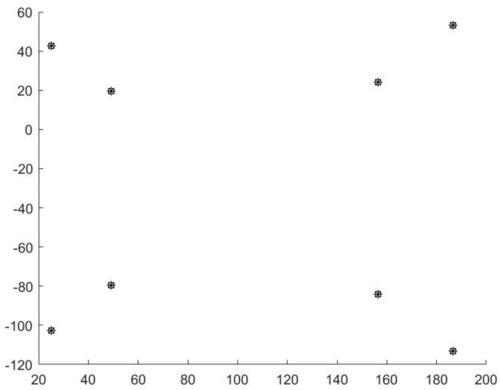
Assignment 4 Description

Yan Miao
260711311

Question 1

Test results:

Using Q1_tester.m




```
Here are the K and K_est (side by side)
  300.0000         0   20.0000  300.0000    0.0000   20.0000
         0  300.0000  -30.0000   -0.0000  300.0000  -30.0000
         0         0    1.0000   -0.0000    0.0000    1.0000

Here are the C and C_est (side by side)
    0.5000    0.5000
    0.5000    0.5000
   -2.0000   -2.0000

Here are the R_y and R_est (side by side)
    0.9659         0    0.2588    0.9659    0.0000    0.2588
         0    1.0000         0   -0.0000    1.0000    0.0000
   -0.2588         0    0.9659   -0.2588   -0.0000    0.9659
```

Using Q1.m

c1.jpg

c2.jpg

Question 2

Case 1: Shifts using K versus R

Discussion:
The green stars show the original “corner” points in the image. The white squares show the transformed points in different positions cause by applying changes to R and K matrices. As we can see from above, we can roughly create similar image shifts by changing R (rotating the camera) and changing K (image plane translation).

Case 2: Shifts using K versus C

Discussion:
The green stars show the original “corner” points in the image. The white squares show the transformed points in different positions cause by applying changes to C and K matrices. As we can see from above, we can roughly create similar image shifts by changing C (moving camera to another position in the world coordinate) and changing K (image plane translation).

Case 3: Expansions using K versus C

Discussion:
The green stars show the original “corner” points in the image. The white squares show the transformed points in different positions cause by applying changes to R and K matrices. As we can see from above, we can roughly create similar image expansion by changing C (moving the camera away from the shelf) and changing K (decrease the focal length).

Question 3

Case 1: Shifts using K versus R

We change matrix K by multiplying a transformation matrix T_K:

$$T\_K = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting matrix K1 is:

$$K1 = \begin{bmatrix} 300 & 0 & 30 \\ 0 & 300 & -30 \\ 0 & 0 & 1 \end{bmatrix}$$

The original K matrix is:

$$K = \begin{bmatrix} 300 & 0 & 20 \\ 0 & 300 & -30 \\ 0 & 0 & 1 \end{bmatrix}$$

Compared to the matrix K1 above, we do the image shifting by changing the principal point from (20,-30) to (30,-30). So all the points in the image shift to the right by 10 units, meaning the image plane translates to the left by 10 units.
In the figure below, we can see the red points shift to the right relative to the original points (black squares).

We change the matrix R by multiplying a transformation matrix T_R:

$$T\_R = \begin{bmatrix} 0.9997 & 0 & 0.0262 \\ 0 & 1.0000 & 0 \\ -0.0262 & 0 & 0.9997 \end{bmatrix}$$

The resulting matrix R1 is:

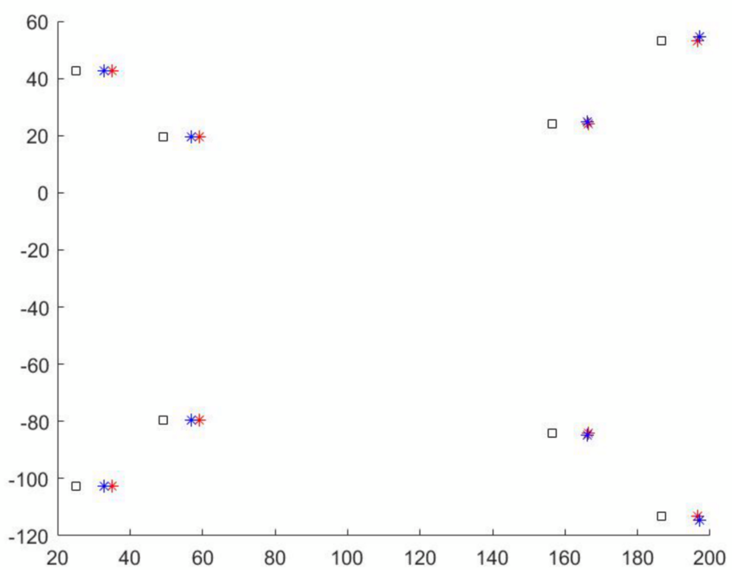
$$R1 = \begin{bmatrix} 0.9588 & 0 & 0.2840 \\ 0 & 1.0000 & 0 \\ -0.2840 & 0 & 0.9588 \end{bmatrix}$$

