{m,t}$, which describes the market distortion. Thus, there will be only an output subsidy, $\tau_{m,t}$ imposed on the oligopolistic renewable energy producer and no carbon tax. In other words, the tax rate on the renewable energy output is:

$$\tau_{m,t} = -\frac{p_{m,t}}{\sigma(s_{m,t})},$$

where $p_{m,t}$ is the price of renewable energy producer m and $s_{m,t}$ is its sales share of total energy sales in the economy. Since τ_m, t is strictly negative, it is always a subsidy.

The solution algorithm for this extension follows the heterogeneous markup case, with the additional step of guessing and updating the renewable energy producer's labor share path. The $\varepsilon_{j,t}$ of oligopolistic coal producers follows the main model's solution, while the $\varepsilon_{m,t}$ of the renewable energy producer is given by:

$$\varepsilon_{m,t} = \left(\frac{\nu \alpha_r \kappa_m N_{0,t}}{1-\alpha-\nu} \frac{\sigma(s_{m,t})-1}{\sigma(s_{m,t})} \right)^{\eta}.$$

5.4.1 Calibration

While we try to follow the calibration targets in Table 1, we need to introduce additional parameters and make adjustments to the baseline ones to incorporate the renewable energy sector to our model. To this end, we use the following calibration summarized in Table 2.

Table 2: Summary of additional and adjusted parameters for the calibration of the renewables in the energy mix extension.

Parameter	**Description**	**Value**
α_r	Relative energy efficiency of renewables	0.6
M	Number of renewable energy producers	2
κ_m	Renewable energy share of producer m	1/2
ν	Output share of energy	0.011825
$\chi_{m,0}$	Marginal cost of renewable energy generation	1,311
g_{χ_m}	Annual growth rate of renewable energy productivity	1.02

We adjust ν to include renewables in the energy sector. We keep $\chi_{m,0}$ and g_{χ_m} at the values used by Golosov et al. (2014). We set α_r to match the relative price ratio of coal and oil in the U.S., while assuming that the relative price between renewables and oil is unity. We use historic price and flow data for coal and oil in the U.S. from U.S. Energy Information Administration (2023a) to obtain the relative price ratio of coal and oil. Finally, we choose the number of renewable energy producers M to match the average markup of solar power generators, firms with NAICS code 221114, in the U.S. in 2011, which was 3, while keeping the elasticity of substitution parameter η constant at baseline value 11.16.

We present the extended model's simulations. Figure 13 shows the simulated optimal carbon tax paths for the lowest, median, and highest productivity coal energy producers in the oligopoly equilibrium when renewables are in the energy mix. Note that, now the optimal tax is not significantly different across producers with different productivities, and are still smaller than, but closer to the Pigouvian tax rate.

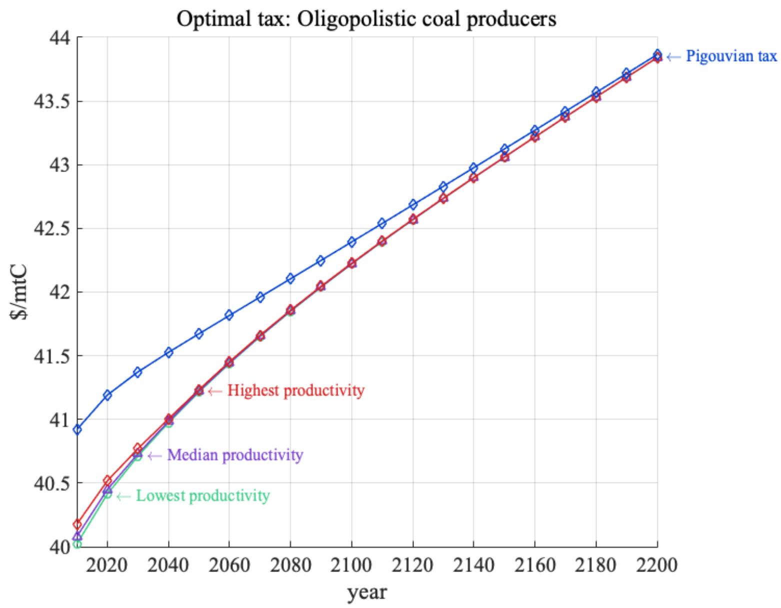

Figure 13: Simulated optimal carbon tax paths for the lowest, median, and highest productivity coal energy producers in the oligopoly equilibrium when renewables are in the energy mix.

