Spin-Orbital Torque(SOT) Simulation

Based on Mumax3

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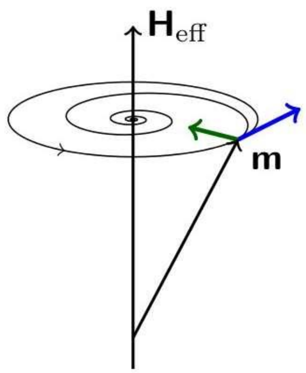
Introduction: Micromagnetic Simulation

- The physics is fully described by the total magnetic energy functional:

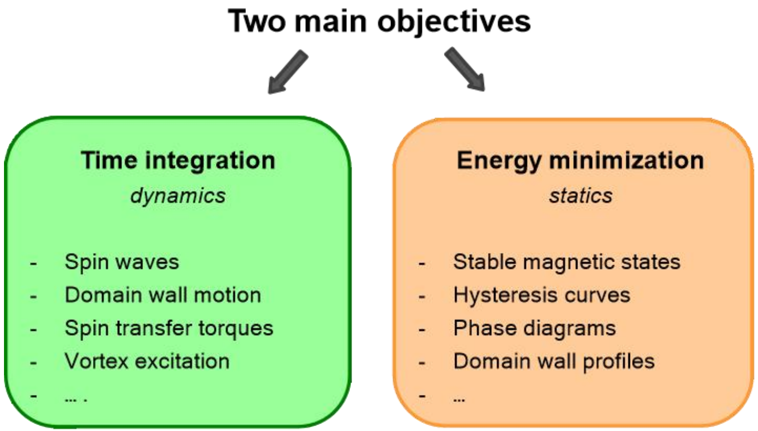
$$E[\boldsymbol{m}] = \int_V \left\{ \underset{\text{Exchange}}{A(\nabla \boldsymbol{m})^2} \underset{\text{Zeeman}}{- \mu_0 \boldsymbol{M} \cdot \boldsymbol{H}_{ext}} - \frac{\mu_0}{2} \boldsymbol{M} \cdot \boldsymbol{H}_{demag} + \begin{matrix} \textit{Crystal anisotropy} \\ \textit{Dzyaloshinskii-Moriya} \\ \textit{Magneto-elasticity} \\ \textit{Higher-order exchange} \\ \dots \end{matrix} \right\} d^3\boldsymbol{r}$$



- 2 Main Objectives
  - Time Intergration--LLG equation

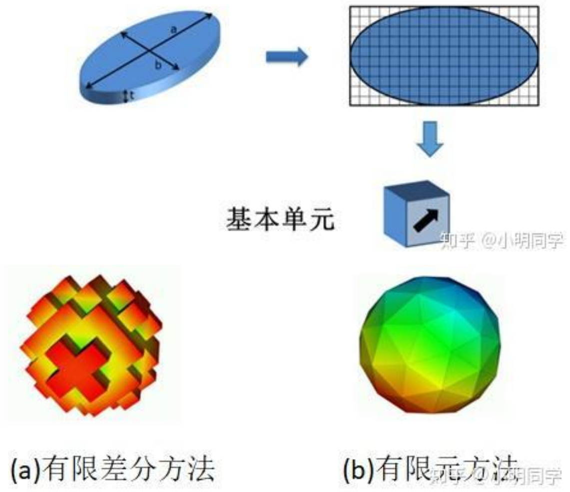
$$\dot{\boldsymbol{m}} = -\frac{\gamma}{1+\alpha^2}[\boldsymbol{m} \times \boldsymbol{H}_{eff} + \alpha \boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{H}_{eff})]$$

  - Energy Minimization --Static

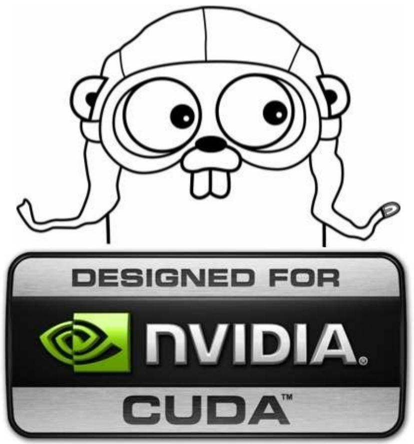


Introduction: Mumax - Opensource,FD

- Free finite-difference based micromagnetic simulation package
- GPU-accelerated *nvidia GPU required*
- Developed at DyNaMat (Ugent) by Arne Vansteenkiste
- Latest official release mumax3.10 *(Aug 13, 2020)*
- Active community *groups.google.com/forum/#!forum/mumax2*
- Documented API *mumax.github.io*
- Open source (GPLv3) *github.com/mumax/3*
- Mainly written in Go
- CUDA C kernels for heavy lifting
- Scripting language + Web GUI
- Well tested (unit tests + NIST standard problems)



Golang

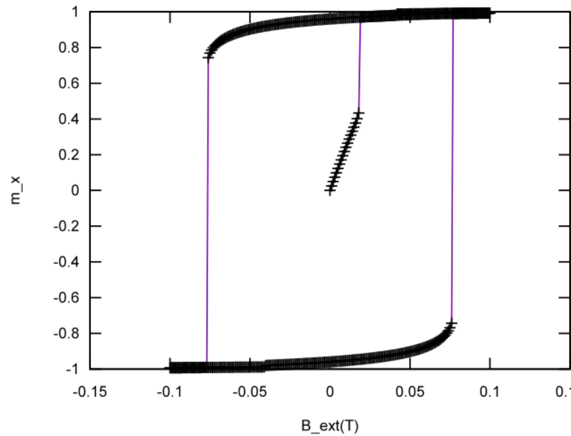


Introduction: Mumax

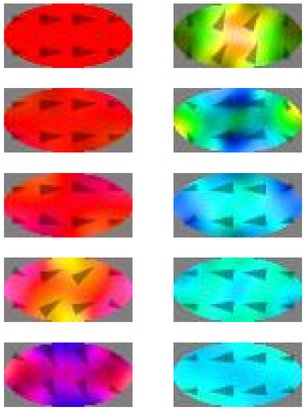
Process of a mumax simulation

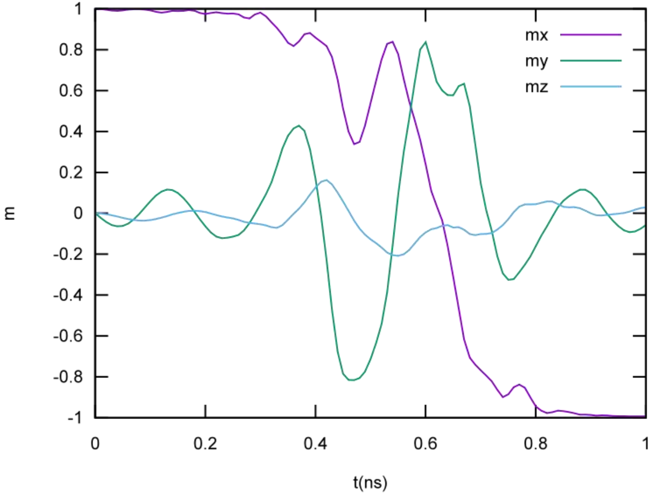
- Discretization
- Set Shape,Region
- Set Material Parameteres/Excitation
- Set Initial Condition
- Set output condition
- Run/Relax/Minize

