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# Les méthodes formelles dans la ville connectée

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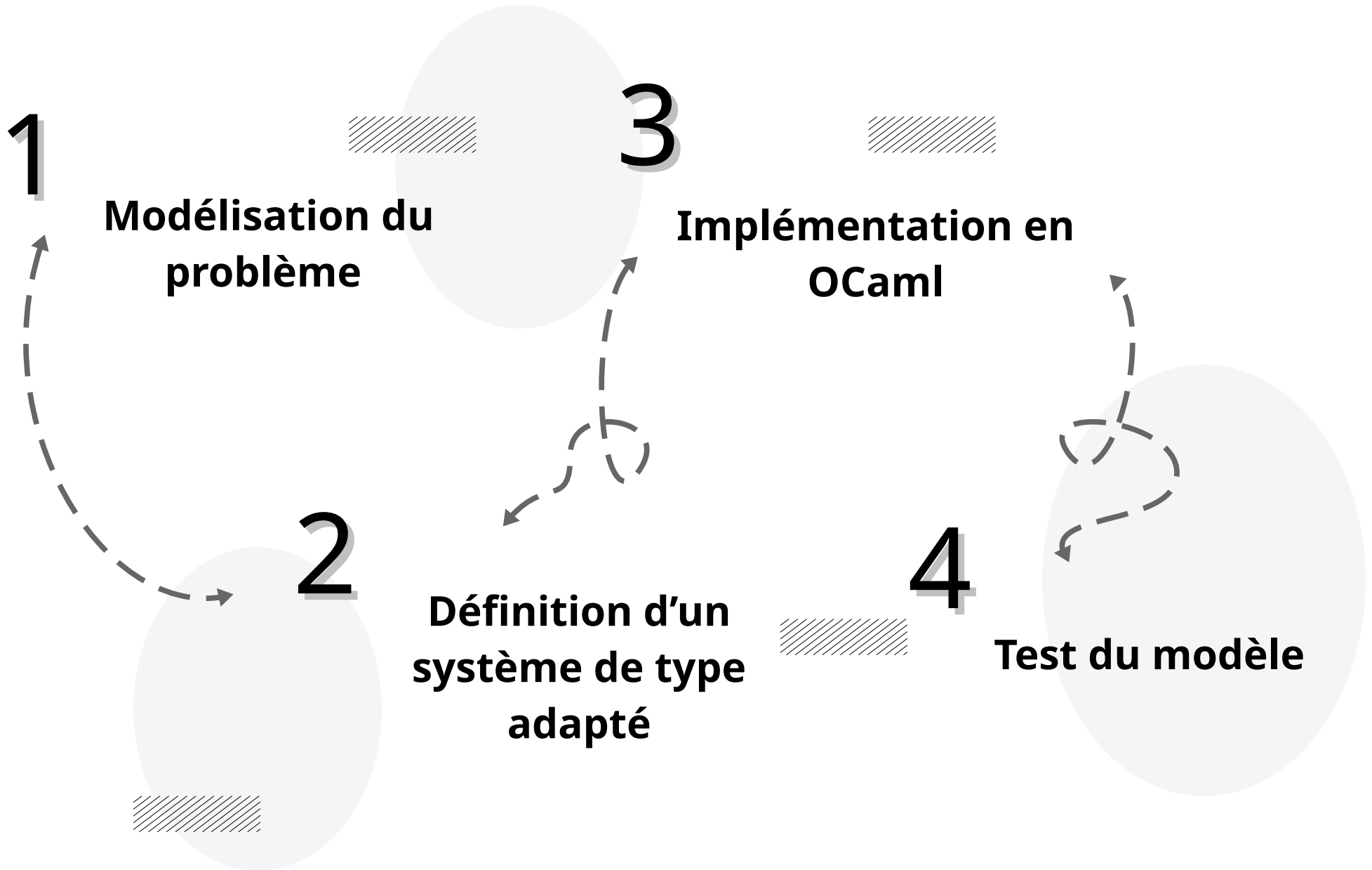
MPI

# Motivation

- Objets connectés
- Cyberattaques
- Language-based security
- Information flow

*En quoi l'approche de Language-based security nous permet de résoudre les problèmes de confidentialité et d'intégrité dans les programmes informatiques ?*





## Modélisation du problème

### Attaquant / pirate

Agent extérieur ayant accès aux résultats du programme

### Non-interférence

Entrées différentes => aucune information sur les variables protégées

### I) Modélisation

II) Résolution avec un système de type

III) Implémentation et exemples

IV) Vérification

### Confidentialité + Intégrité

Données privées /

données publiques



[www.pngegg.com](http://www.pngegg.com)

-  Problème

```

public := 42
while private ≥ 0 do
  skip

;
public := 51

```



## I) Modélisation

II) Résolution avec un système de type

III) Implémentation et exemples

IV) Vérification

### Grammaire du langage :

- Expression  $e ::= n \mid x \mid e1 \odot e2 \mid$
- Commandes  $c ::= x := e \mid \text{skip} \mid$   
if e then c1 else c2 |  
while e do c |  
c1; c2
- Phrases  $p ::= e \mid c$

**Exemple de phrase :**  $a := 5; b := 2; \text{while } a \text{ do } a := a - b$

- I) Modélisation
- II) Résolution avec un système de type
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## Types définis

- Types des données  $\tau ::= L \mid H$
- Types des phrases

$\rho ::= \tau$

$\mid \tau \text{ var}$

$\mid \tau \text{ cmd } n$

Affecte à des variables de type  $\geq \tau$  et se termine en  $n$  étapes

$\mid \tau_1 \text{ cmd } \tau_2$

Affecte à des variables de type  $\geq \tau_1$  et dont la durée d'exécution dépend de variables de type  $\leq \tau_2$