Figure 14 shows that optimal policy paths for the two renewable energy producers, with low and high productivity levels. Since renewable energy use does not result in any carbon emissions, the optimal policy is only the output subsidy to correct for the market distortion. The optimal output subsidies on the renewable energy producers are negative, as expected, and are their absolute size increases with productivity, as shown in Figure 14. However, these subsidies diminish over the long run as the renewable energy sector becomes more productive and thus its output's price decreases.

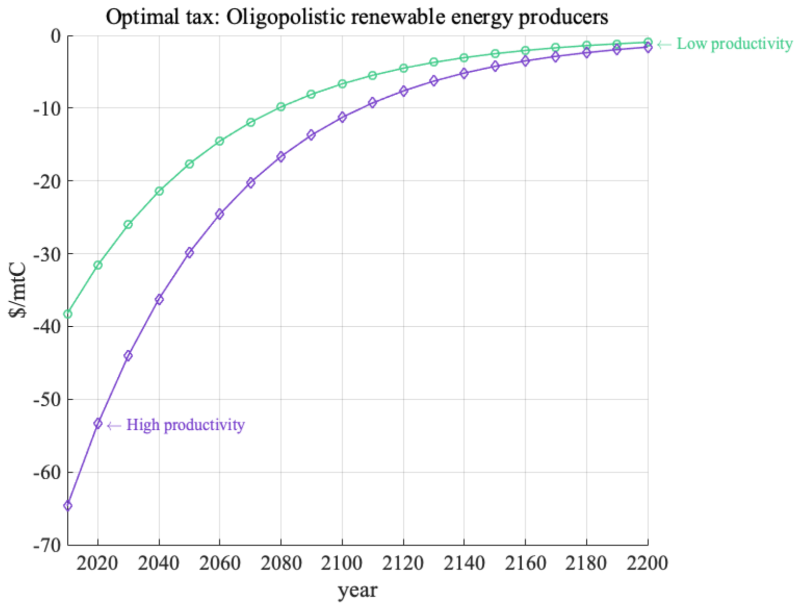

Figure 14: Simulated optimal output subsidy paths for the low and high productivity renewable energy producers in the oligopoly equilibrium.

The renewables in the energy mix extension results imply that the optimal policy is to subsidize the renewable energy producers, while taxing the coal producers at a rate close to the Pigouvian tax, even though coal production is lower than its competitive level. Moreover, the optimal carbon tax on coal is not significantly different across producers with different productivities when ther is an alternative viable carbon-free source of energy.

6 Conclusion

This paper investigates the optimal carbon taxes in an environment where carbon-intensive industries are imperfectly competitive. This exploration unfolds through theoretical and quantitative approaches. From a simple theoretical model, we showed that the optimal carbon tax on an unregulated oligopolistic firm is lower than the Pigouvian tax and depends on the firm's demand elasticity. Moreover, the difference between the optimal tax and the Pigouvian tax increases as the level of competition decreases. This exercise revealed the importance of accounting for market structure when designing carbon taxes.

Immediate next step of this project is to incorporate the endogeneity of firms' entry and exit decisions into the model. This extension is crucial because a carbon tax will influence firms' decisions to enter and exit carbon-intensive industries, giving rise to an endogenous market structure. Recently, some countries have adopted clean energy subsidies rather than a carbon tax. We can use our framework to compare the effectiveness of these two policies in reducing carbon emissions and analyze the implications of these policies on market structure. Other potential extensions include incorporating inflation dynamics, time-inconsistent policy, and nonlinearity in the relationship between energy output and carbon emissions. We leave these extensions for future research.

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Appendix

Appendix A Definitions

A.1 The General Model: Energy Producer j's Problem

We derive the first-order condition for energy producer j's problem with respect to the choice of labor input $N_{j,t}$. First, rewrite the per-period profit maximization problem for energy producer j by substituting its price function from equation (10):

$$\max_{N_{j,t}} \kappa_j p_t E_{j,t}^{1-\frac{1}{\eta}} E_t^{\frac{1}{\eta}} - w_t N_{j,t} \quad \text{s.t. } p_t = \nu \frac{Y_t}{E_t}.$$