More in appendix of this slides



Hysteresis

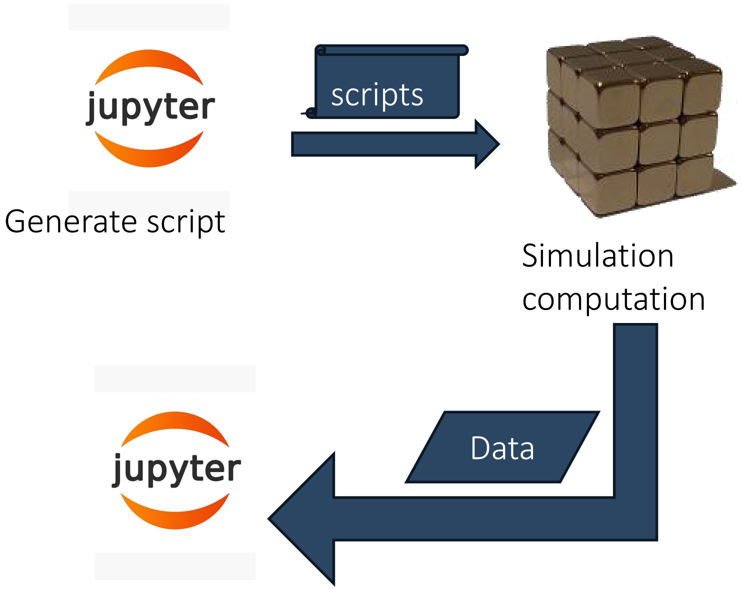




https://mumax.github.io/examples.html

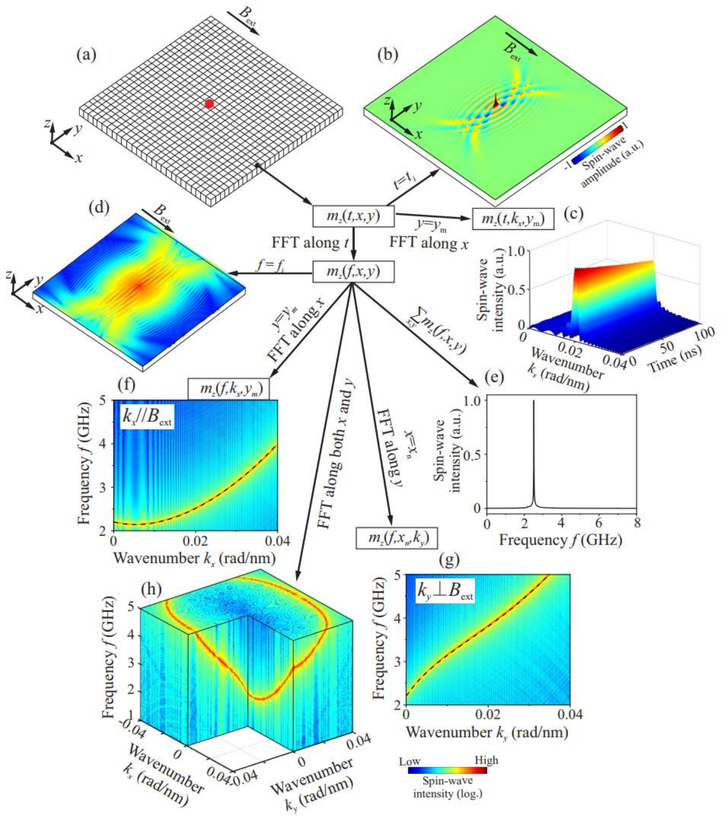
STT MRAM

Introduction: Mumax



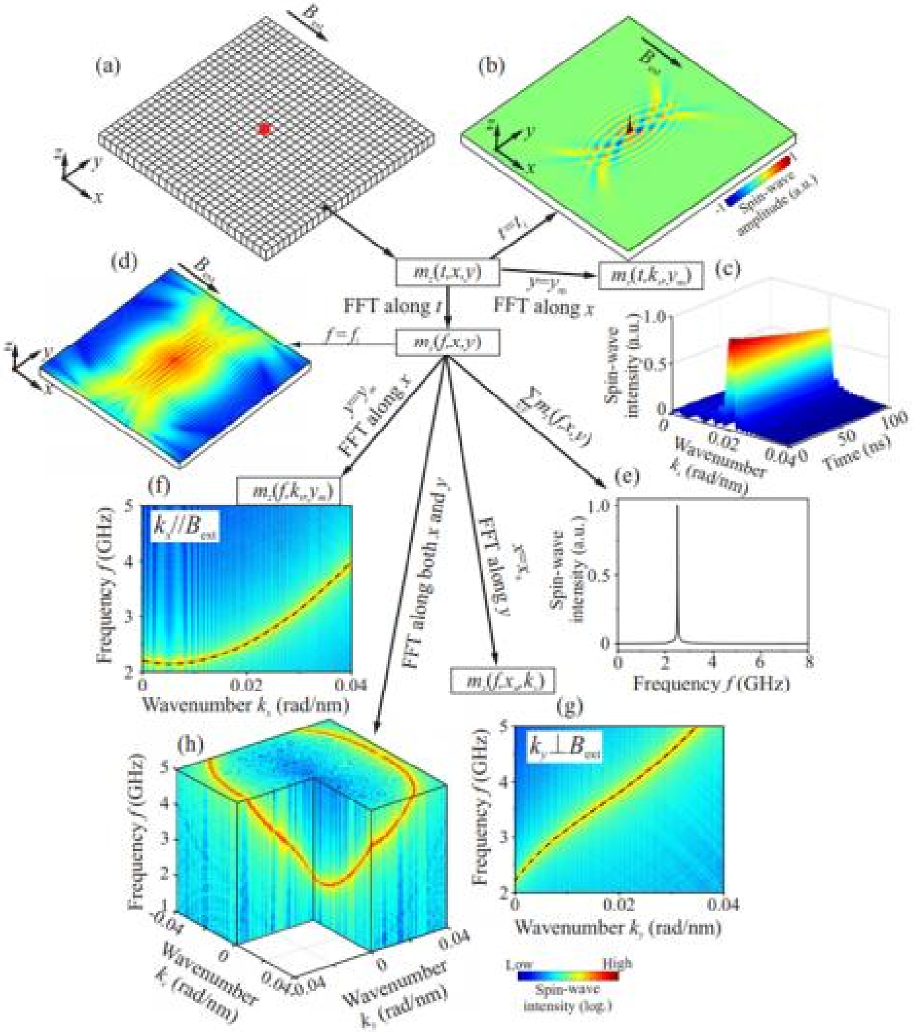
- Data reading
- Data process
- Plot data