### Algorithm 1 Exemple $\tau \text{ cmd } n$

**if**  $x < 0$  **then**

$x := x + x;$

$x := x + x$

**else**

$x := x \times x;$

$x := x \times x$

### Algorithm 2 Exemple $\tau_1 \text{ cmd } \tau_2$

$a := 42;$

**while**  $a$  **do**

$a := a - 1$

## Règles de typage : [6]

I) Modélisation

II) Résolution avec un système de type

III) Implémentation et exemples

IV) Vérification

Règles d'inférence :

$$(R-VAL) \quad \frac{\gamma(x) = \tau \text{ var}}{\gamma \vdash x : \tau}$$

$$(INT) \quad \gamma \vdash n : L$$

$$(SUM) \quad \frac{\gamma \vdash e_1 : \tau, \gamma \vdash e_2 : \tau}{\gamma \vdash e_1 + e_2 : \tau}$$

$$(ASSIGN) \quad \frac{\gamma(x) = \tau \text{ var}, \gamma \vdash e : \tau}{\gamma \vdash x := e : \tau \text{ cmd } 1}$$

$$(SKIP) \quad \gamma \vdash \text{skip} : H \text{ cmd } 1$$

$$(IF) \quad \frac{\begin{array}{l} \gamma \vdash e : \tau \\ \gamma \vdash c_1 : \tau \text{ cmd } n \\ \gamma \vdash c_2 : \tau \text{ cmd } n \end{array}}{\gamma \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 : \tau \text{ cmd } n + 1}$$

$$\frac{\gamma \vdash A \quad \gamma \vdash B}{\gamma \vdash A \wedge B}$$

$$\frac{\begin{array}{l} \gamma \vdash e : L \\ \gamma \vdash c_1 : \tau_1 \text{ cmd } \tau_2 \\ \gamma \vdash c_2 : \tau_1 \text{ cmd } \tau_2 \end{array}}{\gamma \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 : \tau_1 \text{ cmd } \tau_2}$$

$$\frac{\begin{array}{l} \gamma \vdash e : H \\ \gamma \vdash c_1 : H \text{ cmd } H \\ \gamma \vdash c_2 : H \text{ cmd } H \end{array}}{\gamma \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 : H \text{ cmd } H}$$



## Règles de typage : [6]

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

$$\begin{array}{l}
 \text{(WHILE)} \quad \gamma \vdash e : L \\
 \tau_2 \subseteq \tau_1 \\
 \gamma \vdash c : \tau_1 \text{ cmd } \tau_2 \\
 \hline
 \gamma \vdash \mathbf{while} \ e \ \mathbf{do} \ c : \tau_1 \text{ cmd } \tau_2
 \end{array}$$

$$\begin{array}{l}
 \gamma \vdash e : H \\
 \gamma \vdash c : H \text{ cmd } H \\
 \hline
 \gamma \vdash \mathbf{while} \ e \ \mathbf{do} \ c : H \text{ cmd } H
 \end{array}$$

$$\begin{array}{l}
 \text{(COMPOSE)} \quad \gamma \vdash c_1 : \tau_1 \text{ cmd } L \\
 \gamma \vdash c_2 : \tau_1 \text{ cmd } \tau_2 \\
 \hline
 \gamma \vdash c_1; c_2 : \tau_1 \text{ cmd } \tau_2
 \end{array}$$

$$\begin{array}{l}
 \gamma \vdash c_1 : \tau \text{ cmd } H \\
 \gamma \vdash c_2 : H \text{ cmd } H \\
 \hline
 \gamma \vdash c_1; c_2 : \tau \text{ cmd } H
 \end{array}$$

---

```

public := 42
while private ≥ 0 do
  skip
  
```

```

;
public := 51
  
```

---

 Illégale

# Règles de typage : [6]

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

## Relations :

(BASE)	$L \subseteq H$
(CMD)	$\frac{\tau'_1 \subseteq \tau_1, \tau_2 \subseteq \tau'_2}{\tau_1 \text{ cmd } \tau_2 \subseteq \tau'_1 \text{ cmd } \tau'_2}$ $\frac{\tau' \subseteq \tau}{\tau \text{ cmd } n \subseteq \tau' \text{ cmd } n}$ $\tau \text{ cmd } n \subseteq \tau \text{ cmd } L$
(REFLEX)	$\rho \subseteq \rho$
(TRANS)	$\frac{\rho_1 \subseteq \rho_2, \rho_2 \subseteq \rho_3}{\rho_1 \subseteq \rho_3}$
(SUBSUMP)	$\frac{\gamma \vdash p : \rho_1, \rho_1 \subseteq \rho_2}{\gamma \vdash p : \rho_2}$

|  $\tau_1 \text{ cmd } \tau_2$

Affecte à des variables de type  $\geq \tau_1$  et dont la durée d'exécution dépend de variables de type  $\leq \tau_2$

