Simple Numerical Study of Net Sale's Gain from Bundling Price

Weiran Deng, Youngmin Ju, Mahrad Sharifvaghefi, Hayun Song, Jeongsang Yoo

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1 Introduction

To maximize profits, firms are trying innovative marketing techniques to draw in as many customers as they possibly can. One amongst them is a widely used strategy called "bundling." It involves packaging several variety of products into one single product. We see bundled products all the time. Different types of chocolate bars and variety of chips are often bundled together to be sold as one single product. Cosmetic products are also often seen to be packaged into one bundle. Another good example is telecommunications services. Rather than only offering internet, cable, and phone service separately as individual products, telecommunication companies usually package these services into different combinations of products.

For the companies, costs of bundling will likely be low since they already have the individual products all ready to be put together. The real concern is how to set prices for the bundled products. If the bundled product is priced appropriately, the strategy can lead to higher sales, which would then mean higher profits, given the low costs. So then what is the appropriate price? Take the simple case when consumers are only facing two goods, and also a bundle that contains both goods. Obviously, the price of the bundle should be lower than the sum of the prices of the two individual products. If not, consumers are better off buying the two goods individually. If the price of bundle is low enough, then it may induce purchases from some consumers who would not have bought both goods individually. This will certainly be a gain for the companies. However, the low-priced-bundle will also attract a group of consumers who would have been willing to buy the two goods individually. This will be a loss for the companies. Thus, companies must figure out price of the bundle that results in maximum profit, accounting for all the complexities described above.

In this paper, we take the perspective of a firm, and propose a method to try to figure out how to set the appropriate price for a bundle of goods. Specifically, by imposing assumptions on the distribution of values of goods that consumers hold, we generate simulations for different combinations of parameters to examine the net gain from bundling, i.e. the difference between the expected revenue before and after bundling. Our approach takes into account the complexities involved in bundling, e.g. the loss and gain resulting from bundling the products, and therefore offers a practical way to examine benefits of bundling.

The paper is consisted of four sections. Section One introduces the question that we investigated in. In Section Two, the numerical study used for getting empirical results are explained. Section Three shows results and implications. In Section Four, we drew conclusion and mentioned extensions for future studies.

2 Numerical Study

Suppose that we have two goods x_1 and x_2, and their corresponding prices are p_1 and p_2. We have N customers who make decisions about buying these goods based on their utilities. In particular, customer i decides to buy good j if $u^i_{x_j} > p_j$. Moreover, suppose that the customers' utilities are independently drawn from a bivariate normal distribution with identical means and variances. In other words, we have

$$(u^i_{x_1}, u^i_{x_2}) \sim \mathcal{N}\left(\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}, \begin{bmatrix}\sigma^2_{11} & \sigma_{12}\\ \sigma_{21} & \sigma^2_{22}\end{bmatrix}\right),$$

where $\mu_1 = \mu_2 = \mu$ and $\sigma^2_{11} = \sigma^2_{22} = \sigma^2$. It is assumed that a firm knows the distribution function (Normal) from which the customers' utilities are drawn from but has no information about the set of parameters in the distribution function denoted $\theta = \left(\mu, \sigma^2, \sigma_{12}\right)$. Moreover, it is assumed that the firm can only observe customers' decisions about buying the products but not their utilities. Given these observations, the firm first tries to estimate the parameters θ of the distribution function by solving the following system of equations,

$$\mathbb{P}(u^i_{x_1} < p_1 \text{ and } u^i_{x_2} < p_2) = \frac{\sum_{j=1}^{N} I(x^j_1 = 0 \text{ and } x^j_2 = 0)}{N} \tag{1}$$

$$\mathbb{P}(u^i_{x_1} > p_1 \text{ and } u^i_{x_2} < p_2) = \frac{\sum_{j=1}^{N} I(x^j_1 = 1 \text{ and } x^j_2 = 0)}{N} \tag{2}$$

$$\mathbb{P}(u^i_{x_1} > p_1 \text{ and } u^i_{x_2} > p_2) = \frac{\sum_{j=1}^{N} I(x^j_1 = 1 \text{ and } x^j_2 = 1)}{N} \quad , \tag{3}$$

where x^j_i shows whether customer j decides to buy product i or not. Given the estimated parameters $\hat{\theta}$, the firm tries to find the bundling price that maximizes its sales income; i.e.,

$$\begin{aligned} p^* = \arg\max_p \{ & p_1 \cdot \mathbb{P}\left(u_{x_1} > p_1, u_{x_2} < p - p_1\right) + \\ & p_2 \cdot \mathbb{P}\left(u_{x_1} < p - p_2, u_{x_2} > p_2\right) + \\ & p \cdot \mathbb{P}\left(u_{x_1} + u_{x_2} > p, u_{x_1} > p - p_2, u_{x_2} > p - p_1\right)\} \end{aligned}$$

Now, given the value p^*, we can compare sales income of the firm before and after the bundling for various values of μ, σ and σ_{12} to find out how sensitive a gain from bundling of the firm is. In the rest of this study, we are going to consider two different scenarios when θ is known and unknown. Moreover, p_1 and p_2 are set to one in all cases. We have considered 1000 customers for the estimation of θ (old costumers), and 1000 customers for the evaluation of the gains from the bundling price (new customers). All the reported results were computed using 1000 simulations.

