PAI Project Task 0

Goal: posterior probability $P(H_i \mid X)$

↳ Bayes rule: $P(H_i \mid X) = \frac{P(H_i, X)}{P(X)} = \frac{P(X \mid H_i) \cdot P(H_i)}{P(X)}$

$$P(X) = \sum_{i=1}^{3} P(H_i) \cdot P(X \mid H_i)$$

$$P(X \mid H_i) = \prod_{j=1}^{n} P(X_j \mid H_i) \quad \rightsquigarrow X_j \text{ are sampled iid from dist.}$$

Log probabilities:

$$\log P(H_i \mid X) = \log P(X \mid H_i) + \log P(H_i) - \log P(X)$$

$$\log P(X \mid H_i) = \log\left(\prod_{j=1}^{n} P(X_j \mid H_i)\right) = \sum_{j=1}^{n} \log P(X_j \mid H_i)$$

$$\log P(X) = \log\left(\sum_{i=1}^{3} P(H_i) P(X \mid H_i)\right) = \log\left(\sum_{i=1}^{3} e^{\log P(H_i)} \cdot e^{\log P(X \mid H_i)}\right)$$

$$= \log\left(\sum_{i=1}^{3} \exp\left(\log P(H_i) + \log P(X \mid H_i)\right)\right) =$$

$$= LSE\left(\{\log P(H_i) + \log P(X \mid H_i) \mid i \in \{1,2,3\}\}\right)$$

↳ logsumexp