|  $\tau \text{ cmd } n$

Affecte à des variables de type  $\geq \tau$  et termine en  $n$  étapes

# Implémentation en OCaml

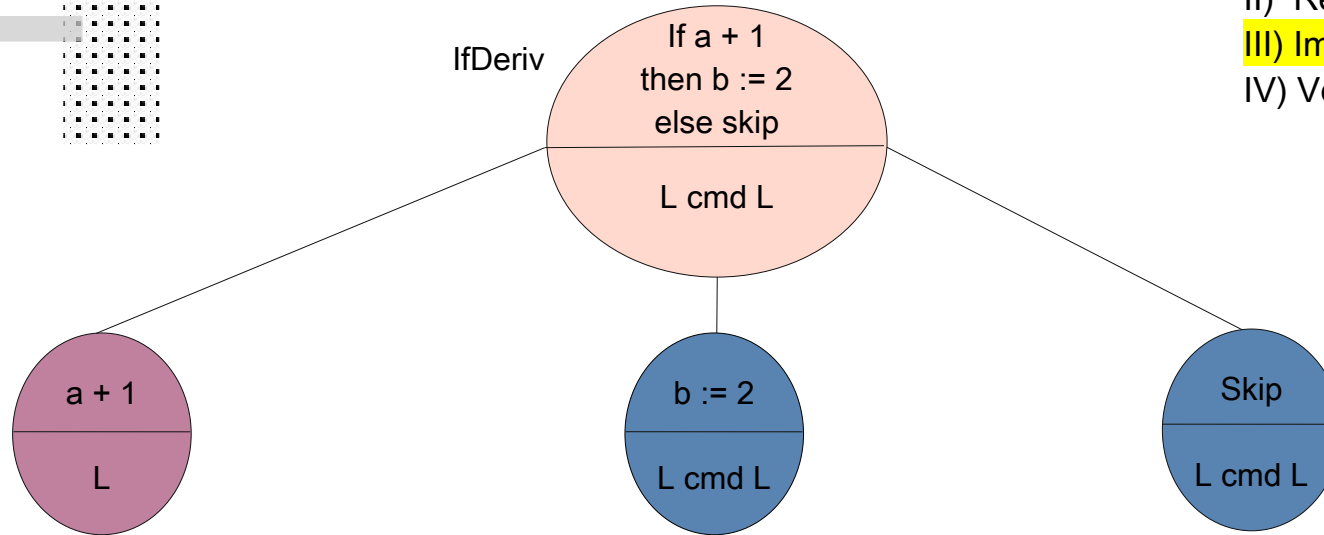
- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

On définit :

- Types des données
- Phrases avec expressions et commandes
- Types des phrases
- Arbre de dérivation
- Fonction type checker

```
type tree =  
  | Empty (* Red *)  
  | VarDeriv of string * phrase_type (* Orange *)  
  | ConstDeriv of int (* Red *)  
  | BinopDeriv of phrase * phrase_type * tree * tree (* Purple *)  
  | SkipDeriv of phrase_type (* Red *)  
  | AssignDeriv of string * phrase * phrase_type * tree * tree (* Light green *)  
  | IfDeriv of phrase * phrase_type * tree * tree * tree (* Pink *)  
  | WhileDeriv of phrase * phrase_type * tree * tree (* Yellow *)  
  | SeqDeriv of phrase * phrase_type * tree * tree (* Dark green *)  
  (* For the subtyping rules : *)  
  | SubDeriv of phrase * phrase_type * phrase_type * tree * tree (* Dark blue *)  
  | SubRules of phrase_type * phrase_type * tree * tree (* Light blue *)
```

## Exemple :



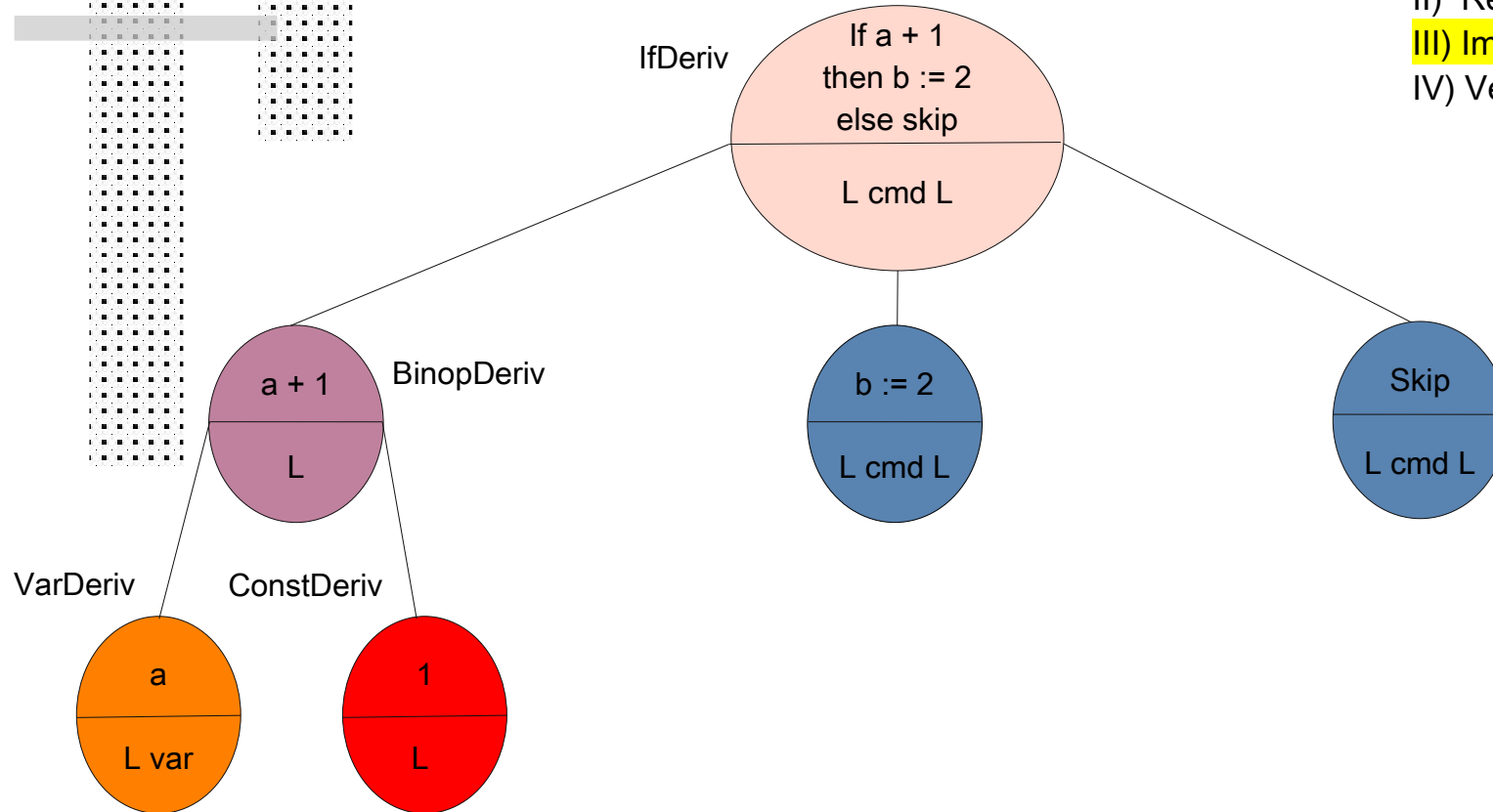
- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