3 Results

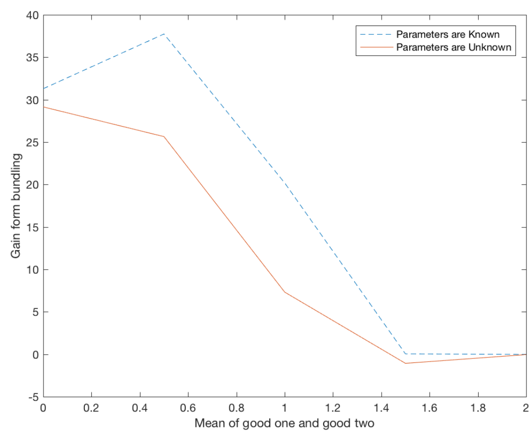
In our benchmark experiment, μ (mean of the utilities' distributions) is set equal to p_1 and p_2, and it is assumed that the utilities of the customers for good one and two are independent of each other ($\sigma_{12} = 0$). In addition, the variance of the marginal distributions is set equal to one ($\sigma^2 = 1$). As can be seen from table 1, a firm can obtain profits from offering the bundling price for good one and two, however, the gain seems to drop sharply if the vector of parameters θ is unknown and needed to be estimated by using equations (1) to (3).

Table 1: Bundling price and gains for the benchmark design

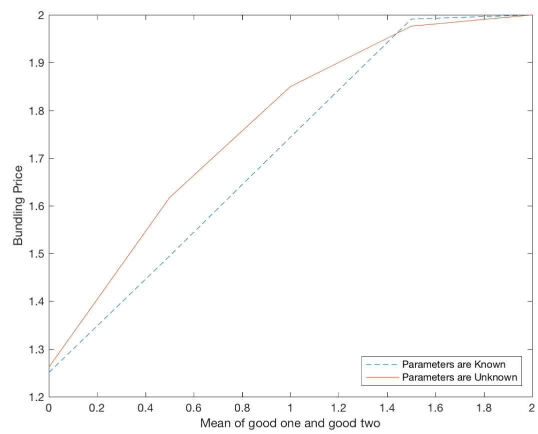
	Bundling Price	**Bundling Gain**
θ is known	1.7437	20.1865
θ is unknown	1.8447	7.6265

$\mu = p_1 = p_2 = 1$, $\sigma^2 = 1$, and $\sigma_{12} = 0$.

In the second experiment, we let μ to change from 0 to 2. As one can expect, a firm can gain more (less) from the bundling, if the average of customers utilities from good one and two is below (above) market prices (p_1 and p_2). The results in figure 1 also indicate that there is a positive relationship between the optimal bundling price and the average of the customers' utilities. It is worth mentioning that the gain from the bundling with unknown parameters is always below the one with the known parameters. In addition, the optimal bundling price from the estimated parameters, seems to be higher than the actual optimal bundling price in which we have knowledge about the parameters.



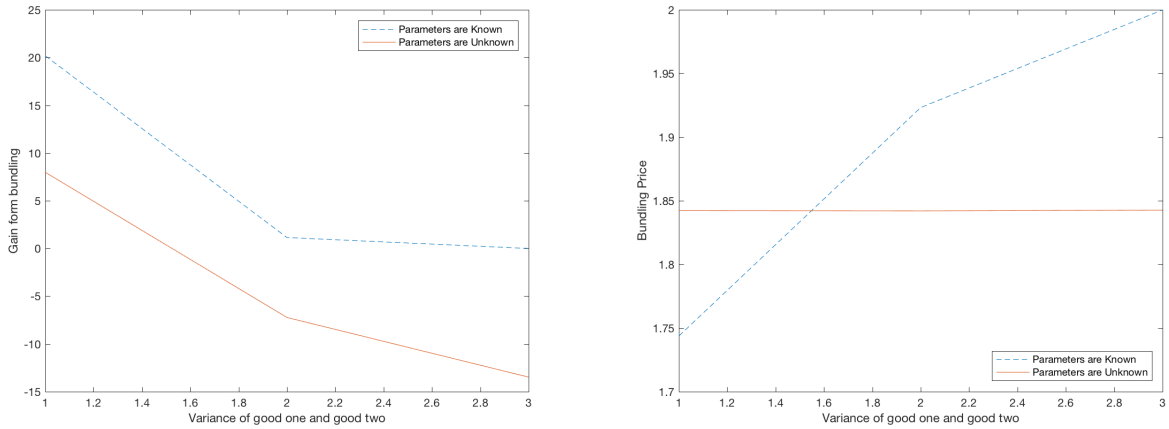
(a) Bundling Gain



(b) Bundling Price

Figure 1: Bundling price and gains for various values of μ

In the third experiment, the importance of utilities' dispersion among the customers on the gain from price bundling is investigated by letting σ^2 to change from 1 to 3. Our results show that as σ^2 increases, the gain from the bundling decreases. In addition, figure 2 shows that the optimal bundling price with unknown θ is constant over σ^2, while the one with known θ is increasing over σ^2. It might be an indicator that equations 1 to 3 are not enough for the identification of the parameter σ^2.