Data analysis roadmap from Qi WANG's thesis



Introduction: Mumax

Figure 3.3: The data analysis flowchart.
(a) Mesh of the simulated thin film. The red circle is the excitation region.
(b) Snapshot of the excited spin waves.
(c) Simulation time as the function of the spin-wave wavenumber *k*x.
(d) Spatial distribution of the spin-wave energy.
(e) Spin-wave frequency spectrum.
(f)Spin-wave dispersion relation for the propagating waves parallel to the external field.
(g) Spin-wave
dispersion relation for the propagating waves perpendicular to the external field. (g). The dashed lines show the analytical results calculated from Eq. 2.37.
(h) Three-dimensional spin-wave dispersion relation
for an arbitrary propagation angle. The figure is taken from [98].



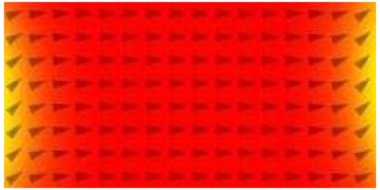
10.13140/RG.2.2.36726.63049

Introduction: Micromagnetic Simulation

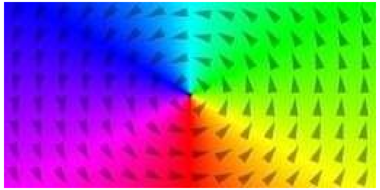
- In ferromagnets, neighboring magnetic moments have the tendency to align
- By Continuum Approx: Magnetization can be described by a continuous vector field

$$\boldsymbol{M}(\boldsymbol{r},t) = M_S(\boldsymbol{r}) \quad \underbrace{\boldsymbol{m}(\boldsymbol{r},\boldsymbol{t})}_{\text{central quantity of intere st}}$$

- picosecond time scale(1e-12s)
- 1nm – 1µm length scale



Quasi-uniform state



Vortex

Néel Skyrmion

Model	Description	Length Scale
Atomic level theory	Quantum mechanical ab initio calculations	<1nm<1nm
Micromagnetic theory	Continuous description of the magnetization	1–1000nm
Domain theory	Description of domain structure	1–1000µm
Phase theory	Description of ensembles of domains	>0.1mm

https://mumax.ugent.be/mumax3-workshop/

Introduction: Spin-Orbital Torque

$$\frac{d\hat{\mathbf{m}}}{dt} = -\gamma\hat{\mathbf{m}} \times \mathbf{H}_{\text{eff}} + \alpha\hat{\mathbf{m}} \times \frac{d\hat{\mathbf{m}}}{dt} + \gamma c_{j,D}\hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \hat{\boldsymbol{\sigma}}) + \gamma c_{j,F}\hat{\mathbf{m}} \times \hat{\boldsymbol{\sigma}}$$

- where $\hat{\mathbf{m}}$ is the unit vector along the magnetization of FM1
- $\hat{\sigma}$ is the unit vector along the spin polarization,
- γ is the gyromagnetic ratio,
- H_{eff} is the effective uniaxial anisotropy field $H_{K,\text{eff}} = 2K_{\text{eff}}/M_s$ in the z direction,.
- α is the damping constant,
- $c_{j,D(F)} = (\hbar\theta_{D(F)}J/2eM_s t_z)$ is the magnitude of DLT(FLT),

$$c_{j,D(F)} = \frac{\hbar\theta_{D(F)}J}{2eM_s t_z}$$

- $\theta_{D(F)}$ is the effective DLT(FLT) efficiency,
- J is the charge current density flowing in the plane (along the x axis),
- e is the electron charge,
- M_S is the saturation magnetization,
- t_z is the thickness of FM1.
- $\xi = \theta_F/\theta_D$

We assume that $\hat{\sigma} = (0, \cos\eta, \sin\eta)$ is a spin polarization direction, because the system is cylindrical symmetry in the $x-y$ plane, and η represents the spin-polarization angle. We express the magnetization vector as $\hat{\mathbf{m}} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$, where $\theta(0 \leq \theta \leq \pi)$ is the

1. Lee, D.-K. & Lee, K.-J. Spin-orbit Torque Switching of Perpendicular Magnetization in Ferromagnetic Trilayers. Sci Rep 10, 1772 (2020).

SOT on Mumax

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \left(\mathbf{H}_{\text{eff}} + \boxed{a_J (\mathbf{m} \times \mathbf{p}) + b_J \mathbf{p}} \right) + \alpha \mathbf{m} \times \dot{\mathbf{m}}$$

- SOT is not explicitly implemented in mumax3
- Solution -> Use the custom fields functionality to add it as an effective field term--**Time Comsuing**
- Equivalent Replacement of Slonczewski spin(SL-STT) --**bettter**

SOT by Adding Terms

$$\frac{d\hat{\mathbf{m}}}{dt} = -\gamma\hat{\mathbf{m}} \times \mathbf{H}_{\text{eff}} + \alpha\hat{\mathbf{m}} \times \frac{d\hat{\mathbf{m}}}{dt} + \gamma c_{j,D}\hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \hat{\boldsymbol{\sigma}}) + \gamma c_{j,F}\hat{\mathbf{m}} \times \hat{\boldsymbol{\sigma}}$$

```
// Define constants
ThetaD      := 0.15
e           := 1.6021766e-19
d           := 1e-9
Ms          := 580e3
hbar        := 1.0545718e-34
p           := Constvector(0,-1,0)
SOTxi       := -2.0 //ThetaF/ThetaD
J_SOT       := abs(-2.e11)
```

```
// Define prefactors aj and bj
ThetaD :=Const(J_SOT*(hbar/2.*alphaHe/d/ms))
ThetaF := Mul(ThetaD,Const(SOTxi))
```

```
// Add damping-like SOT term

dampinglike :=Mul(ThetaD,Cross(m,p))
AddFieldTerm(dampinglike)
AddEdensTerm(Mul(Const(-0.5),Dot(dampinglike,M_full)))
```

```
// Add field-like SOT term

fieldlike:=Mul(ThetaF,p)
AddFieldTerm(fieldlike)
AddEdensTerm(Mul(Const(-0.5),Dot(fieldlike,M_full)))
```