$$\begin{array}{l}
 \gamma \vdash e : L \\
 \gamma \vdash c_1 : \tau_1 \text{ cmd } \tau_2 \\
 \gamma \vdash c_2 : \tau_1 \text{ cmd } \tau_2 \\
 \hline
 \gamma \vdash \text{if } e \text{ then } c_1 \text{ else } c_2 : \tau_1 \text{ cmd } \tau_2
 \end{array}$$

$\gamma = [(a, L \text{ var}); (b, L \text{ var})]$

## Exemple :

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples**
- IV) Vérification

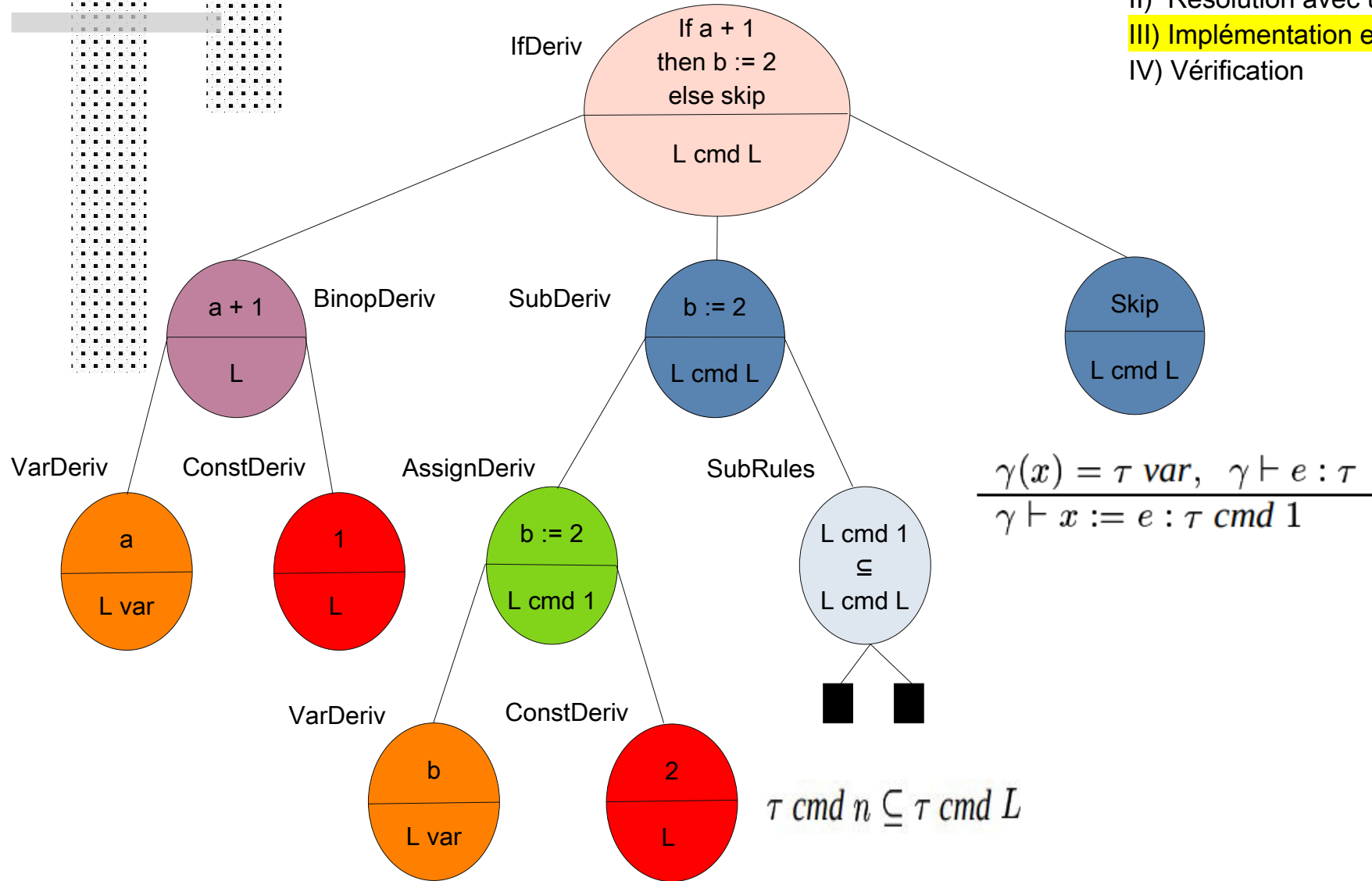


$$\frac{\gamma \vdash e_1 : \tau, \gamma \vdash e_2 : \tau}{\gamma \vdash e_1 + e_2 : \tau}$$

