

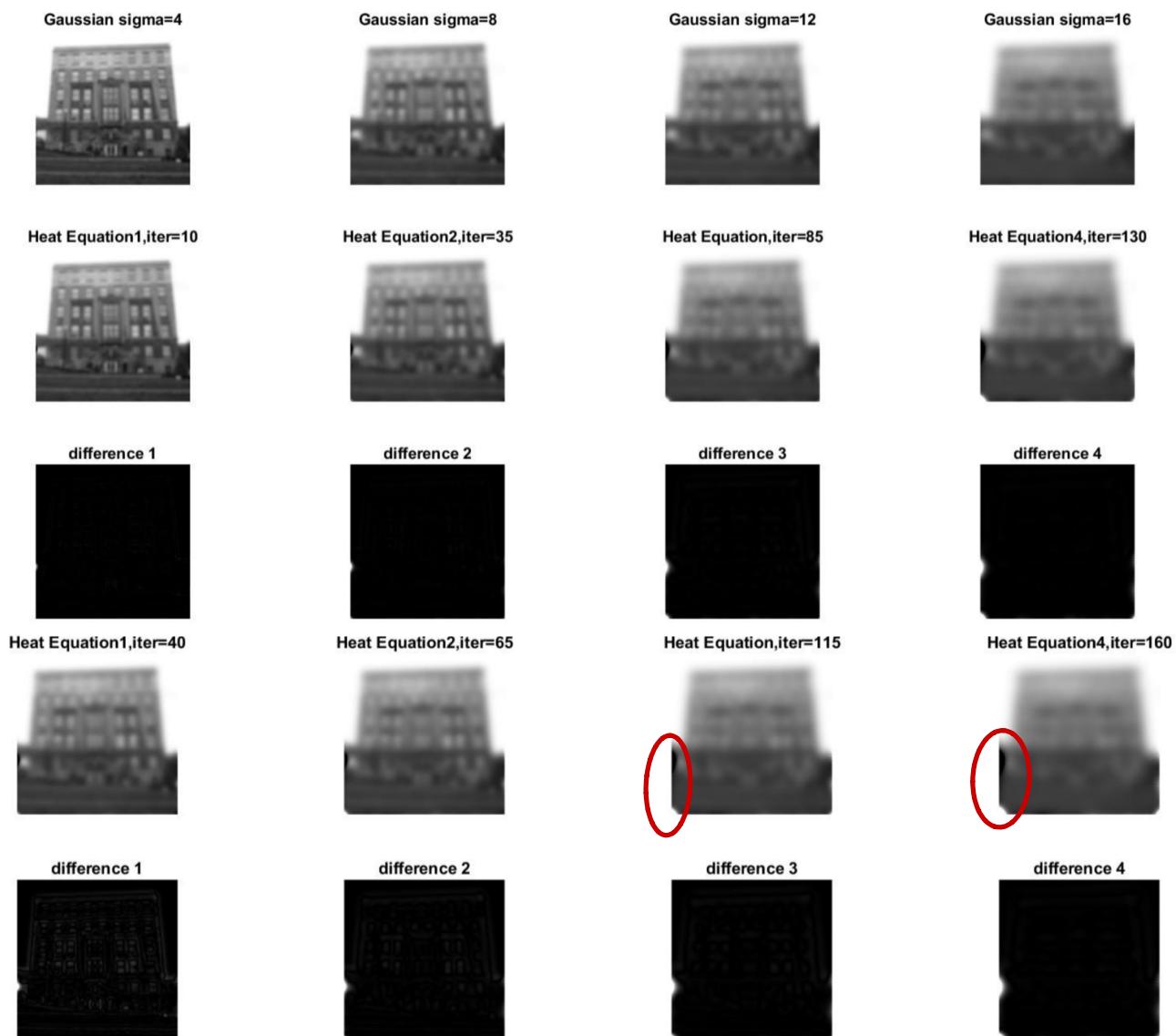
Assignment 2 Description

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Question 1

Compare two results numerically, and discuss any differences you see, and explain why you think these discrepancies occur.

If we compare images smoothed by Gaussian and smoothed using heat equation, we can find some differences on the edge and corner parts of the whole image. To see more clearly, we take the difference of the images. In the output image, we can see white pixels show up in the area with difference while the rest of the image is black.



