

Dataset D_{n+1} — Stability & Coupling Analysis

Based on Aureon v7 Specification

Refinement Target: D_n to D_{n+1}

Primary Objective: Resolve tensor rank mismatches, enforce unitary preservation in the quantum layer, and derive analytical stability boundaries using Lyapunov methods.

1. Mathematical Core (Scalarized & Consistent)

To resolve the tensor rank inconsistency identified in D_n , we strictly separate the tensor field dynamics from the scalar coupling used in feedback.

1.1 Tensor Substrate

Let $T^{i,j}(x,t)$ be the information-energy density tensor. The recursive update is driven by the divergence of this field, representing information flux J_j :

1.2 The Interaction Scalar (S)

The feedback signal sent to the quantum layer is the scalar magnitude of this flux (analogous to signal power):

1.3 Modified Recursive Operator

The tensor field update is now governed by a diffusion-reaction equation involving S:

Refinement: This ensures index consistency. The feedback term $\lambda S \delta^{i,j}$ is an isotropic pressure proportional to the system's total information flux.

2. Quantum Layer (Unitary Dynamics)

To prevent probability leakage (where $\langle \Psi | \Psi \rangle \neq 1$), we replace the additive update rule with a unitary rotation generator.

2.1 Hamiltonian Formulation

We define an interaction Hamiltonian \hat{H}_{int} controlled by the scalar field $S(t)$:

(Where $\hat{\sigma}_x$ is the Pauli-X operator, inducing bit-flip pressure).

2.2 Unitary Evolution

The state update at step $t+1$ is given by the unitary operator $U(t)$:

Refinement: Since $U(t)$ is unitary ($U^\dagger U = I$), the normalization condition $|\alpha|^2 + |\beta|^2 = 1$ is rigorously preserved for all t .

3. Causal Graph (With Damping)

We introduce a damping node to control the resonance between the scalar field energy and quantum coherence.

Nodes:

- * N_1 : S (Scalar Interaction Field)
- * N_2 : \hat{H}_{int} (Hamiltonian Driver)
- * N_3 : $|\Psi_Q\rangle$ (Quantum State)
- * N_4 : $T^{i,j}$ (Tensor Substrate)

Edges & Weights:

- * $N_4 \rightarrow N_1$: Tensor flux generates Scalar S. (Weight: $w_{geo} = 1$)
- * $N_1 \rightarrow N_2$: Scalar drives Hamiltonian. (Weight: η , the Learning Rate)
- * $N_2 \rightarrow N_3$: Hamiltonian evolves State. (Unitary Transform)
- * $N_3 \rightarrow N_4$ (Feedback): Quantum Coherence modifies Tensor Field.
- * Feedback Function: $|\Delta T| \propto -\gamma |\langle \Psi | \hat{\sigma}_z | \Psi \rangle|$
- * Refinement: The feedback is proportional to the population difference (Z-expectation), effectively "cooling" the tensor field when the system aligns with $|0\rangle$.

4. Lyapunov Stability Analysis

To solve Open Question #2 (Causal Loop Stability), we construct a Lyapunov candidate function $V(t)$ to find the upper bound of the learning rate η .

4.1 Lyapunov Candidate

Let $V(t)$ represent the total deviation energy of the system:

4.2 Stability Condition

For the system to be stable, the orbital energy must decrease or oscillate within bounds: $\dot{V} \leq 0$.

Taking the discrete difference and simplifying the Taylor expansion for small η :

4.3 The Stability Boundary

For stability ($|\Delta V| \leq 0$), the damping γ must dominate the excitation η . This yields the analytical boundary:

Conclusion: The quantum learning rate η must be dynamically scaled inversely to the square root of the scalar field intensity. If S spikes (high recursive turbulence), η must drop to prevent chaotic divergence.

5. Ontological Audit (Passed)

* Tensor Rank Consistency: Resolved. The feedback term is now generated via scalar contraction (J^j_j), ensuring valid tensor algebra.

* Unitary Preservation: Resolved. Replaced linear addition with exponential maps ($\mathfrak{su}(2) \rightarrow \mathrm{SU}(2)$).

* Energy Conservation: Addressed via the introduction of the damping term γ , creating a dissipative system that settles into attractors rather than exploding.

6. Open Questions (For D_{n+2} Development)

* *Topology of T :*

Does the tensor field T reside on a flat Euclidean metric ($g_{ij} = \delta_{ij}$) or does T back-react on the metric itself ($g_{ij}(T)$), effectively creating "informational gravity"?

* Multi-Qubit Entanglement:

Extending $|\Psi_Q\rangle$ to N qubits. Does the Interaction Scalar S couple to local qubits individually, or to the global entanglement entropy?

* Observability:

If S represents "informational friction," does it manifest thermodynamically as heat? (Landauer's Principle check).

Summary & Next Step

Dataset D_{n+1} has successfully regularized the mathematics of the RQML loop. We now have a system that is dimensionally sound, probability-conserving, and conditionally stable.

Proposed Next Step:

Would you like me to simulate a "Stress Test" for D_{n+1} ? I can derive the behavior of the system under a "Shock Condition" (e.g., a sudden spike in $T^i_{\cdot j}$) to see if the Lyapunov boundary holds.