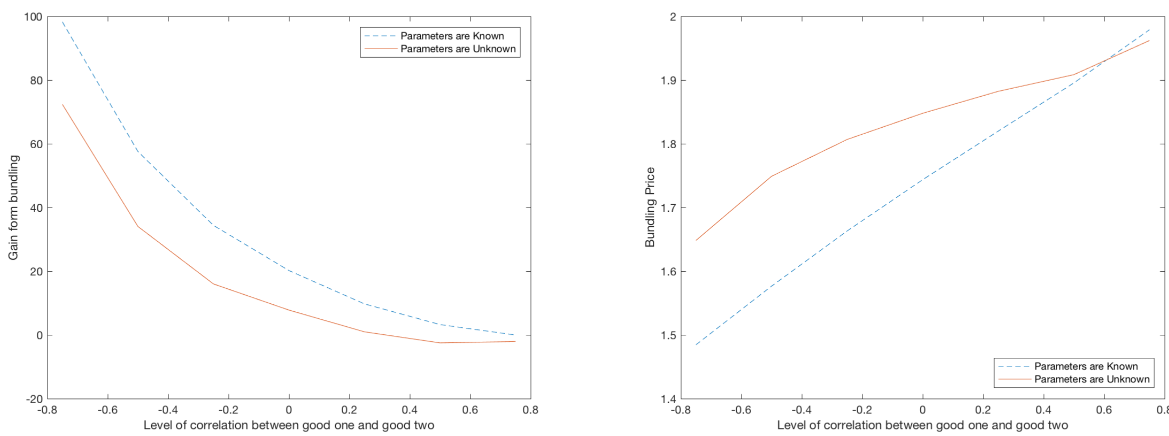


(a) Bundling Gain

(b) Bundling Price

Figure 2: Bundling price and gain for various value of σ^2

In the last experiment, we examine the role of correlation between the utilities from good one and two, on the gain from the bundling, by letting σ_{12} to change from -0.75 to 0.75. Interestingly, figure 3 shows that an increase in the level of positive (negative) correlation between utilities from good one and two, weaken (strengthen) the gain from price bundling. Moreover, it can be seen from the figure, as the correlation between utilities from good one and two increases (from negative -0.75 to 0.75), the optimal bundling price increases.



(a) Bundling Gain

(b) Bundling Price

Figure 3: Bundling price and gains for various values of σ_{12}

The final experiment implies that when the level of positive correlation between utilities of good one and two is high, these goods are considered as strategic complements for the customers. Therefore, these two goods are supposed to be sold under non-bundling prices. Even without having a bundling price, it can cause more profits (gain), which leads more rooms for increasing their bundling price because consumers are eager to buy both goods. The other situation is when both goods are taken as strategic substitutions so that the firm cannot sell both goods at the same time (loss), which leads to a lower bundling price in order to sell both goods.

4 Conclusion

This paper provides a way to set the appropriate price for a bundle of goods for firms. We imposed innocuous distribution assumptions and generated simulation results for the difference between the expected revenue before and after bundling through four cases.

In the benchmark case where we imposed the most strict assumptions, although the firm can get profits from offering the bundling price both good one and two, the final gain shows sharp drop if θ is unknown.

In the second case where μ is varying, a firm can gain more (less) from the bundling, if the average of customers utilities from good one and two is below (above) market prices (p_1 and p_2),

In the third case where σ^2 is varying, we found that the gain from bundling decreases. What's more, optimal bundling price with unknown θ is constant over σ^2, while the one with known θ is increasing over σ^2.

In the fourth case where σ_{12} is varying, we found that an increase in the level of positive (negative) correlation between utilities from good one and two, weaken (strengthen) the gain from price bundling.

The above results show that firms could be more profitable if they were able to get more precise informaiton of consumers' utility distribution, if mean utilities could be lower than price of good 1 and good 2 respectively and do not vary too much. In addition, firms would like to set a relatively higher bundling price if two goods were strategic complements and a relatively lower price if two good were strategic substitutions. We contribute to the understanding of bundling using big data techniques and practical solutions for firms which are intended to use bundling as a strategy to increase their profits.

Admittedly, our study has its limitations. First of all, we only considered s bundle of two goods. Enlarging the number of goods in a bundle will definetely give us more interesting and more practical implications. Second, we made strong assumptions on the parameters of the distribution of consumers' utilities. For instance, we assumed that the expected mean utilities of consuming good 1 and good 2 are exactly the same. The reason we did this is that we do not have enough equations to solve for more unknown parameters. A natural extension of this study is then to set up the parameters in a more general way and solve the values by applying the method on bundles of four goods.