$\gamma = [(a, L \text{ var}); (b, L \text{ var})]$

## Exemple :

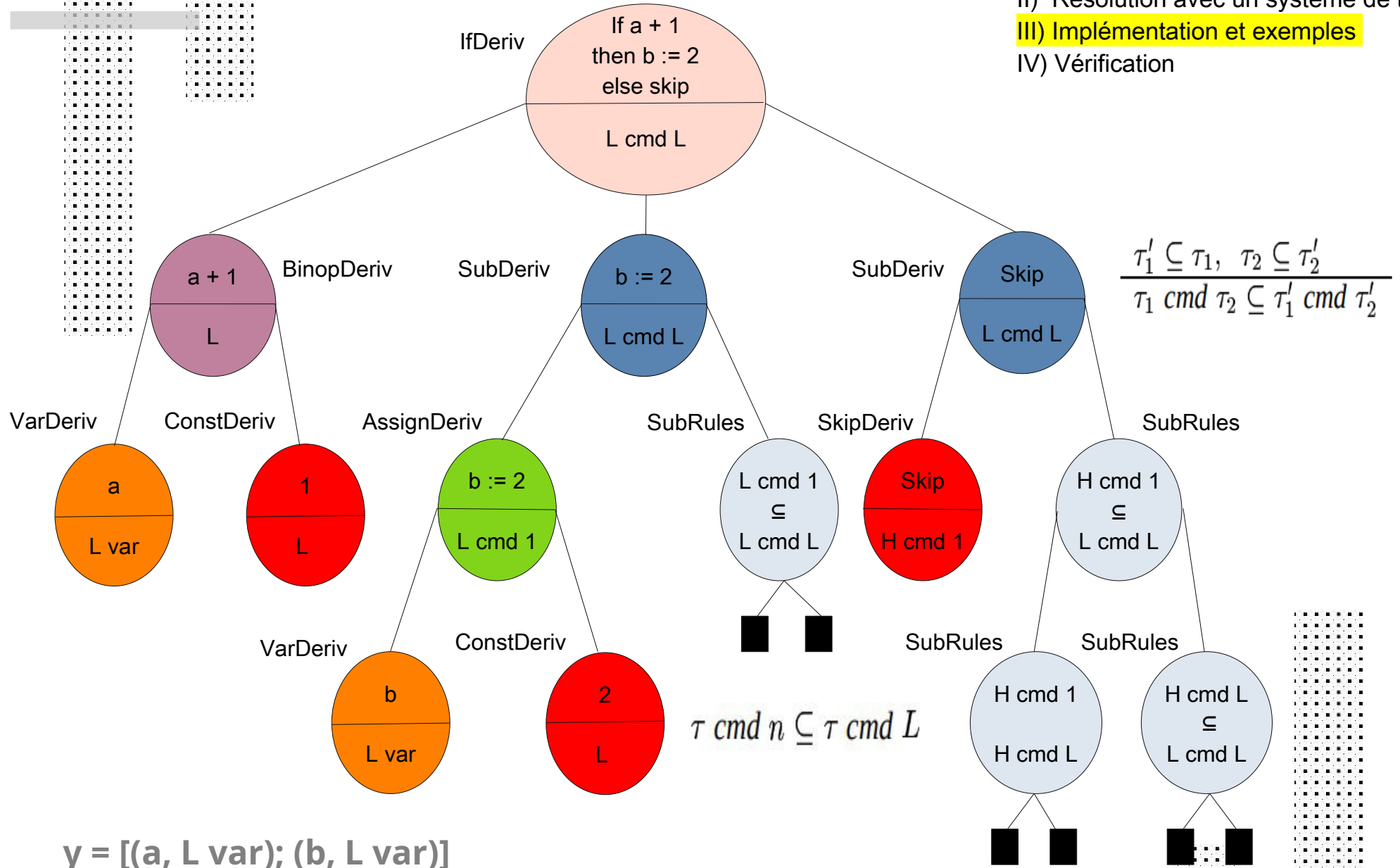
- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification



$\gamma = [(a, L \text{ var}); (b, L \text{ var})]$

## Exemple :

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification



```
utop # check_type ex gamma;;
- : bool = true
```

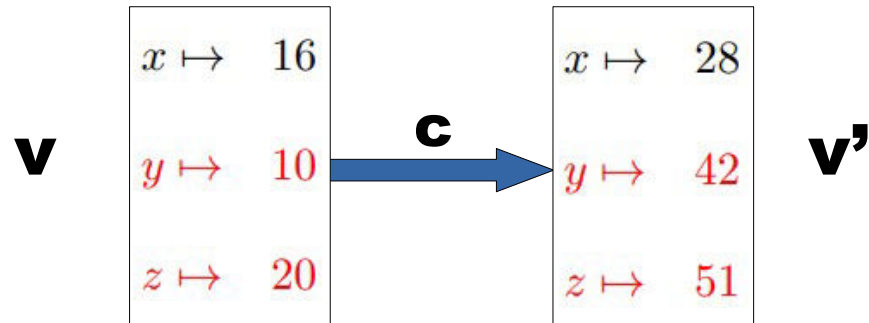
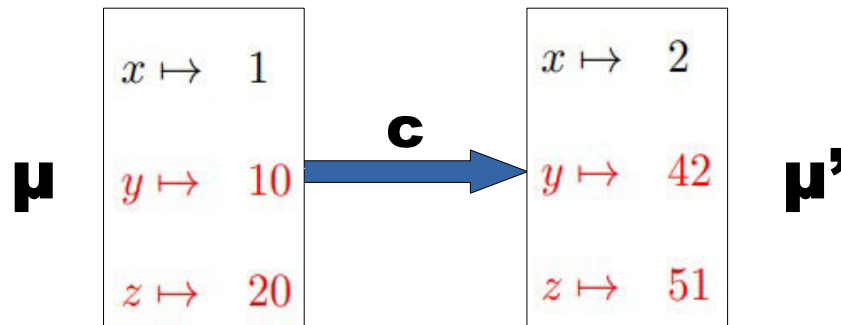
## Résultat :