According to the relation "total time" = $(1/2\sigma^2) = (\# \text{iteration}) \times (\Delta t)$, for a fixed sigma, if we increase the number of iterations, Δt will decrease. As in the above difference images (the dark ones), smoothed by heat equation will be more different from smoothed by Gaussian. The reason of the discrepancy is because:

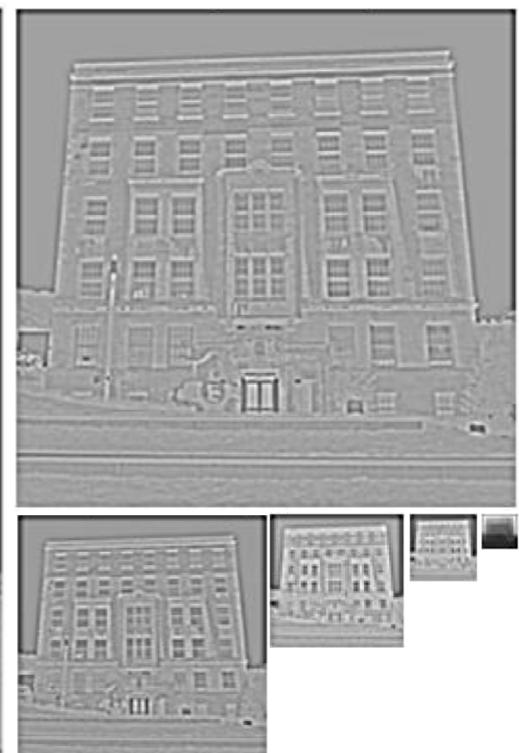
1. Using heat equation to blur the image is a discrete process, so the amount of blurring depends on the number of iterations, the step size Δt and other factors. It cannot guarantee to output the same smoothed image as smoothed by Gaussian.
2. If the intensity difference of neighbor pixels is very big, it may result in a big value of Laplacian, which may lead to an output with area as the circled parts. These parts should have "negative" pixel values which are presented as black pixels. Consequently, we have white pixels in the "difference" images.

Question 2

Part 1: Gaussian Pyramid



Part 2: Laplacian Pyramid



Part 3

How a difference of Gaussians can approximate a Laplacian of a Gaussian

Difference of Gaussians:

$$G(x, y, k\sigma) - G(x, y, \sigma)$$

Heat equation (Laplacian of Gaussian on the right hand side)

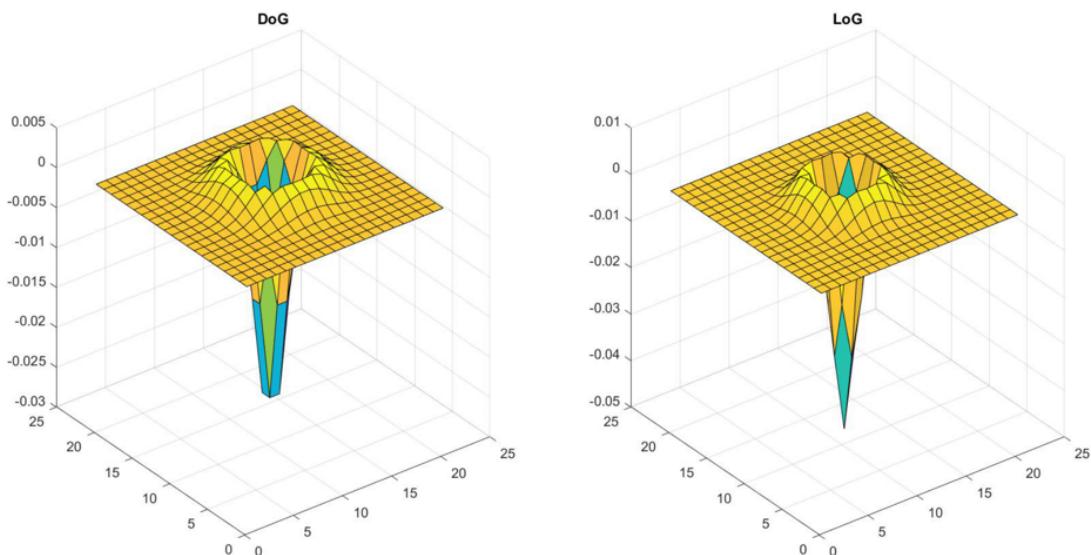
$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G$$