As outlined in Assumption 4, the energy producer j internalizes its influence over the energy composite and price but not the final output and the general price level. Thus, the energy producer j's first-order condition with respect to the choice of labor input $N_{j,t}$ is:

$$\begin{aligned}
\kappa_j \frac{\partial E_{j,t}}{\partial N_{j,t}} \left[\frac{\partial p_t}{\partial E_t} \frac{\partial E_t}{\partial E_{j,t}} E_{j,t}^{1-\frac{1}{\eta}} E_t^{\frac{1}{\eta}} + \frac{1}{\eta} p_t E_t^{\frac{1}{\eta}-1} \frac{\partial E_t}{\partial E_{j,t}} E_{j,t}^{1-\frac{1}{\eta}} + \left(\frac{\eta - 1}{\eta} \right) p_t E_t^{\frac{1}{\eta}} E_{j,t}^{-\frac{1}{\eta}} \right] &= w_t \\
\kappa_j p_t E_t^{\frac{1}{\eta}} E_{j,t}^{-\frac{1}{\eta}} \frac{\partial E_{j,t}}{\partial N_{j,t}} \left[\frac{\partial p_t}{\partial E_t} \frac{p_{j,t} E_{j,t}}{p_t^2} + \frac{1}{\eta} \frac{p_{j,t} E_{j,t}}{p_t E_t} + \left(\frac{\eta - 1}{\eta} \right) \right] &= w_t, \\
p_{j,t} F'_{j,t}(N_{j,t}) \left[\frac{\partial p_t}{\partial E_t} \frac{s_{j,t} E_t}{p_t} + \frac{1}{\eta} s_{j,t} + \left(\frac{\eta - 1}{\eta} \right) \right] &= w_t, \\
\left[\frac{\partial p_t}{\partial E_t} \frac{E_t}{p_t} s_{j,t} + \frac{1}{\eta} s_{j,t} + \left(\frac{\eta - 1}{\eta} \right) \right]^{-1} &= \frac{p_{j,t}}{w_t / F'_{j,t}(N_{j,t})}.
\end{aligned}$$

The right-hand side of the last equation line is price over marginal cost for energy producer j, which is the producer j's markup. The markup expression can be rewritten as $\frac{\sigma(s_{j,t})-1}{\sigma(s_{j,t})}$, where $\sigma(s_{j,t})$ is the price elasticity of energy j demand. Thus,

$$\frac{\sigma(s_{j,t})}{\sigma(s_{j,t}) - 1} = \left[\frac{\partial p_t}{\partial E_t} \frac{E_t}{p_t} s_{j,t} + \frac{1}{\eta} s_{j,t} + \left(\frac{\eta - 1}{\eta} \right) \right]^{-1},$$

which implies that:

$$\sigma(s_j, t) = \left[\frac{1}{\eta}(1 - s_{j,t}) + \frac{E_t / p_t}{|\partial E_t / \partial p_t|} s_{j,t} \right]^{-1},$$

where $\frac{E_t/p_t}{|\partial E_t/\partial p_t|}$ is the absolute value of the inverse price elasticity of energy composite demand. For simplicity, let ρ_t denote the absolute value of the price elasticity of energy composite demand in period t.

A.2 Planning Problem

Building upon the multi-sector neoclassical growth model with an energy sector, we consider the case that the accumulated emissions from coal production result in external damages, which affect the final good production possibilities. The social planner will choose allocations taking this negative externality into account when maximizing aggregate welfare subject to technology, feasibility, and carbon cycle conditions:

$$
\begin{aligned}
\max_{\left\{C_t, K_{t+1}, N_t, N_{0,t}, E_t, \{N_{j,t}, E_{j,t}\}_{j=1}^J, E_t^f, S_t, Y_t\right\}_{t=0}^\infty} \quad & \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \log(C_t) \\
\text{subject to } \forall t, \quad & C_t + K_{t+1} = Y_t + (1-\delta)K_t, \\
& Y_t = \exp[-\gamma_t(S_t - \bar{S})] K_t^\alpha (A_{0,t} N_{0,t})^{1-\alpha-\nu} E_t^\nu, \\
& E_t = \left[\sum_{j=1}^J \kappa_j E_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \\
& E_{j,t} = \chi_{j,t}(N_{j,t}), \ \forall j, \\
& A_{0,t+1} = (1+g_A)A_{0,t}, \\
& \chi_{j,t+1} = (1+g_{j,\chi})\chi_{j,t}, \ \forall j, \\
& C_t, K_{t+1}, E_t, E_{j,t} \geq 0, \ \forall j, \\
& \sum_{j=0}^J N_{j,t} = N_t, \\
& S_t = \sum_{s=0}^{t+T} (1-d_s) E_{t-s}^f, \\
& E_t^f = \sum_{j=1}^J E_{j,t}.
\end{aligned}
$$