The original matrix R is:

$$R = \begin{bmatrix} 0.9659 & 0 & 0.2588 \\ 0 & 1.0000 & 0 \\ -0.2588 & 0 & 0.9659 \end{bmatrix}$$

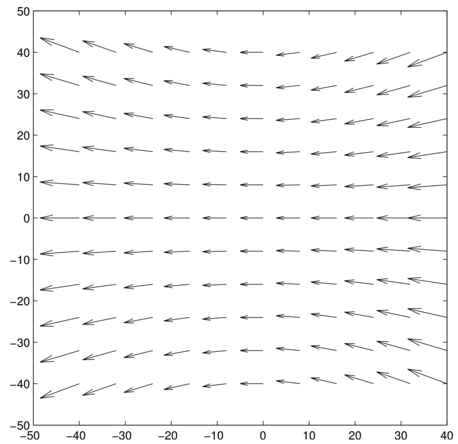
What we are doing is rotating the camera 1.5 degrees about y axis.

How different?
We use blue points to visualize the transformation applied to matrix R. While some blue points are roughly matched with the red points, others are not.



Why different?

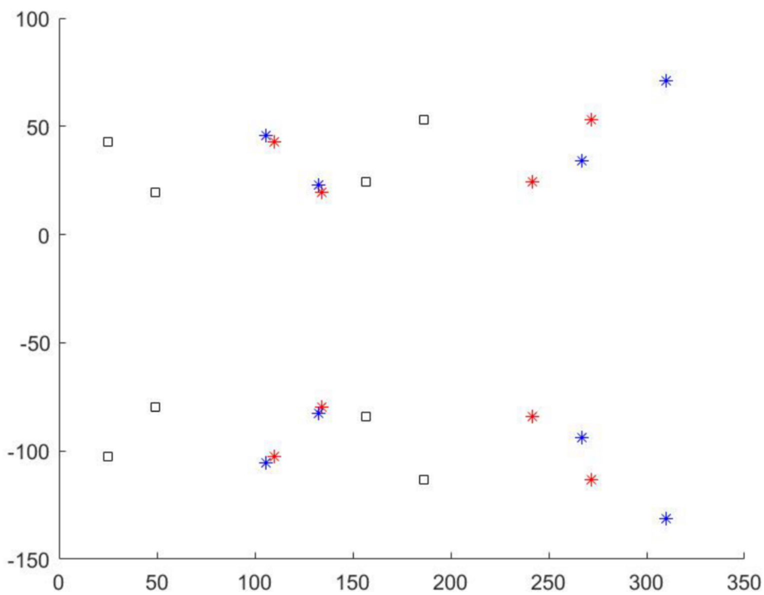
However, it is also clear that the blue and red points did not undergo the exact same shift. This is because we are trying to use camera rotation to create the same effect as image shift. Notice that moving cameras may pan for the case that it is rotating about its y axis. The motion field shows parabolic trajectories caused by rotating camera. So changing R to create similar shifts as changing K only works when the shift is relatively small. (In my example, I only rotate the camera by a really small angle as 1.5 degrees to create the same effect as changing K.) So according to the formulas of image velocity in x and y directions as below, as the image undergoes a greater shift (i.e. greater Ω), if we still want to change R to create a similar shift by increasing the rotation angle, we may only get a distorted and trapezoidal image since the velocities increase as Ω gets greater.
Another reason why the difference shows up is when we move the camera (rotate the camera in this case), we are taking pictures of objects from a different angle. We will do a detailed analysis of this reason in the following cases.



$\Omega = (0, 1, 0)$

$$v_x = f\Omega(1 + (\frac{x}{f})^2)$$
$$v_y = \Omega\frac{xy}{f}$$

The following is what will happen when we try to use a larger camera rotation (15 degrees) to create a greater image shift. Notice the trapezoidal shape (blue points) shows up.



Case 2: Shifts using K versus C

In this part, we do the same change to K by moving the principle point from (20,-30) to (30,-30).
In the figure below, we can see the red points shift to the right relative to the original points (black squares).