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

Non Interférence :

$$\forall \gamma \in F(I, \{L \text{ var}, H \text{ var}\}), \forall c \in \text{commands}, \forall \mu, v \in F(I, N)$$

$$(\exists \rho, \gamma \vdash c : \rho \wedge \mu \sim_{\gamma} v \wedge (c, \mu) \rightarrow^* \mu') \Rightarrow (\exists v', (c, v) \rightarrow^* v' \wedge \mu' \sim_{\gamma} v')$$





# Test du modèle

- I) Modélisation
- II) Résolution avec un système de type
- III) Implémentation et exemples
- IV) Vérification

## Algorithm    Objet connecté

$y = [(y, L \text{ var}); (x, H \text{ var}); (tmp, H \text{ var}); (rest, H \text{ var})]$

Input :  $x$  (private),  $y$  (public)

$tmp := x;$

$\} H \text{ cmd } 1 \subseteq L \text{ cmd } L$

**while**  $y > 0$  **do**

$y := y - 1;$

$\} L \text{ cmd } 1 \subseteq L \text{ cmd } L$

$x := x - 1$

$\} H \text{ cmd } 1 \subseteq L \text{ cmd } L$

$\} L \text{ cmd } L$

;

**if**  $x \leq 0$  **then**

$rest := -x;$

$tmp := tmp + rest;$

$x := 2 \times tmp$

$\} H \text{ cmd } 3 \subseteq L \text{ cmd } H$

**else**

$skip;$

$skip;$

$skip$

$\} H \text{ cmd } 3 \subseteq L \text{ cmd } H$

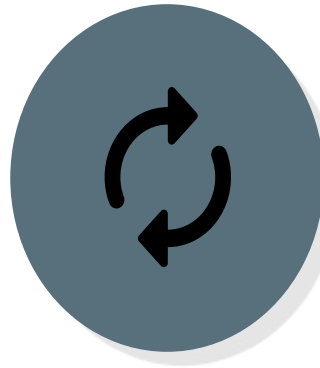
$\} L \text{ cmd } H$

# Bilan



## Assurance

Modèle prouvé correct



## Difficultés

Difficile à automatiser

Taille des formules très grande pour des phrases simples et courtes



## Améliorations possibles

Assistant de preuve



## Annexe

[1] Flemming Nielson, Hanne Riis Nielson : Formal methods, an appetizer :

[2] David Basin : Formal methods for security

[3] Xavier Leroy : Le logiciel, entre l'esprit et la matière : Leçon inaugurale prononcée au Collège de France le jeudi 15 novembre 2018

[4] Vincent Simonet : Flow Caml in a Nutshell

[5] Nevin Heintze, Jon G.Riecke : The Slam Calculus : Programming with Secrecy and Integrity

[6] Geoffrey Smith : A New Type System for Secure Information Flow

[7] Geoffrey Smith : A Type-Based Approach to Program Security



## Code :

```
type data_types = H | L
```

```
type binop =
```

```
    Plus | Minus | Mult | Div | Eq | Neq | Lt | Gt | Leq | Geq
```

```
    | And | Or
```

```
type exp =  Int of int
```

```
           | Var of string
```

```
           | Binop of binop * exp * exp
```

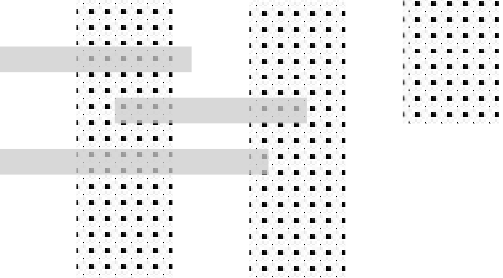
```
type command =  Assign of string * exp
```

```
               | Skip
```

```
               | If of exp * command * command
```

```
               | While of exp * command
```

```
               | Seq of command * command
```



```
type phrase = Exp of exp  
             | Command of command
```

```
type phrase_type = T of data_types  
                  | Var of data_types  
                  | Cmd of data_types * data_types  
                  | Ncmd of data_types * int
```

```
type var_type = string * phrase_type
```

```
let less_or_eq t1 t2 =(t2 = H) || (t1 = L)
```

```
exception Type_error
```

```
exception Phrase_error
```



type tree =

| Empty (\* Red \*)

| VarDeriv of string \* phrase\_type (\* Orange \*)

| ConstDeriv of int (\* Red \*)

| BinopDeriv of phrase \* phrase\_type \* tree \* tree (\* Purple \*)

| SkipDeriv of phrase\_type (\* Red \*)

| AssignDeriv of string \* phrase \* phrase\_type \* tree \* tree (\* Light green \*)

| IfDeriv of phrase \* phrase\_type \* tree \* tree \* tree (\* Pink \*)

| WhileDeriv of phrase \* phrase\_type \* tree \* tree (\* Yellow \*)

| SeqDeriv of phrase \* phrase\_type \* tree \* tree (\* Dark green \*)