Definition. The optimal allocations $\left\{C_t, K_{t+1}, N_{0,t}, E_t, \{N_{j,t}, E_{j,t},\}_{j=1}^J, E_t^f, S_t, S_{1,t}, S_{2,t}, Y_t, \mathrm{SCC}_t\right\}_{t=0}^\infty$ that solve the planning problem need to satisfy the following equations:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}}\right), \tag{19}$$

$$\chi_{j,t} \left[\frac{\nu \kappa_j}{E_t^{\frac{\eta-1}{\eta}} E_{j,t}^{\frac{1}{\eta}}} - \frac{\mathrm{SCC}_t}{Y_t}\right] = \frac{1-\alpha-\nu}{N_{0,t}}, \tag{20}$$

$$C_t = Y_t + K_{t+1}, \tag{21}$$

$$Y_t = \exp[-\gamma_t(S_t - \bar{S})]K_t^{\alpha}(A_{0,t}N_{0,t})^{1-\alpha-\nu}E_t^{\nu},$$

$$E_{j,t} = \chi_{j,t}N_{j,t}, \forall j$$

$$E_t = \left[\sum_{j=1}^{J}\kappa_j E_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$\sum_{j=0}^{J} N_{j,t} = N_t,$$

$$E_t^f = \sum_{j=1}^{J} E_{j,t},$$

$$S_t = S_{1,t} + S_{2,t},$$

$$S_{1,t} = S_{1,t-1} + \psi_L E_t^f,$$

$$S_{2,t} = \psi S_{2,t-1} + \psi_0(1-\psi_L)E_t^f,$$

$$\mathrm{SCC}_t = \sum_{s=0}^{\infty}\beta^s C_t \frac{Y_{t+s}}{C_{t+s}}\gamma_{t+s}(1-d_s), \tag{22}$$

Conditions (19) and (21) are satisfied if and only if the saving rate is constant at $\alpha\beta$. Then by Proposition 1 from Golosov et al. (2014), the optimal SCC_t expression in equation (22) simplifies to Λ_t. Moreover, the ratio SCC_t/Y_t that appears in equation (20) is constant at level $\hat{\Lambda}_t$ as shown in equation (9).

A.2.1 The Solution Algorithm

Assume that energy firms $j = 1, \ldots, J$ are symmetric; thus, they have the same labor productivity and share parameters. To obtain the solution to the planning problem, we use the following algorithm:

1. Guess the path of $N_{0,t}^g$ for $t = 1, \ldots, T$ where $T = 20$ corresponds to 200 years, given that each period is a decade long. A reasonable guess is a constant path for $N_{0,t} = 0.99$ for all t.

2. Set a tolerance level $\epsilon =$ 1e-6 and a maximum number of iterations $M = 1000$.

3. For each period $t = 1, \ldots, T$, solve for $\varepsilon_{j,t}^*$ given by equation (17) using the guessed path of $N_{0,t}^g$, exogenous parameters, and the constant ratio of Λ_t to Y_t derived in equation (9).

4. Using the solved $\varepsilon^*_{j,t}$, solve for the path of energy composite E_t by substituting for $E_{j,t}$ given by equation (16) into equation (8).

5. Given the solved path of E_t, solve for the path of $E_{j,t}$ using equation (16).

6. Using the solved path of $E_{j,t}$ and the exogenous path of technology $\chi_{j,t}$, solve for the path of $N_{j,t}$ using equation (15).

7. Update the path of $N_{0,t}$, call this updated path $N^s_{0,t}$, using the solved path of $N_{j,t}$, i.e., $N_{0,t} = N_t - \sum_{j=1}^{J} N_{j,t}$, given the exogenous path of labor supply, N_t and the symmetry of the energy producers.

8. Finally, calculate the sum of absolute differences between the updated path of $N^s_{0,t}$ and the guessed path of $N^g_{0,t}$, and check if this sum of differences is less than the tolerance level ϵ. If the difference is less than ϵ, the algorithm converges. If the difference is greater than ϵ, then update the guessed path of $N^g_{0,t}$ with the updated path of $N^s_{0,t}$ and repeat steps 3–7 until the algorithm converges or it reaches the maximum number of iterations.