SOT by Equivalent Slonczewski spin transfer torque

$$\tau_{\mathrm{SOT}} = -\frac{\gamma}{(1+\alpha^2)} a_J \left[(1+\xi\alpha)\mathbf{m} \times (\mathbf{m} \times \mathbf{p}) + (\xi - \alpha)(\mathbf{m} \times \mathbf{p})\right]$$

We can use the existing Slonczewski spin transfer torque implementation!

$$P = \frac{2a_J}{\beta} = \alpha_H$$

$$\tau_{SL} = -\frac{\beta\gamma P}{2(1+\alpha^2)} \left[(1+\xi\alpha)(\mathbf{m} \times (\mathbf{m} \times \mathbf{m}_P)) + (\xi - \alpha)\mathbf{m} \times \mathbf{m}_P\right]$$

SOT by Equivalent Slonczewski spin transfer torque

In mumax the parameter is defined as

- SL torque

$$\tau_{SL} = \gamma\beta\frac{\epsilon - \alpha\epsilon'}{1+\alpha^2}(\mathbf{m}\times(\mathbf{m}_P\times\mathbf{m})) - \gamma\beta\frac{\epsilon' - \alpha\epsilon}{1+\alpha^2}\mathbf{m}\times\mathbf{m}_P$$

- beta

$$\beta = \frac{j_z\hbar}{M_{\text{sat}}ed}$$

- epsilon

$$\epsilon = \frac{P\Lambda^2}{(\Lambda^2+1)+(\Lambda^2-1)(\mathbf{m}\cdot\mathbf{m}_P)}$$

Here let the $\Lambda = 1$, that $\epsilon = \frac{P}{2}$

and $\epsilon' = \xi\epsilon$

That SL=

$$-\frac{\beta\gamma P}{2(1+\alpha^2)}[(1+\xi\alpha)(\mathbf{m}\times(\mathbf{m}\times\mathbf{m}_P)) + (\xi-\alpha)\mathbf{m}\times\mathbf{m}_P]$$

Since SOT=

$$\frac{d\hat{\mathbf{m}}}{dt} = -\gamma\hat{\mathbf{m}}\times\mathbf{H}_{\text{eff}} + \alpha\hat{\mathbf{m}}\times\frac{d\hat{\mathbf{m}}}{dt} + \gamma c_{j,D}\hat{\mathbf{m}}\times(\hat{\mathbf{m}}\times\hat{\boldsymbol{\sigma}}) + \gamma c_{j,F}\hat{\mathbf{m}}\times\hat{\boldsymbol{\sigma}} \tag{1}$$

Compare

$$c_{j,D(F)} = \frac{\hbar\theta_{D(F)}J}{2eM_st_z} \tag{1}$$

$$\beta = \frac{j_z\hbar}{M_{\text{sat}}ed} \tag{1}$$

We can find out that

$$\theta_{D/F} = 2c_{j,D/F}/\beta$$

SOT by Equivalent Slonczewski spin transfer torque

In summary to use SL represent equivalent STT we need

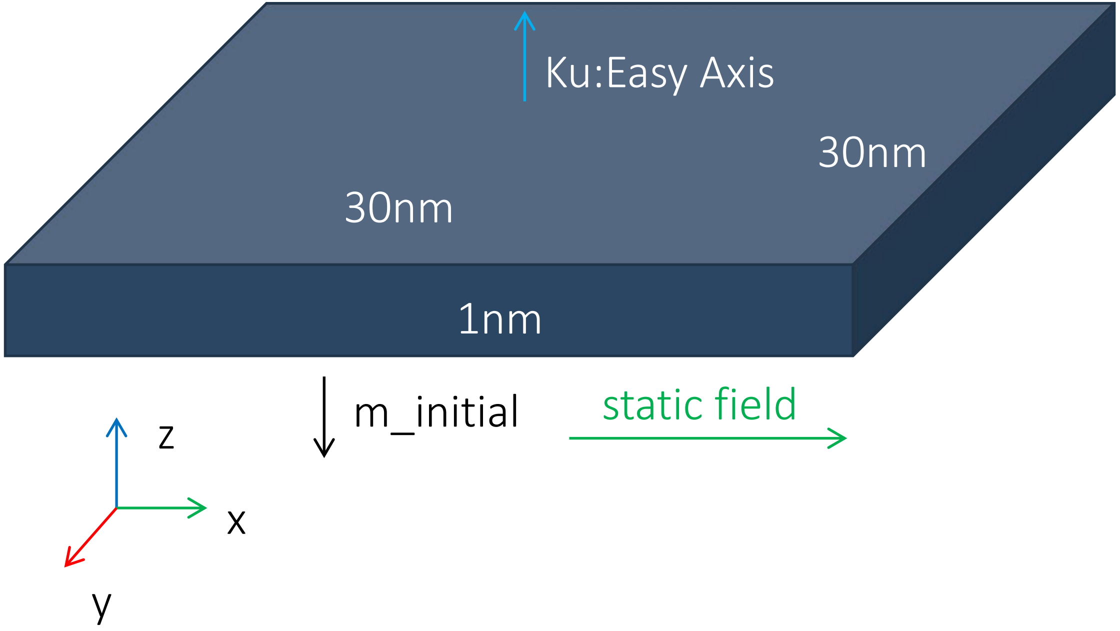
- $$P = \frac{2c_{j,D}}{\beta} = \theta_D$$
- $$\xi = \theta_F / \theta_D$$
- $$\Lambda = 1$$
- $$\epsilon' = \xi\epsilon = \xi\theta_D / 2$$
- spin polarization <=> fixed layer direction in p
- magnetitude of charge current,represent as the current vector in z axis

```
SOTxi  := -2 //thetaF/thetaD
ThetaD:=0.3
Pol=ThetaD
Lambda=1
Epsilonprime = ThetaD/2 * SOTxi
Fixedlayer = vector(0,-1,0) //p
```