We change the matrix C by multiplying a transformation matrix T_C:

$$T\_C = \begin{bmatrix} 1.0000 & -0.0100 & 0.0280 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

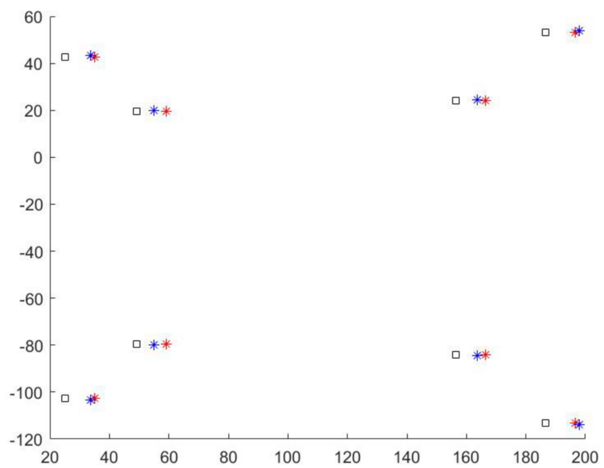
The resulting matrix C1 is:

$$C1 = \begin{bmatrix} 0.4390 \\ 0.5000 \\ -2.0000 \end{bmatrix}$$

The original matrix C is:

$$C = \begin{bmatrix} 0.5000 \\ 0.5000 \\ -2.0000 \end{bmatrix}$$

What we are doing is moving the camera position in the world coordinate from (0.5, 0.5, -2) to (0.4390, 0.5, -2), meaning moving in the negative direction of x axis of the world coordinate by 0.061 units. Notice that to control variables, the camera undergoes translations only in the XY plane, the Z coordinate is not changed (i.e. we do not move the camera either towards or further from the objects)
How different?
We use blue points to visualize the transformation applied to C. We can see that some of the blue points are very near to the red points while the others are relatively distant.



Why different?
Actually, we are looking at different aspects of the same objects because we are placing the camera at a different position. As we mentioned previously, image shifts created by changing K are essentially caused by the translation of image plane. Notice that in this case, the relative position between the camera and the objects stays the same. So the objects presented in the image always have the same relative positions no matter how the image plane translates. But once we move the camera, the spatial relationship between the camera and the objects is changed, which leads to the change in relative positions between objects when they are projected onto the image plane.
From what we have discussed above, using K and C to create similar image shifts only works for small motion of shifts. When the shift gets greater, the discrepancies of using two methods will become more obvious.
So although the positions of the red and blue points are similar, they are indeed different since we cannot recover a image shift caused by image plane translation simply by moving the camera to a different position.

Case 3: Expansions using K versus C

We change matrix K by multiplying a transformation matrix T_K:

$$T\_K = \begin{bmatrix} 0.9000 & 0 & 0 \\ 0 & 0.9000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

The resulting matrix K1 is:

$$K1 = \begin{bmatrix} 270 & 0 & 18 \\ 0 & 270 & -27 \\ 0 & 0 & 1 \end{bmatrix}$$

The original K matrix is:

$$K = \begin{bmatrix} 300 & 0 & 20 \\ 0 & 300 & -30 \\ 0 & 0 & 1 \end{bmatrix}$$

Compared to the matrix K1 above, we do the image expansion by changing the focal length to a multiple of 0.9 of the original value, meaning we are placing the image plane further away from the objects so that the sizes of the objects in the image are 0.9 times the original ones. A side effect of multiplying by T_K is it also changes the principle point. Notice that to control variables, we only let the camera move along the Z axis.
In the figure below, we can see the relative distances between red points decrease to a multiple of 0.9 of the original distances (points presented as black squares).

We change the matrix C by multiplying a transformation matrix T_C:

$$T\_C = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 1.1000 \end{bmatrix}$$

The resulting matrix C1 is:

