Part 1.

1. Gradient Vector $= \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)^T = (a, b)^T$

2. Gradient Vector $= \left(\frac{\partial z}{\partial x_1}, \frac{\partial z}{\partial x_2}, \frac{\partial z}{\partial x_3}, \ldots, \frac{\partial z}{\partial x_N}\right)^T = (a_1, a_2, a_3, \ldots, a_N)^T$

3. $f_x(x, y) = 2A(x - x_0)$

$f_y(x, y) = 2B(y - y_0)$

4. $x^T = (3 \quad 1 \quad 4)$

$y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$

$B^T = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 2 & 4 \end{pmatrix}$

$$X \cdot X = 3\times3 + 1\times1 + 4\times4 = 26$$

$$X \cdot y^T = 3\times2 + 1\times5 + 4\times1 = 15$$

$$X \times y = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$$

$$y \times X = \begin{pmatrix} 15 \end{pmatrix}$$

$$A \times X = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$$

$$B.\mathrm{reshape}(1,6) = \begin{pmatrix} 3 & 5 & 5 & 2 & 1 & 4 \end{pmatrix}$$

5. LLS

$$\frac{\partial L(P)}{\partial m} = 2\sum_{i=1}^{N} -x_i(y_i - (mx_i + b))$$

$$= 2\left[m\sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} x_i(y_i - b)\right]$$

$$\frac{\partial L(P)}{\partial b} = 2\sum_{i=1}^{N} -(y_i - (mx_i + b))$$

$$= 2\left(Nb - \sum_{i=1}^{N}(y_i - mx_i)\right)$$

let $\frac{\partial L(P)}{\partial m} = 0$, $\frac{\partial L(P)}{\partial n} = 0$

$$m = \frac{\sum_{i=1}^{N} y_i(x_i - \bar{x})}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2}$$

$$\because \sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2 = \sum_{i=1}^{N} x_i^2 - (N\bar{x})^2$$

$$= \sum_{i=1}^{N} (x_i^2 - \bar{x})^2 = \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$\sum_{i=1}^{N} y_i(x_i - \bar{x}) = \sum_{i=1}^{N} x_i y_i - N\bar{x}\bar{y} = \sum_{i=1}^{N} x_i y_i - 2N\bar{x}\bar{y} + N\bar{x}\bar{y}$$

$$= \sum_{i=1}^{N} (x_i y_i - x_i\bar{y} - \bar{x} y_i + \bar{x}\bar{y}) = \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

$$\therefore m = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2} = \frac{\frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2} = \frac{cov(x,y)}{var(x)}$$

$$b = \frac{1}{N}\sum_{i=1}^{N} (y_i - m x_i) = \bar{y} - m\bar{x} = \bar{y} - \frac{cov(x,y)}{var(x)}\bar{x}$$

b.

$$X = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1n} & 1 \\ X_{21} & X_{22} & \cdots & X_{2n} & 1 \\ \vdots & & \cdots & & \\ X_{m1} & X_{m2} & \cdots & X_{mn} & 1 \end{pmatrix} \quad W = \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \\ b \end{pmatrix} \quad y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{pmatrix}$$

$$L(P) = \sum_{i=1}^{m} (Y_i - (W_1 X_{i1} + W_2 X_{i2} + \cdots + W_n X_{in} + b))^2$$

Then

$$L(P) = (y - Xw)^T (y - Xw) = y^T y - y^T Xw - w^T X^T y + w^T X^T X w$$

$$\frac{\partial L(P)}{\partial w} = -(y^T X)^T - X^T y + X^T X w + (w^T X^T X)^T$$

$$= -X^T y - X^T y + X^T X w + X^T X w = 2X^T X w - 2X^T y$$

let $\frac{\partial L(P)}{\partial w} = 0$

we have $w = (X^T X)^{-1} X^T Y$