Comparsion

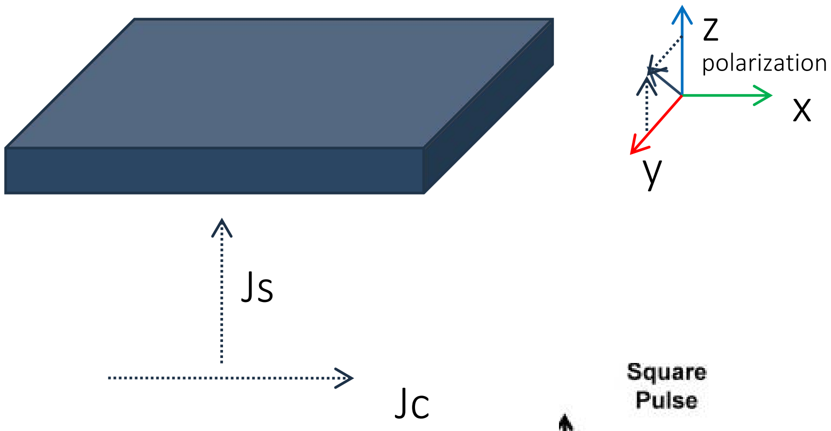
- Adding term increase the complexity of of the script which cause more computation time required
- SL-STT replacement is better with time efficiency.
- Result is same. :-)

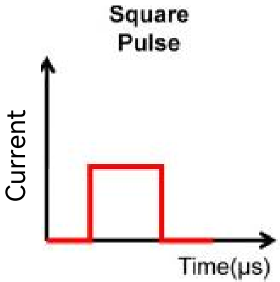
SOT Flip by KJ Lee Data




```
//Specify output format
OutputFormat = OVF2_TEXT

setgridsize(30,30,2) //area of free
layer=900nm2
setcellsize(1e-9,1e-9,0.5e-9)
setpbc(0,0,0)

//set parameter
Msat = 1e6 //1000emu/cm^3=10e6A/m
Aex = 15e-12
Ku1 = 0.8e6 //efective perpendicular anisotropy
constant
//K = 2e6 erg/cm  1st order uniaxial anisotropy
constant (J/m3)
AnisU = vector(0,0,1) // z-direction
alpha = 0.005 //Gilbert damping alpha = 0.005
m=uniform(0,0,-1)
Bdc :=0.03   //External magnetic feld 300Oe on
x direction

//Static field
B_ext = vector(Bdc, 0, 0)
relax()
```

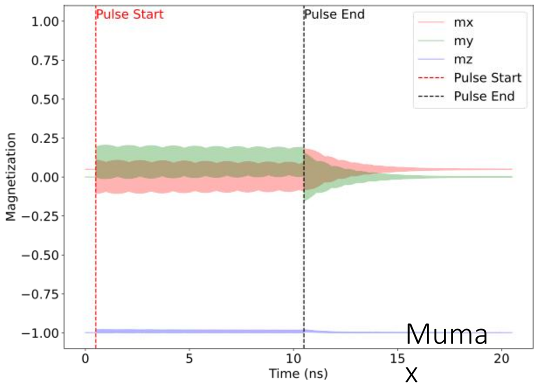
SOT Flip by KJ Lee Data

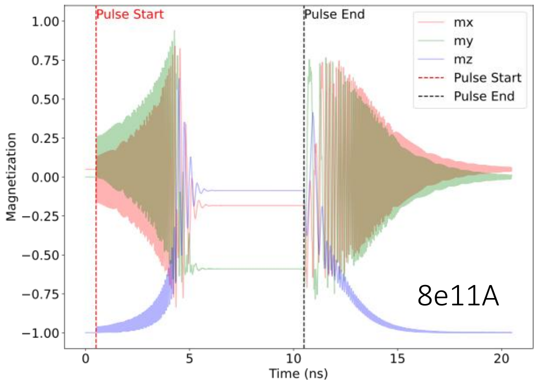


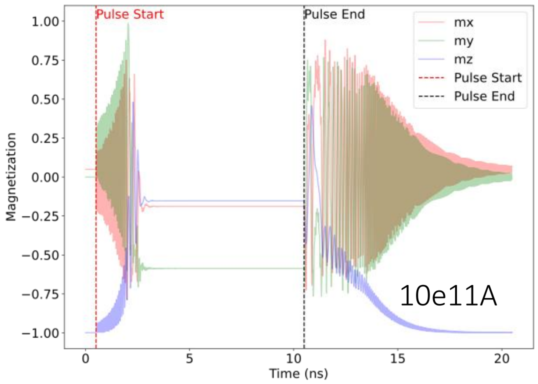


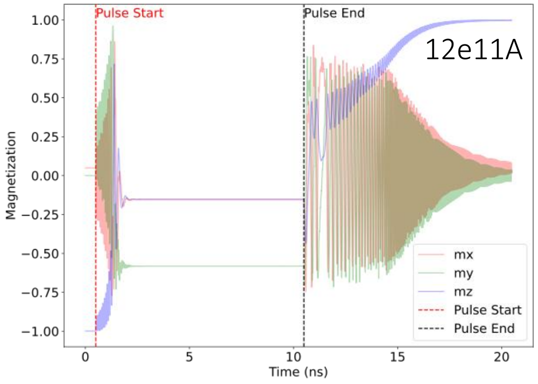

```
//Add SOT
// Define constants
ThetaD := 0.3 //AlphaH equal to the theta jD
SOTxi:=0
eta:={eta}
Pol = ThetaD
Lambda=1
Epsilonprime = ThetaD/2 * SOTxi
Fixedlayer = vector(0,cos(eta),sin(eta)) // p
J=vector(0,0,0)
autosave(m,1e-10)
tableAutosave(1e-11)
run(0.5e-9)
J=vector(0,0,abs({Jc}))
autosave(m,1e-10)
tableAutosave(1e-11)
run({tpulse}*1e-9)
J=vector(0,0,0)
autosave(m,1e-10)
tableAutosave(1e-11)
run(10e-9)
```

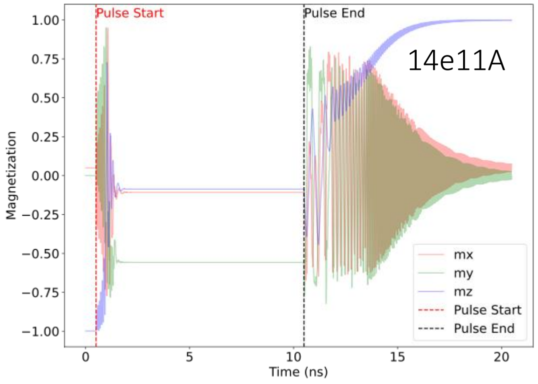
SOT Flip by KJ Lee Data --Current Sweeping (thetaF/thetaD=-2)