(\* For the subtyping rules : \*)

| SubDeriv of phrase \* phrase\_type \* phrase\_type \* tree \* tree (\* Dark blue \*)

| SubRules of phrase\_type \* phrase\_type \* tree \* tree (\* Light blue \*)

```

let extract_phrase_and_type tree =
  match tree with
  | VarDeriv(x, t) -> Exp(Var x), t

  | ConstDeriv(n) -> Exp(Int n), T L

  | BinopDeriv(p, t, _, _) -> p, t

  | SkipDeriv(t) -> Command Skip, t

  | AssignDeriv(x, Exp e, t, _, _) -> Command(Assign(x, e)), t

  | IfDeriv(p, t, _, _, _) -> p, t

  | WhileDeriv(p, t, _, _) -> p, t

  | SeqDeriv(p, t, _, _) -> p, t

  | SubDeriv(p, t1, t2, _, _) -> p, t1

  | _ -> raise Phrase_error

```

```
let rec check_sub_rules tree t1 t2 =
```

```
  match tree with
```

```
  | SubRules(rho1, rho2, Empty, Empty) -> (t1 = rho1 && t2 = rho2) &&
```

```
    (* here we are in the case where we don't introduce another rho3 *)
```

```
    (* we only use base and cmd subtyping rules *)
```

```
  begin
```

```
    match rho1, rho2 with
```

```
    | T L, T H -> true
```

```
    | Cmd(tau1, tau2), Cmd(tau1', tau2') ->  
      (less_or_eq tau1' tau1) && (less_or_eq tau2 tau2')
```

```
    | Ncmd(tau, n), Ncmd(tau', n') when n = n' -> less_or_eq tau' tau
```

```
    | Ncmd(tau, n), Cmd(tau', L) when tau = tau' -> true
```

```
    | _, _ when rho1 = rho2 -> true
```

```
    | _ -> false
```

```
  end
```



```
| SubRules(rho1, rho2, son1, son2) ->
```

```
(t1 = rho1 && t2 = rho2) &&
```

```
( match son1, son2 with
```

```
  | SubRules(t1', t1'', _, _), SubRules(t2', t2'', _, _) ->
```

```
    (t1' = rho1) && (t1'' = t2') && (t2'' = rho2) &&
```

```
    (check_sub_rules son1 rho1 t1'') &&
```

```
    (check_sub_rules son2 t2' rho2)
```

```
  | _ -> false
```

```
)
```

```
| _ -> failwith"problem in the structure of the tree (Subrules)"
```

```
let check_type tree gamma =
```

```
  let rec aux tree expected_phrase expected_type =
```

```
    match tree with
```

```
    | VarDeriv(x, T L) -> (expected_phrase = Exp(Var x))
                          && (expected_type = T L)
                          && (List.mem_assoc x gamma)
                          && (List.assoc x gamma = Var L)
```

```
    | VarDeriv(x, T H) -> (expected_phrase = Exp(Var x))
                          && (expected_type = T H)
                          && (List.mem_assoc x gamma)
                          && (List.assoc x gamma = Var H)
```

```
    | ConstDeriv(n) -> (expected_phrase = (Exp(Int n)))
                      && (expected_type = T L )
```

```

| BinopDeriv(p, t, tree1, tree2) ->
  begin match p with
    | Exp(Binop(_, e1, e2)) ->
      let verify_phrase = p = expected_phrase in
      let verify_type = t = expected_type in
      let type_tree1 = aux tree1 (Exp e1) t in
      let type_tree2 = aux tree2 (Exp e2) t in
      verify_phrase && verify_type && type_tree1 && type_tree2

    | _ -> failwith"problem in the structure of the tree (binop)"
  end

```

```

| SkipDeriv(t) -> t = Ncmd(H, 1)

```

```

| AssignDeriv(x, Exp exp, Ncmd(tau, 1), son_x, son_exp) ->
  (expected_phrase = Command (Assign(x, exp)))
  && (expected_type = Ncmd(tau, 1))
  && (List.mem_assoc x gamma)
  && (List.assoc x gamma = Var tau)
  && (aux son_x (Exp(Var x)) (T tau))
  && (aux son_exp (Exp exp) (T tau))

```

```
| IfDeriv(p, t, tree_exp, tree1, tree2) ->
```

```
begin
```

```
match p with
```

```
Command(If(e, c1, c2)) ->
```

```
( match t with
```

```
| Cmd(H, H) ->
```

```
  let verify_phrase = p = expected_phrase in
```

```
  let verify_type = t = expected_type in
```

```
  let verify_exp = aux tree_exp (Exp e) (T H) in
```

```
  let verify_tree1 = aux tree1 (Command c1) t in
```

```
  let verify_tree2 = aux tree2 (Command c2) t in
```

```
  verify_phrase && verify_type && verify_exp
```

```
  && verify_tree1 && verify_tree2
```

```
  | Cmd(tau1, tau2) ->
```

```
    let verify_phrase = p = expected_phrase in
```

```
    let verify_type = t = expected_type in
```

```
    let verify_exp = aux tree_exp (Exp e) (T L) in
```

```
  let verify_tree1 = aux tree1 (Command c1) t in
```

```
  let verify_tree2 = aux tree2 (Command c2) t in
```

```
  verify_phrase && verify_type && verify_exp
```

```
  && verify_tree1 && verify_tree2
```

```

| Ncmd(tau, n) when n >= 1 ->
let verify_phrase = p = expected_phrase in
let verify_type = t = expected_type in
let verify_exp = aux tree_exp (Exp e) (T tau) in
let verify_tree1 =
    aux tree1 (Command c1) (Ncmd(tau, n - 1)) in
let verify_tree2 =
    aux tree2 (Command c2) (Ncmd(tau, n - 1)) in
verify_phrase && verify_type && verify_exp
&& verify_tree1 && verify_tree2
|_ -> false

)

|_ -> failwith"problem in the structure of the tree (if-else)"