A.3 Decentralized Equilibrium

A.3.1 Competitive Equilibrium

Definition. In a competitive equilibrium, given the exogenous path of taxes $\{\tau_{j,t}\}_{t=0}^{\infty}$, the allocations $\{C_t, K_{t+1}, Y_t, N_t, N_{0,t}, E_t, \{N_{j,t}, E_{j,t}\}_{j=1}^{J}, S_t, S_{1,t}, S_{2,t}, E^f_t\}_{t=0}^{\infty}$, prices $\{q_t, w_t, r_t, p_t, \{p_{j,t}\}_{j=1}^{J}\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$, and profits $\{\Pi_0, \Pi_j, \Pi\}$ that satisfy the set of conditions:

$$\chi_{j,t}\left[\frac{\nu\kappa_j}{E_t^{\frac{\eta-1}{\eta}} E_{j,t}^{\frac{1}{\eta}}} - \frac{\tau_{j,t}}{Y_t}\right] = \frac{1-\alpha-\nu}{N_{0,t}},$$

$$E_t = \left[\sum_{j=1}^{J} \kappa_j E_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$p_{j,t} = \kappa_j p_t E_t^{\frac{1}{\eta}} E_{j,t}^{-\frac{1}{\eta}}, \forall j,$$

$$\frac{1}{C_t} = \beta\frac{1}{C_{t+1}} r_{t+1},$$

$$C_t = Y_t + K_{t+1},$$

$$Y_t = \exp[-\gamma_t(S_t - \bar{S})]K_t^{\alpha}(A_{0,t}N_{0,t})^{1-\alpha-\nu}E_t^{\nu},$$

$$E_{j,t} = \chi_{j,t} N_{j,t},\ \forall j$$

$$\sum_{j=0}^{J} N_{j,t} = N_t,$$

$$E_t^f = \sum_{j=1}^{J} E_{j,t},$$

$$S_t = S_{1,t} + S_{2,t},$$

$$S_{1,t} = S_{1,t-1} + \psi_L E_t^f,$$

$$S_{2,t} = \psi S_{2,t-1} + \psi_0 (1-\psi_L) E_t^f,$$

$$q_t = \beta^t \left(C_t / C_0\right)^{-1},$$

$$r_t = \alpha Y_t / K_t,$$

$$w_t = (1-\alpha-\nu) Y_t / N_{0,t},$$

$$p_t = \nu Y_t / E_t,$$

$$\sum_{t=0}^{\infty} q_t T_t = \sum_{t=0}^{\infty} q_t \sum_{j=1}^{J} \tau_{j,t} E_{j,t},$$

$$\Pi_0 = \sum_{t=0}^{\infty} q_t \left[Y_t - r_t K_t - w_t N_{0,t} - \sum_{j=1}^{J} p_{j,t} E_{j,t} \right],$$

$$\Pi_j = \sum_{t=0}^{\infty} q_t \left[(p_{j,t} - \tau_{j,t}) E_{j,t} - w_t N_{j,t} \right],$$

$$\Pi = \sum_{j=0}^{J} \Pi_j.$$

A.3.2 Equilibrium with an Oligopolistic Energy Sector

Definition. In an oligopoly equilibrium, given the exogenous path of second-best taxes $\{\tau_{j,t}\}_{t=0}^{\infty}$, the allocations $\{C_t, K_{t+1}, Y_t, N_t, N_{0,t}, E_t, \{N_{j,t}, E_{j,t}, s_{j,t}\}_{j=1}^{J}, S_t, S_{1,t}, S_{2,t}, E_t^f\}_{t=0}^{\infty}$, prices $\{q_t, w_t, r_t, p_t, \{p_{j,t}\}_{j=1}^{J}\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$, and profits $\{\Pi_0, \Pi_j, \Pi\}$ satisfy the set of conditions:

$$\chi_{j,t} \left[\frac{\nu \kappa_j}{E_t^{\frac{\eta-1}{\eta}} E_{j,t}^{\frac{1}{\eta}}} \frac{\sigma(s_{j,t}) - 1}{\sigma(s_{j,t})} - \frac{\tau_{j,t}}{Y_t} \right] = \frac{1-\alpha-\nu}{N_{0,t}},$$

$$E_t = \left[\sum_{j=1}^{J} \kappa_j E_{j,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$p_{j,t} = \kappa_j p_t E_t^{\frac{1}{\eta}} E_{j,t}^{-\frac{1}{\eta}}, \forall j,$$

$$s_{j,t} = \frac{p_{j,t} E_{j,t}}{p_t E_t},$$

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} r_{t+1},$$

$$C_t = Y_t + K_{t+1},$$

$$Y_t = \exp[-\gamma_t (S_t - \bar{S})] K_t^{\alpha} (A_{0,t} N_{0,t})^{1-\alpha-\nu} E_t^{\nu},$$