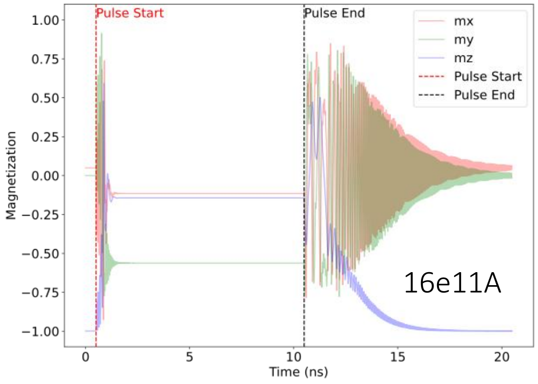






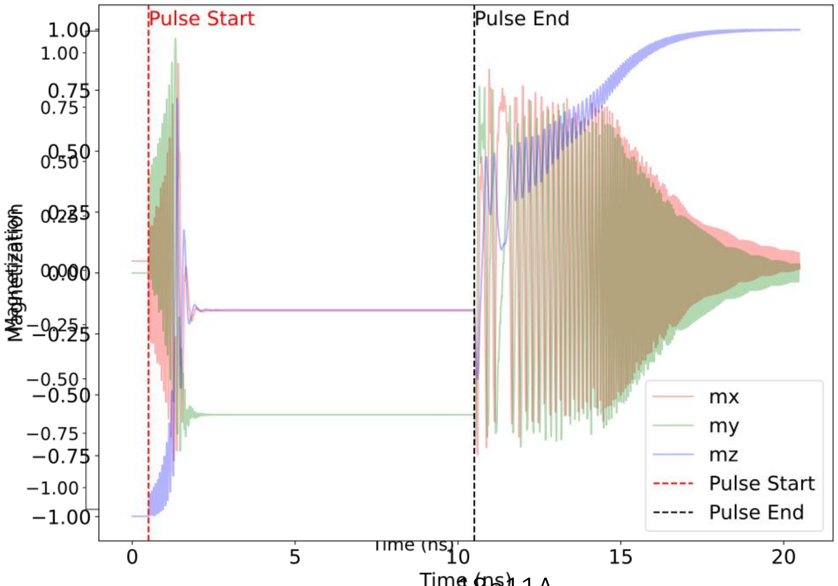






SOT Flip by KJ Lee Data --Current Sweeping

(Continued)



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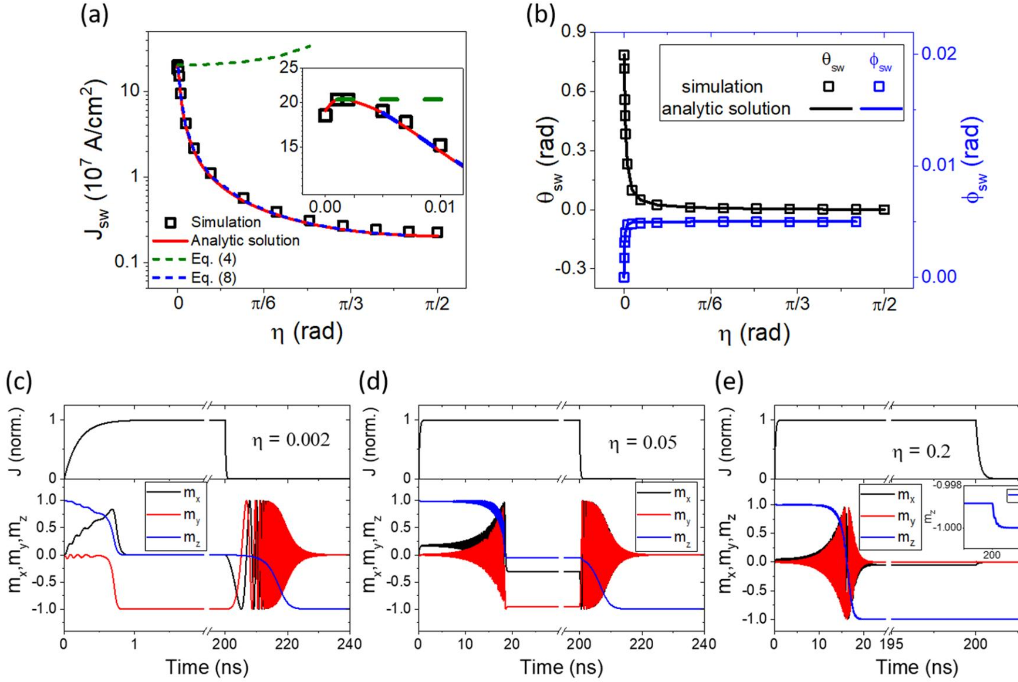
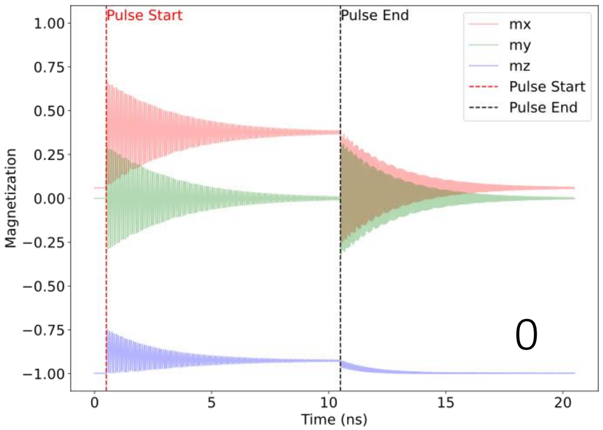
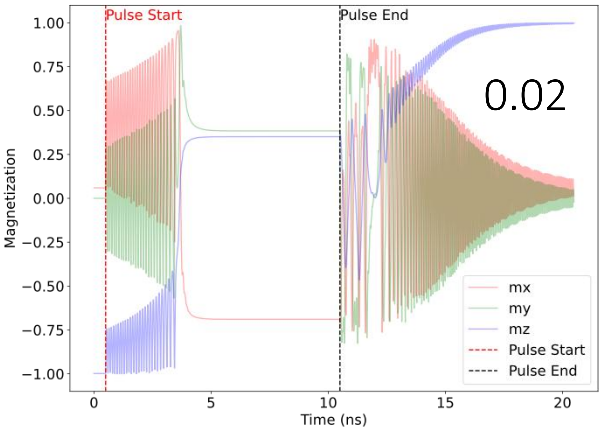
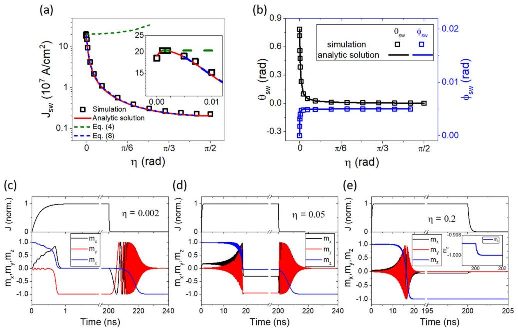
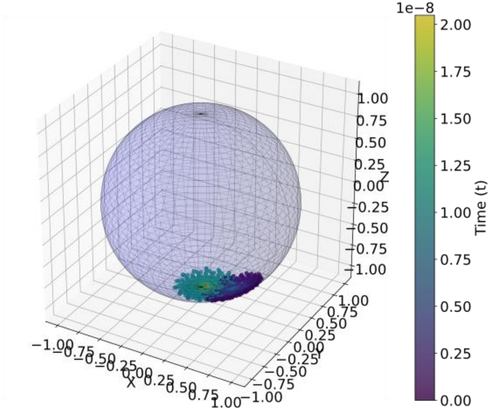