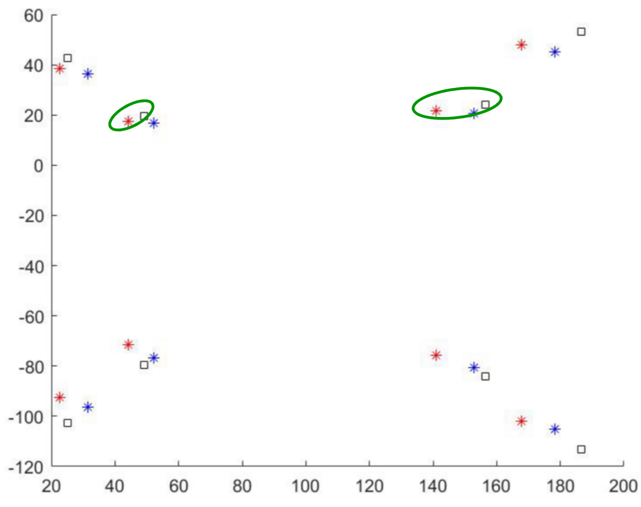
$$C1 = \begin{bmatrix} 0.5000 \\ 0.5000 \\ -2.2000 \end{bmatrix}$$

The original matrix C is:

$$C = \begin{bmatrix} 0.5000 \\ 0.5000 \\ -2.0000 \end{bmatrix}$$

What we are doing is placing the camera at a location that is 1.1 times further away from the "objects" than the original distance. Thus it creates a similar image expansion.

How different?
We use blue points to visualize the transformation applied to C. We can see the blue points are "shrinking" towards the central image whereas the red points are not only "shrinking" but also "shifting" a little. (compare the two parts I circled below)



Why different?
One reason is the change of principle points that leads to a image shift.
The main reason is similar to what we have discussed in the previous case — the different positions of the camera. Unlike the previous cases, in this case, we only move the camera along Z axis. When we perform image expansion by modifying focal length, the camera location remains the same. Thus the size of the objects in the image increase or decrease as a whole part i.e. no changes of relative positions. But as we move the camera to a different position in the world coordinate, the relative positions between camera and objects are changed, leading to the changes of relative positions of objects showed in the image. That is to say, the camera is taking pictures of the objects from a different perspective where the original spatial relationship between objects is changed.

From what we have discussed above, using K and C to create similar image expansions only works for small degree of expansions. When the expansion gets greater, the discrepancies of using two methods will become more obvious.

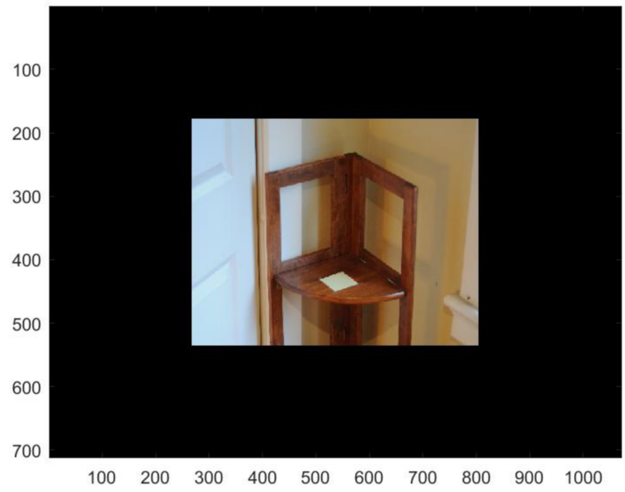
So although the positions of the red and blue points are similar, they are indeed different since we cannot recover a image expansion caused by changing the focal length of the camera simply by moving the camera to a different position.

In conclusion, the major reason behind all these 3 cases of difference is that we CANNOT get the same changes to image by modifying camera intrinsics (K) and by moving camera (change camera extrinsics: R, C) since if we take pictures of the same object at different locations or different angles, we will see new aspects of it, and this is not something that can be done by simply changing some parameters inside the camera.

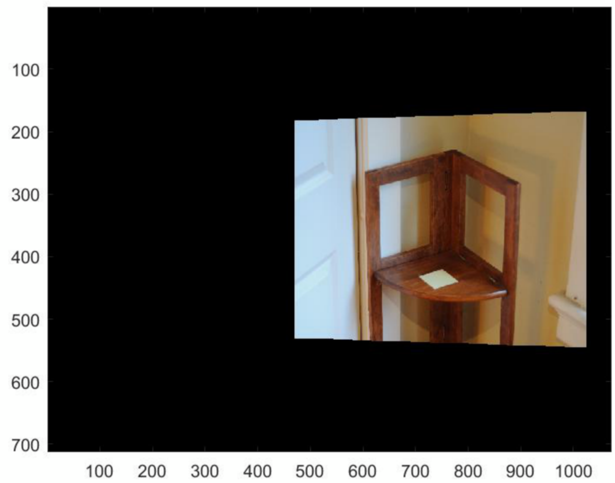
Question 4

The followings are the resulting images after rotating the camera by 0, 10 and 20 degrees.

theta = 0



theta = 10



theta = 20