$$E_{j,t} = \chi_{j,t} N_{j,t}, \forall j$$

$$\sum_{j=0}^{J} N_{j,t} = N_t,$$

$$E_t^f = \sum_{j=1}^{J} E_{j,t},$$

$$S_t = S_{1,t} + S_{2,t},$$

$$S_{1,t} = S_{1,t-1} + \psi_L E_t^f,$$

$$S_{2,t} = \psi S_{2,t-1} + \psi_0 (1 - \psi_L) E_t^f,$$

$$q_t = \beta^t \left(C_t / C_0\right)^{-1},$$

$$r_t = \alpha Y_t / K_t,$$

$$w_t = (1 - \alpha - \nu) Y_t / N_{0,t},$$

$$p_t = \nu Y_t / E_t,$$

$$\sum_{t=0}^{\infty} q_t T_t = \sum_{t=0}^{\infty} q_t \sum_{j=1}^{J} \tau_{j,t} E_{j,t},$$

$$\Pi_0 = \sum_{t=0}^{\infty} q_t \left[Y_t - r_t K_t - w_t N_{0,t} - \sum_{j=1}^{J} p_{j,t} E_{j,t}\right],$$

$$\Pi_j = \sum_{t=0}^{\infty} q_t \left[(p_{j,t} - \tau_{j,t}) E_{j,t} - w_t N_{j,t}\right],$$

$$\Pi = \sum_{j=0}^{J} \Pi_j.$$

Appendix B Markup Calibration

The markup calculation method implemented by De Loecker, Eeckhout, and Unger (2020) is known as the production function approach. Consider a set of heterogeneous firms indexed by $j = 1, \ldots, J$, that produce output in period t according to the production function $Q_{j,t} = f(\Omega_{j,t}, V_{j,t}, K_{j,t})$, where $\Omega_{j,t}$ is the Hicks-neutral productivity term, $V_{j,t}$ is the bundle of variable inputs, and $K_{j,t}$ is the capital stock. They assume that variable inputs adjust frictionlessly within one period (a year in their data), but capital stock is subject to adjustment costs and other frictions. Moreover, the firms take variable input prices as given. Then, the firm j's first-order condition for the cost-minimization problem with respect to the variable input can be adjusted to obtain the following expression for the firm's markup:

$$\mu_{j,t} = \theta^V_{j,t} \frac{P_{j,t} Q_{j,t}}{P^V_{j,t} V_{j,t}}, \tag{23}$$

where $\mu_{j,t}$ is the markup, $\theta^V_{j,t}$ is the elasticity of output with respect to variable input, $P_{j,t}$ is the price of output, and $P^V_{j,t}$ is the price of variable inputs. Thus, the markup can be retrieved from data on the output elasticity, revenue, and cost of variable inputs. We use updated De Loecker, Eeckhout, and Unger's (2020) two-digit industry-level output elasticity estimates provided by Conlon et al. (2023) and Compustat (2023) to construct the markup data for each industry in the energy sector from 1956 to 2020.

Specifically, we use equation (23) to calculate firm-level markups. We calculate the average industry markups as follows:

$$\mu_t = \sum_{j=1}^{J} m_{j,t} \mu_{j,t}, \tag{24}$$

where $m_{j,t}$ is the weight of each firm, which is the share of sales in the industry.

For example, to obtain the average markups of the coal industry, we calculate the firm-level markups using equation (23) for each firm in the coal industry. Then, we average the firm-level markups weighted by the sales shares of each firm in the industry.

Using the aggregate markup data, we replicate our motivational Figure **??** to show the relationship between aggregate industry markups and carbon intensity.