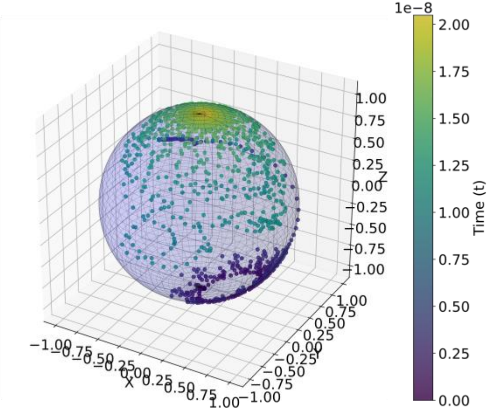
SOT Flip by KJ Lee Data --eta Tilted Polarization(ThetaF/thetaD=0)

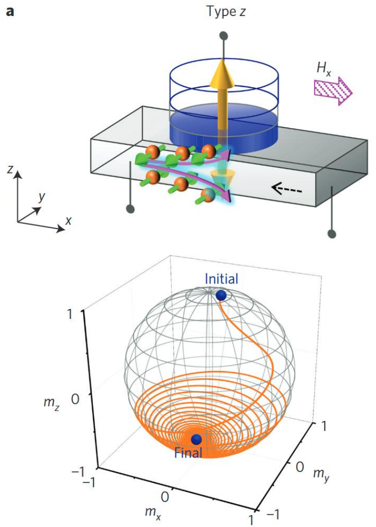




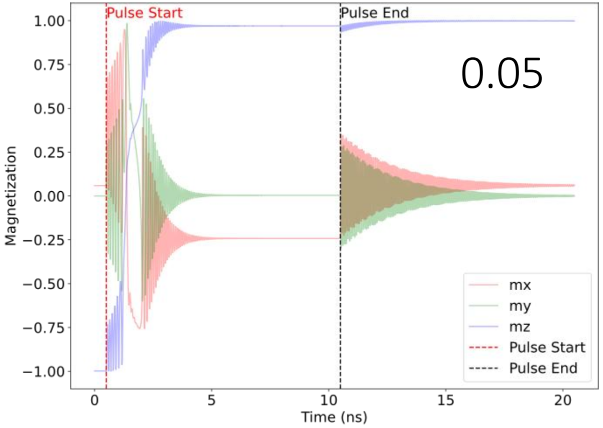


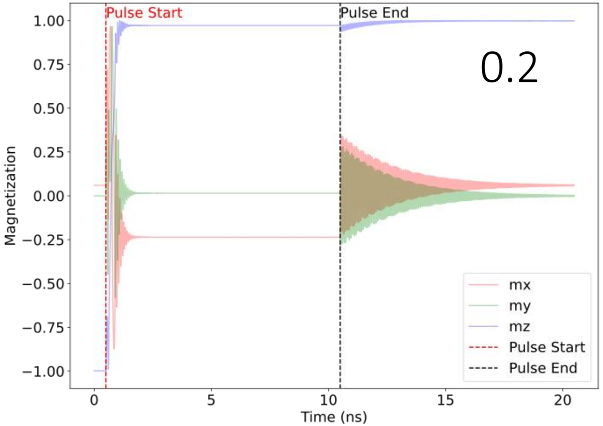


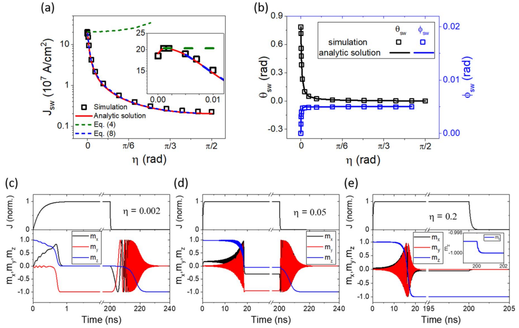


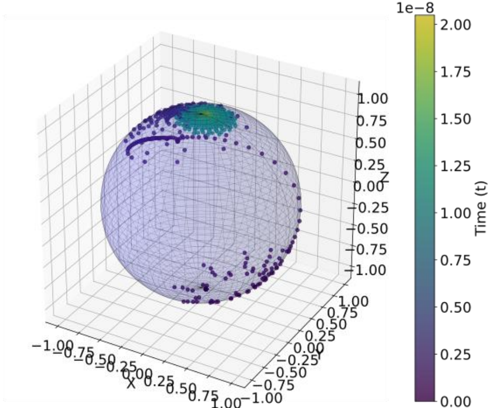


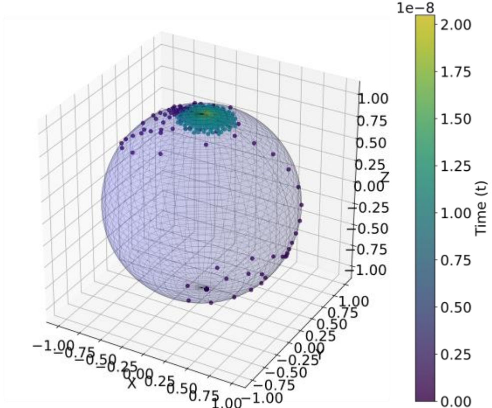
SOT Flip by KJ Lee Data --Tilted Polarization(ThetaF/thetaD=0)

