1 The Stochastic volatility with contemporaneous jumps model

The SVCJ model[1,2] concern with cases where volatility is stochastic and has a property of jumps. Assume that the spot index follows

$$\begin{aligned} dS(t) &= S(t)[\mu_s dt + \sqrt{v} dW_1(t) + J^s], \\ dv(t) &= \kappa(\theta - v)dt + \sigma\sqrt{v} dW_2(t) + dJ^v, \end{aligned} \tag{1}$$

where $S(t)$ is spot price of equity, $\mu_s = r - q - \lambda\xi_s$, $\lambda\xi_s$ is defined by $\lambda\xi_s = e^{r+\delta^2/2}(1 - \nu\rho_J)^{-1} - 1$. ρ_J is the correlation between jumps in returns and variance. the two-dimensional jump process (J^s, J^v) is an $\mathbb{R} \times \mathbb{R}^+$-valued compound Poisson process with intensity $\lambda > 0$. The distribution of the jump size in variance is assumed to be exponential with mean ν. conditional on a jump of size ψ in the variance process, $J^s + 1$ has a log-normal distribution $p(\phi, \psi)$ with the mean in $\log\phi$ being $\gamma + \rho_J\psi$. this gives a bivariate probability density function defined by $p(\phi, \psi) = \frac{1}{\sqrt{2\pi}\phi\delta\nu} e^{-\frac{\psi}{v} - \frac{(\log\phi - \gamma - \rho_J\psi)^2}{2\delta^2}}$. We assume that the price of a European option under the SVCJ model can be obtained as the solution to the PDE

$$\begin{aligned} \frac{\partial u}{\partial \tau} &= \frac{1}{2} v s^2 \frac{\partial^2 u}{\partial s^2} + \rho\sigma v s \frac{\partial^2 u}{\partial s \partial v} + \frac{1}{2}\sigma^2 v \frac{\partial^2 u}{\partial v^2} + (r - q - \lambda\xi_s) s \frac{\partial u}{\partial s} \\ &\quad + \kappa(\theta - v)\frac{\partial u}{\partial v} - (r + \lambda)u \\ &\quad + \lambda \int_0^\infty \int_0^\infty u(s \cdot \phi, v + \psi, \tau) p(\phi, \psi) d\psi d\phi. \end{aligned} \tag{2}$$

The initial condition is followed by $u(s, v, 0) = g(s)$ where g is the payoff function which gives the value of the option at the maturity.

For computational domains, we truncate the unbounded domain to $(s, v, \tau) \in (0, s_{\max}) \times (0, v_{\max}) \times (0, T]$. We impose the boundary conditions $u(0, v, \tau) = e^{-r\tau} g(0)$ and $\frac{\partial u}{\partial v}(s, v_{\max}, \tau) = 0$. Moreover, for integrations, we extend u for $s > s_{\max}$ as $u(s, v, \tau) = 0$. On the boundary $v = 0$, the PDE equation (2) can be posed as a boundary condition.

References

[1] Eraker, Bjłrn. "Do stock prices and volatility jump? Reconciling evidence from spot and option prices." The Journal of Finance 59.3 (2004): 1367-1403.

[2] Zhang, Ying-Ying, et al. "Quadratic finite element and preconditioning for options pricing in the SVCJ model." Journal of Computational Finance, Forthcoming (2011).