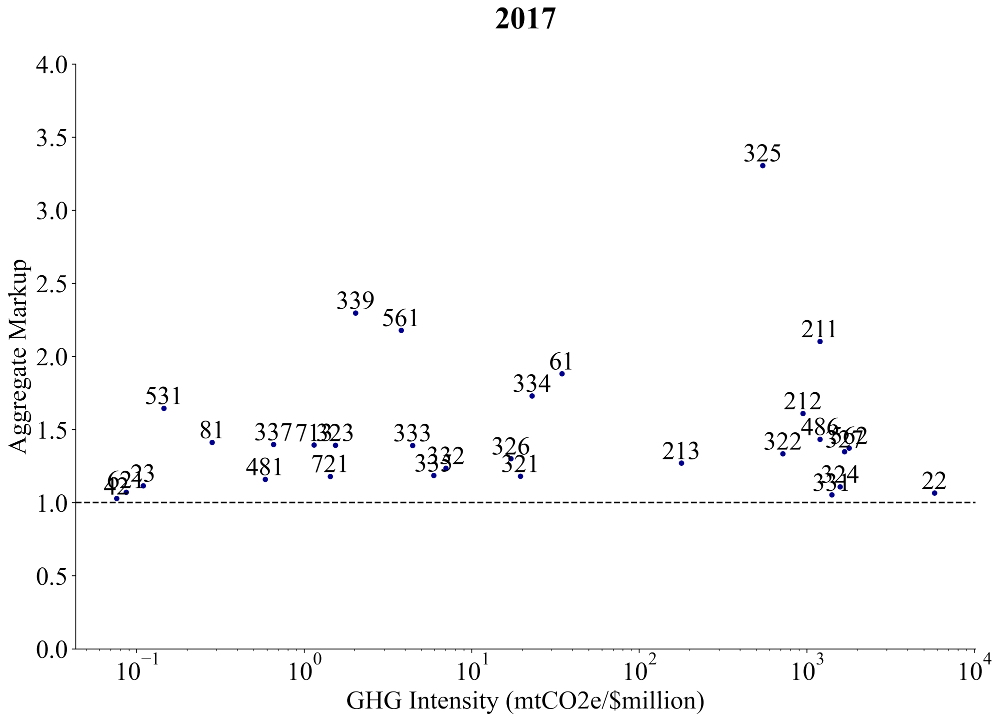

Figure 15: U.S. coal mining industry sales share weighted average markups. Sources: Author's calculations using data from Conlon et al. (2023) and Compustat (2023).

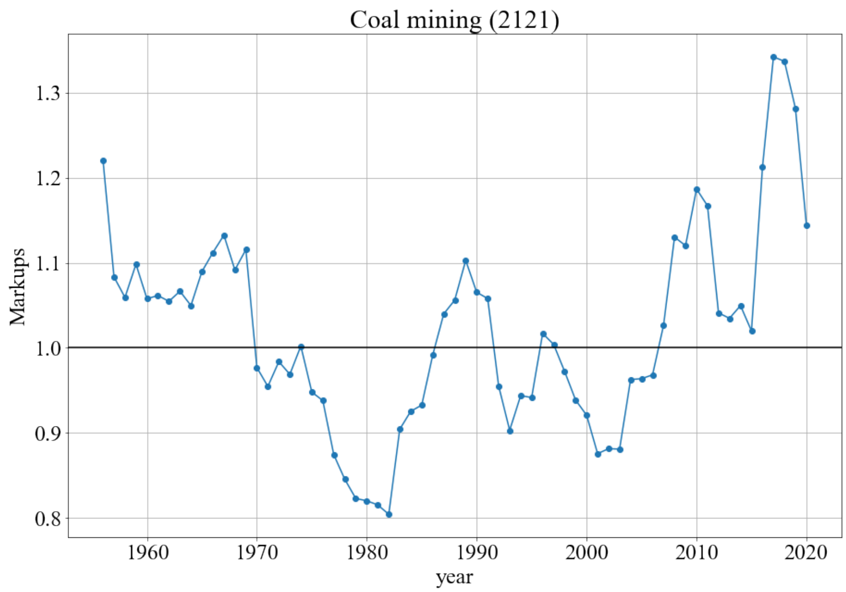

Figure 16: U.S. industry-level sales share weighted average markups against carbon intensity. Sources: Author's calculations using data from Conlon et al. (2023) and Compustat (2023) for average markups and data from U.S. Environmental Protection Agency's (2024) GHGRP and U.S. Bureau of Economic Analysis's (2024c) Industry-Level Production Account (ILPA) for carbon intensity.