```

end

```

| WhileDeriv(p, t, tree_exp, tree_cmd) ->
  begin
    match p with
    | Command (While(e, c)) ->
      ( match t with
        | Cmd(H, H) ->
          let verify_phrase = p = expected_phrase in
          let verify_type = t = expected_type in
          let verify_exp = aux tree_exp (Exp e) (T H) in
          let verify_command = aux tree_cmd (Command c) t in
          verify_phrase && verify_type && verify_exp
          && verify_command

        | Cmd(tau1, tau2) ->
          let verify_phrase = p = expected_phrase in
          let verify_type = t = expected_type in
          let verify_exp = aux tree_exp (Exp e) (T L) in
          let verify_command = aux tree_cmd (Command c) t in
          let comp = less_or_eq tau1 tau2 in
          verify_phrase && verify_type && verify_exp
          && verify_command && comp

        | _ -> false
      )
    )
  | _ -> failwith "problem in the structure of the tree (while)"
  end

```

```
| SeqDeriv(p, t, tree1, tree2) ->
```

```
  begin match p with
```

```
    | Command (Seq(c1, c2)) ->
```

```
      ( match t with
```

```
        | Cmd(tau, H) ->
```

```
          let verify_phrase = p = expected_phrase in
```

```
          let verify_type = t = expected_type in
```

```
          let verify_c1 = aux tree1 (Command c1) t in
```

```
          let verify_c2 = aux tree2 (Command c2) (Cmd(H, H)) in
```

```
          verify_phrase && verify_type && verify_c1
```

```
          && verify_c2
```

```
        | Cmd(tau1, tau2) ->
```

```
          let verify_phrase = p = expected_phrase in
```

```
          let verify_type = t = expected_type in
```

```
          let verify_c1 = aux tree1 (Command c1) (Cmd(tau1, L))
```

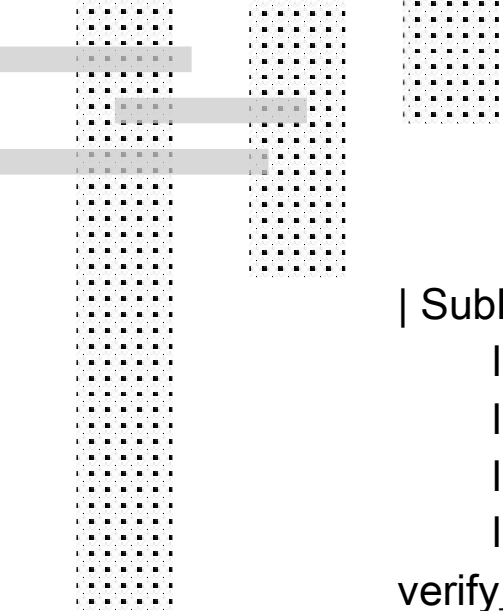
```
          in let verify_c2 = aux tree2 (Command c2) t in
```

```
          verify_phrase && verify_type && verify_c1 && verify_c2
```

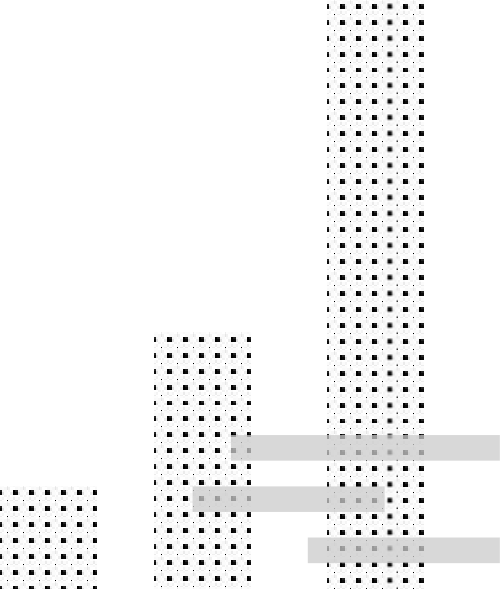
```
        | _ -> false )
```

```
    | _ -> failwith"problem in the structure of the tree (Seq)"
```

```
  end
```



```
| SubDeriv(p, t1, t2, son1, subtype_tree) ->  
  let verify_phrase = p = expected_phrase in  
  let verify_type = t1 = expected_type in  
  let verify_son = aux son1 p t2 in  
  let verify_subrules = check_sub_rules subtype_tree t2 t1 in  
verify_phrase && verify_type && verify_subrules && verify_son  
  
|_ -> false  
  
in let p, t = extract_phrase_and_type tree in aux tree p t
```





```

let tree1 = BinopDeriv(
    Exp(Binop(Plus, Var "a", Int 1)),
    T L,
    VarDeriv("a", T L),
    ConstDeriv(1)
)

```

```

let tree2 = SubDeriv(
    Command (Assign("b", Int 2)),
    Cmd(L, L),
    Ncmd(L, 1),
    AssignDeriv(
        "b",
        Exp(Int 2),
        Ncmd(L, 1),
        VarDeriv("b", T L),
        ConstDeriv(2)
    ),
    SubRules(Ncmd(L, 1), Cmd(L, L), Empty, Empty)
)

```

```

let tree3 = SubDeriv(
    Command Skip,
    Cmd(L, L),
    Ncmd(H, 1),
    SkipDeriv(Ncmd(H, 1)),
    SubRules(
        Ncmd(H, 1),
        Cmd(L, L),
        SubRules(Ncmd(H, 1), Cmd(H, L), Empty, Empty),
        SubRules(Cmd(H, L), Cmd(L, L), Empty, Empty)
    )
)

```

```

let ex = IfDeriv(
    Command(
        If( Binop(Plus, Var "a", Int 1), Assign("b", Int 2), Skip)
    ),
    Cmd(L, L),
    tree1,
    tree2,
    tree3
)

```