Therefore,

$$\sigma \nabla^2 G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

$$\underbrace{G(x, y, k\sigma) - G(x, y, \sigma)}_{\text{Difference of Gaussians}} \approx (k-1) \sigma^2 \underbrace{\nabla^2 G}_{\text{Laplacian of Gaussian}}$$

Visualizing “difference of Gaussians” and “Laplacian of Gaussian”



Part 4

Now that we get a Laplacian Pyramid with 6 levels from part 2, we can detect keypoints in the image. We create a function “find_extrema(up_level, low_level, mid_level, mid_sigma, thre, grad_thre)” to find extrema in a level (i.e. the mid_level).

up_level, low_level and mid_level are 3 input levels from Laplacian Pyramid. And our goal is to find extrema in the “mid_level”.

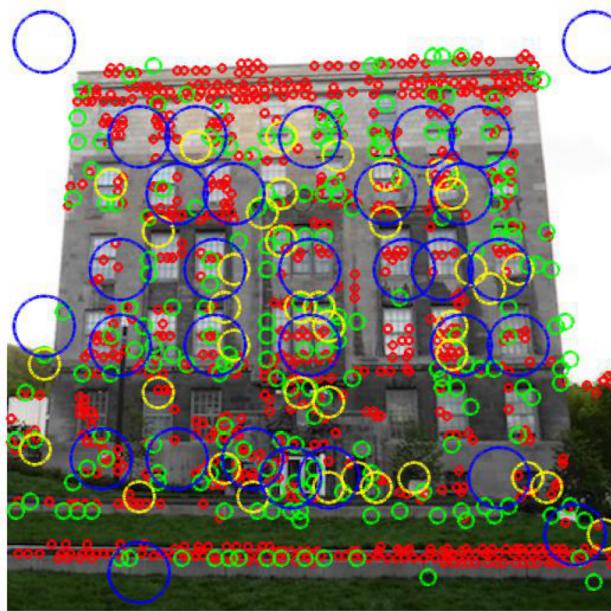
mid_sigma is the corresponding sigma of the middle level. This can let us to return extrema in the form (x,y,sigma).

thre is a low contrast threshold that we set to remove extrema with intensity less than this threshold.

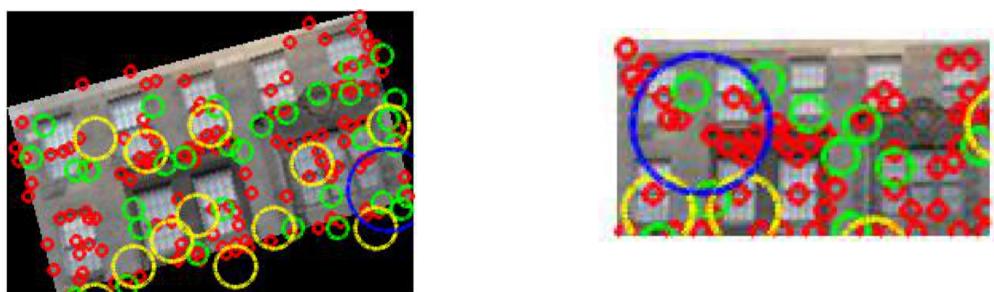
grad_thre is also a threshold. If the intensity differences between a central pixel and its 4 neighboring pixels are not ALL greater than the threshold, the central pixel is not an extrema.

The following is a image with keypoints on it.

(Circles with greater radius indicate that they are keypoints from a level with greater sigma)



To evaluate the quality of SIFT, we do keypoint detection on a scaled and rotated version of a crop of the original image.



As we can see from above, most keypoint locations are present at corresponding locations in the scaled and rotated version. Thus we have a robust keypoint detector.

Question 3

The following is the binary edge image of the original image.



The followings are two output plots ($T=200$). Red dots are inliers of the best model and black dots are outliers of the model.



The criterion for the inliers is:

We first set two thresholds, one for distance, another for angle. Then we traverse through all the edge points. If the Euclidean distance between an edge point and the closest point on the line model is smaller than the distance threshold, we mark it as a candidate. Then we consider the gradient orientation. If the difference between the gradient orientation of the candidate point and the sample point we used to create the line model is less than the angle threshold, then the candidate point becomes an inlier.