Appendix C Monopolistic Competition in the Energy Sector

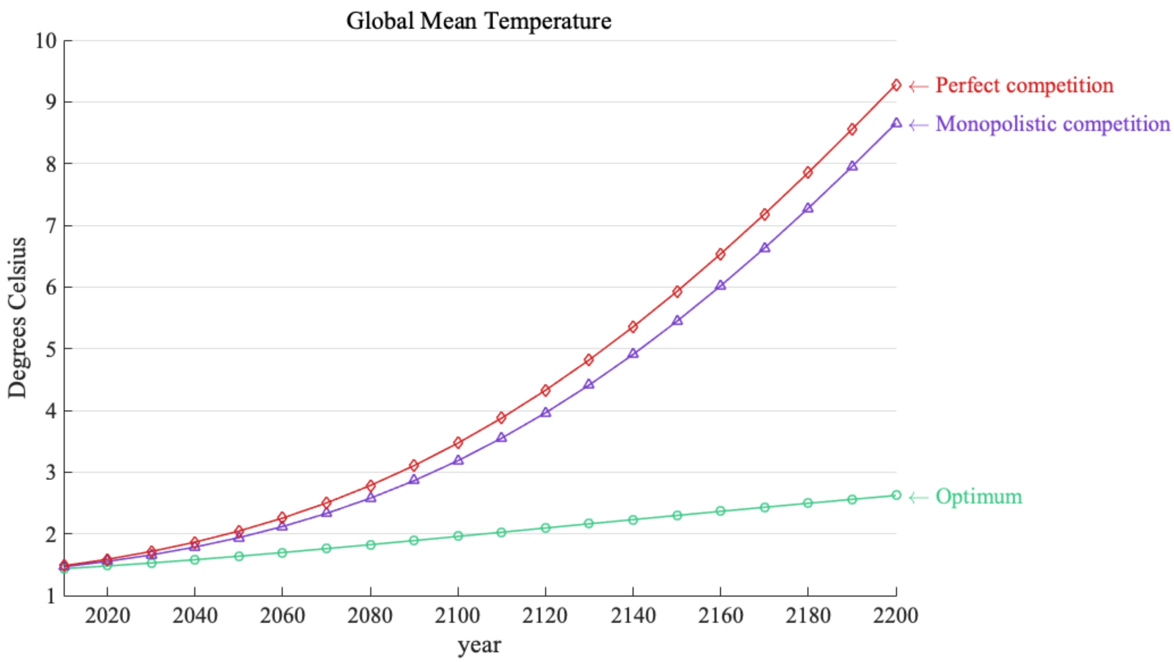

Figure 17: Temperature change

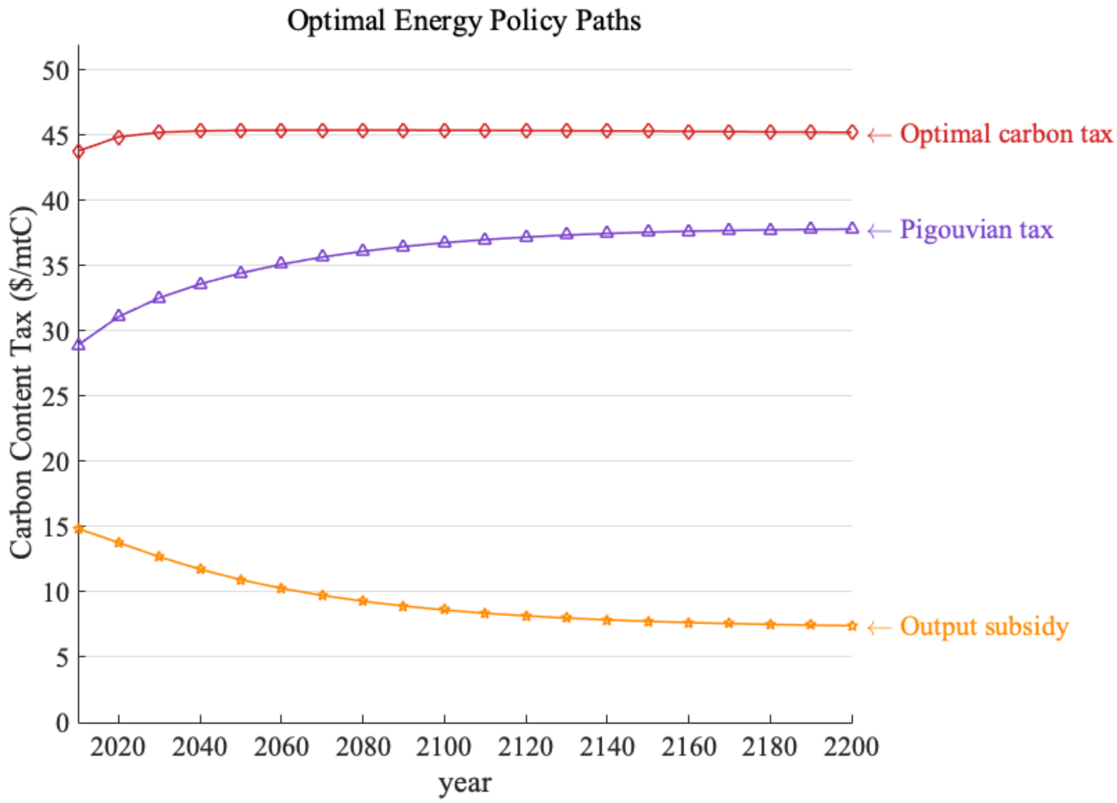

Figure 18: Policy paths