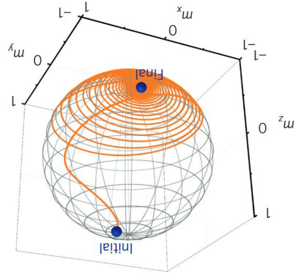


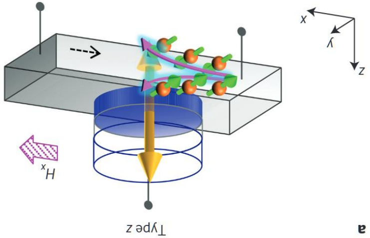












SOT Flip Problem on simulation

1. thetaF/thetaD influence to the flip
   for 15e11Am^-2, for xi=0 cannot flip, but for xi=-2 can flip
   when polarization tilted, the smooth flip happen when xi=0

Reference

SOT relevant

1. Lee, D.-K. & Lee, K.-J. Spin-orbit Torque Switching of Perpendicular Magnetization in Ferromagnetic Trilayers. Sci Rep 10, 1772 (2020).
2. Polley, D. et al. Picosecond spin-orbit torque–induced coherent magnetization switching in a ferromagnet. Sci. Adv. 9, eadh5562 (2023).
3. Shao, Q. et al. Roadmap of Spin–Orbit Torques. IEEE Transactions on Magnetics 57, 1–39 (2021).
4. Fukami, S., Anekawa, T., Zhang, C. & Ohno, H. A spin–orbit torque switching scheme with collinear magnetic easy axis and current configuration. Nature Nanotech 11, 621–625 (2016).

Mumax Relavant

1. Vansteenkiste, A. The design and verification of MuMax3. AIP Advances (2014) doi:10.1063/1.4899186.
2. https://mumax.github.io/
3. https://mumax.ugent.be/mumax3-workshop/
4. https://mumax.github.io/api.html
5. 微磁学模拟|1-微磁学发展和常用软件 - 闲话物理的文章 - 知乎https://zhuanlan.zhihu.com/p/148237322

For access to the code and data related to this project, please visit **https://github.com/ymguo-phy/MagSpinMagnonSim-Pub**

IS THERE A BETTER WAY?

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \left(\mathbf{H}_{\text{eff}} + \boxed{a_J(\mathbf{m} \times \mathbf{p}) + b_J \mathbf{p}} \right) + \alpha \mathbf{m} \times \dot{\mathbf{m}}$$

$$\boxed{\mathbf{m} \times} \dot{\mathbf{m}} = -\gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) - a_J \gamma \mathbf{m} \times (\mathbf{m} \times (\mathbf{m} \times \mathbf{p})) - \gamma b_J \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) + \alpha \mathbf{m} \times (\mathbf{m} \times \dot{\mathbf{m}})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

$$(\mathbf{m} \cdot \dot{\mathbf{m}}) = 0$$

$$(\mathbf{m} \cdot \mathbf{m}) = 1$$

$$\mathbf{m} \times (\mathbf{m} \times (\mathbf{m} \times \mathbf{p})) = -\mathbf{m} \times \mathbf{p}$$

$$\mathbf{m} \times \dot{\mathbf{m}} = -\gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})) + a_J \gamma \mathbf{m} \times \mathbf{p} - \gamma b_J \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) - \alpha \dot{\mathbf{m}}$$

IS THERE A BETTER WAY?

$$\boxed{\mathbf{m} \times \dot{\mathbf{m}}} = -\gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})) + a_J \gamma \mathbf{m} \times \mathbf{p} - \gamma b_J \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) - \alpha \dot{\mathbf{m}}$$

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \left(\mathbf{H}_{\text{eff}} + \boxed{a_J (\mathbf{m} \times \mathbf{p}) + b_J \mathbf{p}} \right) + \alpha \boxed{\mathbf{m} \times \dot{\mathbf{m}}}$$

$$\begin{aligned} (1+\alpha^2)\dot{\mathbf{m}} = & -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha\gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) \\ & -\gamma a_J \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) - \alpha\gamma b_J \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) \\ & -\gamma b_J \mathbf{m} \times \mathbf{p} + \alpha\gamma a_J \mathbf{m} \times \mathbf{p} \end{aligned}$$

Landau-Lifshitz torque

Spin-Orbit torque

$$\boxed{\tau_{\text{SOT}} = -\frac{\gamma}{(1+\alpha^2)} a_J \left[(1+\xi\alpha) \mathbf{m} \times (\mathbf{m} \times \mathbf{p}) + (\xi - \alpha)(\mathbf{m} \times \mathbf{p}) \right]}$$

IS THERE A BETTER WAY?

$$\boxed{\tau_{\mathrm{SOT}} = -\frac{\gamma}{(1+\alpha^2)} a_J \left[(1+\xi\alpha)\mathbf{m}\times(\mathbf{m}\times\mathbf{p}) + (\xi-\alpha)(\mathbf{m}\times\mathbf{p})\right]}$$

Slonczewski STT in mumax3

$$\left\{\begin{aligned} \tau_{SL} &= \gamma\beta\frac{\epsilon-\alpha\epsilon'}{1+\alpha^2}(\mathbf{m}\times(\mathbf{m}_P\times\mathbf{m})) - \gamma\beta\frac{\epsilon'-\alpha\epsilon}{1+\alpha^2}\mathbf{m}\times\mathbf{m}_P \\ \beta &= \frac{j_z\hbar}{M_{\mathrm{sat}}ed} \\ \epsilon &= \frac{P\Lambda^2}{(\Lambda^2+1)+(\Lambda^2-1)(\mathbf{m}\cdot\mathbf{m}_P)} \end{aligned}\right.$$

$$\Lambda = 1 \longrightarrow \epsilon = \frac{P}{2}$$

$$\epsilon' = \xi\epsilon$$

$$\tau_{SL} = -\frac{\beta\gamma P}{2(1+\alpha^2)}\left[(1+\xi\alpha)(\mathbf{m}\times(\mathbf{m}\times\mathbf{m}_P)) + (\xi-\alpha)\mathbf{m}\times\mathbf{m}_P\right]$$

IS THERE A BETTER WAY?

$$\boxed{\tau_{\mathrm{SOT}} = -\frac{\gamma}{(1+\alpha^2)} a_J \left[(1+\xi\alpha)\mathbf{m}\times(\mathbf{m}\times\mathbf{p}) + (\xi-\alpha)(\mathbf{m}\times\mathbf{p})\right]}$$

We can use the existing Slonczewski spin transfer torque implementation!

$$P = \frac{2a_J}{\beta} = \alpha_H$$

$$\boxed{\tau_{SL} = -\frac{\beta\gamma P}{2(1+\alpha^2)} \left[(1+\xi\alpha)(\mathbf{m}\times(\mathbf{m}\times\mathbf{m}_P)) + (\xi-\alpha)\mathbf{m}\times\mathbf{m}_P\right]}$$